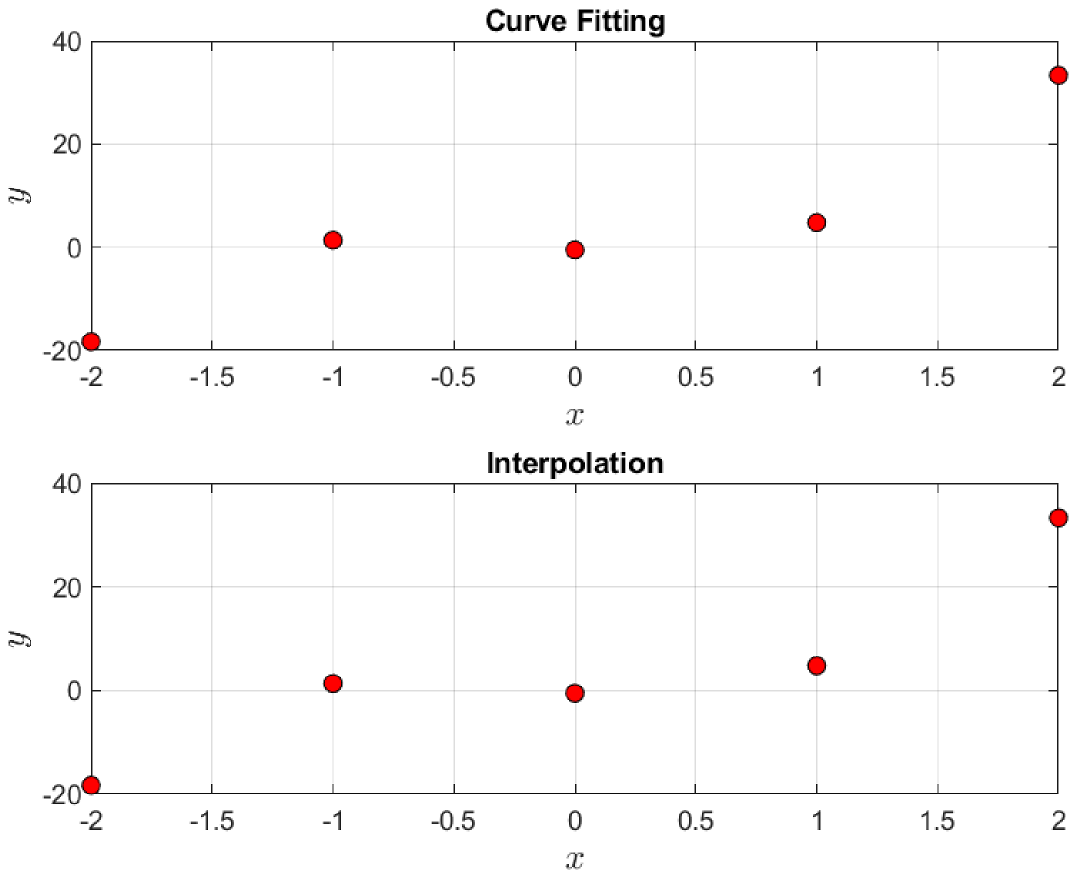

Figure 1: Interpolation vs. curve fitting.

Mathematical Modeling and Simulation | **University of Minnesota**
AEM 3103 | **April 13, 2023**

Practice Problems for Exam #2

1. What is the difference between curve fitting and interpolation? Answer this question by showing what the difference between these to approaches is using the points given in Figure 1

