**AEM 2011 Homework #2** (2.3, 2.4, 2.5) Wednesday, January 25, 2023

Unless otherwise mentioned, these problems should be solvable using a basic calculator. Practice clear communication by showing all work (free body diagrams, algebra, etc). This will be required to receive full credit on any graded problems.

1. A disabled lift has the loading shown at point $B$ and is being supported by a cable pulling along $BC$. Find the required tension in cable $BC$ so that the resultant of the three forces at point $B$ is directed along the lift boom (i.e. member $AB$).


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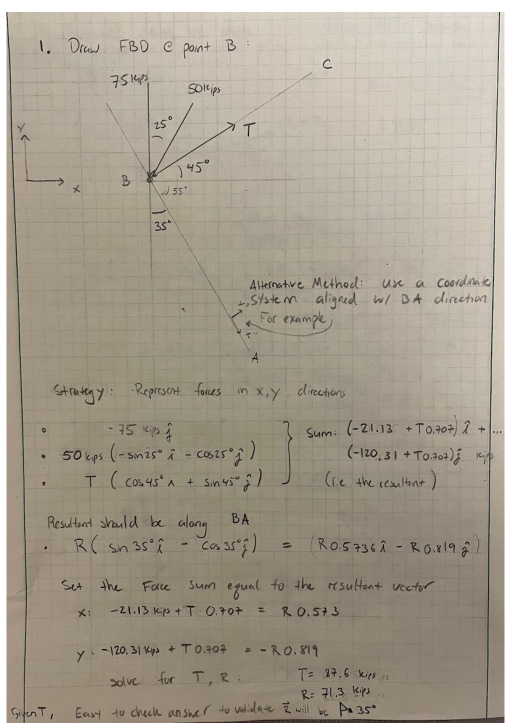
**Solution:**



1. Draw FBD @ point B:



Alternative Method: use a coordinate system aligned w/ BA direction

For example

Strategy: Represent forces in x, y directions

- $-75 \text{ kips } \hat{j}$
- $50 \text{ kips } (-\sin 25° \hat{i} - \cos 25° \hat{j})$
- $T(\cos 45° \hat{i} + \sin 45° \hat{j})$

Sum: $(-21.13 + T0.707)\hat{i} + \ldots$ $(-120.31 + T0.707)\hat{j}$ kips (i.e the resultant)

Resultant should be along BA

- $R(\sin 35° \hat{i} - \cos 35° \hat{j}) = (R0.5736 \hat{i} - R0.819 \hat{j})$

Set the Force sum equal to the resultant vector

x: $-21.13 \text{ kips} + T \cdot 0.707 = R0.573$

y: $-120.31 \text{ kips} + T0.707 = -R0.819$

solve for T, R: $T = 87.6$ kips, $R = 71.3$ kips

Given T, Easy to check answer to validate $\vec{R}$ will be ∠ 35°

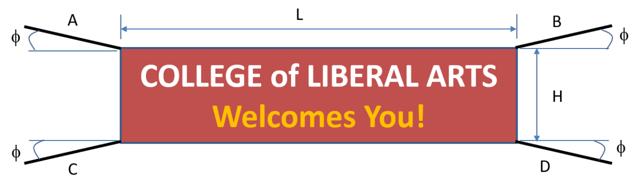
2. The figure below shows a banner hanging on campus along with an idealized diagram for this banner. The banner has length $L = 7.5m$, height $H = 1.5m$, and mass $m = 5kg$. Assume:

   (a) the cables directions are as shown with $\tan(\phi) = \frac{H}{L}$

   (b) the tension in cable A is twice that of cable C

   (c) the tension in cable B is twice that of cable D

   Given these assumptions estimate the tension in each cable so that there is zero net force on the banner.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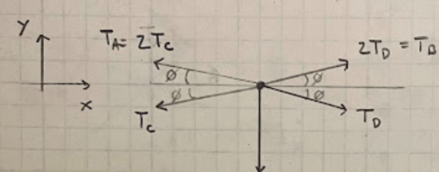
**Solution:**

2. Since only dealing w/ forces, treat banner as a particle.
(or, simply slide forces along line of action to center of banner where "W" acts)

FBD:




$\varnothing = \tan^{-1}\left(\frac{1.5m}{7.5m}\right)$

$= 11.31°$

$W = (5kg)(9.81\frac{m}{s^2})$
$= 49.05N$

$\Sigma F_x$: $2T_D \cos\varnothing + T_D \cos\varnothing - 2T_C \cos\varnothing - T_C \cos\varnothing = 0$

$3T_D - 3T_C = 0$

$T_D = T_C \equiv T$ (i.e. symmetrical)

$\Sigma F_y$: $2T_D \sin\varnothing - T_D \sin\varnothing + 2T_C \sin\varnothing - T_C \sin\varnothing - W = 0$

$T_D \sin\varnothing + T_C \sin\varnothing - W = 0$

$2T\sin\varnothing = W$

$T = \frac{W}{2\sin\varnothing} = 125N$

Thus $T_A = T_B = 250N$
$T_C = T_D = 125N$

3. The space shuttle is headed in a direction that is 80° relative to horizontal and 45° west of north. The rocket thrust acts opposite the direction of travel of the space shuttle. Assume that the $x$, $y$, and $z$ coordinate axes are directed east, up, and south, respectively.

   (a) Draw the space shuttle and the assumed coordinate system.

   (b) Determine the $x$, $y$, and $z$-components of the thrust force.

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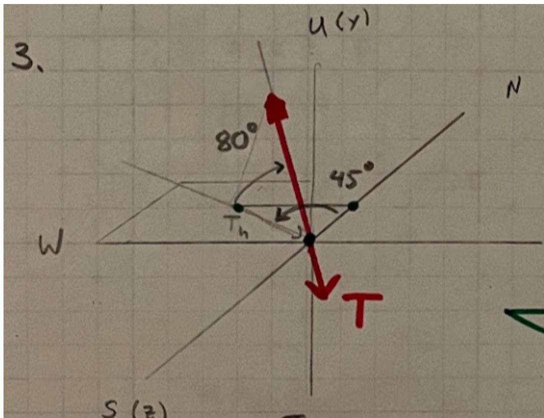
**Solution:**

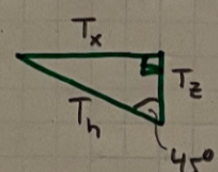
3.




Assume trust of magnitude "T"

$T_y = -T\sin 80°$ (or $-T\cos 10°$)

$T_h = T\cos 80°$




$T_x = T_h \sin 45°$

$T_z = T_h \cos 45°$

Thus

$$\vec{T} = T\left((\cos 80°)\sin 45°\,\hat{i} - \sin 80°\,\hat{j} + \cos 80° \cdot \cos 45°\,\hat{k}\right)$$

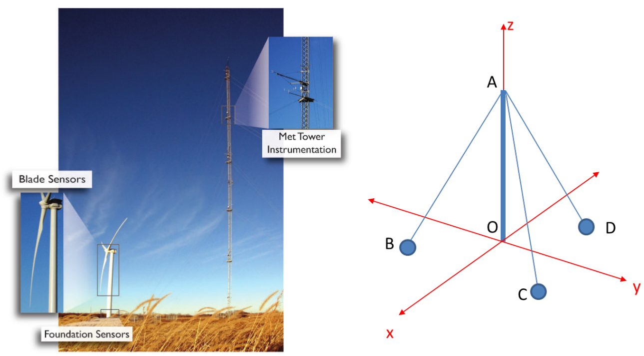
$$= T\left(0.123\,\hat{i} - 0.985\,\hat{j} + 0.123\,\hat{k}\right)$$

4. A wind turbine is installed at the University of Minnesota Eolos Wind Research Field Station. The field station also has a $130m$ ($426ft$) tall meteorological tower with sensors for research purposes. The met tower is supported by cables. For simplicity, assume that the tower has three cables attached to the pin at A and anchored on the ground at points B, C, and D. The locations of the various points are given by:

- $\bar{A} = 130m\mathbf{k}$
- $B = 60m\mathbf{i} - 40m\mathbf{j}$
- $C = 60m\mathbf{i} + 40m\mathbf{j}$
- $D = -40m\mathbf{i} + 30m\mathbf{j}$

Determine the tension in each cable if the met tower exerts an upward vertical force of $4000N$ on the pin at $A$.

