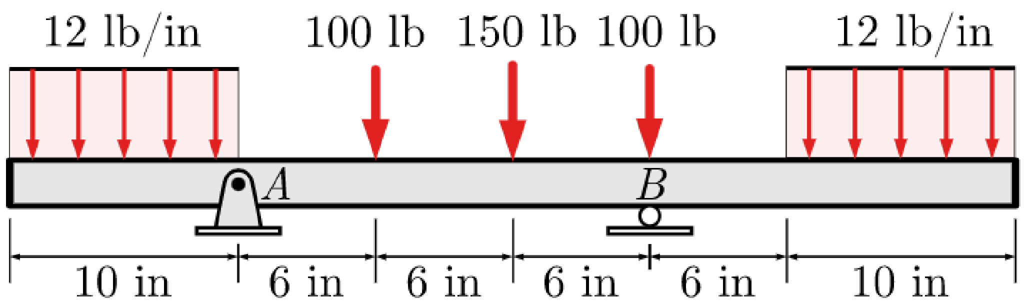
AEM 2011 Homework #8 (5.2 (cont.), 5.3, 5.4) Wednesday, March 15, 2023

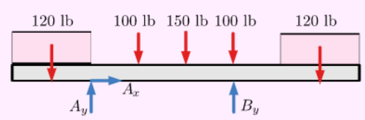
Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. Find the reactions at the supports for the beam shown.



Solution:

Start by drawing a free-body diagram of the beam with the two distributed loads replaced with equivalent concentrated loads. The two distributed loads are $(10\text{ in})(12\text{ lb/in}) = 120\text{ lb}$ each.



Then apply the equations of equilibrium.

$$\begin{aligned}
&\sum M_A = 0 \\
&+(12\text{ lb/in})(10\text{ in})(5\text{ in}) - (100\text{ lb})(6\text{ in}) \\
&-(150\text{ lb})(12\text{ in}) - (100\text{ lb})(18\text{ in}) \\
&+(B_y)(18\text{ in}) - (12\text{ lb/in})(10\text{ in})(29\text{ in}) = 0 \rightarrow \qquad B_y = 393.3\text{ lb}
\end{aligned}$$

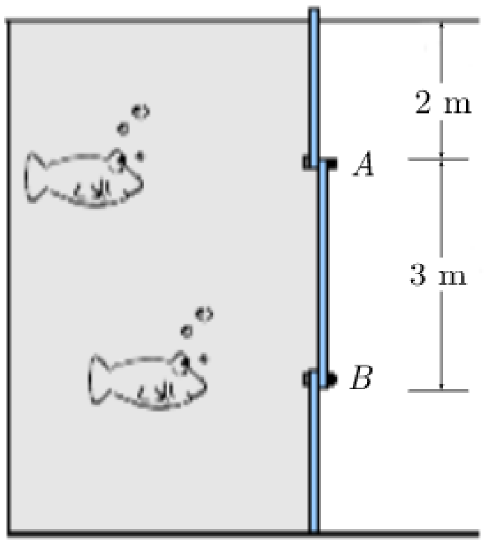
$$\begin{aligned}
&\sum F_y = 0 \\
&-(12\text{ lb/in})(10\text{ in}) + B_y - 100\text{ lb} - 150\text{ lb} \\
&-100\text{ lb} + B_y - (12\text{ lb/in})(10\text{ in}) = 0 \rightarrow \qquad B_y = 196.7\text{ lb}
\end{aligned}$$

$$\sum F_x = 0 \rightarrow \qquad A_x = 0$$

$$A_y = 196.7\text{ lb}, A_x = 0\text{ lb}, B_y = 393.3\text{ lb}$$

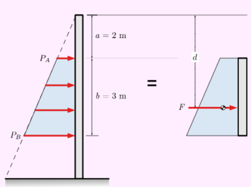
2. An aquarium tank has a $3m \times 1.5m$ window AB for viewing the inhabitants. The tank contains water with a density $\rho = 1000kg/m^3$.

   Find the force of the water on the window, and the location of the equivalent point load.



Solution:

Begin by drawing a diagram of the window showing the load intensity and the equivalent concentrated force.



