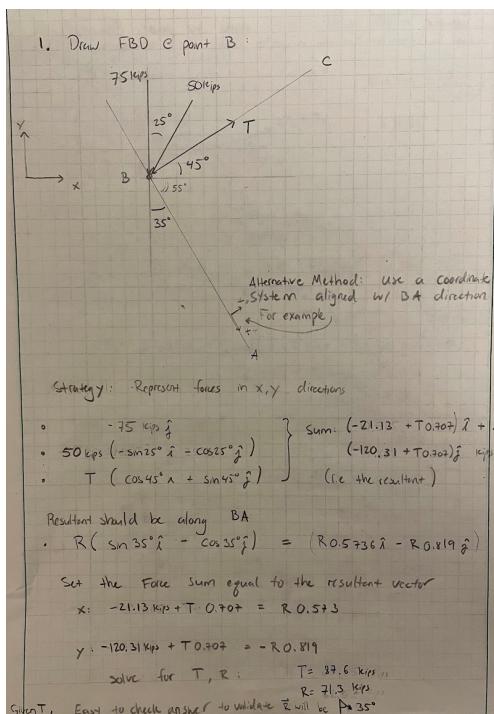


Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. A disabled lift has the loading shown at point *B* and is being supported by a cable pulling along *BC*. Find the required tension in cable *BC* so that the resultant of the three forces at point *B* is directed along the lift boom (i.e. member *AB*).



### Solution:



2. The figure below shows a banner hanging on campus along with an idealized diagram for this banner. The banner has length  $L = 7.5m$ , height  $H = 1.5m$ , and mass  $m = 5kg$ . Assume:

- (a) the cables directions are as shown with  $\tan(\phi) = \frac{H}{L}$
- (b) the tension in cable A is twice that of cable C
- (c) the tension in cable B is twice that of cable D

Given these assumptions estimate the tension in each cable so that there is zero net force on the banner.



### Solution:

2. Since only dealing w/ forces, treat banner as a particle.  
(or, simply "slide" forces along line of action to center of banner  
where "W" acts)  
FBD:

$$\phi = \tan^{-1} \left( \frac{1.5m}{7.5m} \right) = 11.31^\circ$$

$$W = (5\text{kg})(9.81 \frac{\text{m}}{\text{s}^2}) = 49.05\text{N}$$

$$\sum F_x: 2T_D \cos \phi + T_D \cos \phi - 2T_C \cos \phi - T_C \cos \phi = 0$$

$$3T_D - 3T_C = 0$$

$$T_D = T_C \equiv T \quad (\text{i.e. symmetrical})$$

$$\sum F_y: 2T_D \sin \phi - T_D \sin \phi + 2T_C \sin \phi - T_C \sin \phi - W = 0$$

