Research Article

Dynamic pricing of high-speed rail with transport competition

Received (in revised form): 25th August 2011

Kimitoshi Sato[a] and Katsushige Sawaki[b]

[a]*Waseda University, Tokyo, Japan; and* [b]*Nanzan University, Nagoya, Japan*

Kimitoshi Sato is an Assistant Professor at the Graduate School of Finance, Accounting and Low, Waseda University. He received his PhD degrees in Mathematical Sciences and Information Engineering in 2010 from Nanzan University. His research interests include revenue management, supply chain management and mathematical finance.

Katsushige Sawaki is a professor of operations research at the school of Business Administration, Nanzan University. He received his double PhD degrees in Business Administration in 1977 from The University of British Columbia and in Industrial Engineering in 1997 from Kyoto University. He also received the 2007th Award on OR applications from Japanese Society of Operations Research and a finalist award of the 2005 Frantz Edelman from INFORMS. His current research interests include inventory control, revenue management, Markov decision processes and financial engineering. He has written six books and published more than 80 research papers.

Correspondence: Kimitoshi Sato, Graduate School of Finance, Accounting and Law, Waseda University, 1–4–1, Nihombashi, Chuo-ku, Tokyo, 103-0027, Japan

ABSTRACT In recent years, high-speed rails (HSR) have received strong attention in the transport market because of the growing competition with air transport. In this article, we present a revenue management model of dynamic pricing for a competitive route. We suppose that the passengers are allowed to choose among other transport modes and that each transport mode offers the multiple substitutable schedules. In addition, the cancellation, no-show and overbooking are incorporated in our model. Using the multinomial logit model to describe the customer's discrete choice, we derive an optimal pricing policy so as to maximize the expected total revenue for substitutable schedules for HSR. Furthermore, we obtain some analytical properties for the optimal price. Finally, we present several numerical results to show the effect of competition on pricing strategies.

Journal of Revenue and Pricing Management (2012) **11,** 548–559. doi:10.1057/rpm.2011.29;
published online 14 October 2011

Keywords: dynamic pricing; discrete choice model; high-speed rail

INTRODUCTION

In recent years, High-Speed Rail (HSR), with a speed of over 300 km per hour, has been introduced globally because HSR emits less CO_2 than do cars and airplanes. In countries such as France, Germany, Japan and Korea, HSR has gained a leading market share in the medium to long distance transport markets. Reflecting the trend, the competition between HSR and Air invites many researchers to analyze pricing policies from a point of view of the revenue management (RM) (Park and Ha, 2006; Clever and Hansen, 2008; Adler *et al*, 2010, and

Román *et al*, 2007). Under the competitive environment, the implemented pricing systems differ from HSR company to company. It can be classified into three types of selling method. (1) A ticket price is predetermined between origin and destination stations and it stays unchanged throughout the selling period. HSR companies in Japan, China and Taiwan are using this type of selling method. (2) Ticket prices are predetermined, but different prices are set depending on the day of week and time zone. This type of method is used by HSR companies in Korea, Germany and America. (3) A price is adjusted based on the sales volume in real time. It is called dynamic pricing and HSR companies in France and the United Kingdom are adapting this method. For airlines, the dynamic pricing is adapted in several low-cost carriers. The question of how such competition affects the dynamic pricing policy on the company's revenue is an important issue to the research area for RM problems. There are very few papers in the area of railway passenger RM problem. Armstrong and Meissner (2010) provide an overview of literatures for both passenger and freight rail RM and list the fundamental differences between the airlines and passenger rail. Sibdari *et al* (2008) havedeveloped some pricing policies for a multi-product RM problem and analyzed the optimal policies by using Amtrak's sales data.

