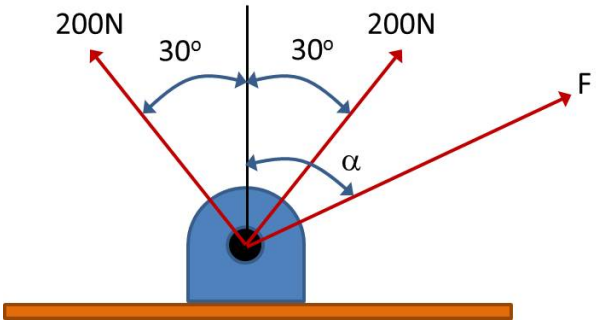
AEM 2011 Final Exam Monday, December 16

You may only use a simple calculator that does not communicate in any wireless manner. Pagers, cell phones, and all other communications devices must be turned off during the quiz. All work (free body diagrams, algebra, etc) is required to receive full credit.

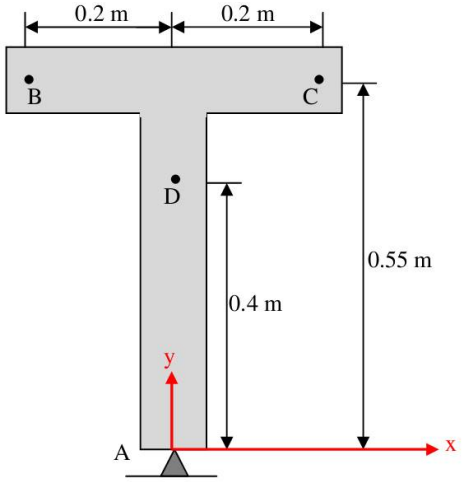
1. Three forces are applied to a bracket as shown in the diagram below.

   (a) (5 points) What is the resultant force on the bracket if $\alpha = 90^o$ and $F = 100N$?

   (b) (5 points) Assume $F = 200N$. Determine the range of angles α for which the resultant force on the bracket has magnitude less than $200N$.



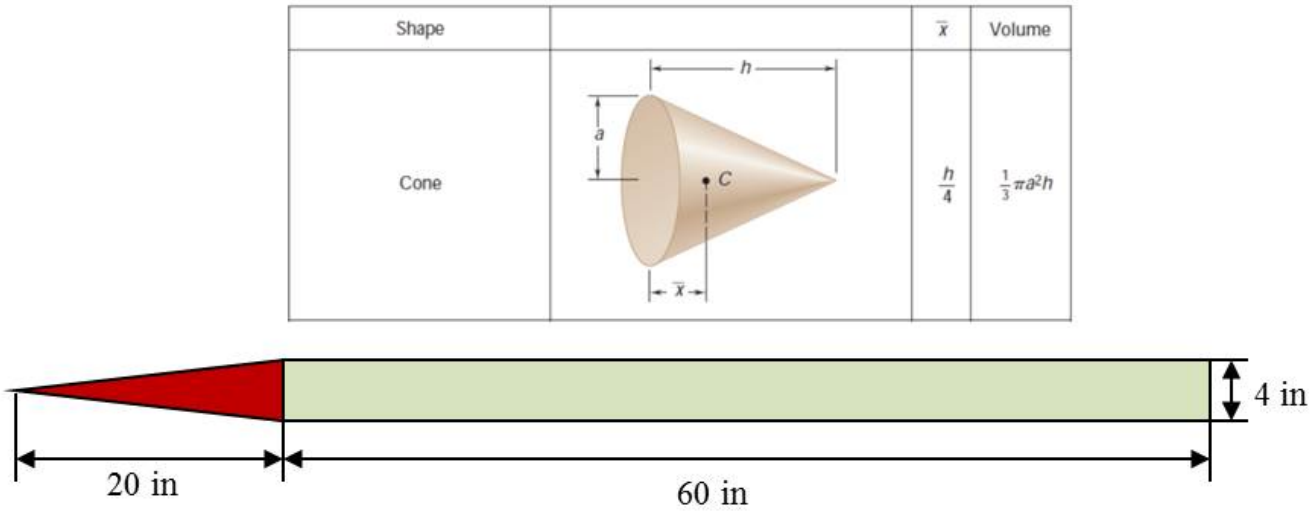
2. (15 points) A T-shaped bracket is shown in the figure below. The bracket is supported by a frictionless ball joint at point A. Assume there are also cables attached at points B and C. Both cables are directed in the positive z direction (out of the page). The bracket has a weight of 80 N which acts in the negative y direction at the center of gravity D. Finally, the bracket experiences two additional external forces: $\mathbf{F_B} = -700N\mathbf{k}$ applied at point B, and $\mathbf{F_C} = -100N\mathbf{k}$ applied at point C (both forces act into the page). Assume the bracket is in static equilibrium as shown. Find the tension in both cables and the reaction at A.



3. The airframe of a rocket consists of a hollow cylindrical body tube and a solid conical nosecone. You may ignore the fins. The body tube has an outer diameter of $4in$, an inner diameter of $3.875in$, and a length of $60in$. The nosecode has a diameter of $4in$ at the base and a height of $20in$. The body tube is composed of fiberglass with specific weight $\gamma_f = 0.065\ lb/in^3$ and the nosecone is composed of injection-molded plastic with specific weight $\gamma_p = 0.043\ lb/in^3$.

   (a) (10 points) Calculate the center of gravity (c.g.) of the rocket airframe. Give the c.g. position in inches from the nosecone tip.

   (b) (5 points) Will the c.g. change if the body tube is solid instead of hollow (but still composed of the same material)? If so, calculate the c.g. position in this case.