The pressure at the top and the bottom of the window are

$$P_A = \rho\, g(2\text{ m}) = 19620\text{ N/m}^2$$
$$P_B = \rho\, g(5\text{ m}) = 49050\text{ N/m}^2$$

Since the loading is linear, the average pressure acting on the window is

$$\begin{aligned} P_{ave} &= (P_A + P_B)/2 \\ &= 34300\text{ N/m}^2 \end{aligned}$$

The total force acting on the window is the average pressure times the area of the window

The total force acting on the window is the average pressure ti... window

$$\begin{aligned} F &= (P_{ave})(3\text{ m} \times 1.5\text{ m}) \\ &= 155\text{ kN} \end{aligned}$$

This force may also be visualized as the volume of a trapezoidal prism with a 1.5 m depth into the page.

The line of action of the equivalent force passes through the centroid of the trapezoid, which may be calculated using composite areas, see Section 7.5.

Dividing the trapezoid into a triangle and a rectangle and measuring down from the surface of the tank, the distance to the equivalent force is

$$d = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$

$$d = \frac{[P_A(3\text{ m})](3.5\text{ m}) + \left[\frac{1}{2}(P_B - P_A)(3\text{ m})\right](4\text{ m})}{[P_A(3\text{ m})] + \left[\frac{1}{2}(P_B - P_A)(3\text{ m})\right]}$$

$$d = 3.71\text{ m}$$

If you prefer, you may use the formula from the Centroid table to locate the centroid of the trapezoid instead.

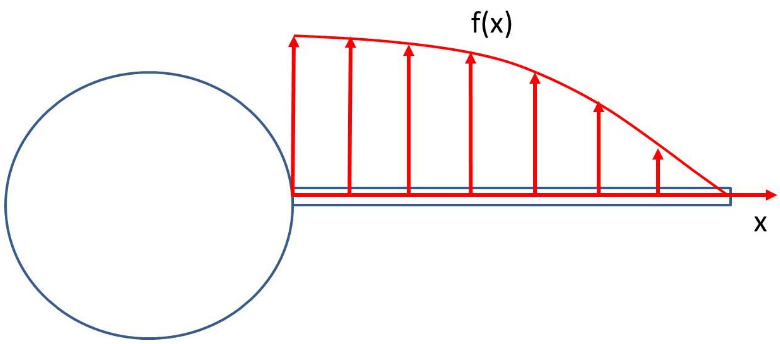
3. The lift force generated on an aircraft wing is distributed along the span of the wing. Generally the distributed force is larger near the body of the aircraft and decreases toward the wingtip. A simple diagram showing the aircraft body (viewed from the front) and one wing is shown below. Treat the wing as rigidly attached to the aircraft body and assume the distributed lift force is given by an ellipitical profile for $x \leq 3$m:

$$f(x) = \sqrt{9 - x^2} N/m$$

Find:

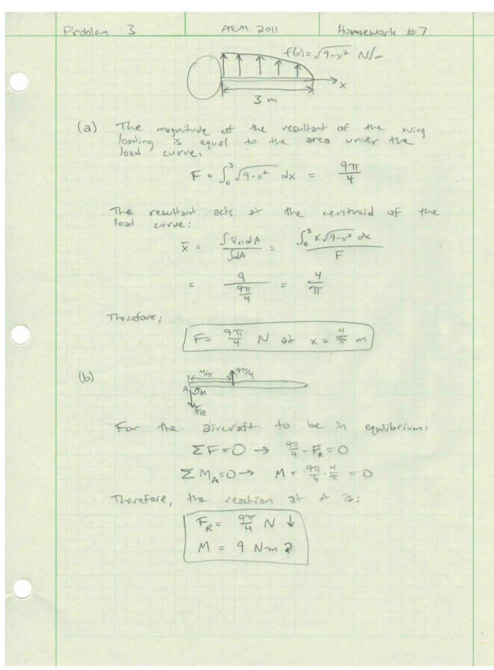
(a) The magnitude and location of the resultant of the distributed load.

(b) The support reaction applied by the aircraft body on the aircraft wing at $x = 0$.



Simplified diagram of wing loading.