Many customers take into account ticket prices and their schedules when they make out an itinerary by using the internet. Thus, the demand for a seat depends not only on its price, but also on the price of a similar schedule for a different mode. In this article, we study an RM problem under the competition for different transportation modes with multiple substitutable schedules. The objective is to maximize the sum of the revenue for all schedules of a mode. We also take into account a passenger who cancels their reservation or no-shows, and are overbooked by the company. The demand for each schedule depends on the price and non-price characteristics of all transportations, such as the comfort, total trip time and frequency. We use the multinomial logit (MNL) model to describe the customers' choice behavior. The model can be formulated as a dynamic programming problem in discrete time to derive an optimal dynamic pricing policy.

The basic problem of dynamic pricing has been considered in the existing literature. An extensive review of this work is given in Talluri and van Ryzin (2004). Recently, many researchers focus on and take into consideration multiple product and competition into their model. Zhang and Cooper (2009) consider a dynamic pricing problem for multiple substitutable flights between the same origin and destination, and taking into account customer choice among the flights. Dong *et al* (2009) study a dynamic pricing and inventory control of substitute products for a retailer who faces a long supply lead-time and a short selling season. Li and Chen (2009) also study a multi-product dynamic pricing model with a time-dependent customer arrival by using MNL. They formulate this problem as an optimal stochastic intensity control problem in continuous time. Furthermore, there are a few dynamic pricing models in the literature that incorporate competition. Currie *et al* (2008) consider the two-competitor situation and incorporate a probabilistic function of customer demand, which is influenced by the prices offered by the company and the competitor, and the time remaining until the end of the selling period. They give simple conditions that ensure the uniqueness of a solution. Lin and Sibdari (2009) formulate a game-theoretic model to describe real-time dynamic price competition between firms that sell substitutable products. They show the existence of Nash equilibrium under the assumption that the real-time inventory levels of all firms are public information. And then, they evaluate the heuristic policy under different information structures numerically. Levin *et al* (2009) consider the problem that the oligopolistic firms sell differentiated perishable goods to multiple finite segments of strategic consumers who are aware of dynamic pricing according to the changes of the price. We incorporate the competition in the multi-product setting along Dong *et al* (2009). In addition, we extend Lin and Sibdari (2009) model into the model where including substitute schedules.

The rest of this article is organized as follows: The following section outlines all assumptions and formulates the problem as a dynamic programming model. In the next section, we analyze the model and determine an optimal pricing policy. In the subsequent section, we provide some numerical results demonstrating some properties of the optimal policy. Conclusions are finally drawn in the last section.

MODEL

We assume that there are n types of transportation modes (for example, railway, airplane, expressway bus and so on) between a single origin and destination pair, and each transit agency provides an alternative schedule in a day. We do not allow an HSR customer to stop over or board between the departure and destination station. We describe a schedule j for mode i as a service (i, j) and denote the capacity by c_{ij}. Let m_i be the number of alternative schedules for mode i, and t_j^i be the time period of the end of selling season for a service (i, j). For example, if a mode i represents HSR, then $j=1$ and $j=m_i$ correspond to the first train in the morning and the last train in the night, respectively. The selling season moves backward in time indexed by $t \in [T, 0]$ $(T>0)$, that is, $t=0$ and $t=T$ correspond to the end and the beginning of the selling season, $0=tm_i<\cdots<t_2^i<t_1^i<\cdots<T$.

Each period has at most one customer arrival with probability λ, and each arriving customer requires no more than one seat. Then, one of the following events occurs at each stage: (1) A customer arrives and decides whether or not to buy a ticket for a service (i, j), (2) A customer cancels his/her reservation holding at the moment, (3) No customer arrives.

Assumption 1: Each mode keeps track of the real-time number of seats available in all other modes.

This assumption is called the perfect information assumption and contains other dynamic pricing models with competition (Levin *et al*, 2009; Lin and Sibdari, 2009). Lin and Sibdari (2009) develop a heuristic policy that requires only the initial inventory levels of the other companies. Xu and Hopp (2006) develop an optimal policy when all companies can know the information of the aggregate inventory for all the modes in the real time. They establish the weak perfect Bayesian equilibrium for the pricing game.

