AEM 2011 Homework #4 (3.3, 3.4) Wednesday, February 8, 2023

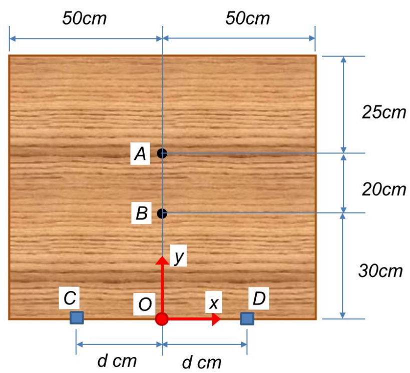
Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. A researcher is building a mount to experimentally test a motor for an uninhabited aerial vehicle (UAV). To build the mount she must insert a screw in a plank of wood at point A as shown in the diagram below. The plank is secured to the workbench at points C and D by clamps. Each clamp is capable of providing a force up to 50N tangential to the plank. Assume that the forces provided by the clamps act along the y-direction.

   (a) The researcher first places the clamps at $d = 20cm$. She uses a power screw-driver which exerts a clockwise torque of 30 N-m on the plank. What is the value of the force couple and the corresponding force that has to be provided by the clamps so that the plank remains in equilibrium?

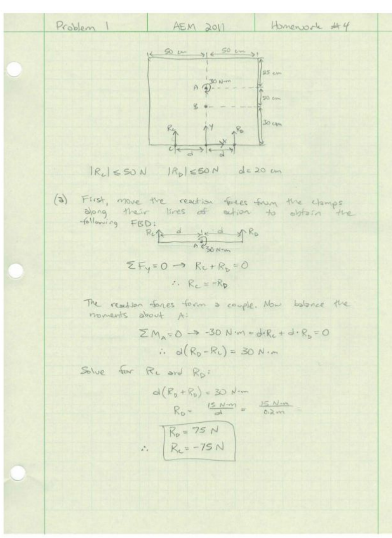
   (b) Will the clamps give way (yield) in this situation? In other words, is the force calculated in (a) greater than the 50N limit? If so, calculate the minimum value of distance d so that the clamps will not give way.

   (c) Would your answers change if she inserts the screw at point B instead of point A. Explain briefly.



Top-down view of experimental motor mount

Solution:


Problem 1 | AEM 2011 | Homework #4

$|R_C| \leq 50\,N \quad |R_D| \leq 50\,N \quad d = 20\text{ cm}$

(a) First, move the reaction forces from the clamps along their lines of action to obtain the following FBD:

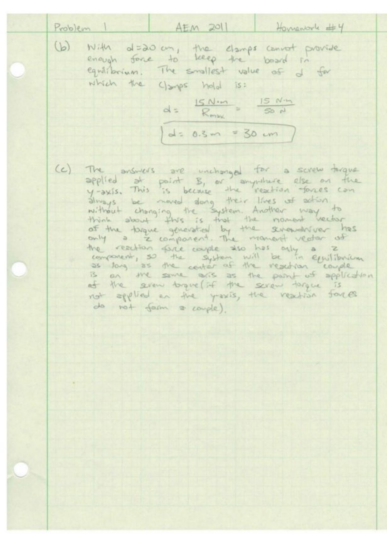
$$\Sigma F_y = 0 \rightarrow R_C + R_D = 0$$
$$\therefore R_C = -R_D$$

The reaction forces form a couple. Now balance the moments about A:

$$\Sigma M_A = 0 \rightarrow -30\text{ N}\cdot\text{m} - d\cdot R_C + d\cdot R_D = 0$$
$$\therefore d(R_D - R_C) = 30\text{ N}\cdot\text{m}$$

Solve for R_C and R_D:

$$d(R_D + R_D) = 30\text{ N}\cdot\text{m}$$
$$R_D = \frac{15\text{ N}\cdot\text{m}}{d} = \frac{15\text{ N}\cdot\text{m}}{0.2\text{ m}}$$
$$\therefore \boxed{R_D = 75\text{ N} \quad R_C = -75\text{ N}}$$


Problem 1 | AEM 2011 | Homework #4

(b) With $d = 20$ cm, the clamps cannot provide enough force to keep the board in equilibrium. The smallest value of d for which the clamps hold is:

$$d = \frac{15\text{ N}\cdot\text{m}}{R_{max}} = \frac{15\text{ N}\cdot\text{m}}{50\text{ N}}$$
$$\boxed{d = 0.3\text{ m} = 30\text{ cm}}$$

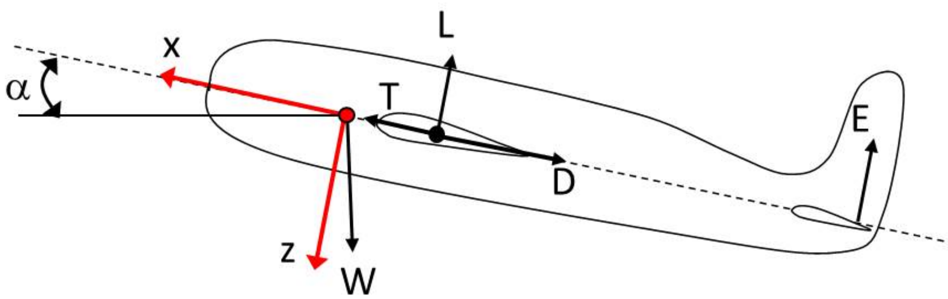
(c) The answers are unchanged for a screw torque applied at point B, or anywhere else on the y-axis. This is because the reaction forces can always be moved along their lines of action without changing the system. Another way to think about this is that the moment vector of the torque generated by the screwdriver has only a z component. The moment vector of the reaction force couple also has only a z component, so the system will be in equilibrium as long as the center of the reaction couple is on the same axis as the point of application of the screw torque (if the screw torque is not applied on the y-axis, the reaction forces do not form a couple).

