AEM 2011 Homework #13 (8.1 (cont.), 8.2A) Wednesday, April 19, 2023

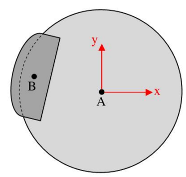
Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. Disc brakes are the most common type of brakes used in modern automobiles. They work by having a disc fixed to each wheel of the vehicle, and a hydraulic caliper which applies a force to the disc. The friction between the brake pads and the disc will slow down and ultimately stop the rotation of the wheel.

   The diagram below shows a disc brake and caliper assembly. The disc rotates about point A, and when the brakes are actuated, a force of $-N\mathbf{k}$ (into the page) is applied to the front of the disc, and a force of $+N\mathbf{k}$ (out of the page) is applied to the back of the disc. Both forces act at point B, which is given by $-10\mathbf{i} + 3\mathbf{j}$ centimeters. The coefficients of friction between the brake pads and disc are $\mu_s = 0.5$ and $\mu_k = 0.4$.

   (a) When the brakes are fully actuated, the force N reaches its maximum value of $20kN$. If the car is at rest and the brakes are fully actuated, what is the maximum moment that can be supplied by the axle before the brakes fail and the car begins to move?

   (b) When the car is coasting down a steep hill, a moment of 200 $N \cdot m$ is generated about the axle. If light brake pressure is applied, the car will move down the hill at a constant speed instead of accelerating. What is the force N that the brakes must apply for the car to move at a constant speed?

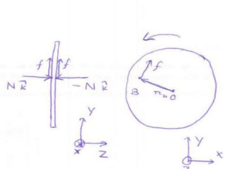


Solution:

HW 11 Solutions

Problem - 1

As friction force always opposes, it will act in the transverse direction ($\perp$ to the radial direction) as shown



a) N = 20 kN

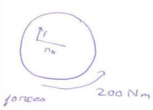
For each side
friction force f = μ_s N (μ_s because the car is at rest)
= 0.5 x 20 kN
= 10 kN

Total friction force = 2f = 20 kN = F

Moment = $\vec{r} \times \vec{F}$
|Moment| = |r| x |F| x Sin(θ) [As r and F are perpendicular to each other. => Sin θ = 90°]

= |r| x |F|
= |-10i+3j| x 20kN
= $\sqrt{109}$ cm x 20 kN
= 208.8 x (10^{-2} m) x (10^{3} N)
= 2088 Nm (opposing the motion

(b) As the car moves with a constant speed =>
Wheel is in equilibrium



=> Moment provided by frictional forces is equal to 200 N-m

=> 200 = | $\vec{r} \times \vec{F}$ |
= | r | x | $\vec{F}$ |
= $\sqrt{109}$ cm x | 2 x μ_K x N |

(We are using μ_k because car is moving)

200 = $\sqrt{109}$ x 2 x 0.4 x N x 10^{-2} Nm

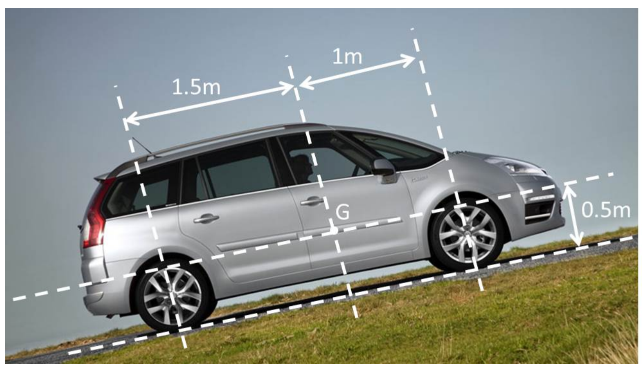
=> $\boxed{N = 2394.5 \text{ N}}$

2. Imagine that you are a material scientist working for a popular car tire manufacturer. Your job involves applying concepts from tribology (the study of friction) to decide on the material that will be used to manufacture the car tires. In this problem, you will design material-level properties, in particular the coefficients of static and kinetic friction between the rubber and the road, given higher-level requirements on the car. The car, shown in the figure below, has a mass of 1,000kg. The center of gravity of the car is located 0.5m from the road, 1m from the forward axle, and 1.5m from the rear axle. In answering the following questions, you may assume that the car is two-dimensional, i.e. you do not have to split the forces on the front/rear tires since there are two sets of front/rear tires.

