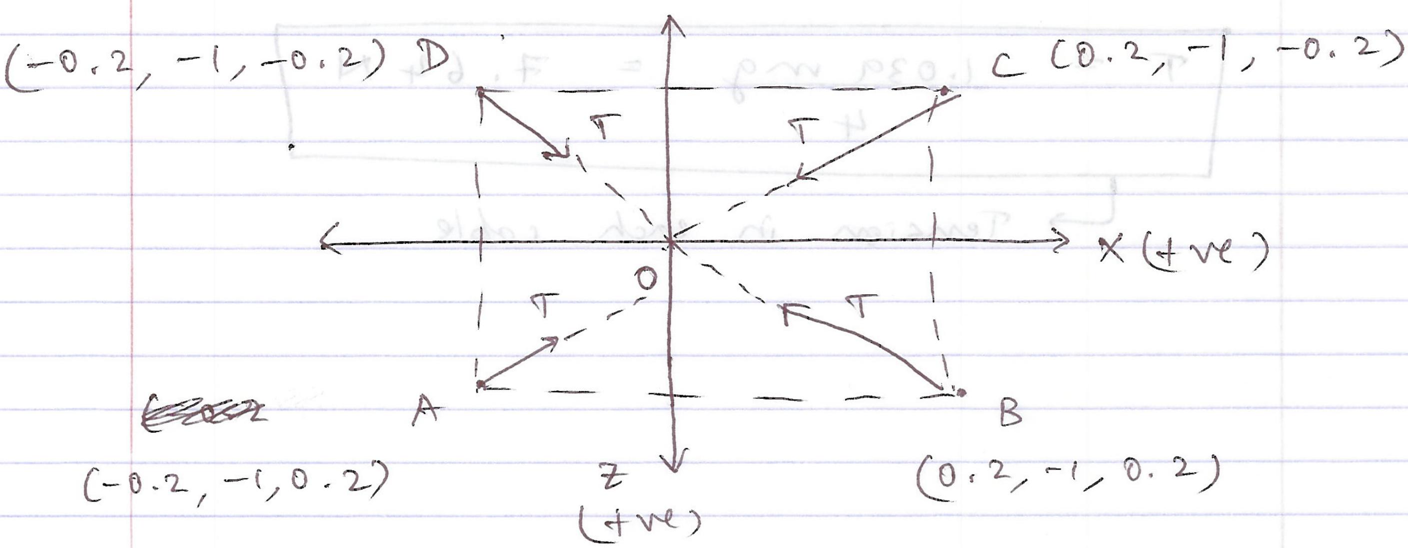
AEM 2011 : EXAM # 1 : SOLUTIONS

PROBLEM 1

$m = 3\,kg \Rightarrow W = mg = 3 \times 9.81\ N$



TOP - VIEW

From the symmetry of the problem, the tensions in each of the four cables have the same magnitude. The x and z components of the tensions cancel out between themselves. Only the Y components of the tensions are balanced by an external force, i.e., the weight.

Consider for example the tension along AO

$\overline{OA} = \overline{r}_A = (-0.2, -1, 0.2)$

$\overline{AO} = -\overline{OA} = (0.2, 1, -0.2)$

$$\overline{T}_{AO} = T.\hat{AO} = T \frac{(0.2, 1, -0.2)}{\sqrt{0.2^2 + 1^2 + 0.2^2}}$$

②

$$\bar{T}_{AO} \cdot \hat{j} = T \times \frac{1}{\sqrt{0.2^2+1^2+0.2^2}} = \frac{T}{1.039}$$

y-component of $\bar{T}_{AO}$

$$\frac{4T}{1.039} = W \Leftarrow \Sigma F_y = 0$$

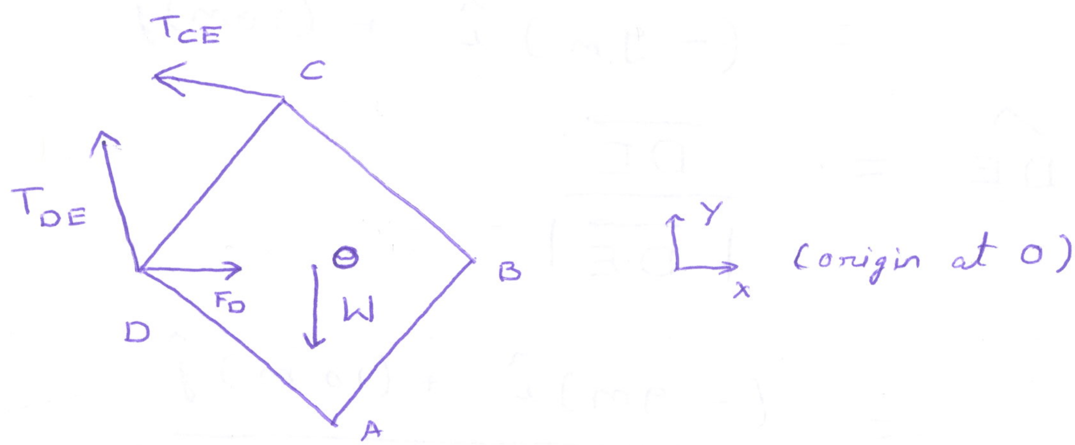
$$\boxed{T = \frac{1.039\, mg}{4} = 7.64\ N}$$

