AEM 2011 Homework #9 (6.1, 6.2) Wednesday, March 22, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. Book problems:
   (a) 6.5
   (b) 6.51

   Additional Practice Problems: 6.11, 6.22, 6.45, 6.61

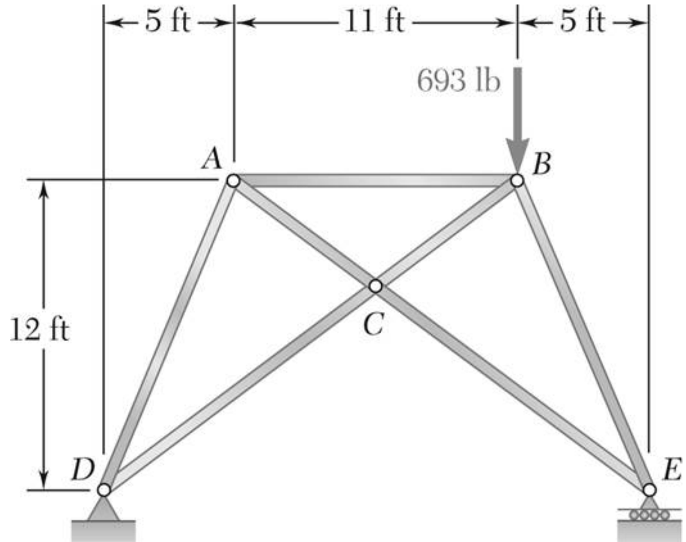
   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

6.5 $F_{BD} = 0$, $F_{AC} = 5.00$ kips C, $F_{AD} = 13.00$ kips T, $F_{CD} = 30.0$ kips C, $F_{DF} = 5.00$ kips T, $F_{CF} = 32.5$ kips T, $F_{CE} = 17.50$ kips C, $F_{EF} = 0$

6.51 $F_{DE} = 25.0$ kips T, $F_{DF} = 13.00$ kips C

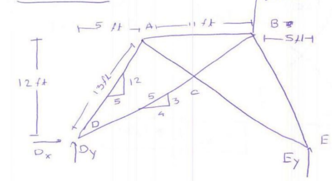
2. While constructing the new recreational-center building, the construction company erected a temporary platform as shown in the figure. The equipment kept on the platform at point B weighs 693 lb. The platform can be modeled as a truss. Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.



Temporary Platform

Solution:

Problem - 2



Reactions

$\Sigma F_x = 0 \Rightarrow \boxed{D_x = 0}$

$\Sigma M_E = 0 \Rightarrow D_y \times 21 - 693 \times 5 = 0$

$\Rightarrow \boxed{D_y = 165\ Lb}$

$\Sigma F_y = 0 \Rightarrow E_y = 693 - 165 \Rightarrow \boxed{E_y = 528 Lb}$

Joint D

T_{AD} T_{CD}

$\Sigma F_x = 0 \Rightarrow \frac{5}{13} T_{AD} + \frac{4}{5} T_{DC} = 0$

$\Sigma F_y = 0 = \frac{12}{13} T_{AD} + \frac{3}{5} T_{DC} + 165 = 0$

$\Rightarrow \boxed{T_{AD} = -260 Lb}$ $\boxed{T_{DC} = 125 Lb}$

Joint E

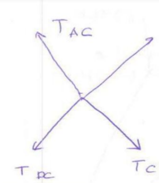
T_{BE} T_{CE}

$\Sigma F_x = 0 \Rightarrow -\frac{5}{13} T_{BE} - \frac{4}{5} T_{CE} = 0$

$\Sigma F_y = 0 \Rightarrow \frac{12}{13} T_{BE} + \frac{3}{5} T_{CE} + 528 = 0$

$\Rightarrow \boxed{T_{BE} = -832 Lb}$ $\boxed{T_{CE} = 400 Lb}$

t C



$T_{BC} =$
T_{AC}

$\boxed{c = 400 Lb}$, $\boxed{T_{BC} = 125 Lb}$

T_{AB}

T_{AC}

$\Sigma F_x = 0$

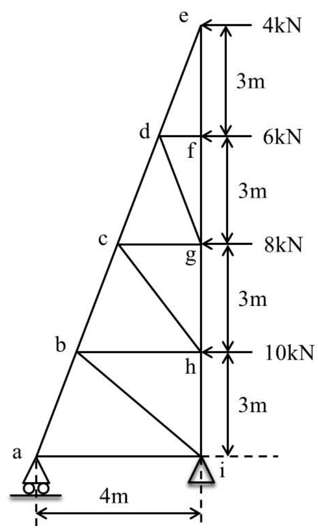
$\Rightarrow \frac{5}{13}(260) + \frac{4}{5}(400) +$

