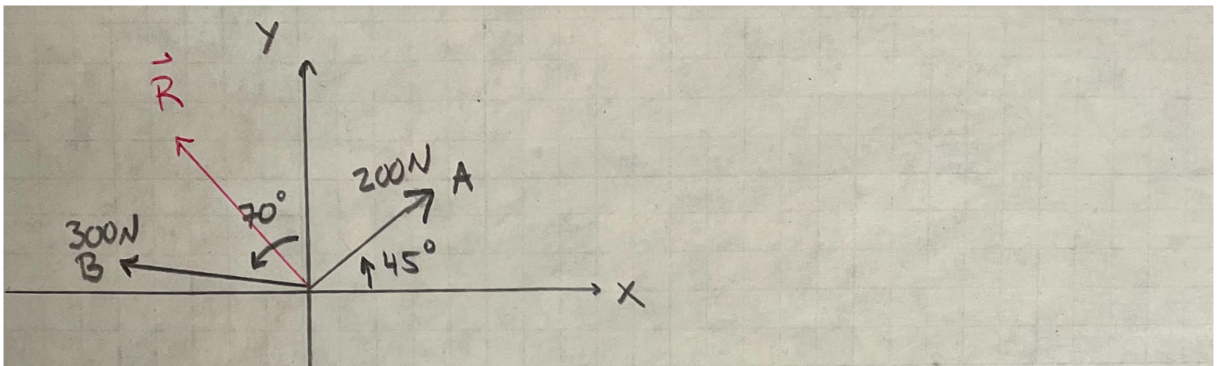
AEM 2011 Homework #1 Tuesday, January 17, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. Vector $\mathbf{A} = 200N\angle 45°$ counterclockwise from the x axis, and vector $\mathbf{B} = 300N\angle 70°$ counterclockwise from the y axis.

   Draw the vectors and find the resultant $\mathbf{R} = \mathbf{A} + \mathbf{B}$ by addition of scalar components.

Solution:



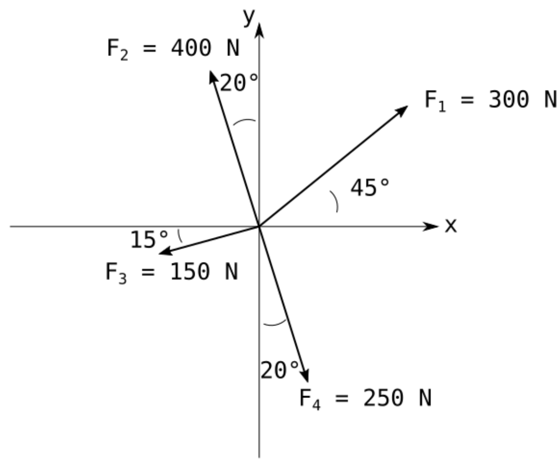
Vector representation:

$\vec{A} = (200\cos 45°\,\hat{\imath} + 200\sin 45°\,\hat{\jmath})$ N

$\vec{B} = (-300\sin 70°\,\hat{\imath} + 300\cos 70\,\hat{\jmath})$ N

$\vec{R} = \vec{A} + \vec{B} = -140.5\,\hat{\imath} + 244.0\,\hat{\jmath}$ N

2. Resolve each of the four vectors into x-components and y-components.

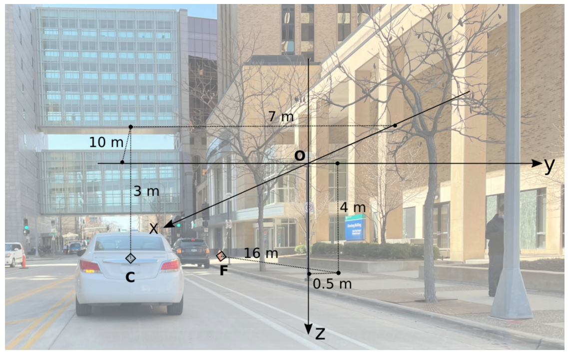


Solution:

Name	Magnitude	X-comp	Y-comp
F_1	300 N	$300 \cos 45°$	$300 \sin 45$
F_2	400	$-400 \sin 20°$	$400 \cos 20°$
F_3	150	$-150 \cos 15°$	$-150 \sin 15°$
F_4	250	$250 \sin 20°$	$-250 \cos 20$

