AEM 2011 Homework #10 (6.3, 6.4, 7.1) Wednesday, March 29, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. Book problems:
   (a) 6.75
   (b) 6.93
   (c) 6.141

   Additional Practice Problems: 6.79, 6.99, 6.123, 6.143, 7.8, 7.12, 7.26

   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

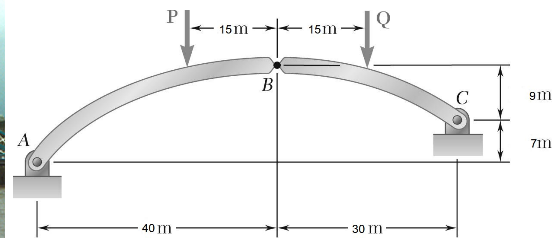
6.75 $F_{BD} = 1750$ N (C), $C_x = 1400$ N left, $C_y = 700$ N down

6.93 $A_x = 176.3$ lb left, $A_y = 60.0$ lb down, $G_x = 56.3$ lb right, $G_y = 510$ lb up

6.141 $C = 4.65$ kips right, $E = 6.14$ kips $40.7°$ down from the left

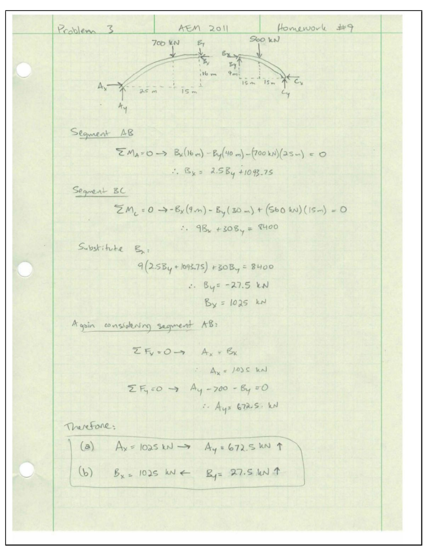
2. A Twin-Leaf Bascule bridge is a bridge where the two leafs (sections) of the bridge can rotate upwards to let the height clearance below the bridge to increase to allow ships to pass below. Consider a parabolic Twin Leaf Bascule bridge as shown below. Assume that in a closed position, the joint between the two leafs is a hinge joint. The axis of the three-hinge arch ABC is a parabola with the vertex at B. Knowing that $P = 700$ kN and $Q = 560$ kN, determine:

- The components of the reaction at A.
- The components of the force exerted at B on segment AB.



Bascule Bridge (left) Simplified Diagram (right)

Solution:



Problem 3 AEM 2011 Homework #9

Segment AB

$\Sigma M_A = 0 \rightarrow B_x(16\text{ m}) - B_y(40\text{ m}) - (700\text{ kN})(25\text{ m}) = 0$

$\therefore B_x = 2.5B_y + 1093.75$

Segment BC

$\Sigma M_C = 0 \rightarrow -B_x(9\text{ m}) - B_y(30\text{ m}) + (560\text{ kN})(15\text{ m}) = 0$

$\therefore 9B_x + 30B_y = 8400$

Substitute B_x:

$9(2.5B_y + 1093.75) + 30B_y = 8400$

$\therefore B_y = -27.5$ kN

$B_x = 1025$ kN

Again considering segment AB:

$\Sigma F_x = 0 \rightarrow A_x = B_x$

$\therefore A_x = 1025$ kN

$\Sigma F_y = 0 \rightarrow A_y - 700 - B_y = 0$

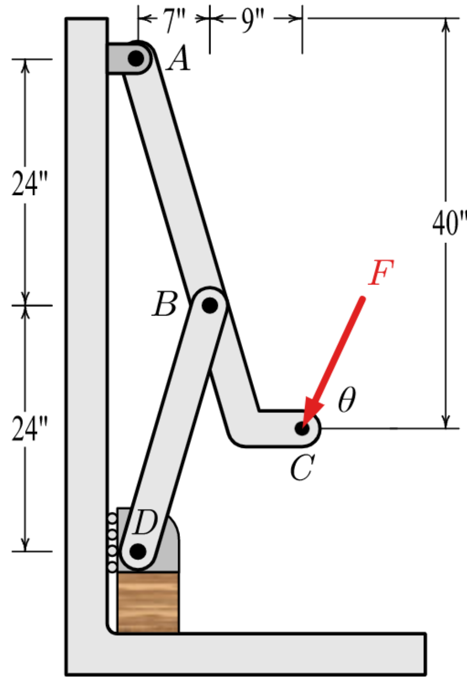
$\therefore A_y = 672.5$ kN

Therefore:

(a) $A_x = 1025$ kN $\rightarrow$ $A_y = 672.5$ kN $\uparrow$

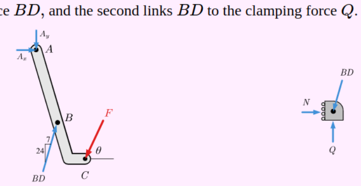
(b) $B_x = 1025$ kN $\leftarrow$ $B_y = 27.5$ kN $\uparrow$

3. A toggle clamp is shown below. Knowing that angle $\theta = 60°$, find the vertical clamping force acting on the piece at D and the magnitude of the force exerted on member ABC at pin B in terms of force F applied to the clamp arm at C.



Solution:

For this problem, we need two free-body diagrams. The first links the input force F to the link force BD, and the second links BD to the clamping force Q.



(a) FBD I **(b)** FBD II

Figure 6.6.18.

We will assume the two-force member BD is in compression based on the physical situation. The forces acting on the link, lever and roller are all directed along a line-of-action defined by a 7-24-25 triangle. Similar triangles gives

$$BD_x = \left(\frac{7}{25}\right) BD$$
$$BD_y = \left(\frac{24}{25}\right) BD.$$

Applying $\sum M = 0$ at A to the free-body diagram of the lever gives BD in terms of F.

$$\begin{aligned}
\text{FBD I:} \quad \Sigma M_A &= 0 \\
BD_x(24) + BD_y(7) - F_x(40) - F_y(16) &= 0 \\
\left(\frac{7}{25} BD\right)(24) + \left(\frac{24}{25} BD\right)(7) &= (F\cos 60°)(40) + (F\sin 60°)(16 \\
13.44 BD &= 33.86F \\
BD &= 2.52F
\end{aligned}$$

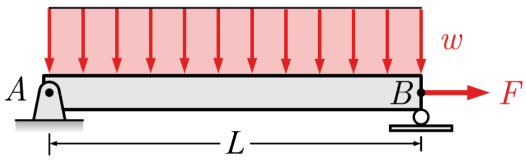
The positive sign on the answer reveals that our assumption that member BD was in compression was correct.

Applying $\sum F_y = 0$ to the free-body diagram of the roller will give Q in terms of F.

$$\begin{aligned}
\text{FBD II:} \quad \Sigma F_y &= 0 \\
Q - BD_y &= 0 \\
Q &= \frac{24}{25} BD \\
&= \frac{24}{25}(2.52F) \\
&= 2.42F
\end{aligned}$$

While you could certainly find A_x, A_y and N using other equilibrium equations they weren't asked for and we don't bother to find them.

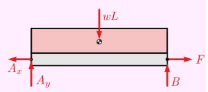
4. A beam of length L is supported by a pin at A and a roller at B and is subjected to a horizontal force F applied to point B and a uniformly distributed load over its entire length. The intensity of the distributed load is w with units of [force/length].



Solution:

1. *Find the external reactions.*

   Begin by drawing a free-body diagram of the entire beam, simplified by replacing the distributed load w with an equivalent concentrated load at the centroid of the rectangle.



   The magnitude of the equivalent load W is equal to the "area" under the rectangular loading curve.

$$W = w(L)$$

   Then apply and simplify the equations of equilibrium to find the external reactions at A and B.