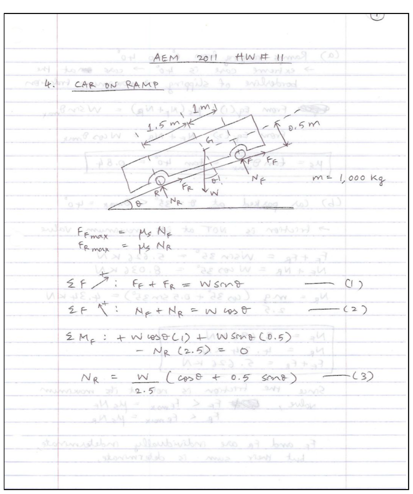
   (a) The car company is especially concerned about the maximum steepness of the slopes on which its cars can be parked. Choose the coefficient of static friction such that the cars can be parked on ramps ranging in inclination from 0° to 40°.

   (b) Given that the car is parked on a ramp of inclination 35°, find the normal reactions and friction forces on both the front and rear tires.

   (c) The car company wants to ensure that if the car started sliding down the ramp, due to an external disturbance, it would not continue to slide down. For a 35° ramp, choose the lowest possible value for the coefficient of kinetic friction such that a car disturbed from its parked position returns to a complete stop.



Solution:



AEM 2011 HW # 11

4. CAR ON RAMP

$m = 1{,}000$ Kg

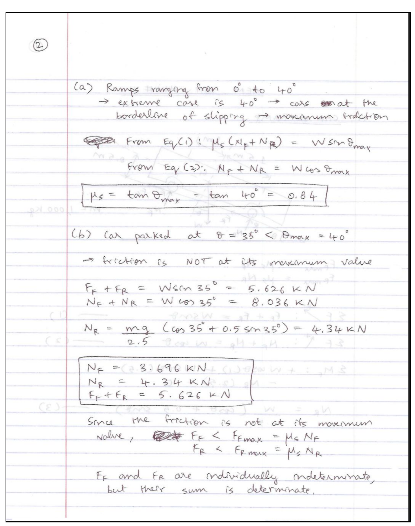
$F_{F\,max} = \mu_s N_F$
$F_{R\,max} = \mu_s N_R$

$\Sigma F \nearrow^{+}: \quad F_F + F_R = W \sin\theta$ —— (1)

$\Sigma F \nwarrow^{+}: \quad N_F + N_R = W \cos\theta$ —— (2)

$\Sigma M_F: \; + W\cos\theta\,(1) + W\sin\theta\,(0.5) - N_R\,(2.5) = 0$

$N_R = \dfrac{W}{2.5}\left(\cos\theta + 0.5\sin\theta\right)$ —— (3)



(2)

(a) Ramps ranging from 0° to 40°
→ extreme case is 40° → cars at the borderline of slipping → maximum friction

From Eq (1): $\mu_s (N_F + N_R) = W \sin\theta_{max}$

From Eq (2): $N_F + N_R = W\cos\theta_{max}$

$\boxed{\mu_s = \tan\theta_{max} = \tan 40^\circ = 0.84}$

(b) Car parked at $\theta = 35^\circ < \theta_{max} = 40^\circ$

→ friction is NOT at its maximum value

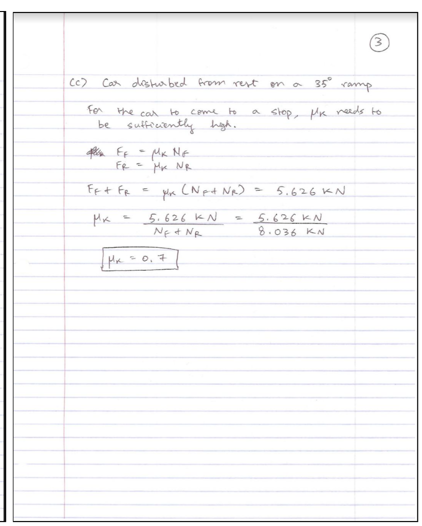
$F_F + F_R = W \sin 35^\circ = 5.626$ KN
$N_F + N_R = W\cos 35^\circ = 8.036$ KN

$N_R = \dfrac{mg}{2.5}\left(\cos 35^\circ + 0.5 \sin 35^\circ\right) = 4.34$ KN

$$\boxed{\begin{aligned} N_F &= 3.696 \text{ KN} \\ N_R &= 4.34 \text{ KN} \\ F_F + F_R &= 5.626 \text{ KN} \end{aligned}}$$

Since the friction is not at its maximum value, $F_F < F_{F\,max} = \mu_s N_F$
$F_R < F_{R\,max} = \mu_s N_R$

F_F and F_R are individually indeterminate, but their sum is determinate.



(3)

(c) Car disturbed from rest on a 35° ramp

For the car to come to a stop, μ_K needs to be sufficiently high.