2. Consider the coordinate system (red lines) attached to the aircraft center of gravity (red dot) depicted in the diagram below. The positive y-axis, not shown, points into the page (to the right-side of the pilot). The aircraft weight W acts downward at the center of gravity. The lift L and drag D forces are generated by the airflow over the wing and act at the center of pressure (black dot) in the negative z and negative x directions, respectively. The center of pressure is located at $\bar{r}_{CP} = -d\mathbf{i}$. The engine thrust T is also assumed to act at the center of pressure in the positive x direction.

   (a) Assume that all forces balance along the x direction. What lift force L is required to ensure that all forces balance along the z direction? Your answer should be expressed in terms of W and α. Assume $E = 0$.

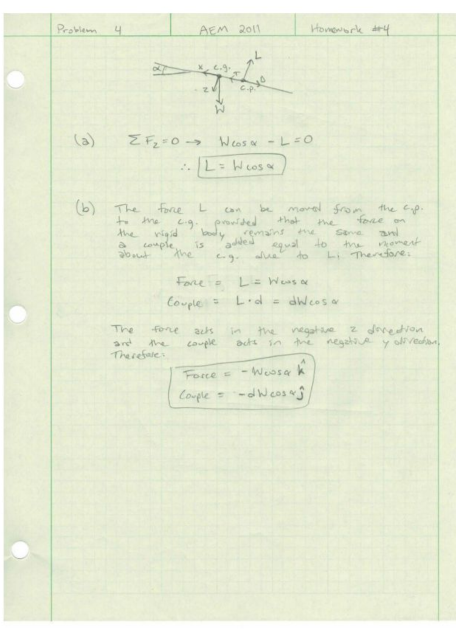
   (b) Replace the lift force L computed in the previous part with a force-couple system at the center of gravity. Your answer should be expressed in terms of W , α, and d. Assume $E = 0$.

   (c) The tail surface on an aircraft is needed to prevent the aircraft from rotating about the y-axis. E is the force generated by the flow over the tail surface of the aircraft. This force acts in the negative z direction at $\bar{r}_E = -11d\mathbf{i}$. What forces L and E are required to ensure that the system is equivalent to zero force and zero moment about the center of gravity?



Side-view of aircraft

Solution:



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(a) $\Sigma F_z = 0 \rightarrow W\cos\alpha - L = 0$

$\therefore \boxed{L = W\cos\alpha}$

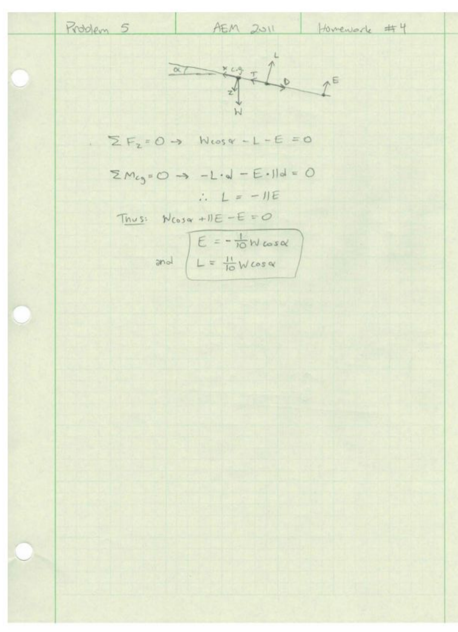
(b) The force L can be moved from the c.p. to the c.g. provided that the force on the rigid body remains the same and a couple is added equal to the moment about the c.g. due to L. Therefore:

Force = $L = W\cos\alpha$

Couple = $L \cdot d = dW\cos\alpha$

The force acts in the negative z direction and the couple acts in the negative y direction. Therefore:

$$\boxed{\begin{aligned} \text{Force} &= -W\cos\alpha\,\hat{k} \\ \text{Couple} &= -dW\cos\alpha\,\hat{j} \end{aligned}}$$



Problem 5 | AEM 2011 | Homework #4

$\Sigma F_z = 0 \rightarrow W\cos\alpha - L - E = 0$

$\Sigma M_{cg} = 0 \rightarrow -L \cdot d - E \cdot 11d = 0$

$\therefore L = -11E$

Thus: $W\cos\alpha + 11E - E = 0$

and $\boxed{\begin{aligned} E &= -\tfrac{1}{10}W\cos\alpha \\ L &= \tfrac{11}{10}W\cos\alpha \end{aligned}}$

3. Book problems:
   (a) 3.81
   (b) 3.92
   (c) 3.96
   (d) 3.116
   (e) 3.124

   Additional Practice Problems: 3.73, 3.83, 3.98, 3.105, 3.114, 3.126

   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

3.81: (a) $\bar{F}_B = 500N$ pointing $60.0°$ right-and-down from horizontal, $\bar{M}_B = 86.2N \cdot m$ clockwise (b) $\bar{A} = 689N$ up, $\bar{B} = 1150N$ pointing $77.4°$ right-and-down from horizontal.

3.92: $P_{min} = 300N$ pointing $30.0°$ right-and-up from the horizontal

3.96: $\bar{M}_C = ((77.4)\mathbf{i} + (61.5)\mathbf{j} + (106.8)\mathbf{k})N \cdot m$

3.116: (a) the rivet is $0.365m$ above G (b) the rivet is $0.227m$ right of G

3.124: $\bar{R} = -(420N)\mathbf{i} - (50N)\mathbf{j} - (250N)\mathbf{k}$ and $\bar{M}_A^R = (30.8N \cdot m)\mathbf{j} - (22.0N \cdot m)\mathbf{k}$