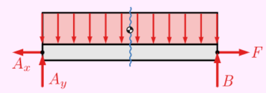
$$\begin{aligned} \Sigma M_A &= 0 \\ -(wL)(\cancel{L}/2) + (B)\cancel{L} &= 0 \\ B &= wL/2 \end{aligned}$$

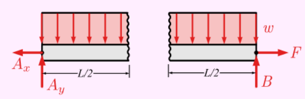
$$\begin{aligned} \Sigma F_x &= 0 \\ -A_x + F &= 0 \\ A_x &= F \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= 0 \\ A_y - wL + B_y &= 0 \\ A_y &= wL - wL/2 \\ &= wL/2 \end{aligned}$$

2. *Cut the beam.*

   Cut the beam at the point of interest and separate the beam into two sections. Notice that as the beam is cut in two, the distributed load w is cut as well. Each of these distributed load halves will support equivalent point loads of $wL/2$ acting through the centroid of each cut half.





3. *Add the internal forces.*

   At each cut, a shear force, a normal force, and a bending moment will be exposed, and these need to be included on the free-body diagram.

   At this point, we don't know the actual directions of the internal forces, but we do know that they act in opposite directions. We will assume that they act in the positive sense as defined by the standard sign convention.

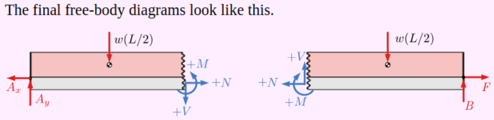
   Axial forces are positive in tension and act in opposite directions on the two halves of the cut beam.

   Positive shear forces act down when looking at the cut from the right, and up when looking at the cut from the left. An alternate definition of positive shears is that the positive shears cause clockwise rotation. This definition is useful if you are dealing with a vertical column instead of a horizontal beam.

   Bending moments are positive when the moment tends to bend the beam into a smiling U-shape. Negative moments bend the beam into a frowning shape.

   For vertical columns, positive bending moments bend a beam into a C shape and negative into a backward C-shape.

   The final free-body diagrams look like this.



   Horizontal beams should always have assumed internal loadings in these directions at the cut, indicating that you have assumed *positive* shear, *positive* normal force and *positive* bending moments at that point.

4. *Solve for the internal forces.*

   You may uses either FBD to find the internal forces using the techniques you have already learned. So, with a standard xy coordinate system, forces to the right or up are positive when summing forces and counter-clockwise moments are positive when summing moments.

Using the left free-body diagram and substituting in the reactions, we get:

$$\begin{aligned} \Sigma F_x &= 0 \\ -A_x + N &= 0 \\ N &= A_x \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= 0 \\ A_y - wL/2 - V &= 0 \\ V &= wL/2 - wL/2 \\ V &= 0 \end{aligned}$$

$$\begin{aligned} \Sigma M_{\text{cut}} &= 0 \\ (wL/2)(L/4) - (A_y)(L/2) + M &= 0 \\ M &= -wL^2/8 + wL^2/4 \\ M &= wL^2/8 \end{aligned}$$

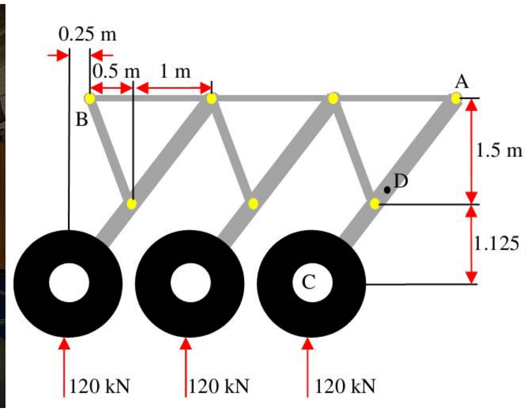
Using the right side free-body diagram we get:

$$\begin{aligned} \Sigma F_x &= 0 \\ -N + F &= 0 \\ N &= F \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= 0 \\ V - wL/2 + B_y &= 0 \\ V &= wL/2 - B_y \\ V &= wL/2 - wL/2 \\ V &= 0 \end{aligned}$$

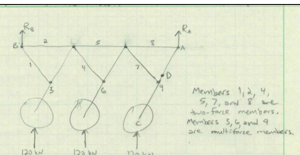
$$\begin{aligned} \Sigma M\_ &= 0 \\ -M - (L/4)(wL/2) + (L/2)(B_y) &= 0 \\ M &= -WL^2/8 + wL^2/4 \\ M &= WL^2/8 \end{aligned}$$

5. The rear landing gear mechanism from the Airbus A400M military transport aircraft is shown below. The simplified diagram provided shows the external forces acting on the mechanism during the final phase of a landing. Each of the three sections of the mechanism have identical dimensions. The mechanism is connected to the aircraft with a fixed support at point A and a roller support at point B. Determine the internal forces at point D, which is the midpoint of member AC.