$\Rightarrow \boxed{F_{AB} = -420\ Lb}$

3. Considering the truss shown in Fig. 2, answer the following questions:

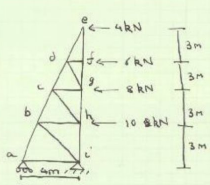
   (a) Show that the truss is stable (*i.e. static*).

   (b) Find the fices in members cd, cg, and gh.



Schematic of Truss

Solution:



ⓐ Show that truss is stable

ⓑ Find forces in members cd, cg & gh of this truss.

Solu^n:

ⓐ $m = 15$

$n = 9$

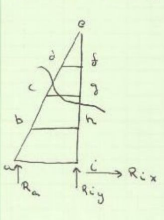
$\therefore m = 2n - 3$

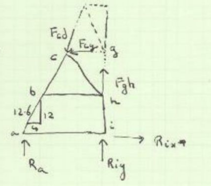
$15 = 2(9) - 3$

$15 \equiv 15$

$\therefore$ The truss is stable.

ⓑ A cut is passed thru cd, cg & gh.





For equilibrium,

$\Sigma M_i = 0 \qquad -R_a(4) + 10(3) + 8(6) + 6(9) + 4(12) = 0$

$\therefore R_a = 45\,kN$

$\Sigma F_y = 0 \qquad \therefore R_a - R_{iy} = 0$

$\therefore R_{iy} = R_a = 45\,kN$

$\Sigma F_x = 0 \qquad R_{ix} - 10 - 8 - 6 - 4 = 0$

$\therefore R_{ix} = 28\,kN$

Find length cg ?

$$\frac{l_{cg}}{6} = \frac{4}{12} \Rightarrow l_{cg} = 2m$$

Now let us look at the FBD of the lower part of the truss (the cut section)

$\Sigma M_c = 0$

$$-45(2) + F_{gh}(2) - 10(3) + 28(6) - 45(2) = 0$$

$$\therefore F_{gh} = 21\,kN\,(T)$$

$\Sigma F_y = 0 \qquad \therefore 45 - \frac{12}{12.6}F_{cd} + F_{gh} - 45 = 0$

$$\therefore F_{cd} = 22.1\,kN\,(c)$$

$\Sigma F_x = 0$

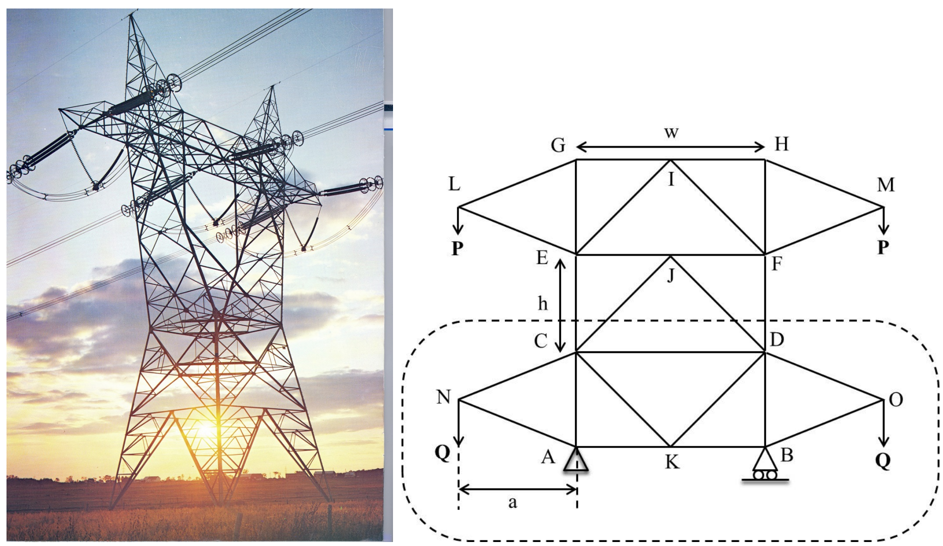
$$\therefore \frac{-4}{12.6}F_{cd} - F_{cg} - 10 + 28 = 0$$

$$\therefore F_{cg} = 11\,kN\,(c)$$

4. Electrical transmission towers can be modeled as trusses, as shown in the figure. Although they are actually three-dimensional space trusses, in this problem, they can be approximated as the standard two-dimensional truss. Consider a simplified section of the transmission tower, as shown, with the following dimensions and forces: $w = 1m$, $h = 0.5m$, $a = 0.8m$, $P = 800N$, and $Q = 1200N$. Assume that sections AC, CE, and EG all have the same height h. You may also make use of the fact that the truss is symmetric about a vertical axis passing through points I, J, and K. Compute the following:

   (a) The reaction forces at supports A and B.

