A non-communicating calculator is allowed. Full credit will only be given if all steps used are clearly communicated (free body diagrams, algebra, etc).

1. (25 points) The Endless Bridge is an iconic cantilever style bridge located in Minneapolis. The bridge extends 178 feet from the face of the theater and stands approximately 55 ft above the ground. The empty weight of the bridge is estimated at 25,000 lb. We will model the bridge as a horizontal beam extending out from a wall.

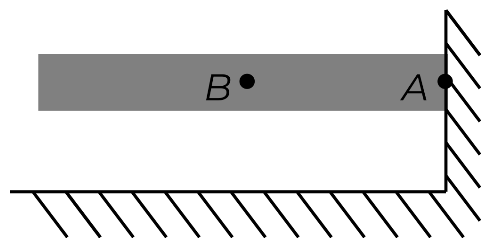
   (a) Draw the Free-Body Diagram (FBD) for the bridge including reactions at the face of the building (A), the weight applied at the mid-span (B), and relevant dimensions.

   (b) Add an additional force to the FBD for the weight of N visitors. Assume the total force of all visitors is applied 170 ft from the wall and the average visitor weights 150 lbs.

   (c) Calculate the reaction at A to maintain static equilibrium, give your answer in terms of N visitors.

   (d) Calculate the maximum visitor capacity for the bridge (N_{max}) such that it does not exceed the maximum safe moment of support A, given as a magnitude of 2.47×10^7lb-ft.

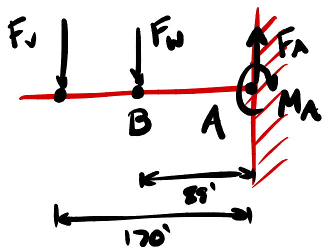
(a) The Endless Bridge *(source: www.guthrietheater.org)*



(b) Simplified Model

a)

b)



c) $\Sigma F_y = F_A - F_v - F_B$

$\begin{cases} F_v = 150N \text{ lb} \\ F_B = 25{,}000 \text{ lb} \end{cases}$

$\Rightarrow \boxed{F_A = 150N + 25{,}000} \uparrow$

$\Sigma M_A = -M_A + F_v(170) + F_B(89)$

$\Rightarrow \boxed{M_A = 25500N + 2{,}225{,}000}$ ↺

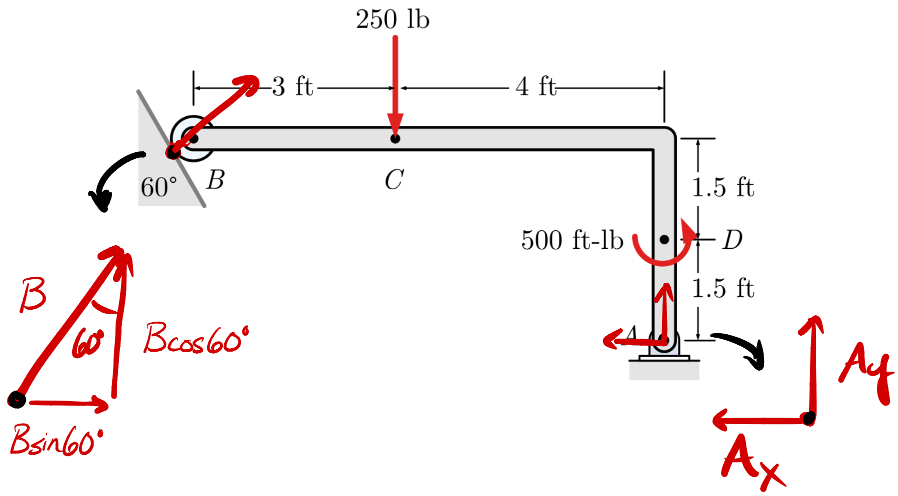
(note sign + direction on FBD)

d) $M_A \leq 2.47 \times 10^7 = 25{,}500N + 2{,}225{,}000$

$$\Rightarrow N_{max} = \frac{2.47 \times 10^7 - 2{,}225{,}000}{25{,}500}$$

$$\boxed{N_{max} = 881 \text{ visitors}}$$

2. (25 points) The L-shaped body is supported by a roller at B and a frictionless pin at A. The body supports a 250 lb vertical force at C and a 500 lb-ft couple-moment at D. Determine the reactions at A and B.



Force Sum

$$\Sigma F_x = B\sin 60° - A_x = 0$$

$$\Rightarrow A_x = B\sin 60° \quad (1)$$

$$\Sigma F_y = B\cos 60° + A_y - 250 = 0$$

$$\Rightarrow A_y = 250 - B\cos 60° \quad (2)$$

$$\Sigma M_A = 500 + 250(4) - B(7\cos 60° + 3\sin 60°)$$

$$\Rightarrow B = \frac{500 + 250(4)}{7\cos 60° + 3\sin 60°} = 245.979$$

$$\boxed{\begin{aligned} B &= 246 \text{ lb } 30° \\ A_x &= 213 \text{ lb } \leftarrow \\ A_y &= 127 \text{ lb } \uparrow \end{aligned}}$$

or $A = 248 \text{ lb}$ ~31°

3. (25 points) A force $\mathbf{F} = (-100\mathbf{j} - 700\mathbf{k})$ lbs is applied at the point B on the bucket of a skid-steer loader. Given:

- $\mathbf{r}_{O\to B} = (-4\mathbf{i} + 2\mathbf{j} + 0.2\mathbf{k})$ ft
- $\mathbf{r}_{O\to A} = (-0.2\mathbf{i} - 0.2\mathbf{j} + 1\mathbf{k})$ ft
- $\mathbf{r}_{A\to A'} = (-8\mathbf{i})$ ft

