AEM 2011 Homework #3 (3.1, 3.2) Wednesday, February 1, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. Force $\bar{F}$, which has a magnitude of 4 kN, acts along a line passing through points D and D'. The grid units are in m and counter-clockwise moments are positive. Determine the moment of the force about points A, and B using the definition of the moment: $M = F \cdot d_{\perp}$.

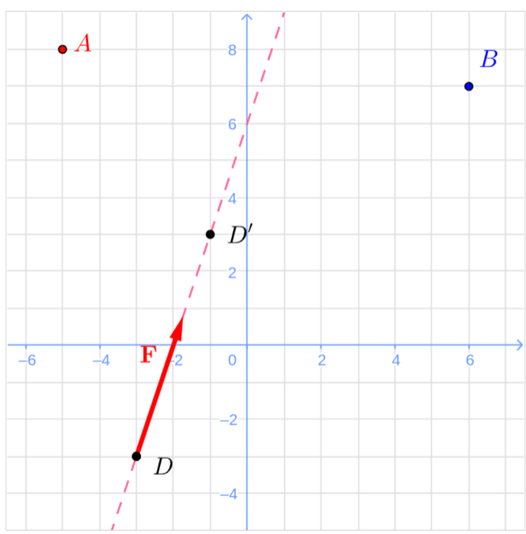
   (a) Find the length of the line segment from A to D (or any other point on the line of action of $\bar{F}$).
   - $d =$

   (b) Find the angle ($<= 90°$) between segment $\bar{AD}$ and the line of action of $\bar{F}$.
   - $\theta_A =$

   (c) Find the perpendicular distance between point A and the line of action of force $\bar{F}$.
   - $d_{\perp} =$

   (d) Use the definition of a moment to computer M_A.

   (e) In the same way, determine the moment of the force $\bar{F}$ about point B.



Solution:

(a) $d = 11.18m$

(b) $\theta_A = 28.74°$

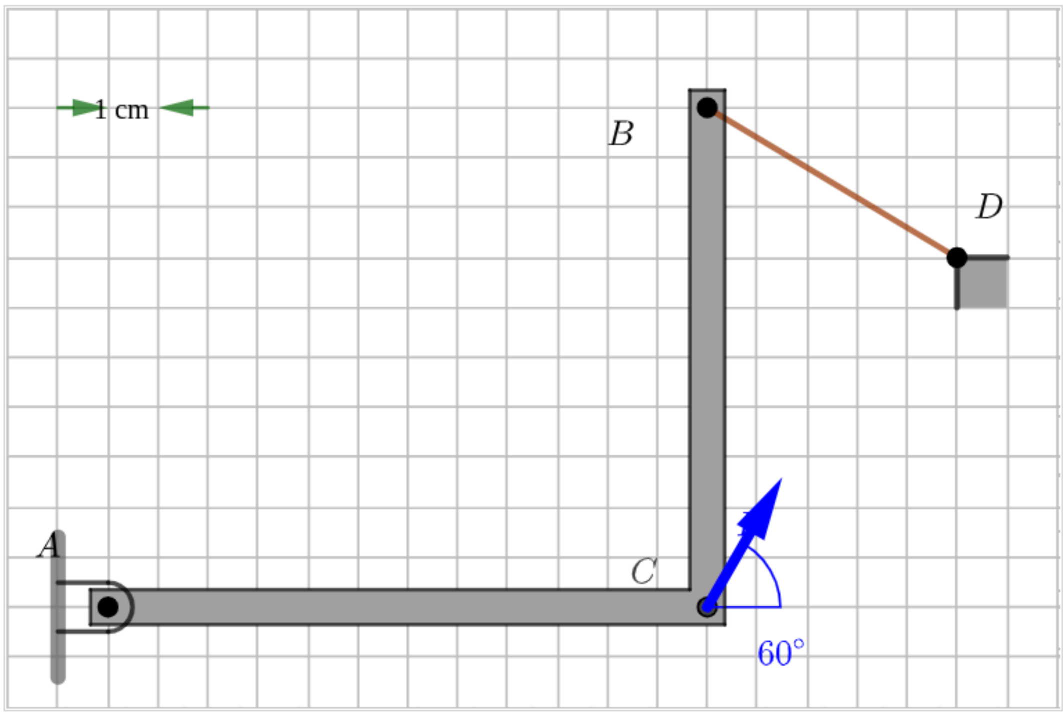
(c) $d_{\perp} = 5.376$

(d) $M_A = 21.5m \cdot kN$

(e) $M_B = 21.5m \cdot kN$

2. An L-shaped bracket is supported by a frictionless pin at A and a cable between points B and D. Determine the tension in the cable required for equilibrium when a $70N$ force $\bar{F}$ is applied to the bracket as shown at point C.