   (b) The forces in all members enclosed by the dashed region shown in the figure.



Electrical Transmission Tower

Solution:

①

PROBLEM 4

$w = 1\ m$, $h = 0.5\ m$, $a = 0.8\ m$
$P = 800 N$, $Q = 1200N$

SOLUTION

First, we need to establish that this truss is statically determinate and stable.

no. of members $= 27 = m$
no. of joints $= 15 = n$

$\boxed{m = 2n - 3 \text{ is satisfied}}$

(a) Reactions at A, B

Since none of the external forces are in the horizontal direction, the support at A only exerts a vertical force.

$\cancel{A_y + B_y}$, $\Sigma F_y = A_y + B_y - 2(P+Q) = 0$
Due to symmetric loading, $\boxed{A_y = B_y = 2KN}$

$\boxed{A_y = B_y = 2KN \uparrow}$

(b) The dashed region encloses the following members: AN, NC, AC, AK, CK, CD, CE, CJ and their symmetric counterparts. In the following analysis, we will assume that all members are in tension. (Compression will present itself with

②

a negative sign on the force.)

Bolt A

$\Sigma F_x: -T_{AN} \sin\theta + T_{AK} = 0$
$\Sigma F_y: T_{AC} + A_y + T_{AN} \cos\theta = 0$
$\cancel{T_{AC} = A_y = 2000 N}$
$T_{AC} = -A_y - T_{AN}\cos\theta = -1400N$

$\theta = \tan^{-1}\left(\frac{2a}{h}\right) = 72.65°$

Bolt N

$\Sigma F_x: T_{NC}\sin\theta + T_{AN}\sin\theta = 0$
$\Rightarrow T_{NC} = -T_{AN}$

$\Sigma F_y: T_{NC}\cos\theta - T_{AN}\cos\theta - Q = 0$
$\Rightarrow T_{AN} = \frac{Q}{-2\cos\theta} = -2012 N$

$T_{NC} = 2012 N$

$T_{AK} = T_{AN}\sin\theta = -1920.45 N$

Bolt K

$\Sigma F_x: -T_{AK} + T_{KB} = 0$
$T_{KB} = -1920.45N$

$\Sigma F_y: (T_{CK} + T_{KD})\cos\alpha = 0$
From symmetry, $T_{CK} = T_{KD} = 0$

By the same argument, $T_{CJ} = T_{DJ} = 0$
and $T_{EI} = T_{FI} = 0$

$T_{CE} = T_{AC} + T_{NC}\cos\theta$

③

Bolt C

$\Sigma F_x: -T_{NC}\sin\theta + T_{CD} = 0$

$T_{CD} = 1920.45 N$

$\Sigma F_y: T_{CE} - T_{AC} - T_{NC}\cos\theta = 0$

$T_{CE} = \cancel{-1400 N}\ -800N$

$T_{CJ} = 0$, T_{CD}, $T_{CK} = 0$, T_{NC}, T_{AC}

Summary:

$T_{AN} = -2012 N \Rightarrow$ compression
$T_{NC} = 2012 N \Rightarrow$ tension
$T_{AC} = \cancel{-2000}\ -1400 N \Rightarrow$ compression
$T_{AK} = -1920.45 N \Rightarrow$ compression
$T_{CK} = 0$
$T_{CD} = 1920.45 N \Rightarrow$ tension
$T_{CE} = \cancel{-1400}\ -800 N \Rightarrow$ compression
$T_{CJ} = 0$

Follow up question?

If the members CJ, DJ, EI, FI, CK, DK have zero force, why do you think they are present on the transmission tower?

