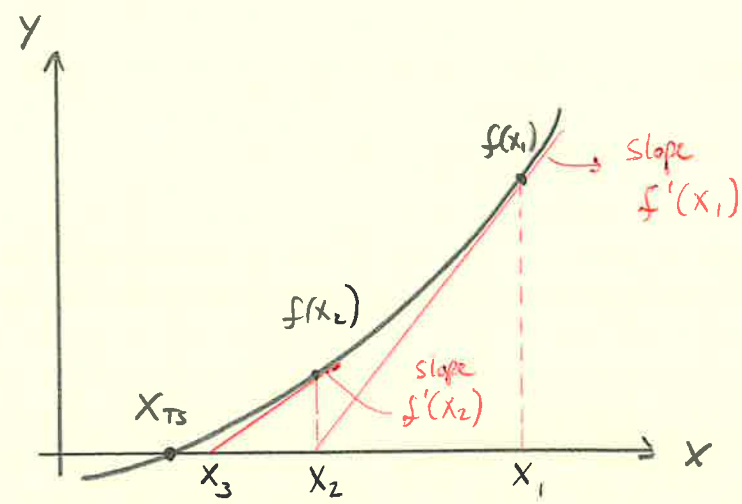
Newton's Method

- open method
- find numerical solution to $f(x)=0$ where
  - $f(x)$ continuous
  - $f(x)$ differentiable
  - initial guess is "close" (x_1)



Iteration Formula:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_2 = x_1 - \left(\frac{1}{f'(x_1)}\right) f(x_1)$$

or

$$f'(x_1) = \frac{f(x_1) - 0}{x_1 - x_2}$$

Notice, we need both $f(x)$ & $f'(x)$.

(Can also be derived using Taylor series expansion of $f(x)$ about a point x_r where $f(x_r)=0$ is assummed.)

Algorithm:

1) Choose x_1 as initial guess
2) for $i = 2, \ldots$ until stop condition, apply iteration formula.

- Newton's method generally works well.
- Problems occur when $f'(x)$ is near zero near solution ($f(x)=0$)
- The requirement for $f'(x)$ to be available may be cumbersome. Alternatives:
  - compute $f'(x)$ numerically
  - use secant method

    similar, but uses two points in vicinity of $f(x)=0$ to estimate new solution.