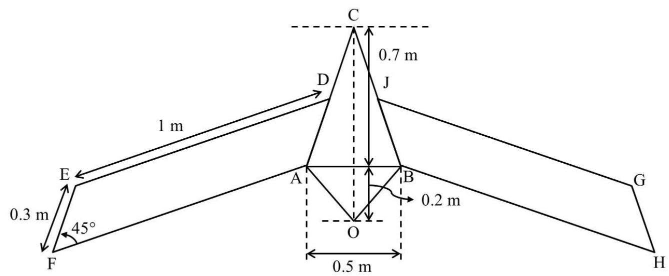
AEM 2011 Homework #7 (5.1, 5.2) Wednesday, March 1, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. The UAV lab of the AEM department acquired the Body Freedom Flutter (BFF) aircraft. This aircraft serves as an experimental platform for studying the interaction of aerodynamics, flexible structures, and control systems. Shown below are a photograph and a schematic of the planform (top-view) of the BFF.

   One of the key challenges in experimental aircraft research is locating the center of gravity. For the purpose of this homework, we will treat the BFF as a flat-plate whose mass is uniformly distributed. The wings of the BFF can be assumed to be parallelograms, while the centerbody of the BFF can be assumed to be composed of two triangles. Treating point O in the schematic as the origin, locate the center of gravity of the BFF aircraft.

   (Hint: Aircraft have a plane of symmetry passing through their longitudinal axis, i.e. their left and right sides are mirror images of each other.)



(left) BFF Photograph (credit: Brian Taylor) (right) BFF Schematic

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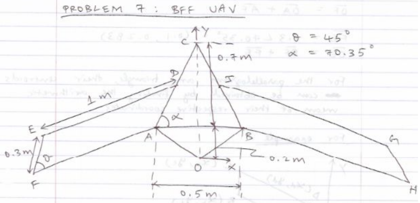
Solution:

①

AEM 2011 : HW# 6 : PROBLEM 7

PROBLEM 7 : BFF UAV



From the geometry the following vector can be derived

$\overline{OC} = (0, 0.9)$

$\overline{OA} = (-0.25, 0.2)$ | $\overline{OB} = (0.25, 0.2)$

$\overline{OC} = (0, 0.9)$

$\overline{OD} = (-0.15, 0.483)$ | $\overline{OJ} = (0.15, 0.483)$

$\overline{OF} = (-1.154, -0.228)$ | $\overline{OH} = (1.154, -0.228)$

$\overline{OE} = (-1.054, 0.055)$ | $\overline{OG} = (1.054, 0.055)$

(mirror image) ⟷ (mirror image)

$\overline{AD} = 0.3 \angle 70.35^\circ = (0.1, 0.283)$

$\overline{OD} = \overline{OA} + \overline{AD}$

$\cancel{\overline{FA} = 1 \angle(\alpha - \theta)}$

②

$\overline{FA} = 1 \angle(\alpha - \theta) = (0.904, 0.428)$

$\overline{AF} = (-0.904, -0.428)$

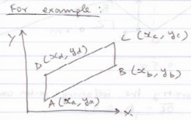
$\overline{OF} = \overline{OA} + \overline{AF}$

$\overline{FE} = 0.3 \angle 70.35^\circ = (0.1, 0.283)$

$\overline{OE} = \overline{OF} + \overline{FE}$

For the parallelogram and triangle, their centroids ~~the~~ can be obtained by finding the arithmetic mean of their respective coordinates.

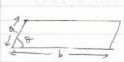
For example :

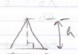


The above parallelogram has its centroid at :

$$(\bar{x}, \bar{y}) = \left(\frac{x_a + x_b + x_c + x_d}{4}, \frac{y_a + y_b + y_c + y_d}{4} \right)$$

A similar property holds for the triangle. (NOTE: for the triangle you would divide by 3, not 4)





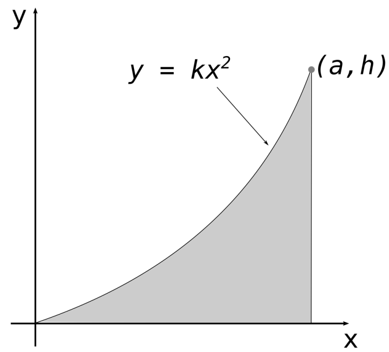
Area = $ab \sin\theta$ Area = $0.5\, bh$

③

Region	Area (m^2)	$\bar{x}$ (m)	$\bar{y}$ (m)
FEDA	0.2121	−0.652	0.1275
JGBH	0.2121	0.652	0.1275
ABC	0.175	0	0.4333
ABO	0.05	0	0.1333

$$\boxed{\begin{aligned} \bar{X} &= \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i} = 0 \text{ m} \\ \bar{Y} &= \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = 0.2104 \text{ m} \end{aligned}}$$

2. Use direct integration to find the centroid of the area shown in terms of a and h.



Solution:

Look up prabolic spandrel in the *centroid of common shapes* figure/table (12th ed: Fig. 5.8A (12ed).

3. Book problems:
   (a) 5.32
   (b) 5.34
   (c) 5.49
   (d) 5.55
   (e) 5.56

   Additional Practice Problems: 5.15, 5.18, 5.25, 5.36, 5.40

   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

5.32 (a) 0.513a (b) 0.691a

- *hint: solve for $\bar{y}$ as a function of (a, k, h). To maximize $\bar{y}$ over h, we take the derivative with respect to h and set that to zero. Solve for h where it satisfies $h < a$. Plug in k values and check that it is the maximizing h.*

5.34 $\bar{x} = 2a/3$, $\bar{y} = h/3$

5.49 $\bar{x} = 0.236L$, $\bar{y} = 0.454a$

5.55 $V = \pi^2 R r^2$, $A = 2\pi^2 R r$

5.56 (a) $V = \frac{8}{15}\pi a h^2$ (b) $V = \frac{1}{2}\pi a^2 h$