$$T_D \sin \phi + T_C \sin \phi - W = 0$$

$$2T \sin \phi = W$$

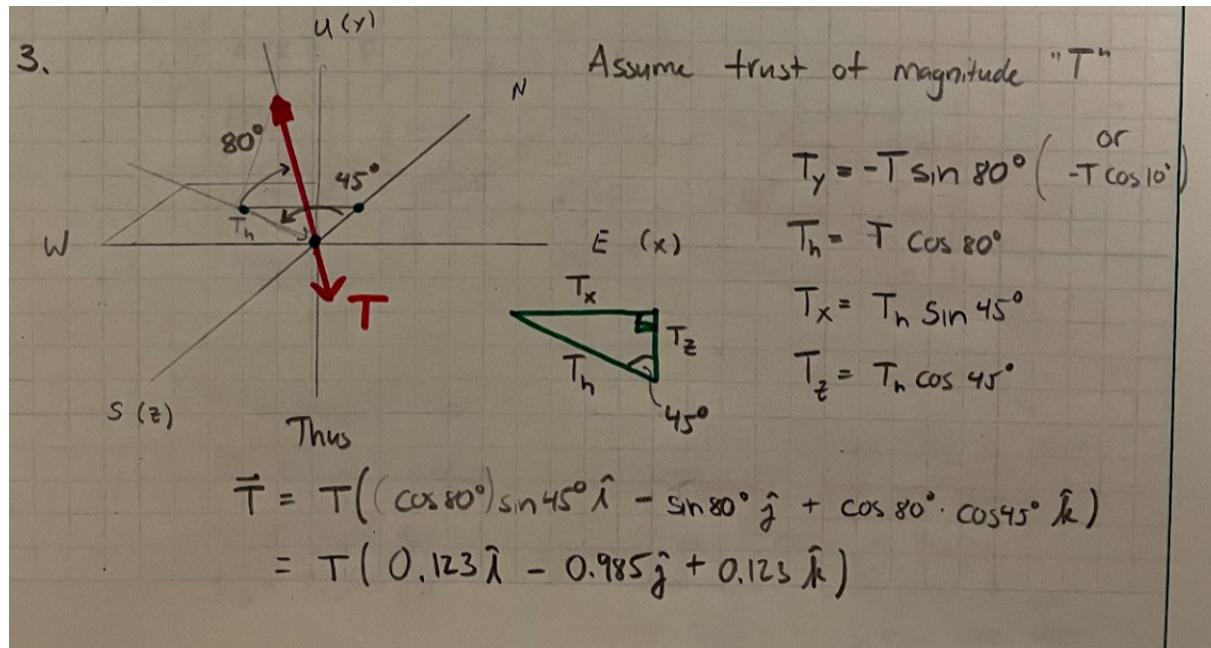
Thus  $T_A = T_B = 250\text{N}$   
 $T_C = T_D = 125\text{N}$

$$T = \frac{W}{2 \sin \phi} = 125\text{N}$$

3. The space shuttle is headed in a direction that is  $80^\circ$  relative to horizontal and  $45^\circ$  west of north. The rocket thrust acts opposite the direction of travel of the space shuttle. Assume that the  $x$ ,  $y$ , and  $z$  coordinate axes are directed east, up, and south, respectively.

- (a) Draw the space shuttle and the assumed coordinate system.  
 (b) Determine the  $x$ ,  $y$ , and  $z$ -components of the thrust force.

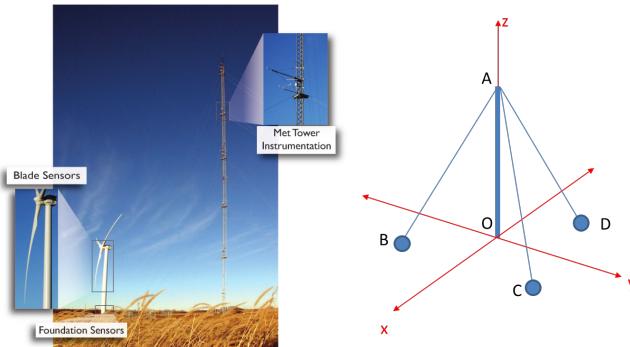
**Solution:**



4. A wind turbine is installed at the University of Minnesota Eolos Wind Research Field Station. The field station also has a 130m (426ft) tall meteorological tower with sensors for research purposes. The met tower is supported by cables. For simplicity, assume that the tower has three cables attached to the pin at A and anchored on the ground at points B, C, and D. The locations of the various points are given by:

- $\vec{A} = 130m\mathbf{k}$
- $\vec{B} = 60m\mathbf{i} - 40m\mathbf{j}$
- $\vec{C} = 60m\mathbf{i} + 40m\mathbf{j}$
- $\vec{D} = -40m\mathbf{i} + 30m\mathbf{j}$

Determine the tension in each cable if the met tower exerts an upward vertical force of 4000N on the pin at A. Reflect on the tensions computed. What does this say about the supports provided by the assumed cable structure?



Left: Eolos Field Station (Image courtesy of [eolos.umn.edu](http://eolos.umn.edu)), Right: Simplified diagram

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### Solution:

4. Pin A has 4 forces acting on it.

$\vec{F}_p = 4kN \hat{\mathbf{k}}$

$\vec{T}_B = T_B \hat{\mathbf{u}}_{AB}$

$\vec{T}_C = T_C \hat{\mathbf{u}}_{AC}$

$\vec{T}_D = T_D \hat{\mathbf{u}}_{AD}$

Find unit vectors to complete vector representation

$\vec{AB} = \vec{B} - \vec{A} = (60\hat{\mathbf{i}} - 40\hat{\mathbf{j}} - 130\hat{\mathbf{k}}) \text{ m}$