3. The City of Rochester installs a sensor atop a lamppost to monitor walking, biking, and car traffic patterns. To validate the installation, you want to compare the sensor output with measurements made using an alternate reference sensor, as shown in the image. Write out the position vector from the sensor origin to the:

- Fire Hydrant ($\mathbf{r}_{OF}$)
- Car Emblem ($\mathbf{r}_{OC}$)



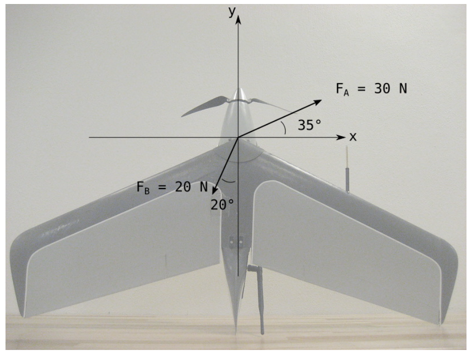
Solution:

Vectors :

Fire Hydrant $\quad \vec{r}_{OF} = 16\,\text{m}\,\hat{i} + 0.5\,\text{m}\,\hat{j} + 4\,\text{m}\,\hat{k}$

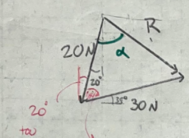
$\vec{r}_{OC} = -10\,\text{m}\,\hat{i} - 7\,\text{m}\,\hat{j} + 3\,\text{m}\,\hat{k}$

4. The wing-to-body connection of a survey UAV is tested by simultaneously applying the two forces shown. Using the graphical method of adding vectors, find the resultant of the two forces $\bar{\mathbf{F}}_A$ and $\bar{\mathbf{F}}_B$.



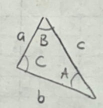
Solution:

Draw vectors graphically:



$(90° - 20° - 35°) = 35°$

Law of Cosines:



$c^2 = a^2 + b^2 - 2ab\cos C$

apply law of cosines to get magnitude

$R^2 = 20^2 + 30^2 - 2(20)(30)\cos 35°$

so $R = 17.8\ N$

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

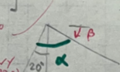
now apply law of sines to get direction

$$\frac{\sin 35°}{R} = \frac{\sin \alpha}{30\ N}$$

$$\alpha = \sin^{-1}\left[\left(\frac{30N}{17.8N}\right)\sin 35°\right]$$

$= \cancel{75.2°}$ or $104.9°$

supplementary angle $(180 - 75.2)$



Law of sines has ambiguity between angle (θ) & supplementary angle $(180° - \theta)$.

We could check 3rd angle to show it should be 104.9° so sum of angles sum to 180°.

Lets show answer as angle down from horizontal (β).

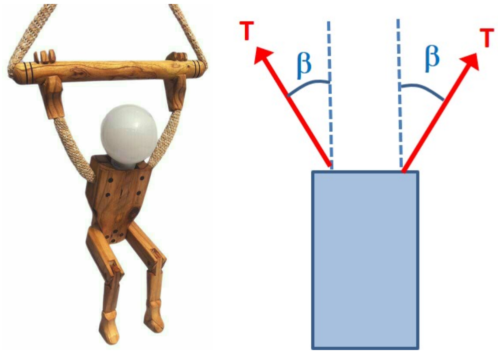
$\beta = 90° - (\alpha - 20°)$

$= \cancel{34.8°}$

$= 5.1°$

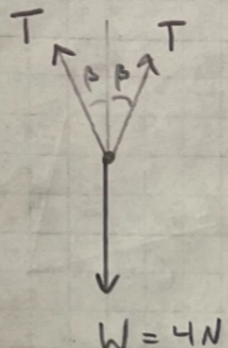
so R is 17.8 N ↘ $\cancel{34.8°}$ 5.1°

5. The figure below shows a hanging robot lamp and a simplified diagram of the hanging robot. The weight of the robot is $W = 4N$. The tension T in each robot arm acts at an angle $\beta = 25°$ from vertical. What is the tension T so that the total vertical force from the two arms balances the downward weight W of the robot?



