Numerical Solutions : When to Stop?

- May never get to exact solution $f(x) = 0$
- Decide when estimate is good enough
  (or, when to stop)

Notation:

X_{TS} : true (exact) solution such that $f(X_{TS}) = 0$

X_{NS} : numerical approximate solution s.t. $f(x_{NS}) = \varepsilon$

where ε is a small number.

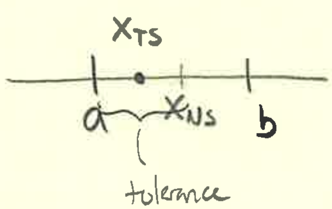
Ideally, look at True error: $X_{TS} - X_{NS}$

But usually X_{TS} is not known. Thus can't compute.

Tolerance in $f(x)$: $\left| f(x_{TS}) - f(x_{NS}) \right| = \left| 0 - f(x_{NS}) \right| = |\varepsilon|$

Look at $|f(x_{NS})|$ as a indicator of how far x_{NS} is from x_{TS}

Tolerance in Solution: If using a bracketing method, then $[a, b]$ bounds the solution.



If $x_{NS} = \dfrac{a+b}{2}$ (midpoint), then

tolerance $= \left| \dfrac{b-a}{2} \right|$

Relative Error: True: $\left| \dfrac{x_{TS} - x_{NS}}{x_{TS}} \right|$

But x_{TS} unknown.

Estimated: $\left| \dfrac{x_{NS}^{(n)} - x_{NS}^{(n-1)}}{x_{NS}^{(n-1)}} \right|$

iteration n

prior iteration

Finally, watch the # of iterations. Stop after exceeding a threshold.

Summary:

Decide when to stop iterations by monitoring any (or all) of:

- Tolerance in $f(x)$
- Tolerance in solution
- Estimated relative error
- # of iterations completed.