$|\vec{AB}| = 142.65 \text{ m}$

$\hat{\mathbf{u}}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = 0.404\hat{\mathbf{i}} - 0.267\hat{\mathbf{j}} - 0.874\hat{\mathbf{k}}$

Similarly:

$\hat{\mathbf{u}}_{AC} = 0.404\hat{\mathbf{i}} + 0.267\hat{\mathbf{j}} - 0.874\hat{\mathbf{k}}$

$\hat{\mathbf{u}}_{AD} = -0.232\hat{\mathbf{i}} + 0.215\hat{\mathbf{j}} + 0.933\hat{\mathbf{k}}$

$\sum \vec{F} = 0 = \vec{F}_p + \vec{T}_B + \vec{T}_C + \vec{T}_D$

$$\begin{bmatrix} 0 \\ 0 \\ -4kN \end{bmatrix} = \begin{bmatrix} 0.404 & 0.104 & -0.267 \\ -0.232 & 0.404 & 0.215 \\ 0.199 & 0.374 & -0.933 \end{bmatrix} \begin{bmatrix} T_B \\ T_C \\ T_D \end{bmatrix}$$

Solve by inverting matrix

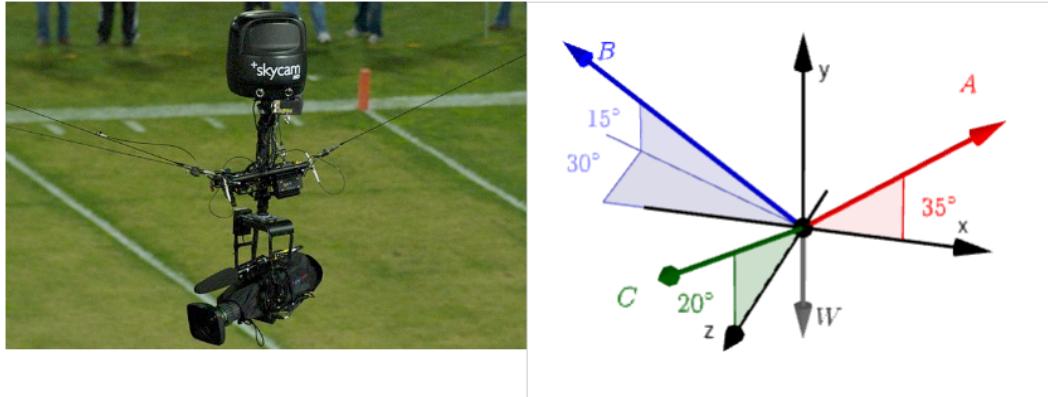
$T_B = 1.94 \text{ kN}$

$T_C = -0.11 \text{ kN}$

$T_D = 2.57 \text{ kN}$

This shows  $T_C$  is not "pulling" that is "pushing" (i.e. cable is in compression). But cables can't push & so this support arrangement won't work.

5. The skycam at Stanford University Stadium has a mass of  $20\text{kg}$  and is supported by three cables as shown. Assuming that it is currently in equilibrium, find the tension in each of the three supporting cables.



**Solution:**

Write vector representation for forces:

$$\vec{W} = -(20\text{kg})(9.81 \frac{\text{m}}{\text{s}^2}) \hat{j} \\ = -196.2 \text{ N} \hat{j}$$

$$\vec{T}_A = T_A (\cos 35^\circ \hat{i} + \sin 35^\circ \hat{j}) = T_A (0.819 \hat{i} + 0.5736 \hat{j})$$

$$\vec{T}_C = T_C (\sin 20^\circ \hat{j} + \cos 20^\circ \hat{k}) = T_C (0.342 \hat{j} + 0.9397 \hat{k})$$

$$\vec{T}_B : \quad y\text{-comp: } T_B \sin 15^\circ$$

$$x\text{-proj: } T_B \cos 15^\circ$$

$$x\text{-comp: } -T_B \cos 15^\circ \cos 30^\circ$$

$$z\text{-comp: } -T_B \cos 15^\circ \sin 30^\circ$$

$$\text{Finally: } \vec{T}_B = T_B (-0.8365 \hat{i} + 0.2588 \hat{j} - 0.483 \hat{k})$$

$$\text{Force equilibrium: } \sum \vec{F} = \vec{W} + \vec{T}_A + \vec{T}_B + \vec{T}_C = 0$$

