AEM 2011 Homework #5 (4.1, 4.2) Wednesday, February 15, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

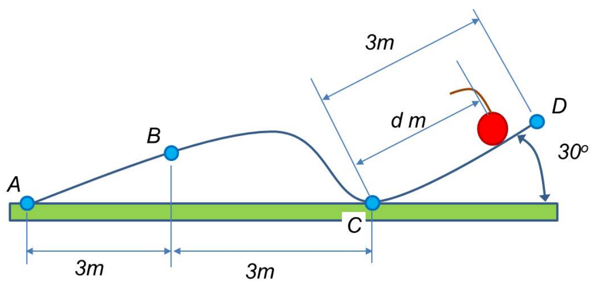
1. The engineers constructing the famous cherry and spoon sculpture in Minneapolis were required to carry out a structural analysis in case of an earthquake. They conducted the analysis by assuming that after an earthquake the foundation was not able to provide any rigid support to the structure. This means that the structure is just resting on the ground. A picture and diagram for the situation is shown below. Assume that:

   - The mass of section ABC is 100 kg and the weight acts at point B.
   - The mass of section CD is 80 kg and the weight acts at the mid-point of CD.
   - The mass of the cherry is 100 kg and the cherry is placed at a distance d from the point C.
   - Point C acts as a hinge while A acts as roller support.

   Complete the following analyses:

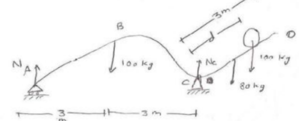
   (a) Find the maximum value of distance d such that the system does not topple about point C.

   (b) The ground at A can only support a load of 300 N. What is the minimum value of distance d such that the normal reaction force at point A does not exceed 300 N?



Solution:

The system can be represented as



$\sum M_C = 0$

$\Rightarrow -N_A(6) + 100g \times 3 - 80 \times \cancel{3} \cos 30^\circ \times g \times 1.5$
$- 100 \times d \cos 30^\circ \times g = 0$

A) For toppling $N_A = 0$

$\Rightarrow 100 g \times 3 - 80 \times \frac{3}{2} \cos 30\, g - 100\, d \cos 30\, g = 0$

$\Rightarrow d = \dfrac{300 - 80 \times \frac{3}{2} \times \frac{\sqrt{3}}{2}}{100 \times \frac{\sqrt{3}}{2}}$