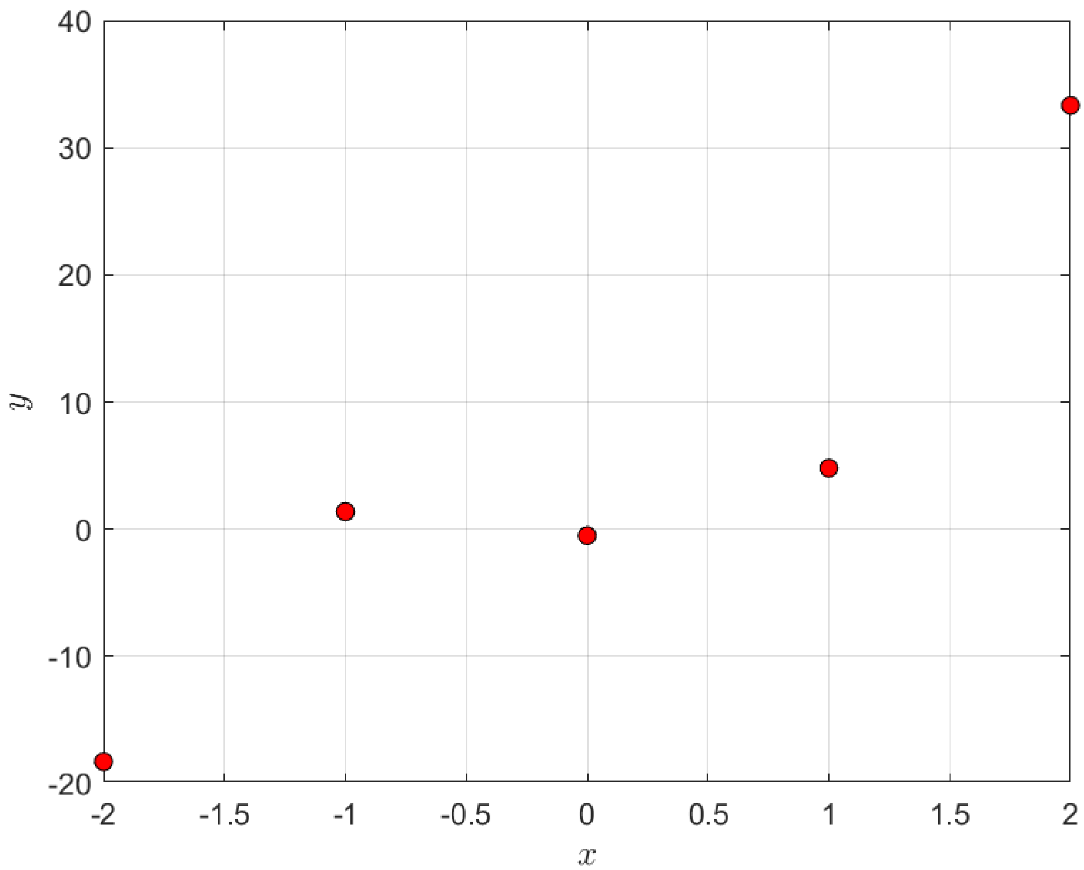

Figure 2: Least squares curve fitting.

2. *Least Squares Curve Fitting*: For the data shown in Figure 2, determine the coefficient of a third order polynomial that fits the given data. Note, it will be difficult to do this problem by hand, so show clearly how your would set up in MATLAB to solve.

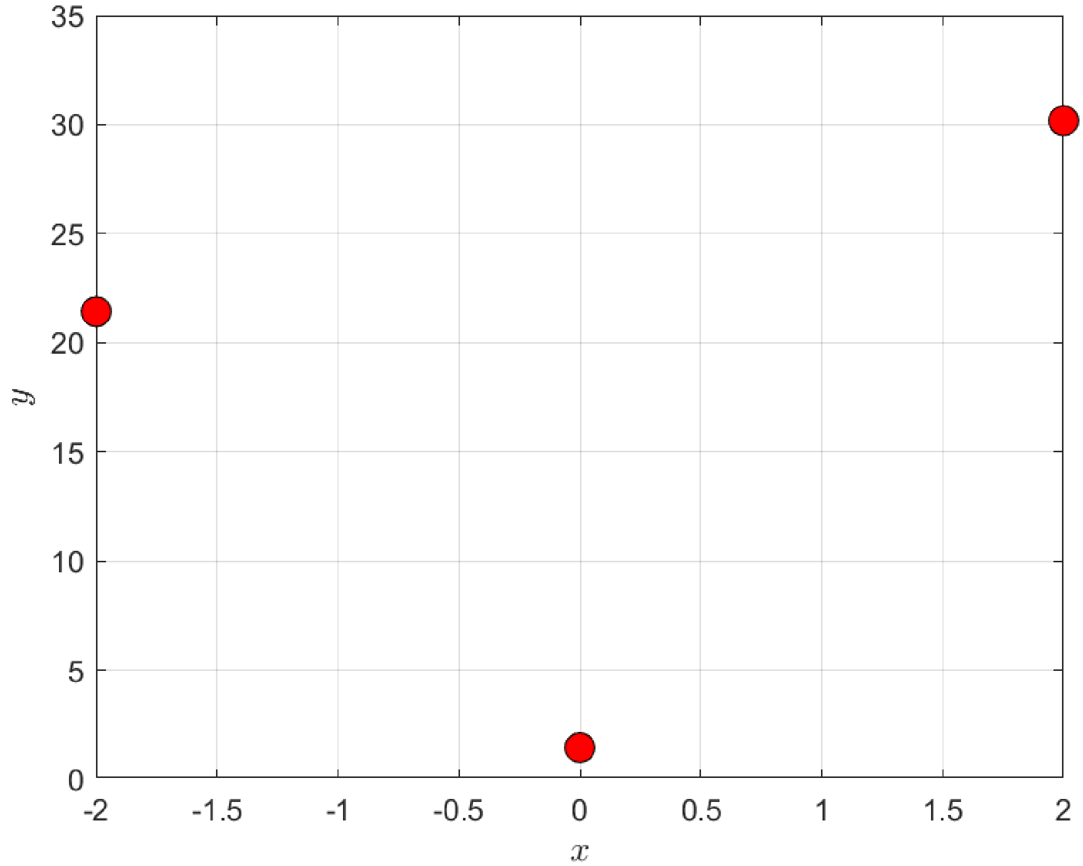

Figure 3: Lagrangian interpolation.

3. *Interpolation*: For the data shown in Figure 3, use Lagrangian interpolation to determine the value of $f(x)$ at $x = 1$.