Tension in each cable

Problem – 2

a) Free body diagram

Let's call drag force = F_D



b) $\overrightarrow{CE} = \vec{r}_E - \vec{r}_C$

$= [-10\,\hat{i}\,m + 10\,\hat{j}\,m] - [\hat{j}\,m]$

$= (-10\,m)\,\hat{i} + (9\,m)\,\hat{j}$

$\Rightarrow \widehat{CE} = \dfrac{\overrightarrow{CE}}{|\overrightarrow{CE}|} = \dfrac{(-10m)\hat{i} + (9m)\hat{j}}{\sqrt{(10m)^2 + (9m)^2}}$

$$\boxed{\widehat{CE} = -0.743\,\hat{i} + 0.669\,\hat{j}}$$

Similarly

$$\overrightarrow{DE} = \overrightarrow{r_E} - \overrightarrow{r_D}$$

$$= [-10\,\hat{i}\,m + 10\,\hat{j}\,m] - [-\hat{i}\,m]$$

$$= (-9m)\,\hat{i} + (10m)\,\hat{j}$$

$$\widehat{DE} = \frac{\overrightarrow{DE}}{|\overrightarrow{DE}|}$$

$$= \frac{(-9m)\,\hat{i} + (10\,m)\,\hat{j}}{\sqrt{(-9m)^2 + (10m)^2}}$$

$$\boxed{\widehat{DE} = (-0.669)\,\hat{i} + (0.743)\,\hat{j}}$$

c) For calculating drag, consider the following equilibrium equation

$$\Sigma M_E = 0$$

$$\Rightarrow (\overrightarrow{r}_{EO} \times \overrightarrow{W}) + (\overrightarrow{r}_{ED} \times \overrightarrow{F_D}) = 0$$

$$\Rightarrow (10m\,\hat{i} - 10m\,\hat{j}) \times (-mg)\,\hat{j}$$

$$+ (9m\,\hat{i} - 10m\,\hat{j}) \times F_D\,\hat{i} = 0$$

$$\Rightarrow m\left[-10mg\,\hat{k} + 0 + 10F_D\,\hat{k}\right] = 0$$

$$\Rightarrow 10\left[F_D - mg\right] = 0$$

$$\Rightarrow F_D = mg = 100 \times 9.8$$

$$\Rightarrow \boxed{F_D = 980\ N}$$

→ For calculating tensions, lets consider two Force equations

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0$$

$$\rightarrow \Sigma F_x = 0$$

$$\Rightarrow F_D + T_{CE} \times (-0.743) + T_{DE} \times (-0.669) = 0$$

$$\Rightarrow \boxed{0.743\,T_{CE} + 0.669\,T_{DE} = 980} \quad \ldots (1)$$

$$\rightarrow \Sigma F_y = 0$$

$$\Rightarrow -W + T_{CE}(0.669) + T_{DE}(0.743) = 0$$

$$\Rightarrow \boxed{0.669\,T_{CE} + \cancel{0.736}\ 0.743\,T_{DE} = 980} \quad \ldots (2)$$

Solving 1, 2 simultaniously

$$\boxed{\begin{aligned} T_{CE} &= 694 \text{ N} \\ T_{DE} &= 694 \text{ N} \end{aligned}}$$

PROBLEM 3

$m = 10\,\mathrm{Kg}$

$\bar{r}_D = (-1, 0, 0.5)$

$\bar{F}_D = -mg\hat{j} = (0, -98.1, 0)$

(a) $\boxed{\bar{M}_o = \bar{r}_D \times \bar{F}_D = (49.05\hat{i} + 0\hat{j} + 98.1\hat{k})\ \mathrm{Nm}}$

(b) $\boxed{(\bar{M}_o \cdot \hat{i})\hat{i} = 49.05\hat{i}}$

(c) $\bar{F}_D$ produces a positive moment about the x-axis. The reaction forces at O and A produce zero moment about the x-axis (because they intersect the x-axis). Hence to balance the ~~mom~~ positive moment produced by $\bar{F}_D$ about the x-axis, the reactions at B and C need to produce negative moment about the x-axis. This is only possible if the z-components of $\bar{F}_B$ and and $\bar{F}_C$ are positive.

$\boxed{\text{Mathematically: } \bar{F}_B \cdot \bar{k} > 0 \text{ and } \bar{F}_C \cdot \bar{k} > 0}$