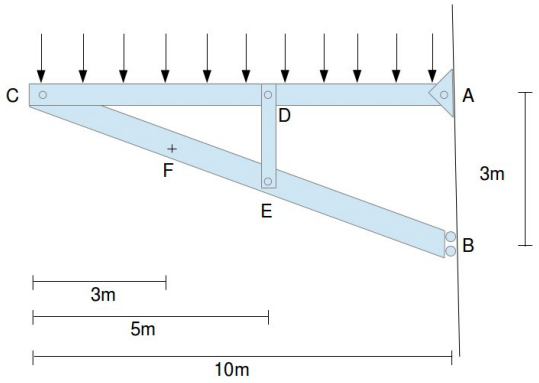
4. Consider the frame shown in the figure below. The frame is connected to the wall by a roller joint at B and a pin joint at A. In addition, there are pin joints at C, D and E. Finally, there is a constant load of 60N/m in the "downward" direction on the member AC.

   (a) (5 points) Compute the reaction forces at A and B.

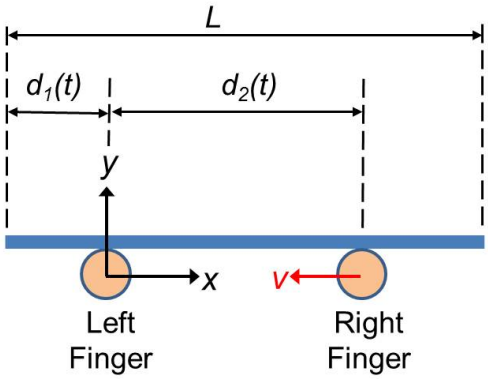
   (b) (5 points) Determine the force in the member DE.

   (c) (10 points) Compute the axial force, shear force, and bending moment at point F.

   Clearly mention the direction of the forces and/or moments in each case.



5. The bar shown below has length $L = 2$m and mass $m = 2$ kg. Gravity acts in the $-y$ direction. The bar is supported by two fingers placed under the bar. The left finger remains stationary at the origin of the coordinate system. The right finger moves with a constant speed of $v > 0$ in the $-x$ direction and its position as a function of time is $d_2(t)$. The left end of the bar is located at $d_1(t)$ in the $-x$ direction. The coefficients of static and kinetic friction between the bar and each finger are given by $\mu_s = 0.75$ and $\mu_k = 0.5$.



(a) (5 points) What is the location of the center of gravity when the right finger is located at $d_2(t)$ and the left end of the bar is located is located at $d_1(t)$?

(b) (5 points) What are the reaction forces on the bar at $t = 0$ if $d_1(0) = 0$m and $d_2(0) = 1.5$m? What is the velocity of the bar at this instant?

(c) (5 points) What are the reaction forces on the bar at $t = 0$ if $d_1(0) = 0.5$m and $d_2(0) = 1.5$m? What is the velocity of the bar at this instant?

(d) (5 points) Assume the bar and fingers start at $d_1(0) = 0$m and $d_2(0) = 1.5$m. The right finger moves with a constant speed of $v > 0$ in the $-x$ direction until it touches the left finger. Will the bar: (i) tip over clockwise, (ii) tip over counter-clockwise, or (iii) remain on top of the fingers? If the bar remains on top of the fingers then specify the final location $d_1(t)$ for the left end of the bar. Briefly justify your answers.

6. The beam shown in the left diagram below has length $L = 4m$. The beam is supported by a pin joint at A and a roller joint at B. There are two point loads acting in the negative y direction. The beam cross-section, shown in the right diagram below, is a hollow rectangle with the given dimensions.

   (a) (3 points) What are the reaction forces at A and B?

   (b) (5 points) Sketch the shear force $V(z)$ and bending moment $M(z)$ as a function of the beam position, z. You may neglect the beam cross-section dimensions in performing these calculations.

   (c) (4 points) What is the moment of inertia of the beam cross-section about the x-axis?

   (d) (4 points) What is the largest stress in tension and where does it occur on the beam?

   (e) (4 points) The beam is made of A36 steel which has a yield stress of $\sigma_y = 250 \times 10^6\ N/m^2$. Will the beam yield (i.e. deform) for the given loads? Justify your answer.

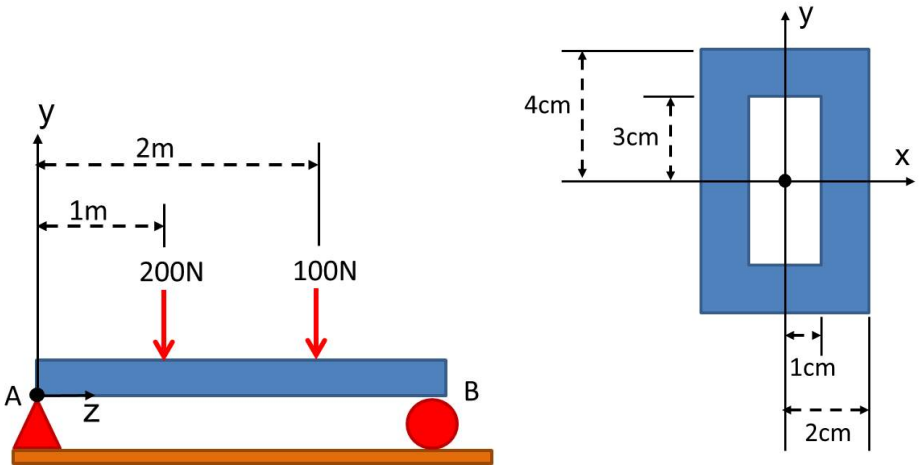


Figure 1: Loaded beam (Left) and beam cross-section (Right)