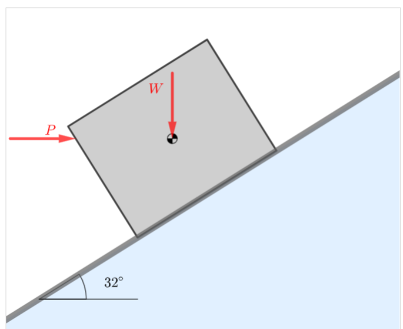
AEM 2011 Homework #12 (8.1) Wednesday, April 12, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. A 360 lbs horizontal force P is applied to a 180 lbs box resting on an 32° incline. The line of action of P passes through the center of gravity of the box. The box is 5 ft wide × 4 ft tall, and the coefficents of friction between the box and the surface are $\mu_s = 0.1$ and $\mu_k = 0.080$.

   (a) Determine the magnitude of the friction force acting on the box.

   (b) Is the box:
   - In Equilibrium
   - Sliding up the incline
   - Sliding down the incline
   - Tipping



Solution:

- $F = 27.47$ lbs
- The box is *Sliding up the incline*

Advice

Establish a coordinate system parallel and perpendicular to the incline, and draw a free body diagram of the box. At this point we don't know whether the box is in equilibrium or the direction of the frictional force, so we will provisionally assume that the box is in equilibrium and that the frictional force acts up the incline. A assumed friction force acting up the incline implies that the box is sliding down the incline.

Calculate the force require to hold the box in equilibrium

$$\begin{aligned} \sum F_x &= 0 \\ F &= -P_x + W_x \\ &= -P\cos\theta + W\sin\theta \\ &= -360\cos 32^\circ + 180\sin 32^\circ \\ &= -305.3 + 95.39 \\ F &= -209.9 \text{ lbs} \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0 \\ N &= P_y + W_y \\ &= P\sin\theta + W\cos\theta \\ &= 360\sin 32^\circ + 180\cos 32^\circ \\ &= 190.8 + 152.6 \\ N &= 343.4 \text{ lbs} \end{aligned}$$

Calculate the available frictional force

$$F_s = \mu_s N = (0.1)\,(343.4) = 34.34 \text{ lbs}$$

$$F_k = \mu_k N = (0.080)\,(343.4) = 27.47 \text{ lbs}$$

Draw Conclusions

Since the absolute value of the required force F is greater than the available static friction force F_s, the box is sliding. The magnitude of the actual friction force is $F = F_k = 27.47$ lbs.

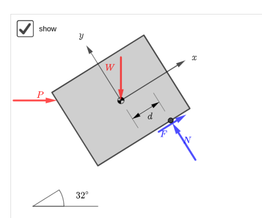
Since the sign on the required frictional force F is negative, the assumption that friction acts up the incline is incorrect. The actual force acts down the incline, which means that the box being pushed up the incline.

Check that no tipping occurs.

The resultant frictional and normal forces can be considered to act at a single point. The location of this point can be found by performing a moment equilibrium about a convenient point. We choose the center of gravity, where the lines of action of the weight and the load P intersect, and use the actual value of the frictional force, which is F if the box is in equilibrium, F_k if the box is sliding. If distance d is greater than half the width of the box, it tips.

$$\begin{aligned} \sum M_C &= 0 \\ N(d) &= F(h/2) \\ d &= \frac{Fh}{2N} \\ &= \frac{(27.47)(4)}{2343.4} \\ &= 0.16 \text{ ft} \end{aligned}$$

Since $|d| <= 2.5$ ft, the block is not tipping.

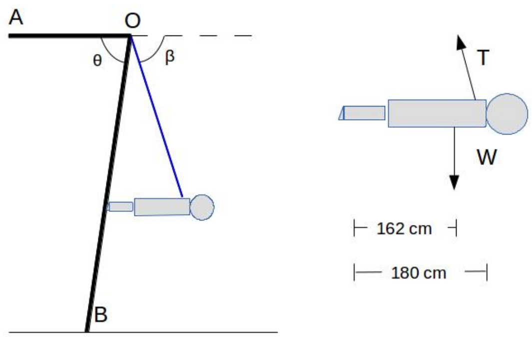


2. Professor Seiler is trying his hand at rappelling. He has chosen a cliff (AOB) which is inclined to the vertical as shown in the figure. It is known that $\theta = 80$ degrees and $\beta = 65$ degrees. It is also know that Professor Seiler weighs 750 N. His CG is located at 162 cm from his feet and he is holding the rope with his hands which are at a distance of 180 cm from his feet as shown on the right. Let the coefficient of friction between his feet and the cliff wall be μ_s.

   Find:

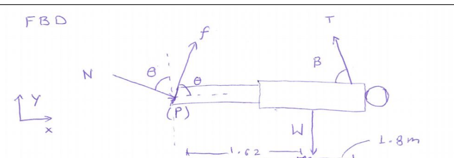
   (a) Tension in the rope.

   (b) The minimum value of μ_s so that he does not slip and remains in equilibrium.



Solution:

Q8 FBD



Lets name the point of contact between the wall and the feet as P

a) $\Sigma M_P = 0$

$\Rightarrow -1.62 \times W + 1.8 \times T \sin\beta = 0$

$\Rightarrow T = \dfrac{1.62 \times W}{1.8 \times \sin(\beta)}$

$\Rightarrow \boxed{T = 744.78\,N}$

(b) Here, we will first calculate f, N ~~for μ~~ such that the system remains in equilibrium.

$\Sigma F_Y = 0$

$\Rightarrow T \sin\beta + f \sin\theta - W - N\cos\theta = 0$

$\Rightarrow 675 + 0.9848 f - 750 - 0.1736 N = 0$

$\Rightarrow \boxed{0.9848 f - 0.1736 N = 75}$

$\Sigma F_x = 0$

$\Rightarrow N \sin\theta + f \cos\theta - T\cos\beta = 0$

$\Rightarrow \boxed{0.1736 f + 0.9848 N = 314.7576}$

$\Rightarrow$ Solving these two equations

$\boxed{\begin{aligned} f &= 128.5\ N \\ N &= 296.96\ N \end{aligned}}$

$\Rightarrow$ The minimum μ which can sustain the above system is $\boxed{\mu_{min} = \dfrac{f}{N} = 0.43}$