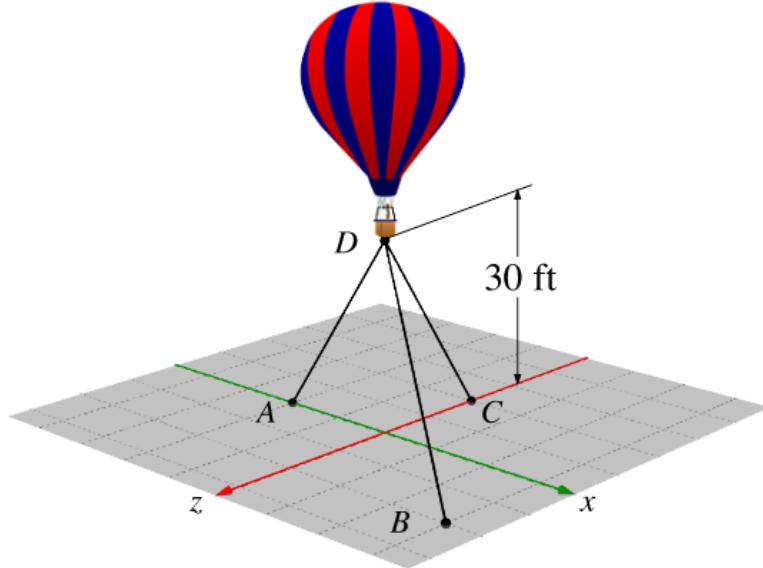
$$\begin{bmatrix} 0 \\ 196.2 \text{ N} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.819 & -0.8365 & 0 \\ 0.5736 & 0.2588 & 0.342 \\ 0 & -0.483 & 0.9397 \end{bmatrix} \begin{bmatrix} T_A \\ T_B \\ T_C \end{bmatrix}$$

$$\text{Solve: } T_A = 196.4 \text{ N}$$

$$T_B = 192.3 \text{ N}$$

$$T_C = 98.8 \text{ N}$$

6. A hot air balloon 30ft above the ground is tethered by three cables as shown in the diagram. If the balloon is pulling upwards with a force of 900lb, what is the tension in each of the three cables? The grid lines on the ground plane are spaced 10ft apart.



**Solution:**

6. We need vector representations for all forces:

$$\vec{F}_o = 900 \text{ lb} \hat{j}$$

$$\vec{T}_A = T_A (\hat{u}_{DA}) = T_A (-0.5547 \hat{x} - 0.832 \hat{j})$$

$$\vec{T}_B = T_B (\hat{u}_{DB}) = T_B (0.6396 \hat{x} - 0.6396 \hat{j} + 0.4264 \hat{k})$$

$$\vec{T}_C = T_C (\hat{u}_{DC}) = T_C (-0.8321 \hat{j} - 0.5547 \hat{k})$$

Force Equilibrium:  $\sum \vec{F} = 0 \Rightarrow \vec{F}_o + \vec{T}_A + \vec{T}_B + \vec{T}_C$

$$\begin{bmatrix} 0 \\ -900 \text{ lb} \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{u}_{DA} & \hat{u}_{DB} & \hat{u}_{DC} \end{bmatrix} \begin{bmatrix} T_A \\ T_B \\ T_C \end{bmatrix}$$

Solve:  $T_A = 463.6 \text{ lb}$   
 $T_B = 402.0 \text{ lb}$   
 $T_C = 309.0 \text{ lb}$

7. Book problems:

- (a) 2.50
- (b) 2.69
- (c) 2.83
- (d) 2.67

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**Solution:**

2.50:  $179.3N < P < 669N$

2.69: (a)  $1293N$  (b)  $2220N$

2.83: (a)  $F_z = -184.0N$ ,  $F_y = -62.0N$  (b)  $\theta_x = 67.6^\circ$ ,  $\theta_y = 107.2^\circ$

2.67: (a)  $300lb$  (b)  $300lb$  (c)  $200lb$  (d)  $200lb$  (e)  $150.0lb$