Tip: Since the object is in equilibrium $\sum M_A = 0$ *implies that the opposing moments must have equal magnitudes.*

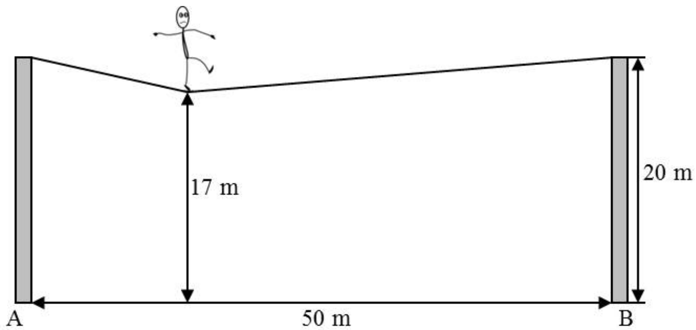


Solution:

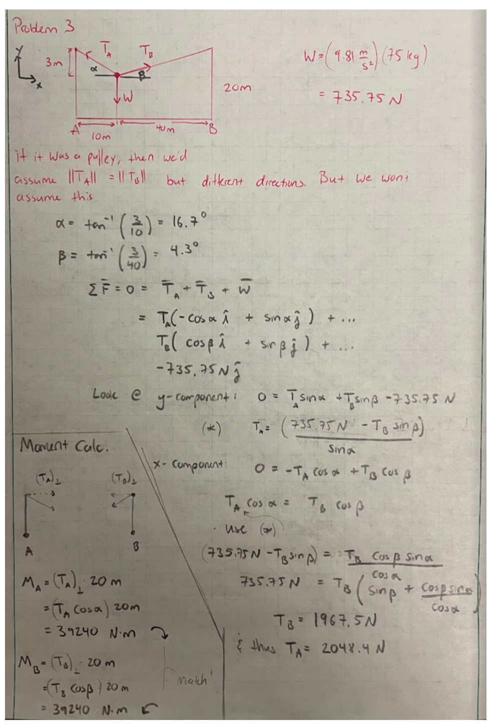
(a) $M_F = 727.5N \cdot cm$

(b) $T = 49.32N$

3. A 75 kilogram tightrope walker is attempting a walk across a 50 meter span 20 meters above the ground. When he is 20% across the span, the rope is deflected such that he is 17 meters above the ground. Find the tension in the rope both ahead of and behind the walker, and the moments about points A and B (where the support posts meet the ground). Ignore the mass of the rope itself.



Solution:



Problem 3

$W = \left(9.81 \frac{m}{s^2}\right)(75\,kg)$

$= 735.75\,N$

If it was a pulley, then we'd assume $\|T_A\| = \|T_B\|$ but different directions. But we won't assume this

$\alpha = \tan^{-1}\left(\frac{3}{10}\right) = 16.7^\circ$

$\beta = \tan^{-1}\left(\frac{3}{40}\right) = 4.3^\circ$

$\Sigma \bar{F} = 0 = \bar{T}_A + \bar{T}_B + \bar{W}$

$= T_A(-\cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}) + \ldots$

$T_B(\cos\beta\,\hat{i} + \sin\beta\,\hat{j}) + \ldots$

$-735.75\,N\,\hat{j}$

Look @ y-component: $0 = T_A \sin\alpha + T_B \sin\beta - 735.75\,N$

(*) $T_A = \frac{(735.75\,N - T_B \sin\beta)}{\sin\alpha}$

x-component: $0 = -T_A\cos\alpha + T_B\cos\beta$

$T_A \cos\alpha = T_B \cos\beta$

use (*)

$(735.75\,N - T_B \sin\beta) = \frac{T_B \cos\beta \sin\alpha}{\cos\alpha}$

$735.75\,N = T_B\left(\sin\beta + \frac{\cos\beta \sin\alpha}{\cos\alpha}\right)$

$T_B = 1967.5\,N$

& thus $T_A = 2048.4\,N$

Moment Calc.

$(T_A)_\perp$ $(T_B)_\perp$

A B

$M_A = (T_A)_\perp \cdot 20\,m$

$= (T_A \cos\alpha)\,20\,m$

$= 39240\,N \cdot m$

$M_B = (T_B)_\perp \cdot 20\,m$

$= (T_B \cos\beta)\,20\,m$

$= 39240\,N \cdot m$

match!