$\boxed{d = 2.26\text{ m}}$

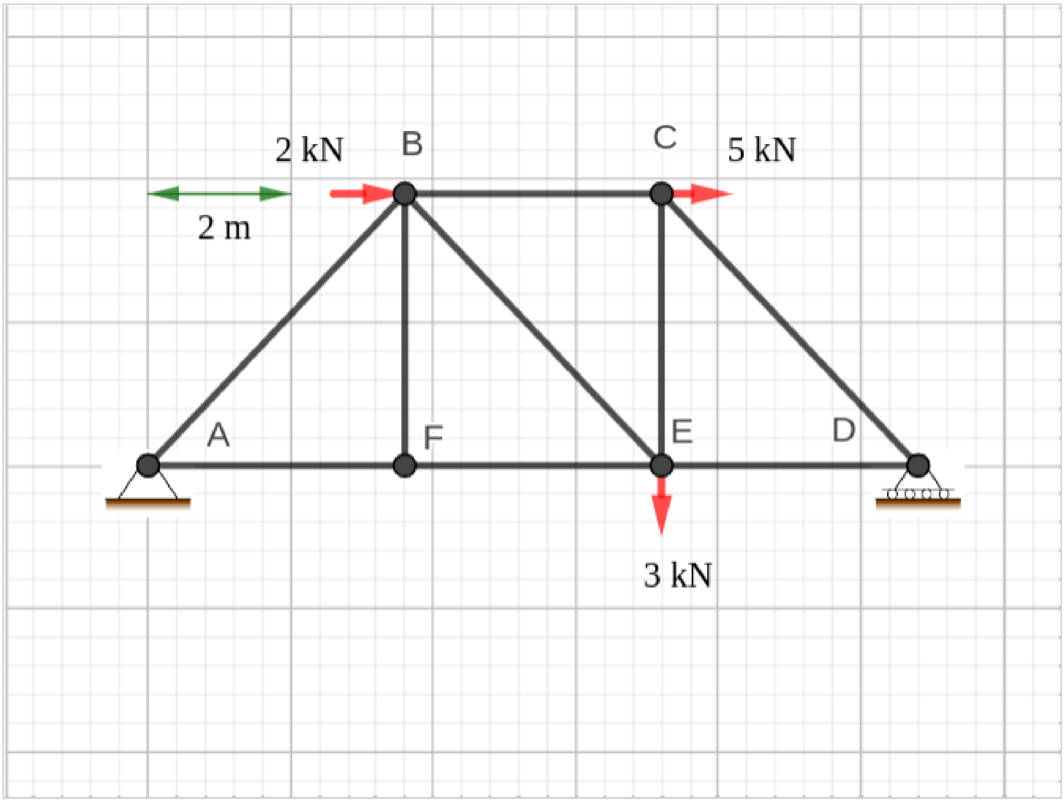
B) $N_A = 300\text{N}$

$\Rightarrow 6 \times 300 + 300g - 120 \cos 30^\circ g$
$- 100 d \cos(30^\circ)\, g = 0$

$\Rightarrow d = \dfrac{300g - 1800 - 120 \cos 30^\circ g}{100 \cos 30^\circ g}$

$\boxed{d = 0.143}$

2. The truss shown consists of three sections 3.6 m wide and 3.8 m tall, subjected to the loads shown. Determine the reactions at the pin and the roller.



Solution: $D = 4.463kN$ up, $A_x = 7kN$, $A_y = 1.463kN$ down

3. A large rocket is assembled and taken to the launch pad in a horizontal orientation. To prepare it for launch, it must be hoisted into a vertical position at the launch tower using an electric winch. The initial geometry is shown in the left diagram below. The rocket has a bracket at its base which is connected to the ground via a frictionless pin at the origin O. The cable first contacts the winch spool at point B located at $(L/10)\mathbf{i} + L\mathbf{j}$ (assume B does not move as θ changes). The cable from the winch is connected to the rocket's center of gravity, located initially (when $\theta = 0°$) at $-L\mathbf{i}$. Assume that the hoisting action begins instantaneously and is carried out at a constant speed (such that both the net force and net moment acting on the rocket are always zero). The geometry after the rocket has been hoisted to an angle θ is shown in the right diagram below.

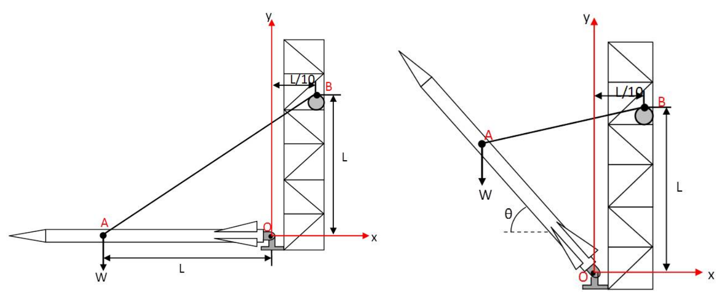
(a) If the rocket weighs 20,000 lbs and $L = 24ft$, find the tension in the cable and the reaction at the origin when $\theta = 40°$.

(b) For what value of θ is the tension in the cable T the highest? Also compute the maximum tension T_{max}.

(c) You are the engineer in charge of sizing the parts for the winch. You may choose from a selection of three motors two spools:

- Motor A costs $4,000 and has a maximum torque of 25,000 lb-ft
- Motor B costs $5,000 and has a maximum torque of 35,000 lb-ft
- Motor C costs $6,000 and has a maximum torque of 50,000 lb-ft
- Spool X costs $1,000 and has a diameter of 2 ft
- Spool Y costs $2,000 and has a diameter of 3 ft

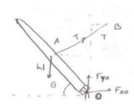
Find the cheapest viable combination of motor and spool for the winch.



