بنام خدا

Runge-Kutta Methods

Remember general form for single-step explicit numerical methods:

$$x_{k+1} = x_k + h$$

$$y_{k+1} = y_k + (\text{slope}) \cdot h$$

2nd Order Runge-Kutta : Uses 2 points to compute slope.

3rd " " " : " 3 points " " "

4th " " " : " 4 points " " "

and so on.

Sometimes called the classical Runge-Kutta method.

Higher Order → Higher accuracy but requires extra function evaluations.

2^{nd} Order Runge-Kutta Method (RK)

Assume: $\frac{dy}{dx} = f(x,y)$

$$y_{k+1} = y_k + \underbrace{(c_1 \cdot k_1 + c_2 \cdot k_2)}_{\text{"Slope"}} \cdot h$$

where $k_1 = f(x_k, y_k)$

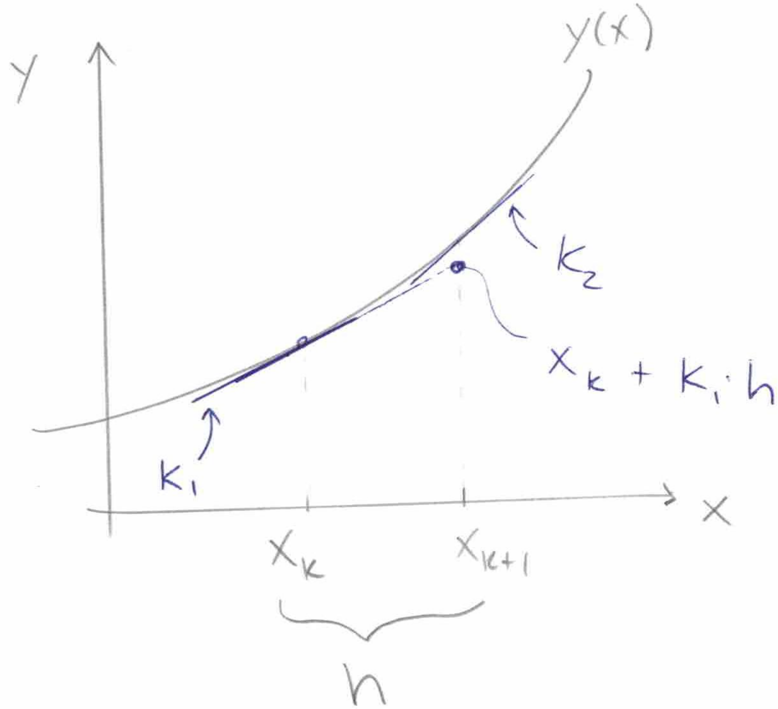
$$k_2 = f(x_k + a_2 \cdot h,\ y_k + b_{21} \cdot k_1 \cdot h)$$

Pick c_1, c_2, a_2, b_{21}.

If $c_1 = c_2 = \frac{1}{2}$ and $a_2 = b_{21} = 1$, then

2^{nd} Order RK matchs Modified Euler method

$$y_{k+1} = y_k + \frac{1}{2}\Big(f(x_k, y_k) + \ldots$$
$$f(x_k + h,\ y_k + k_1 \cdot h)\Big) \cdot h$$



To compute k_2, we approximat the exact "y_{k+1}" w/ "$y_k + h \cdot k_1$"

4th Order Runge-Kutta

(Book eqns 10.86 10.87)

$$y_{k+1} = y_k + \overbrace{\frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)h}^{\text{"slope"}}$$

where

$$k_1 = f(x_k, y_k)$$

$$k_2 = f\left(x_k + \tfrac{1}{2}h,\ y_k + \tfrac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_k + \tfrac{1}{2}h,\ y_k + \tfrac{1}{2}k_2 h\right)$$

$$k_4 = f\left(x_k + h,\ y_k + k_3 \cdot h\right)$$