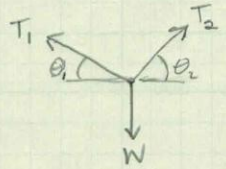
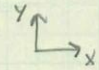
Problem 1 | AEM 2011 | Exam 1

FBD:





Sum forces in x and y directions:

$$\sum F_x = 0 \rightarrow -T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0$$

$$\sum F_y = 0 \rightarrow T_1 \sin\theta_1 + T_2 \sin\theta_2 - W = 0$$

From the problem description:

$$\theta_1 = \tan^{-1}\frac{0.5}{0.8} = 32.01^\circ \qquad \theta_2 = \tan^{-1}\frac{0.5}{0.4} = 51.34^\circ$$

$$W = 2\,kg \cdot 9.81\,m/s^2 = 19.62\ N$$

Therefore:

$$-0.848\,T_1 + 0.625\,T_2 = 0$$

$$\therefore\ T_1 = 0.737\,T_2$$

$$0.530\,T_1 + 0.781\,T_2 = 19.62$$

Use substitution for T_1:

$$0.391\,T_2 + 0.781\,T_2 = 19.62$$

$$\therefore\ T_2 = 16.74\ N$$

Back-substitute:

$$T_1 = 0.737\,T_2 = 12.34\ N$$

Therefore:

$$\boxed{T_{AB} = 12.34\ N \qquad T_{BC} = 16.74\ N}$$

Problem 2 | AEM 2011 | Exam 1

(a) The moment about the origin due to $\vec{F}_A$ is:

$$\vec{M}_{O,A} = \vec{r}_A \times \vec{F}_A = (5\hat{\imath} + 20\hat{\jmath}) \times (-10{,}000\,\hat{k})$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & 20 & 0 \\ 0 & 0 & -10{,}000 \end{vmatrix} = \begin{vmatrix} 20 & 0 \\ 0 & -10{,}000 \end{vmatrix}\hat{\imath} - \begin{vmatrix} 5 & 0 \\ 0 & -10{,}000 \end{vmatrix}\hat{\jmath}$$

$$= \boxed{-200{,}000\,\hat{\imath} + 50{,}000\,\hat{\jmath} \text{ N-m}}$$

(b) The moment about the origin due to $\vec{F}_B$ is:

$$\vec{M}_{O,B} = \vec{r}_B \times \vec{F}_B = (-5\hat{\imath} + 20\hat{\jmath}) \times (-10{,}000\,\hat{k})$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -5 & 20 & 0 \\ 0 & 0 & -10{,}000 \end{vmatrix} = \begin{vmatrix} 20 & 0 \\ 0 & -10{,}000 \end{vmatrix}\hat{\imath} - \begin{vmatrix} -5 & 0 \\ 0 & -10{,}000 \end{vmatrix}\hat{\jmath}$$

$$= \boxed{-200{,}000\,\hat{\imath} - 50{,}000\,\hat{\jmath} \text{ N-m}}$$

(c) If the turbine is in static equilibrium, then the moments due to both forces must be cancelled out by the reaction at the origin:

$$\Sigma M_x = 0 \rightarrow -200{,}000 - 200{,}000 + M_{R,x} = 0$$

$$\therefore M_{R,x} = 400{,}000$$

$$\Sigma M_y = 0 \rightarrow 50{,}000 - 50{,}000 + M_{R,y} = 0$$

$$\therefore M_{R,y} = 0$$

Therefore, the reaction moment is:

$$\boxed{\vec{M}_R = 400{,}000\,\hat{\imath} \text{ N-m}}$$

Problem 2 | AEM 2011 | Exam 1

(a) <u>Fast/intuitive approach:</u>

Even with scaling, the y-components of the moments due to the forces on the turbine blades will cancel and the reaction moment will have an x-component only. The magnitude of the x-component is:

$$M_{R,x} = (2\cdot\|\vec{F}_A\| + 2\cdot\|\vec{F}_B\|)(3\cdot 20\text{ m})$$
$$= 2{,}400{,}000$$

Therefore: $\boxed{\vec{M}_R = 2{,}400{,}000\,\hat{\imath}\ \text{N-m}}$

<u>Longer/mathematical approach:</u>

Recompute the moments about the origin with the scaled conditions:

$$\vec{M}_{o,A} = (15\hat{\imath} + 60\hat{\jmath})\times(-20{,}000\hat{k}) = -1{,}200{,}000\hat{\imath} + 3{,}000{,}000\hat{\jmath}\ \text{N-m}$$

$$\vec{M}_{o,B} = (-15\hat{\imath} + 60\hat{\jmath})\times(-20{,}000\hat{k}) = -1{,}200{,}000\hat{\imath} - 3{,}000{,}000\hat{\jmath}\ \text{N-m}$$

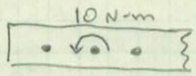
Find reaction moment:

$$\Sigma M_x = 0 \rightarrow -1{,}200{,}000 - 1{,}200{,}000 + M_{R,x} = 0$$
$$\therefore M_{R,x} = 2{,}400{,}000$$
$$\Sigma M_y = 0 \rightarrow 3{,}000{,}000 - 3{,}000{,}000 + M_{R,y} = 0$$
$$\therefore M_{R,y} = 0$$

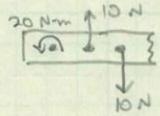
Therefore: $\boxed{\vec{M}_R = 2{,}400{,}000\,\hat{\imath}\ \text{N-m}}$

Problem 3 | AEM 2011 | Exam 1

Since the system in Figure 1 is in static equilibrium, any equivalent system will also be in equilibrium. Note that in all cases (a) through (f), the forces F_1, F_2, and F_3 are the same as in Figure 1. Therefore, the conditions for equilibrium are based on the left half of Figure 1, i.e. zero net force and a net moment of 10 N-m about the center dot:



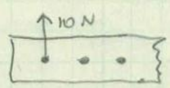
Now evalute each case:

(a) 

$\Sigma F_y = 10 - 10 = 0$ ✓

$\Sigma M = 20$ N-m $- (10\ N)(1\ m) = 10$ N-m ✓

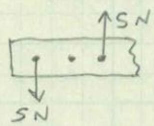
Therefore, (a) is in equilibrium.

(b) 

$\Sigma F = 10$ N ✗

$\Sigma M = -(10\ N)(1\ m) = -10$ N-m ✗

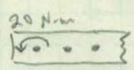
Therefore, (b) is not in equlibrium.

(c) 

$\Sigma F = 5 - 5 = 0$ ✓

$\Sigma M = (5\ N)(1\ m) + (5\ N)(1\ m) = 10$ N-m ✓

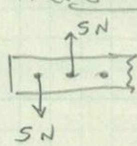
Therefore, (c) is in equilibrium.

(d) 

$\Sigma F = 0$ ✓

$\Sigma M = 20$ N-m ✗

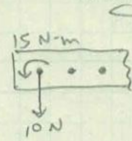
Therefore, (d) is not in equilibrium.

(e) 

$\Sigma F = 5 - 5 = 0$ ✓

$\Sigma M = (5\ N)(1\ m) = 5$ N-m ✗

Therefore, (e) is not in equilibrium.

(f) 

$\Sigma F = -10$ N ✗

$\Sigma M = 15$ N-m $+ (10\ N)(1\ m) = 25$ N-m ✗

Therefore, (f) is not in equilibrium.