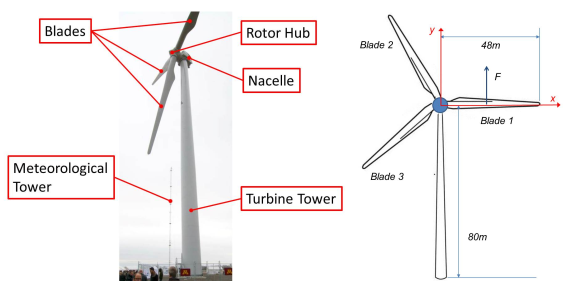
4. In HW2 you analyzed the cable tensions for a meteorological tower at the University of Minnesota Eolos Wind Research Field Station. The field station also has a 2.5MW Clipper Liberty Wind Turbine. The nacelle of this turbine is located at an elevation of 80m and the rotor radius is $R = 48m$. The power produced by a wind turbine is given by $P = \frac{1}{2}\rho A v^3 C_p(W)$ where $\rho = 1.225(kg/m^3)$ is the density of air, $A = \pi R^2$ is the circular area of the rotor plane (m^2), v is the wind speed (m/s) and C_p (unitless) is the efficiency of the wind turbine.

   (a) What is C_P if the turbine generates $P = 2.5MW$ of power when the wind speed is $v = 11m/sec$?

   (b) The diagram (right) shows a right-handed coordinate system attached to the rotor hub. The wind flowing past the blades causes a force to be distributed along each blade. For Blade 1 the effect is equivalent to a single force F acting vertically at mid span ($x = 24m$). Assume a similar force acts perpendicular to the other blades at midspan. What is the total moment about the center of the rotor hub, denoted M_{hub}, in terms of this force F?

   (c) The blades rotate at approximately $\omega = 1.6rad/s$ about the horizontal rotor axis (z-axis). The power captured by a wind turbine can also be calculated as $P = M_{hub}\omega$. What is the effective force F on each blade if the turbine is producing $2.5MW$ of power?

   (d) Suppose you plan to design a turbine to operate off-shore where the wind speed will be $v = 22m/sec$. Assume this new turbine has the same efficiency C_P (calculated in part A), rotational speed ω and radius R as the old turbine. You may also assume that the air density will remain the same. What is the power generated by this new off-shore turbine? What is the effective force on the blades of this new turbine? What is the impact on the design of the new turbine blades?



Left: Clipper Liberty Turbine, Right: Diagram of turbine rotor

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Solution:

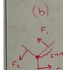
Given: · elevation 80m
· Rotor radius 48m
· Power produced by wind: $P = \frac{1}{2}\rho A v^3 C_p(\omega)$
$\rho = 1.225\ (kg/m^3)$
$A = \pi R^2$

(a) Given $P = 2.5\,MW$ when $v = 11\ m/s$, find C_p

$$2.5 \times 10^6\,W = \frac{1}{2}\left(1.225\,\frac{kg}{m^3}\right)\left(\pi \cdot 48^2\,m^2\right)\left(11\,\frac{m}{s}\right)^3 C_p$$

$C_p = 0.42$ or 42% efficient

(b) Compute moment about origin (hub)



$M = (F_1) \cdot 24m + (F_2) \cdot 24m + (F_3) \cdot 24m$

assume $F_1 = F_2 = F_3 = F$

$M_{hub} = 72F$

(c) $\omega = 1.6\ rad/s$ Power captured $P = M_{hub} \cdot \omega$

Say $P = 2.5 \times 10^6\,W$ & use eqn from (b)

$2.5 \times 10^6\,W = (72 \cdot F) \cdot 1.6\,\frac{rad}{s}$

$F = 21701\ N$ on each blade.

(d) $V = 22\ m/s$ $R = 48\ m$ $\rho = 1.225\ kg/m^3$

$C_p = 0.42$ $\omega = 1.6\ rad/s$

$P = \frac{1}{2}\rho A v^3 C_p(\omega) = 20\,MW$ which we know is $= M_{hub} \cdot \omega$

$= 72F \cdot \omega$

so $F = 173600\ N$

Notice: doubled speed but force went up 8x!

5. Book problems:
    (a) 3.26
    (b) 3.37
    (c) 3.53
    (d) 3.60

   Additional Practice Problems: 3.2, 3.15, 3.31, 3.35, 3.41, 3.46

   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

3.26: $(1200i - 1500j - 900k)N \cdot m$

3.37: $38.7°$

3.53: $283lb$

3.60: $-9.5N \cdot m$