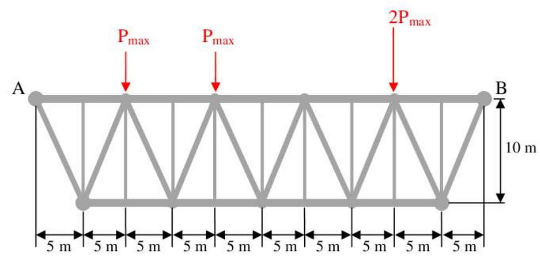
Hint: Think redundancy.
What if member AC fails? Do you think the truss will be stable? What changes would you make to the truss to ensure that

...the truss remains stable? After AC has failed, would the forces in CK and DK still remain zero? Why or why not?

5. The Stone Arch Bridge in Minneapolis consists of 21 stone arch spans and one steel truss span. The steel truss span replaced two of the original stone arches when the St. Anthony Falls lock and dam system was built. Under maximum loading conditions, the truss experiences the forces shown in the diagram below. Assume:

   - horizontal members can withstand a maximum of 3600 kN in tension and 1200 kN in compression
   - diagonal members can withstand a maximum of 1200 kN in tension and 400 kN in compression
   - vertical members can withstand a maximum of 300 kN in tension and 100 kN in compression.

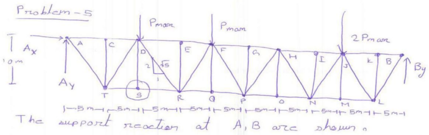
   Find the value of P_{max} such that the forces remain within the specified tension and compression limits. The truss is attached to the bridge with a fixed support at A and a roller support at B.



Stone Arch Bridge (left) and free body diagram of steel truss span (right)

Solution:

Problem - 5



The support reaction at A, B are shown.

Considering the entire truss. For equilibrium

$\Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$ . . . . (1)

$\Sigma F_y = 0 \Rightarrow P_{max} + P_{max} + 2P_{max} - (A_y + B_y) = 0$

$\Rightarrow \boxed{A_y + B_y = 4P_{max}}$ . . . . (2)

$\Sigma M_B = 0 \Rightarrow A_y \times 50 - P_{max} \times 40 - P_{max} \times 30$

$- 2P_{max} \times 10 = 0$

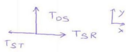
$\Rightarrow \boxed{A_y = \frac{9 P_{max}}{5}}$ . . . . . (3)

Substituting (3) in (2)

$\boxed{B_y = \frac{11 P_{max}}{5}}$

Now, Lets consider joint S. FBD

For equilibrium $\Sigma F_y = 0$



$\Rightarrow \boxed{T_{DS} = 0}$

And $\Sigma F_x = 0 \Rightarrow \boxed{T_{ST} = T_{SR}}$

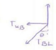
Similarly force in all the vertical members = 0

$\Rightarrow \boxed{T_{CT} = T_{DS} = T_{ER} = T_{FQ} = T_{GP} = T_{HO} = T_{IN} = T_{JM} = T_{KL} = 0}$

(We will ignore these members)

Now, We will consider a joint one by one

Joint B



$B_y = \frac{11 P_{max}}{5}$

$\Sigma F_x = 0$

$-T_{KB} - T_{BL} \times \frac{\cancel{2}\,1}{\sqrt{5}} = 0$

$\tan\theta = \frac{1}{2}$

$\Rightarrow \boxed{T_{KB} = -\frac{\cancel{2}T_{BL}}{\sqrt{5}}}$

$\Sigma F_y = 0 \Rightarrow B_y - T_{BL}\frac{2}{\sqrt{5}} = 0$

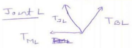
$\Rightarrow T_{BL} = \frac{\sqrt{5}}{2} \times \frac{11}{5} P_{max}$

$\Rightarrow \boxed{T_{BL} = \frac{11\sqrt{5}}{10} P_{max}}$

$\Rightarrow \boxed{T_{KB} = -\frac{11}{10} P_{max}}$

Joint K $\boxed{T_{JK} = T_{KB} = -\frac{11}{10} P_{max}}$

Joint L



$\Sigma F_y = 0$

$\Rightarrow T_{BL} \times \frac{2}{\sqrt{5}} + T_{JL} \times \frac{2}{\sqrt{5}} = 0$