A400M Rear Landing Gear (left) Simplified Diagram (right)

Solution:



Members 1, 2, 4, 5, 7, and 8 are two-force members. Members 3, 6, and 9 are multiforce members.

To find the internal forces at point D in member 9 (member AC), we must find either the force in member 7, or the force in member 8 and the reaction at A. First, the reactions at A and B are found from the FBD of the full landing gear:

$$\Sigma M_A = 0 \rightarrow -R_B(4.5) - (120)(4.75) - (120)(3.25) - (120)(1.75) = 0$$

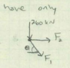
$$\therefore \underline{R_B = -260 \text{ kN}}$$

$$\Sigma F_y = 0 \rightarrow R_B + R_A + 360 = 0$$

$$\therefore \underline{R_A = -100 \text{ kN}}$$

At point B, we have only two unknowns:

Point B:



$$\theta = \tan^{-1}\frac{1.5}{0.5} = 71.57°$$

$$\Sigma F_y = 0 \rightarrow -260 - F_1\sin\theta = 0$$

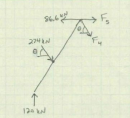
$$\therefore F_1 = -274 \text{ kN} \quad (274 \text{ kN in compression})$$

$$\Sigma F_x = 0 \rightarrow F_2 + F_1\cos\theta = 0$$

$$\therefore F_2 = 86.6 \text{ kN} \quad (\text{tension})$$

Now, we can take the FBD of member 3 and we will have two unknowns, the forces in members 4 and 5.

Member 3:



$$\Sigma F_y = 0 \rightarrow -274\sin\theta - F_4\sin\theta + 120 = 0$$

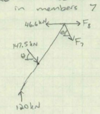
$$\therefore F_4 = -147.5 \text{ kN} \quad (147.5 \text{ kN in compression})$$

$$\Sigma F_x = 0 \rightarrow 274\cos\theta - 86.6 + F_4\cos\theta + F_5 = 0$$

$$\therefore F_5 = 46.6 \text{ kN} \quad (\text{tension})$$

We can now take the FBD of member 6 to find the forces in members 7 and 8:

Member 6:



$$\Sigma F_y = 0 \rightarrow -147.5\sin\theta - F_7\sin\theta + 120 = 0$$

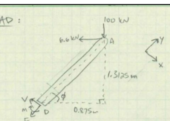
$$\therefore F_7 = -21.0 \text{ kN} \quad (21 \text{ kN in compression})$$

$$\Sigma F_x = 0 \rightarrow 147.5\cos\theta - 46.6 + F_7\cos\theta + F_8 = 0$$

$$\therefore F_8 = 6.6 \text{ kN} \quad (\text{tension})$$

Finally, we can take the FBD of either section AD or CD to find the internal forces at point D.

Section AD:



$$\text{Let } \phi = \tan^{-1}\frac{1.3125}{0.875} = 56.3°$$

$\underline{\Sigma F_x = 0}$

$$-6.6\sin\phi + 100\cos\phi - V = 0$$

$$\therefore \boxed{V = 50.0 \text{ kN}}$$

$\underline{\Sigma F_y = 0}$

$$6.6\cos\phi - 100\sin\phi - F = 0$$

$$\therefore \boxed{F = -86.9 \text{ kN}}$$

$\underline{\Sigma M_A = 0}$

$$-V\left(\sqrt{0.875^2 + 1.3125^2}\right) - M = 0$$

$$\therefore \boxed{M = -78.9 \text{ kN}}$$