$F_F = \mu_K N_F$
$F_R = \mu_K N_R$

$F_F + F_R = \mu_K (N_F + N_R) = 5.626$ KN

$\mu_K = \dfrac{5.626 \text{ KN}}{N_F + N_R} = \dfrac{5.626 \text{ KN}}{8.036 \text{ KN}}$

$\boxed{\mu_K = 0.7}$

3. Race tracks and speedways are designed with turns that are banked at an angle. Banked turns help cars execute the turn at higher speeds as compared to flat turns. In the figure below, a car is shown executing a 25° banked turn. Since this is actually a problem in dynamics, we will simply assume that a centripetal force F_C acts in the horizontal direction on the car. This centripetal force has the following expression:

$$F_C = m\frac{V^2}{R}$$

where, m is the mass of the car, V is the speed of the car, and R is the radius of curvature of the turn. The coefficient of static friction between the tires and the road is $\mu_s = 0.8$.

Under the constraint that a car should not slide along the banked turn, find the percentage increase in the maximum possible speed at which a car can execute a banked turn as compared to a flat turn.

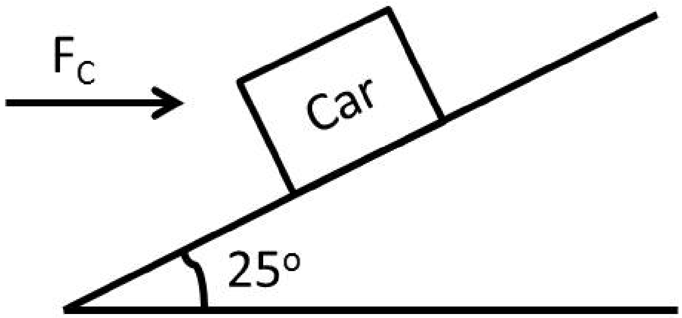
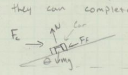


Diagram of car executing banked turn (car is driving into the page).

Solution:

Race tracks are designed with turns that are banked at an angle. Most speedways have turns that are at an angle of 25°.

Find the effect that a 25° banking angle has on the velocity they can complete the turn with.



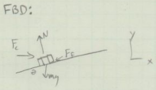
Given: $F_c = m\frac{V^2}{R}$

R = const, m = const,

$\mu = .8$

Sol'n)

FBD:



$\Sigma F_x \Rightarrow F_c = F_f \cos\theta + N \sin\theta$

$\Sigma F_y \Rightarrow mg = N\cos\theta - F_f \sin\theta$

$f_f = \mu N$

For Banked turn, $\theta = 25°$:

$\Sigma F_x \Rightarrow m\frac{V^2}{R} = \mu N \cos 25 + N \sin 25$ (1)

$\Sigma F_y \Rightarrow mg = N\cos 25 - \mu N \sin 25$ (2)

Solve (2) For N: $N = \left(\frac{mg}{\cos 25 - \mu \sin 25}\right)$ (3)

insert (3) into (1)

$$m\frac{V_B^2}{R} = \frac{\mu mg \cos 25}{\cos 25 - \mu \sin 25} + \frac{mg \sin 25}{\cos 25 - \mu \sin 25}$$

$$\frac{V_B^2}{R} = 2.02g$$

For unbanked turn, $\theta = 0$

$\Sigma F_x \Rightarrow F_c = N\mu$ (1)

$\Sigma F_y \Rightarrow mg = N$ (2)

(2) → (1)

$m\frac{V^2}{R} = mg\mu = .8g$

For Banked turn, $V_B = \sqrt{2.02Rg} = 1.42\sqrt{Rg}$

For unbanked, $V = \sqrt{.8Rg} = .894\sqrt{Rg}$

Therefore a banked turn at 25° allows the drivers to take the turn at a speed 1.588 times faster than if it were flat.

4. Book problems:
   (a) 8.48
   (b) 8.56

   Additional Practice Problems: 8.15, 8.17, 8.23, 8.52

   The quiz problem will not be selected from these additional practice problems. However, these exercises contain important elements of the course and similar problems may appear on the exam.

Solution:

8.48 (a) 283 N to left, (b) $B_x = 413N$ to left, (c) $B_y = 480N$ down

8.56 (a) 28.1°, (b) 728 N at 14.04° up and to the right from the horizontal

8.15 For impending tipping, the reaction is at C. And being a 3-force body, the reaction must pass through the intersection of P and W. (a) $\mu_s = 0.485$ (found by solving for ϕ_s) (b) $P = 255N$