Reflect on the tensions computed. What does this say about the supports provided by the assumed cable structure?




Left: Eolos Field Station (Image courtesy of `eolos.umn.edu`), Right: Simplified diagram

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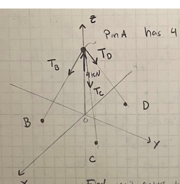
**Solution:**

4.




Pin A has 4 forces acting on it.

$\vec{F}_+ = (4kN)\hat{k}$

$\vec{T}_B = T_B \cdot \hat{u}_{AB}$

$\vec{T}_C = T_C \cdot \hat{u}_{AC}$

$\vec{T}_D = T_D \cdot \hat{u}_{AD}$

Find unit vectors to complete vector representation

$\vec{AB} = \vec{B} - \vec{A} = (60\hat{i} - 40\hat{j} - 130\hat{k})\ m$

$|\bar{AB}| = 148.66\ m$

$\hat{u}_{AB} = \dfrac{\vec{AB}}{|\bar{AB}|} = 0.404\hat{i} - 0.269\hat{j} - 0.874\hat{k}$

Similarly:

$\hat{u}_{AC} = 0.404\hat{i} + 0.269\hat{j} - 0.874\hat{k}$

$\hat{u}_{AD} = -0.287\hat{i} + 0.215\hat{j} - 0.933\hat{k}$

$\Sigma\vec{F} = 0 = \vec{F}_+ + \vec{T}_B + \vec{T}_C + \vec{T}_D$

$$\begin{bmatrix} 0 \\ 0 \\ -4kN \end{bmatrix} = \begin{bmatrix} \underbrace{\begin{matrix} 0.404 \\ -0.269 \\ -0.874 \end{matrix}}_{u_{AB}} & \underbrace{\begin{matrix} 0.404 \\ 0.269 \\ -0.874 \end{matrix}}_{u_{AC}} & \underbrace{\begin{matrix} -0.287 \\ 0.215 \\ -0.933 \end{matrix}}_{u_{AD}} \end{bmatrix} \begin{bmatrix} T_B \\ T_C \\ T_D \end{bmatrix}$$

Solve by inverting matrix

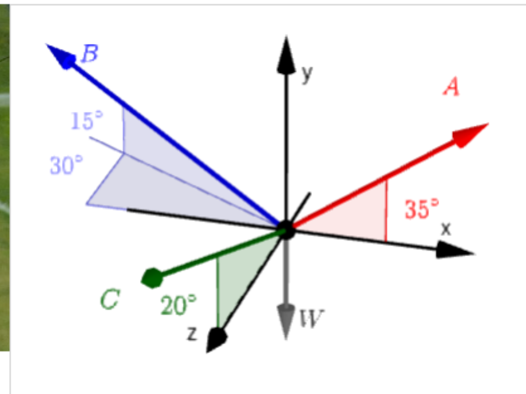
$T_B = 1.94\ kN$

$T_C = -0.11\ kN$

$T_D = 2.57\ kN$

This shows $T_C$ is not "pulling" but is "pushing" (i.e cable is in compression). But cables can't push & so this support arrangement won't work.

5. The skycam at Stanford University Stadium has a mass of $20kg$ and is supported by three cables as shown. Assuming that it is currently in equilibrium, find the tension in each of the three supporting cables.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**Solution:**

Write vector representation for forces:

$$\vec{W} = -(20kg)\left(9.81\ \tfrac{m}{s^2}\right)\hat{j}$$
$$= -196.2\ N\ \hat{j}$$

$$\vec{T}_A = T_A(\cos 35^\circ \hat{\imath} + \sin 35^\circ \hat{j}) = T_A(0.819\hat{\imath} + 0.5736\hat{j})$$

$$\vec{T}_C = T_C(\sin 20^\circ \hat{j} + \cos 20^\circ \hat{k}) = T_C(0.342\hat{j} + 0.9397\hat{k})$$