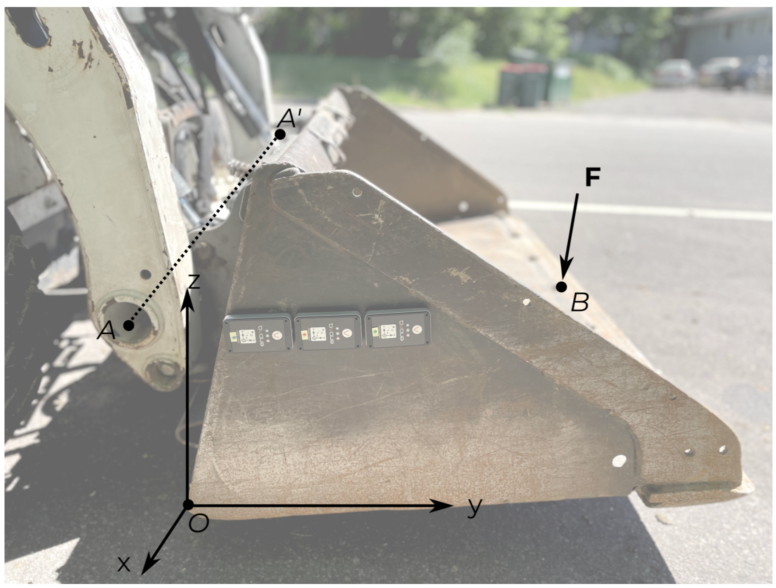
Find:

(a) The position $\mathbf{r}_{A\to B}$ (*hint:* $\mathbf{r}_{O\to B} - \mathbf{r}_{O\to A}$)

(b) The moment about point A ($\mathbf{M}_A$) caused by force $\mathbf{F}$

(c) If we move force $\mathbf{F}$ to point A, and add a couple $\mathbf{M}_A$ (calculated in part b), then we will have an equivalent force-couple system. True or False? Where can we apply $\mathbf{M}_A$?

(d) The component of the moment $\mathbf{M}_A$ about an axis passing from A to A'? (*hint: look at the direction of* $\mathbf{r}_{A\to A'}$)



a) $-3.8\hat{\imath} + 2.2\hat{\jmath} - 0.8\hat{k}$

b) $\vec{M}_A = \vec{r}_{A\to B} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3.8 & 2.2 & -0.8 \\ 0 & -100 & -700 \end{vmatrix} = \boxed{-1620\hat{\imath} - 2660\hat{\jmath} + 380\hat{k}}$

c) true, $\vec{M}_A$ can be applied anywhere

d) $\vec{M}_A' = \vec{M}_A \cdot \dfrac{\vec{r}_{A\to A'}}{\|\vec{r}_{A\to A'}\|}$ $\boxed{= -1620\hat{\imath}}$

↳ only need unit vector (i.e. parallel component)

4. (25 points) An engineering student is using the lessons of statics to perfect a side-kick. The kick can impact the wooden board at any of the angles shown (given with respect to the vertical).

   (a) Given an objective, select the best kick angle and briefly justify your answer.

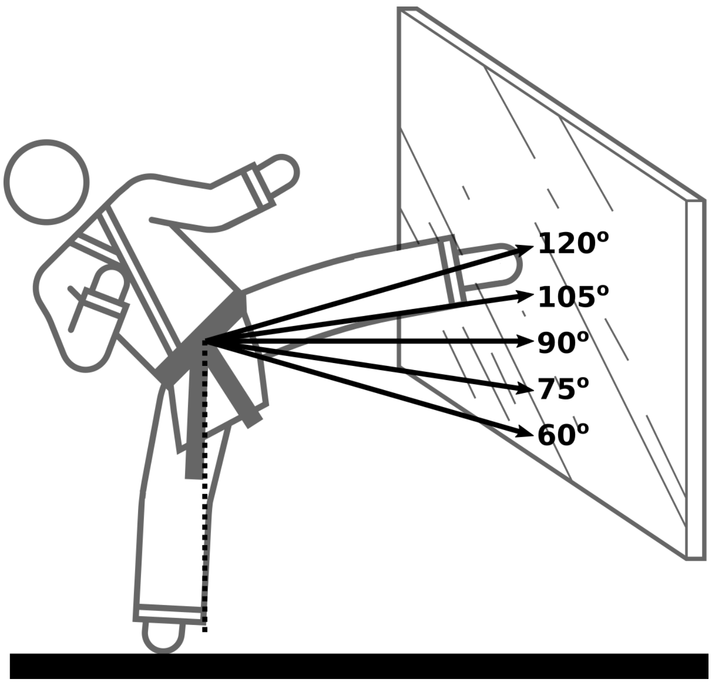
      i. Breaking the board

      ii. Sending the board flying up

      iii. Sending the board crashing down

   (b) The kicking action is happening while on the sidewalk and wearing shoes with good grip. What reactions are imparted by the ground on their standing foot? Draw them for the 2D scenario.

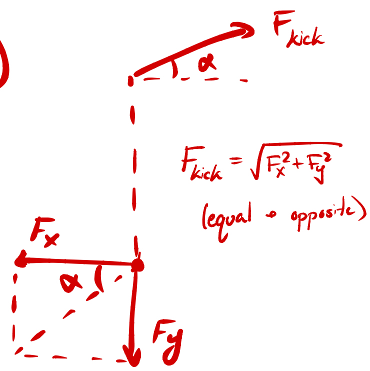
   (c) They want a powerful kick at the board but without falling over. Which angles would let them leverage the ground reaction to maximize the kicking force?



ai) 90°, assuming board is weakest in perpendicular and all kicks have same magnitude

aii) 120°   aiii) 60°

b)



c) 120° gives a larger vertical component enabling a larger horizontal force, (consider how friction changes as F_y increases)