Solution:

Problem 6

FBD at an angle θ



$\Sigma M_o = 0$

$\Rightarrow \vec{OB} \times \vec{T} + \vec{OA} \times \vec{W} = 0$

Now $\boxed{\vec{OB} = \frac{L}{10}\hat{i} + L\hat{j}}$

Unit vector in T direction

$= \frac{\vec{AB}}{|\vec{AB}|} = \frac{(\frac{L}{10}\hat{i} + L\hat{j}) - (-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j})}{|(\frac{L}{10}\hat{i} + L\hat{j}) - (-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j})|}$

$= \frac{L(\frac{1}{10} + \cos\theta)\hat{i} + L(1 - \sin\theta)\hat{j}}{(L^2(\frac{1}{10} + \cos\theta)^2 + L^2(1 - \sin\theta)^2)^{1/2}}$

$\Rightarrow \vec{T} = T \times \left[\frac{L(\frac{1}{10} + \cos\theta)\hat{i} + L(1 - \sin\theta)\hat{j}}{(L^2(\frac{1}{10} + \cos\theta)^2 + L^2(1 - \sin\theta)^2)^{1/2}}\right]$

At $\theta = 40°$, $L = 24$ ft,

$\vec{T} = T \times \left[\frac{20.78\hat{i} + 8.57\hat{j}}{\sqrt{20.78^2 + 8.57^2}}\right]$

$\boxed{\vec{T} = T \times 0.92\hat{i} + T\,0.38\hat{j}}$

$\Rightarrow \vec{OB} \times \vec{T} = 0.912T - 22.08T$

$\boxed{\vec{OB} \times \vec{T} = -21.17T\,\hat{k}}$

2nd Term

$= \vec{OA} \times \vec{W}$

$\vec{OA} = -L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j} = -18.38\hat{i} + 15.43\hat{j}$

$\vec{W} = -W\hat{j} = -20000\hat{j}$

$\Rightarrow \vec{OA} \times \vec{W} = 367600\,\hat{k}$

$\Rightarrow (-21.17T + 367600)\hat{k} = 0$

$\Rightarrow \boxed{T = 17364.2 \text{ lb}}$

B) We can minimize the general expression for T with respect to θ, but we can also find the desired angle using intuition.

We can see that as θ increases from 0 to higher values, the moment of the weight (which has to be countered by Tension T) is decreasing.

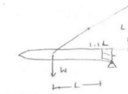
Also, the angle between $\vec{OA}$ and $\vec{T}$ is getting closer to 90°, so a lesser tension is required to produce same moment

Thus, we can conclude that as θ increas, T decreases.

So T is maximum for θ = 0°

which is the initial condition (diagram on left)

Solving



$\Sigma M_o = 0$

$\Rightarrow WL - T\left(\frac{L}{\sqrt{L^2 + (1.1L)^2}}\right)L = 0$

$\Rightarrow T = \frac{W}{L}\sqrt{L^2 + (1.1L)^2}$

$= \frac{20000 \times 24 \times \sqrt{1^2 + 1.1^2}}{24}$

~~$= 71357$~~ ~~$T_{max} = 71357.3$ lb~~

$\boxed{T_{max} = 29732.14 \text{ lb}}$

c) Torque $\geq T \times \frac{d}{2}$

$\Rightarrow T \leq \frac{2 \times \text{Torque}}{d}$

$\Rightarrow$ To support high Tension T, d should be lower (which is also cheaper)

$\boxed{\text{We choose X spool.}}$

$\Rightarrow \text{Torque} \geq \frac{29732.14 \times 2}{2}$

$\text{Torque} \geq 29732.14$

$\boxed{\text{We choose Motor B}}$

Total cost = $1000 + $5000
= $6000

4. Book problems:
   (a) 4.5
   (b) 4.18
   (c) 4.69

   Additional Practice Problems: 4.7, 4.13, 4.23, 4.28, 4.70, 4.84, 4.90

   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

4.5 (a) $34.0kN$ up (b) $4.96kN$ up

4.18 230 lb

4.69 (a) 499 N (b) 457 N 26.6° from the horizontal pointing up and to the left