$\vec{T}_B$ :

y-comp: $T_B \sin 15^\circ$

xz-proj: $T_B \cos 15^\circ$

x-comp: $-T_B \cos 15^\circ \cos 30^\circ$

z-comp: $-T_B \cos 15^\circ \sin 30$

Finally : $\vec{T}_B = T_B(-0.8365\hat{\imath} + 0.2588\hat{j} - 0.483\hat{k})$

Force equilibrium: $\Sigma \vec{F} = \vec{W} + \vec{T}_A + \vec{T}_B + \vec{T}_C = 0$

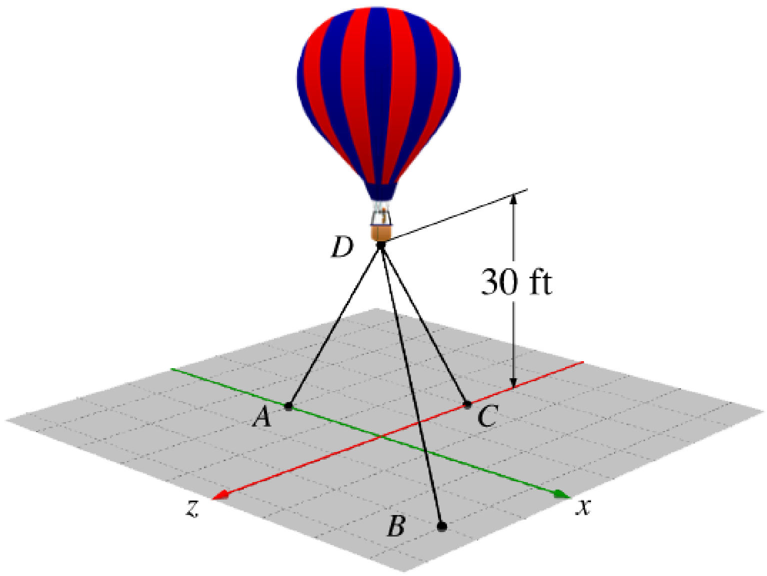
$$\begin{bmatrix} 0 \\ 196.2N \\ 0 \end{bmatrix} = \begin{bmatrix} 0.819 & -0.8365 & 0 \\ 0.5736 & 0.2588 & 0.342 \\ 0 & -0.483 & 0.9397 \end{bmatrix} \begin{bmatrix} T_A \\ T_B \\ T_C \end{bmatrix}$$

Solve : $T_A = 196.4\ N$

$T_B = 192.3N$

$T_C = 98.8\ N$

6. A hot air balloon $30ft$ above the ground is tethered by three cables as shown in the diagram. If the balloon is pulling upwards with a force of $900lb$, what is the tension in each of the three cables? The grid lines on the ground plane are spaced $10ft$ apart.

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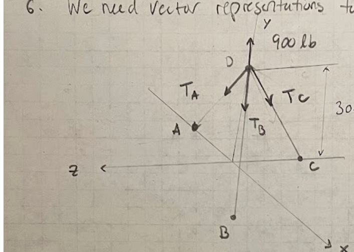
**Solution:**

6. We need vector representations for all forces:




$\vec{DA} = (-20\hat{i} - 30\hat{j})$ ft

$\vec{DB} = (30\hat{i} - 30\hat{j} + 20\hat{k})$ ft

$\vec{DC} = (-30\hat{j} - 20\hat{k})$ ft

$\vec{F}_B = 900lb\,\hat{j}$

$\vec{T}_A = T_A(\hat{u}_{DA}) = T_A(-0.5547\hat{i} - 0.832\hat{j})$

$\vec{T}_B = T_B \cdot \hat{u}_{DB} = T_B(0.6396\hat{i} - 0.6396\hat{j} + 0.4264\hat{k})$

$\vec{T}_C = T_C \cdot \hat{u}_{DC} = T_C(-0.8321\hat{j} - 0.5547\hat{k})$

Force Equilibrium:

$\sum \vec{F} = 0 = \vec{F}_B + \vec{T}_A + \vec{T}_B + \vec{T}_C$

$$\begin{bmatrix} 0 \\ -900\,lb \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{u}_{DA} & \hat{u}_{DB} & \hat{u}_{DC} \end{bmatrix} \begin{bmatrix} T_A \\ T_B \\ T_C \end{bmatrix}$$

Solve: $T_A = 463.6\ lb$

$T_B = 402.0\ lb$

$T_C = 309.0\ lb$