Solution:



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$f(x) = \sqrt{9-x^2}$ N/m

3 m

(a) The magnitude of the resultant of the wing loading is equal to the area under the load curve:

$$F = \int_0^3 \sqrt{9-x^2}\,dx = \frac{9\pi}{4}$$

The resultant acts at the centroid of the load curve:

$$\bar{x} = \frac{\int \bar{x}_{el}\,dA}{\int dA} = \frac{\int_0^3 x\sqrt{9-x^2}\,dx}{F}$$

$$= \frac{9}{\frac{9\pi}{4}} = \frac{4}{\pi}$$

Therefore;

$$\boxed{F = \frac{9\pi}{4} \text{ N at } x = \frac{4}{\pi} \text{ m}}$$

(b)

$4/\pi$ $9\pi/4$ A M F_R

For the aircraft to be in equilibrium:

$$\Sigma F = 0 \rightarrow \frac{9\pi}{4} - F_R = 0$$

$$\Sigma M_A = 0 \rightarrow M + \frac{9\pi}{4} \cdot \frac{4}{\pi} = 0$$

Therefore, the reaction at A is:

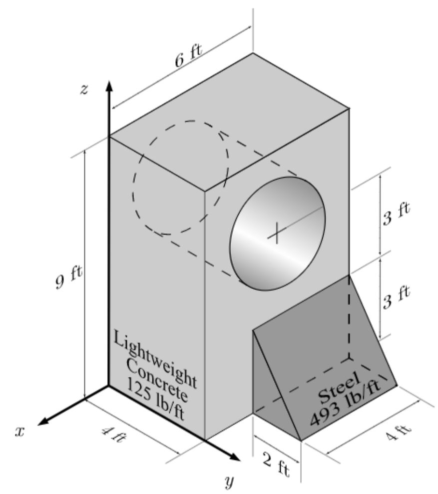
$$\boxed{\begin{array}{l} F_R = \frac{9\pi}{4} \text{ N} \downarrow \\ M = 9 \text{ Nm} \end{array}}$$

4. A composite solid consists of a rectangular block of lightweight concrete and a triangular wedge of steel with dimensions as shown. The rectangular block has a $2ft$ radius circular hole, centered and drilled through its full depth, perpendicular to the front and back faces.

   Assume:

   - $\gamma_C = 125lb/ft^3$
   - $\gamma_S = 493lb/ft^3$

   Find the center of mass of this composite solid.



Solution:

Table 7.5.4.

Part	V_i [ft^3]	γ [lb/ft^3]	W_i [lb]	$\bar{x}_i$ [ft]	$\bar{y}_i$ [ft]	$\bar{y}_i$ [ft]	$W_i\,\bar{x}_i$ [lb-ft]	$W_i\,\bar{y}_i$ [lb-ft]	$W_i\,\bar{z}_i$ [lb-ft]
block	216	125	27000	-3	2	4.5	-81000	54000	121500
hole	-50.27	125	-6283	-3	2	6	18850	-12566	-37699
wedge	12	493	5916	-4	4.67	1	-23664	27608	5916
			26633				-85814	69042	89717

$$\bar{x} = \frac{\sum W_i\,\bar{x}_i}{\sum V_i} = \frac{-85814\ \text{ft}^3}{26633\ \text{ft}^2} = -3.22\ \text{ft}$$

$$\bar{y} = \frac{\sum W_i\,\bar{y}_i}{\sum V_i} = \frac{69042\ \text{ft}^3}{26633\ \text{ft}^2} = 2.59\ \text{ft}$$

$$\bar{z} = \frac{\sum W_i\,\bar{z}_i}{\sum V_i} = \frac{89717\ \text{ft}^3}{26633\ \text{ft}^2} = 3.37\ \text{ft}$$

We have actually found the coordinates of the center of gravity, but since g is constant they are also coordinates of the center of mass.

$$\bar{x} = -3.22\ \text{ft}$$
$$\bar{y} = 2.59\ \text{ft}$$
$$\bar{z} = 3.37\ \text{ft}$$

5. Book problems:

   (a) 5.76

   (b) 5.97

   (c) 5.111

   Additional Practice Problems: 5.73, 5.74, 5.86, 5.102, 5.109

   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

5.76 B = 150.0 lb up, C = 5250 lb up

5.97 $21h/16$ above the vertex of the cone

5.111 $\bar{X} = 17.00in$, $\bar{Y} = 15.68in$, $\bar{Z} = 14.16in$