Solving Nonlinear Equations

Linear equation : solve analytically

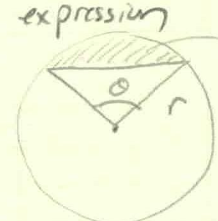
Non-Linear equations : some : solve analytically

e.g. $x^2 - 2x - 2$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2}$$

$$= \frac{2 \pm \sqrt{12}}{2}$$

most : not possible to compute analytical expression



A_s area of segment of circle

e.g $A_s = \frac{1}{2} r^2 (\theta - \sin\theta)$

given A_s & r, find θ.

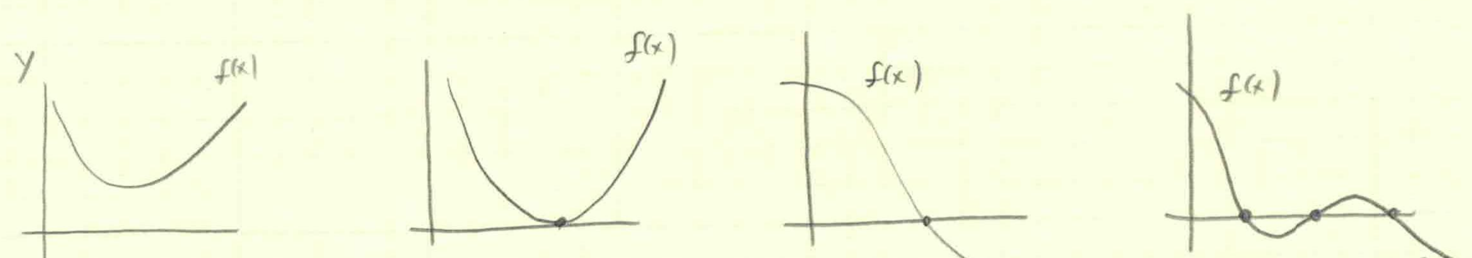
$\Rightarrow$ "solve" numerically!

Not exact. Must define close enough.

	# of equations	# of variables	
Cases:	Single equation	single Variable	Discuss first
	System of equations	More than one variable	Discuss Next

Solving Nonlinear : Scalar Function of One Variable

- Can always be written as: $f(x) = 0$
- solution is called a "root"
- An equation can have 3 outcomes:

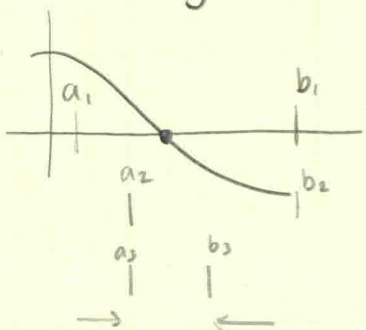


NO SOLN | ONE SOLN | ONE SOLN | MULTIPLE SOLN

- Strategy for Numerical Solution:
  1) Start w/ an approximate guess
  2) Use strategy to iterate & improve guess
  3) End when close enough or max iterations reached

- Many Algorithms exist
- Fall into two groups

Bracketing methods

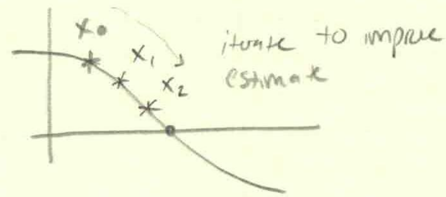


narrow in on solution

* Bisection

  Regula Falsi

Open methods



* Newton's
* Secant

  Fixed-point Iteration

* Plan to discuss

Question: Is root finding the same as minimizing a function?

A: a root finding problem could be solved w/ a minimization tool. But it should be checked that the soln "solves" the root. The opposite is not true.

Define a problem to solve

$$y = x \tan\theta - \frac{1}{2} \frac{x^2 g}{V_0^2} \frac{1}{\cos^2\theta} + h_Q$$

(See fig in problem 3.29)

Given: $V_0 = 50$ ft/s

$x = 60$ ft

$h_Q = 6.5$ ft

$g = 32.2$ ft/s^2

Find θ when $y = 7$ ft

Not easy (impossible?) to write analytic expression. Solve numerically. Write as $f(x) = 0$

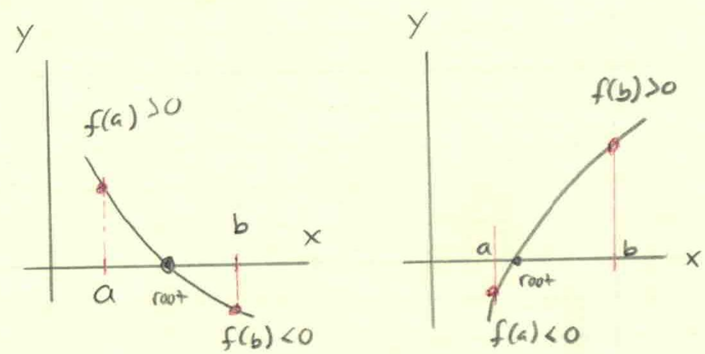
Find roots of expression

$$f(x) = x \tan\theta - \frac{1}{2} \frac{x^2 g}{V_0^2} \frac{1}{\cos^2\theta} + h_Q - 7 = 0$$

Bisection Method

- Bracketing Method
- Requires suppling interval $[a, b]$ where solution exists & function is continuous

  If true, then $f(a) \cdot f(b) < 0$



Algorithm

1) Choose interval $[a, b]$ (e.g. plot it)

   → check $f(a)\, f(b) < 0$

2) Calculate numerical soln using midpoint

$$x_{NS_1} = \frac{(a+b)}{2}$$

3) Check if root is between $\underbrace{[a, x_{NS1}]}_{f(a)\, f(x_{NS1}) < 0}$ or $\underbrace{[x_{NS1}, b]}_{f(a)\, f(x_{NS1}) > 0}$

4) Select the subinterval, update $[a, b]$, return to step 2.

   Iterate until stop condition reached.

Question: What are good stop conditions?