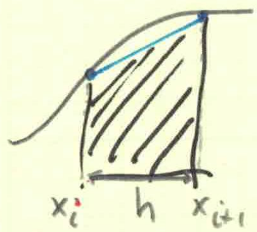
(cont.) Numerical Integration: Approximate Integrand w/ 2nd or 3rd order polynomial.

$$I(f) = \int_a^b f(x)\,dx$$

of subinterval

Recall Trapezoidal method: approximates integrand by straight line




$\Rightarrow$ 1st order polynomial

$x_i$ $x_{i+1}$ $x_{i+2}$

Better approximation of integrand of subinterval:

Simpson's 1/3 method $\Rightarrow$ use quadratic polynomial

$p(x) = \underline{c_2} x^2 + \underline{c_1} x + \underline{c_0}$ || Require $p(x_i) = f(x_i)$, $p(x_{i+1}) = f(x_{i+1})$, $p(x_{i+2}) = f(x_{i+2})$ ] 3 eqns

Simpson's 3/8 method $\Rightarrow$ use cubic polynomial

$p(x) = \underline{c_3} x^3 + \underline{c_2} x^2 + \underline{c_1} x + \underline{c_0}$ || Similar requirement to pass through the 4 points.

Derivation: ① Derive integrand for a single set of subintervals (two or three adjacent values)
→ compute coefficients of polynomial.
→ integrate the polynomial exactly

② Write integral over entire $[a, b]$ interval as sum of the above.

③ Simplify expression.

Result: (eqn. 9.19)

Assumes equally spaced subintervals.

2nd order

# of intervals

$$I(f) \approx \frac{h}{3}\left[ f(a) + 4\sum_{i=2,4,6}^{N} f(x_i) + 2\sum_{j=3,5,7}^{N-1} f(x_j) + f(b)\right]$$

Simpson's ⅓ method → # of subintervals must be even.

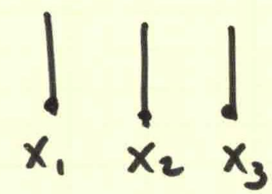
3rd order

$$I(f) = \frac{3h}{8}\left[ f(a) + 3\sum_{i=2,5,8}^{N-1} \left[ f(x_i) + f(x_{i+1})\right] + 2\sum_{j=4,7,10}^{N-2} f(x_j) + f(b)\right]$$

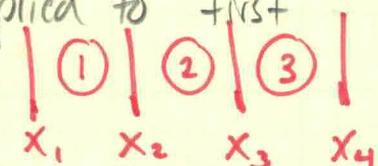
Simpson's 3/8 method → # of sub intervals must be divisible by 3.

Handling any discrete $f(x)$ w/ $N$ subintervals:

$N$ even: Simpsons 1/3 method




$N$ odd: Simpson's 3/8 method applied to first 3 subintervals.




$(N-3)$ is now even: apply Simpson's 1/3 method.

Estimated Accuracy:

midpoint method $O(h^2)$

trapezoidal Method $O(h^2)$

Simpson's 1/3 method $O(h^4)$

Simpson's 3/8 method $O(h^4)$