$\Rightarrow \boxed{T_{JL} = -\frac{11\sqrt{5}}{10} P_{max}}$

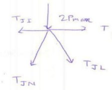
$\Sigma F_x = 0$

$T_{ML} = T_{BL} \times \frac{1}{\sqrt{5}} - T_{JL} \times \frac{1}{\sqrt{5}}$

$\boxed{T_{ML} = \frac{11}{5} P_{max}}$

Joint M $\boxed{T_{NM} = T_{ML} = \frac{11}{5} P_{max}}$

Joint J



$\Sigma F_y = 0$

$\Rightarrow \cancel{\boxed{T_{JN} = \frac{11\sqrt{5}}{10} P_{max}}}$

$-2P_{max} + \frac{11\sqrt{5}}{10} \times \frac{2}{\sqrt{5}} P_{max}$

$- T_{JN} \times \frac{2}{\sqrt{5}} = 0$

$\Rightarrow T_{JN} = \frac{P_{max}}{5} \times \frac{\sqrt{5}}{2}$

$\Rightarrow \boxed{T_{JN} = \frac{\sqrt{5}}{10} P_{max}}$

$\Sigma F_x = 0 \Rightarrow T_{JK} + T_{JL} \times \frac{1}{\sqrt{5}} - T_{JI}$

$- T_{JN} \times \frac{1}{\sqrt{5}} = 0$

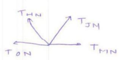
$\Rightarrow T_{JI} = -\frac{11}{10} P_{max} - \frac{11}{10} P_{max}$

$- \frac{1}{10} P_{max}$

$\Rightarrow \boxed{T_{JI} = -\frac{23}{10} P_{max}}$

Joint I $\boxed{T_{HI} = T_{JI} = \frac{-23}{10} P_{max}}$

Joint N



$\Sigma F_y = 0 \Rightarrow \cancel{\boxed{T_{HN} = -\frac{\sqrt{5}}{10} P_{max}}}$ $\boxed{T_{HN} = -\frac{\sqrt{5}}{10} P_{max}}$

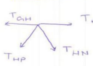
$\Sigma F_x = 0$ $T_{ON} = T_{MN} + T_{JN} \times \frac{1}{\sqrt{5}} - T_{HN}\frac{1}{\sqrt{5}}$

$= \frac{11}{5} P_{max} + \frac{1}{10} P_{max} + \frac{1}{10} P_{max}$

$\Rightarrow \boxed{T_{ON} = \frac{24}{10} P_{max}}$

Joint O $\boxed{T_{OP} = T_{ON} = \frac{24}{10} P_{max}}$

Joint H



$\Sigma F_y = 0$

$\Rightarrow \boxed{T_{HP} = \frac{\sqrt{5}}{10} P_{max}}$

$\Sigma F_x = 0 = T_{GH} = T_{HI} + T_{HN} \times \frac{1}{\sqrt{5}}$

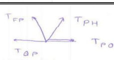
$- T_{HP} \times \frac{1}{\sqrt{5}}$

$\Rightarrow T_{GH} = -\frac{23}{10} P_{max} - \frac{P_{max}}{10} - \frac{P_{max}}{10}$

$\boxed{T_{GH} = \frac{-25 P_{max}}{10}}$

Joint G $\boxed{T_{FG} = T_{GH} = \frac{-25 P_{max}}{10}}$

Joint P



$\Sigma F_y = 0$

$\Rightarrow \boxed{T_{FP} = -\frac{\sqrt{5}}{10} P_{max}}$

$\Sigma F_x = 0 \Rightarrow T_{QP} = T_{OP} + T_{PH} \times \frac{1}{\sqrt{5}}$

$- T_{FP} \times \frac{1}{\sqrt{5}}$

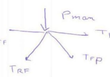
$= \frac{24}{10} P_{max} + \frac{1}{10} P_{max}$

$+ \frac{1}{10} P_{max}$

$\Rightarrow \boxed{T_{QP} = \frac{26}{10} P_{max}}$

Joint Q $\boxed{T_{RQ} = T_{PQ} = \frac{26}{10} P_{max}}$

Joint F



$\Sigma F_y = 0$

$\Rightarrow -T_{RF}\frac{2}{\sqrt{5}} - T_{FP} \times \frac{2}{\sqrt{5}}$

$- P_{max} = 0$

~~$\Rightarrow T_{RF} = P_{max}$~~

$T_{RF} \times \frac{2}{\sqrt{5}} = -P_{max} + \frac{2}{10} P_{max}$

$= -\frac{8}{10} P_{max}$

$\Rightarrow \boxed{T_{RF} = -\frac{4\sqrt{5}}{10} P_{max}}$

~~Joint E~~ ~~$T_{DE} = T_E$~~

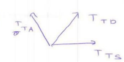
$\Sigma F_x = 0 \Rightarrow T_{EF} = T_{FG} + T_{FP} \times \frac{1}{\sqrt{5}} - T_{RF}\frac{1}{\sqrt{5}}$