Solution:

Model as all applied at a single point (body of robot)



$\beta = 25^\circ$

We need $T\big|_{vertical} + T\big|_{vertical} = W$

now $T\big|_{vertical} = T\cos\beta$

so $T\cos\beta + T\cos\beta = 4N$

$2T\cos\beta = 4N$

$T = \dfrac{2N}{\cos\beta}$

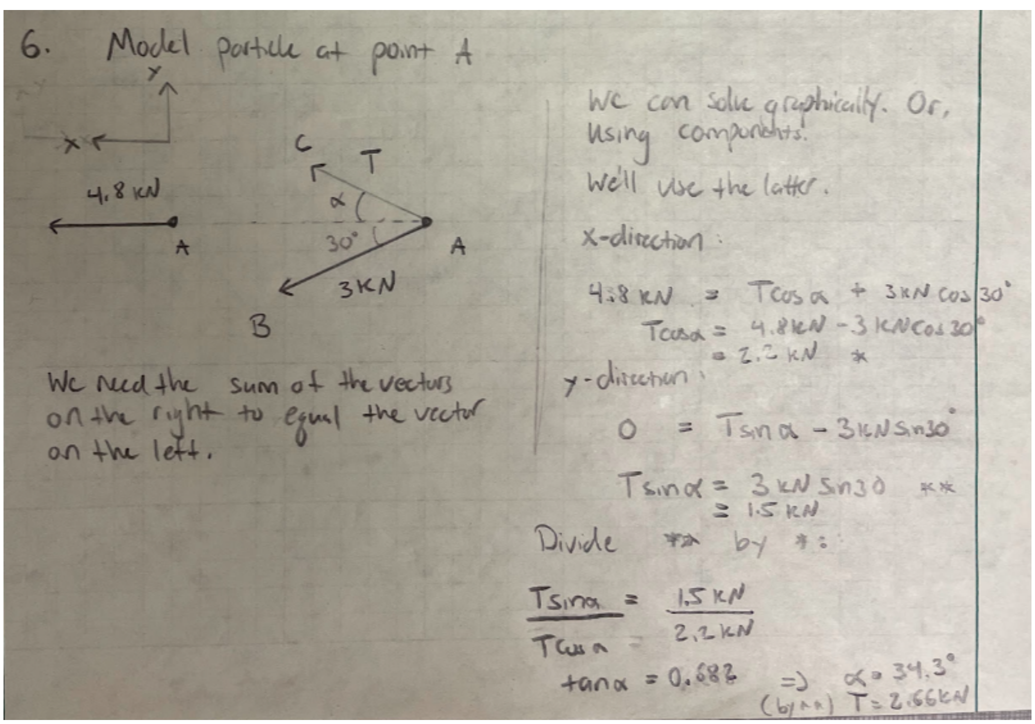
$= 2.2N$

so $T = 2.2N$

6. A skid-steer loader breaks down at a job site. Two tow ropes are attached pulling along AB and AC. The tension in rope AB is $3kN$. The goal is to pull along AC such that a $4.8-kN$ force is applied horizontally at point A, thereby pulling the skid-steer loader safely away from the job site.

Determine the tension and direction (α) required to pull along AC to achieve this.

Solution:



6. Model particle at point A

y

x

4.8 kN

A

C

T

α

30°

A

3 kN

B

We need the sum of the vectors on the right to equal the vector on the left.

We can solve graphically. Or, using components!

We'll use the latter.

x-direction:

$4.8\ kN = T\cos\alpha + 3\ kN\cos 30°$

$T\cos\alpha = 4.8\ kN - 3\ kN\cos 30°$

$= 2.2\ kN$ *

y-direction:

$0 = T\sin\alpha - 3\ kN\sin 30°$

$T\sin\alpha = 3\ kN\sin 30$ **

$= 1.5\ kN$

Divide ** by *:

$\dfrac{T\sin\alpha}{T\cos\alpha} = \dfrac{1.5\ kN}{2.2\ kN}$

$\tan\alpha = 0.682 \Rightarrow \alpha = 34.3°$

(by **) $T = 2.66\ kN$