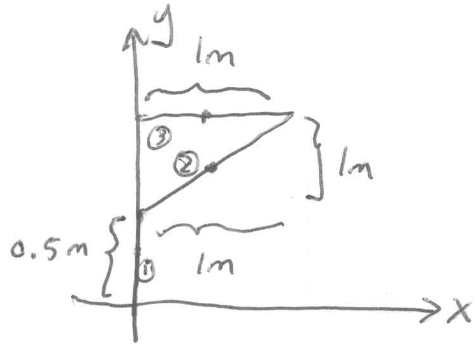
1



The centroids of all 3 links are at the geometric center of the link

(a) $(x_1, y_1) = (0, 0.25)$ m

(b) (x_2, y_2) $(0.5, 1.0)$ m

(c) $(x_3, y_3) = (0.5\text{m}, 1.5)$ m

(d) Total Length, $L = \frac{1}{2} + \sqrt{2} + 1 = 1.5 + \sqrt{2}$
≈ 2.91m

(e) $x_c L = x_1 L_1 + x_2 L_2 + x_3 L_3$

$x_c (2.91) = 0 \cdot \frac{1}{2} + 0.5 \cdot \sqrt{2} + 0.5 \cdot 1 = \frac{1}{2} + \frac{\sqrt{2}}{2} \approx 1.21$

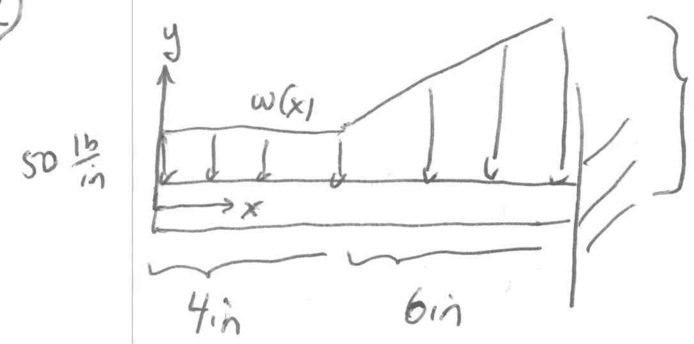
$$\boxed{x_c \approx 1.21/2.91 = 0.42}$$

$y_c L = y_1 L_1 + y_2 L_2 + y_3 L_3$

$y_c (2.91) = (0.125)(0.5) + (1)(\sqrt{2}) + (1.5)(1)$
$= \sqrt{2} + 1.6125 \approx 3.04$

$\rightarrow$ $$\boxed{y_c \approx 3.04/2.91 \approx 1.04}$$

(2)



$$w(x) = \begin{cases} 50 & 0<x\le 4 \\ 50 + \frac{50}{6}(x-4) & 4<x\le 10 \end{cases}$$

(A) Total Force $F = \int_0^{10} w(x)\,dx = \int_0^4 50\,dx + \int_4^{10} 50 + \frac{50}{6}(x-4)\,dx$

$$= 50x\Big|_{x=0}^{4} + \left[50x + \frac{50}{12}(x-4)^2\right]_{x=4}^{10}$$

$$= 200 + [(500+150) - (200)]$$

$$= 650\ \text{lb.}$$

(B) Resultant Location $F\bar{x} = \int_0^{10} w(x)\,x\,dx = \int_0^4 50x\,dx + \int_4^{10} 50x + \frac{50}{6}(x-4)x\,dx$

$$= \int_0^4 50x\,dx + \int_4^{10} \frac{50}{6}x^2 + \frac{100}{6}x \quad dx$$

$$= 25x^2\Big|_{x=0}^{4} + \left[\frac{50}{18}x^3 + \frac{100}{12}x^2\right]_{x=4}^{10}$$

$$= 400 + \left[\left(\frac{5\times 10^4}{18} + \frac{10^4}{12}\right) - \left(\frac{3200}{18} + \frac{1600}{12}\right)\right]$$