$= -\frac{25 P_{max}}{10} - \frac{1}{10} P_{max} + \frac{4}{10} P_{max}$

$\boxed{T_{EF} = -\frac{22 P_{max}}{10}}$

Joint C $\boxed{T_{CA} = T_{CD} = -\frac{9}{10} P_{max}}$

Joint T



$\Sigma F_y = 0$

$\Rightarrow \boxed{T_{TA} = \frac{9\sqrt{5}}{10} P_{max}}$

We can check our calculations by calculating T_{TS} and comparing it with previous value.

$\Sigma F_x = 0 \Rightarrow T_{TA} \times \frac{1}{\sqrt{5}} = T_{TS} + T_{DT} \times \frac{1}{\sqrt{5}}$

$\Rightarrow \frac{9}{10} P_{max} = \frac{18}{10} P_{max} - \frac{9}{10} P_{max}$

$\Rightarrow \boxed{\frac{9}{10} P_{max} = \frac{9}{10} P_{max}}$

$\Rightarrow$ Our solution is correct

~~Joint A~~

Now Diagonal elements

$T_{BL} = \frac{11\sqrt{5}}{10} P_{max}$ (T)

$T_{JL} = \frac{11\sqrt{5}}{10} P_{max}$ (C)

$T_{JN} = \frac{\sqrt{5}}{10} P_{max}$ (T)

$T_{NH} = \frac{\sqrt{5}}{10} P_{max}$ (C)

$T_{HP} = \frac{\sqrt{5}}{10} P_{max}$ (T)

$T_{PF} = \frac{\sqrt{5}}{10} P_{max}$ (C)

$T_{RF} = \frac{4\sqrt{5}}{10} P_{max}$ (C)

$T_{RD} = \frac{4\sqrt{5}}{10} P_{max}$ (T)

$T_{TD} = \frac{9\sqrt{5}}{10} P_{max}$ (C)

$T_{TA} = \frac{9\sqrt{5}}{10} P_{max}$ (T)

Horizontal elements

$T_{KB} = \frac{11}{10} P_{max}$ (C)	$T_{JK} = \frac{11}{10} P_{max}$ (C)
$T_{ML} = \frac{11}{5} P_{max}$ (T)	$T_{NM} = \frac{11}{5} P_{max}$ (T)
$T_{IJ} = \frac{23}{10} P_{max}$ (C)	$T_{HI} = \frac{23}{10} P_{max}$ (C)
$T_{ON} = \frac{24}{10} P_{max}$ (T)	$T_{OP} = \frac{24}{10} P_{max}$ (T)
$T_{GH} = \frac{25}{10} P_{max}$ (C)	$T_{FG} = \frac{25}{10} P_{max}$ (C)
$T_{QP} = \frac{26}{10} P_{max}$ (T)	$T_{RQ} = \frac{26}{10} P_{max}$ (T)
$T_{EF} = \frac{22}{10} P_{max}$ (C)	$T_{DE} = \frac{22}{10} P_{max}$ (C)
$T_{RS} = \frac{18}{10} P_{max}$ (T)	$T_{TS} = \frac{18}{10} P_{max}$ (T)
$T_{CD} = \frac{9}{10} P_{max}$ (C)	$T_{AC} = \frac{9}{10} P_{max}$ (C)

Most critical

$\frac{26}{10} P_{max} \le 3600$ KN $\Rightarrow P_{max} \le 1.38 \times 10^3$ KN

$\frac{25}{10} P_{max} \le 1200$ KN $\Rightarrow P_{max} \le 480$ KN

$\frac{11\sqrt{5}}{10} P_{max} \le \cancel{1200}\ 1200$ KN $\Rightarrow P_{max} \le \cancel{121.96}\ 487.86$ KN

$\frac{11\sqrt{5}}{10} P_{max} \le \cancel{100}\ 400$ KN $\Rightarrow P_{max} \le \cancel{40.655}\ 162.62$ KN

$\Rightarrow$ ~~$P_{max} \le 40.655$ KN~~ $\Rightarrow \boxed{P_{max} \le 162.62 \text{ KN}}$