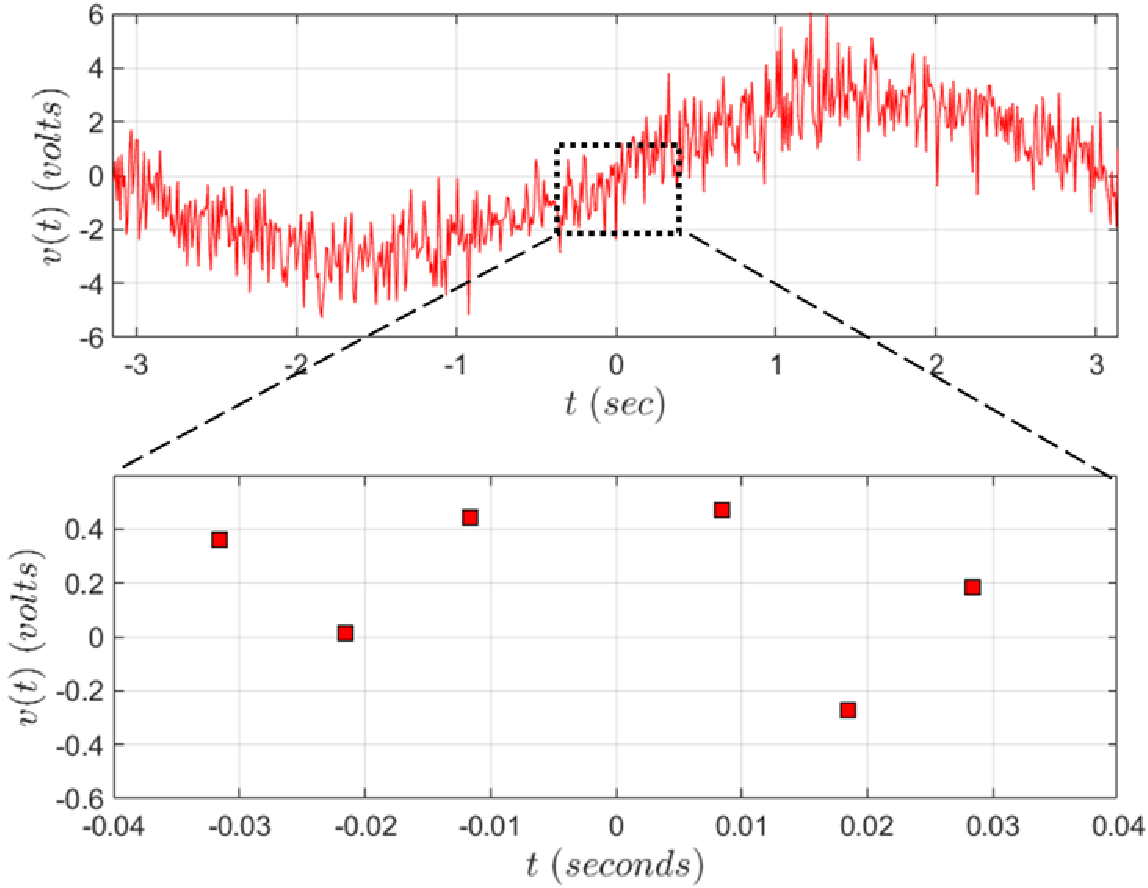

Figure 4: Time history of voltage $v(t)$ accorss an electronic component.

4. Numerical Differentiation: The upper plot shown in in Figure 4 is experimental data collected by measuring the voltage across some electronic component. The lower figure is a zoom-in around $t = 0$ seconds. Explain how you would use a forward difference to generate an accurate estimate of the derivative of $v(t)$. Then calculate and report the value of your estimate for the derivative.

   Hint: This is an engineering problem and not a mathematics problem. Getting the correct answer depends on you thinking the way an engineer would think about a problem like this encountered in practice.

5. *Numerical Integration*: Evaluate the definite integral below using (i) The rectangular, (ii) Trapezoidal and (3) Simpson's 3/8 rule using a step size $h = 0.3142$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x + 2}}$$

6. What is the advantage of the Runge-Kutta method over the Euler method when it comes to solving ODEs numerically? What is the advantage of Euler's method over the Runge-Kutta approach?

7. *Solving ODEs numerically*: Consider the following ordinary differential equation:

$$\begin{aligned} y'' &= \frac{1}{2}\left(x + y + y' + 2\right) \\ y(0) &= 0 \\ y'(0) &= 0 \end{aligned}$$

Solve this differential equation numerically using Euler's method for $0 \leq x \leq 5$ and a step size $h = 1$. Show your work clearly and report the value of y and y' at each of the six point in the interval $0 \leq x \leq 5$.

8. *Solving ODEs numerically*: Consider the following ordinary differential equation:

$$\begin{aligned} y' &= x + y \\ y(0) &= 0 \end{aligned}$$

Solve this differential equation numerically using the Runge-Kutta method for $0 \le x \le 5$ and a step size $h = 1$. Show your work clearly and report the value of y and y' at each of the six point in the interval $0 \le x \le 5$.