$\Rightarrow$ $F\bar{x} = \cancel{3300}\ 3700$

$\bar{x} = 3700/650 = 5.69\ m$

(2) An alternative is to split the load into two pieces:

$$w(x) = w_1(x) + w_2(x)$$

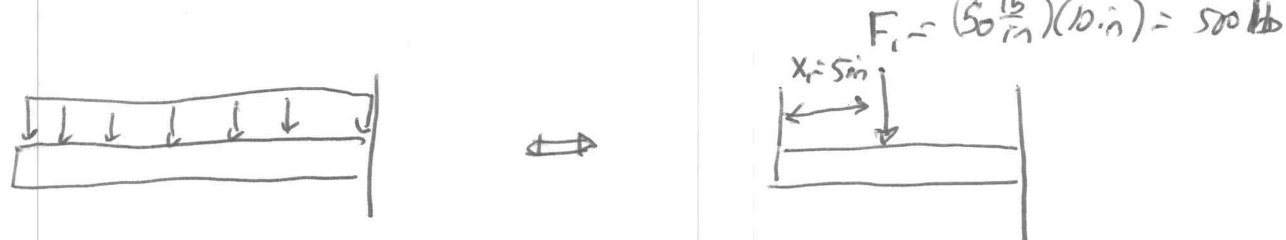
where $w_1(x) = 50 \text{ lb/in}$ for $0 \le x \le 10$

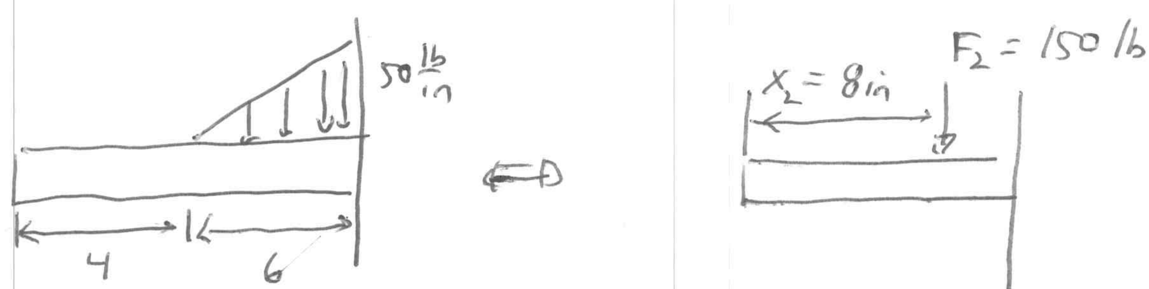
rectangular load

$$w_2(x) = \begin{cases} 50 \text{ lb/in } (x-4) & \text{for } 4 \le x \le 10 \\ 0 & \text{for } 0 \le x \le 4 \end{cases}$$

triangular load

$w_1(x)$





(Equivalent force for triangular load acts 2/3 of the way along the distance)

(A) Total Force $F = F_1 + F_2 = 650 \text{ lb}$

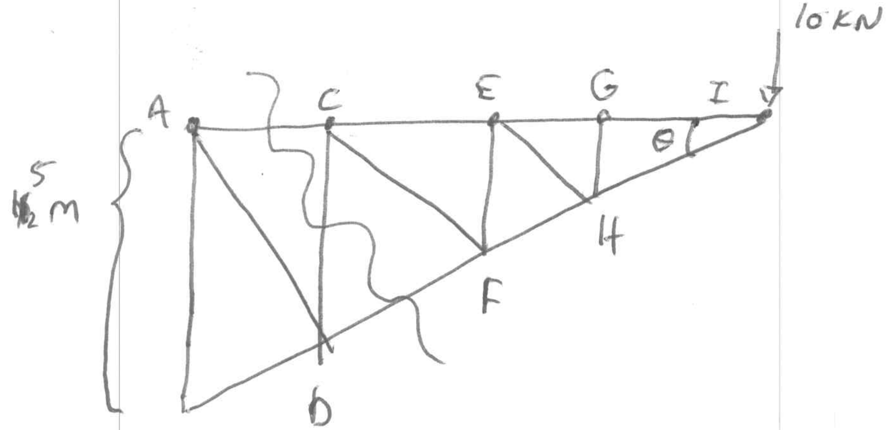
(B) Resultant Location $\bar{x} F = x_1 F_1 + x_2 F_2$

$$= (5)(500) + (8)(150)$$

$$= 2500 + 1200 = 3700$$

$$\Rightarrow \bar{x} = 3700/650 = 5.69 \text{ in}$$

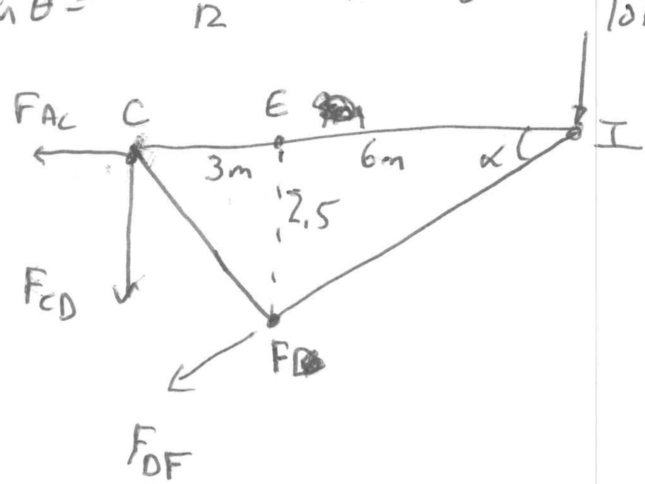
(3)



3m between (A,C), (C,E), (E,G), and (G,I)

(a) $\tan\theta = \frac{5}{12} \Rightarrow \theta = 0.395\,\text{rad} = 22.6^{\circ}$

(b)



① Sum Forces in x (horizontal) direction $\qquad -F_{AC} - F_{DF}\cos\theta = 0$

② Sum Forces in y (vertical) direction $\qquad -F_{CD} - 10\,kN - F_{DF}\sin\theta = 0$

③ Sum Moments about C

$$0 = (-10kN)(9m) + (3\vec{i} - 2.5\vec{j}) \times (-F_{DF}\cos\theta\,\vec{i} - F_{DF}\sin\theta\,\vec{j})$$

Use $\theta = 22.6^{\circ}$ $\boxed{F_{DF} = -26\ N}$ $\boxed{\text{(Thus DF is in compression)}}$

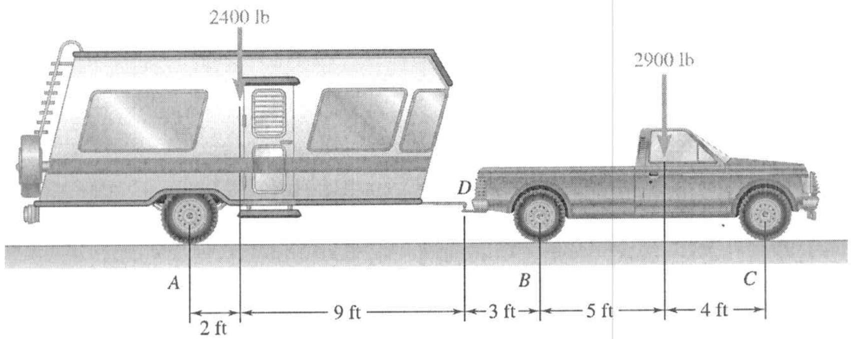
Sub back into ② to show $\boxed{F_{CD} = 0}$

Note! If you sum moments about the point I then you immediately obtain $F_{CD} = 0$.

The exam only asked for
(a) FBDs of the truck and trailer
(b) The reaction force at D

PROBLEM 6.95

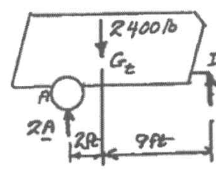
A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at D. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.



SOLUTION

(*a*) Free body: Trailer:

(We shall denote by A, B, C the reaction at one wheel.)



$$+\circlearrowleft \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + D(11 \text{ ft}) = 0$$

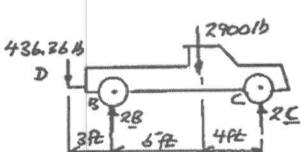
$$D = 436.36 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad 2A - 2400 \text{ lb} + 436.36 \text{ lb} = 0$$

$$A = 981.82 \text{ lb}$$

$\mathbf{A} = 982 \text{ lb}\uparrow$ ◀

Free body: Truck.



$$+\circlearrowleft \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) - (2900 \text{ lb})(5 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = 732.83 \text{ lb}$$

$\mathbf{C} = 733 \text{ lb}\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2900 \text{ lb} + 2(732.83 \text{ lb}) = 0$$

$$B = 935.35 \text{ lb}$$

$\mathbf{B} = 935 \text{ lb}\uparrow$ ◀

(*b*) Additional load on truck wheels.

Use free body diagram of truck without 2900 lb.

$$+\circlearrowleft \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = -72.73 \text{ lb}$$

$\Delta C = -72.7 \text{ lb}$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2(72.73 \text{ lb}) = 0$$

$$B = 290.9 \text{ lb}$$

$\Delta B = +291 \text{ lb}$ ◀