Let $f_{ij}(t)$ be the price for a service (i, j) at time period t. We define an $n \times m_{\bar{n}}$ price matrix $\boldsymbol{f}_t = \{f_{ij}(t)\}$, where $\bar{n} = \max\{i\colon\ m_i, i=1, \ldots, n\}$ is the largest number of frequency for all modes. Denoting the ith row of $\boldsymbol{f}_t$ by $\boldsymbol{f}_t^i$, a pricing policy of mode i for the entire selling season is given by $\boldsymbol{f}^i = \{\boldsymbol{f}_t^i,\ 0 \leqslant t \leqslant T\}$. We set $f_{ij}(t) = \infty$ if $t<t_j^i$ or $m_i<j$ for all i and j. Let $x_{ij}(t)$ be the number of bookings for a service (i, j) in period t. Assume that $\boldsymbol{x}_t$ and $\boldsymbol{x}_t^i$ are the $n \times m_{\bar{n}}$ matrix and ith row of $\boldsymbol{x}_t$, respectively. After the current number of bookings $\boldsymbol{x}_t$ is monitored, the company i sets prices $\boldsymbol{f}^i = \{f_{ij}(\boldsymbol{x}_t, t),\ j=1, \ldots, m_i\}$ for pair $(i, j) \in \Gamma(\boldsymbol{x}_t^i)$, with $\Gamma(\boldsymbol{x}_t^i) \equiv \{1 \leqslant j \leqslant m_i \mid c_{ij} + \theta_{ij} - x_{it}(t) \geqslant 0\}$ where θ_{ij} is the number of overbooking for service (i, j). If a service (i, j) has no seats in period t, $j \notin \Gamma(\boldsymbol{x}_t^i)$, we set $f_{ij}(t) = \infty$.

Let $p_{ij}(\boldsymbol{f}_t)$ and $p_0(\boldsymbol{f}_t)$ be the probability that a passenger arrives in period t, chooses a service (i, j) and the no-purchase probability, respectively. Note that

$$p_0(\boldsymbol{f}_t) + \sum_{i=1}^{n} \sum_{j=1}^{m_i} p_{ij}(\boldsymbol{f}_t) = 1 \quad \text{for all } t \quad (1)$$

and $p_{ij}(\boldsymbol{f}_t) \geqslant 0$ for all i and j. The probability can be calculated by using the MNL model in the next section.

Next, we suppose that the following two assumptions hold, in order to establish the existence of an equilibrium for the multiple-period game.

Assumption 2: Each $f_{ij}(t)$ is defined on $A_i = [0, \bar{f}_{ij}(t)]$ where $\bar{f}_{ij}(t)$ is the maximum admissible value of $f_{ij}(t)$ and $p_{ij}(\boldsymbol{f}_t)|_{f_{ij}(t)=\bar{f}_{ij}(t)} = 0$.

Assumption 3: If there exist the multiple equilibrium, each mode selects an equilibrium.

The booked seats can be canceled before the departure. In order to take into account time-dependent cancellation refund, we consider the expected refund (Subramanian *et al*, 1999). The refund is paid at the time when a reservation is canceled rather than at the time of service. Let $\alpha_{ij}(t)$ and η_{ij} denote the cancellation rate in period t and the no-show rate in departure period t_j^i for service (i, j), respectively. Note that $\alpha_{ij}(t) = 0$ if $t < t_j^i$. As at most one request or cancellation can occur at each stage, we have

$$\lambda + \sum_{i=1}^{n} \sum_{j=1}^{m_i} \alpha_{ij}(t) + q_0(t) = 1 \qquad (2)$$

where $q_0(t)$ is the no-show probability. Let $\gamma_{ij}(l)$ be the refund rate for a passenger who cancels a service (i, j) in period l. The refund for a passenger who bought a ticket in period t and cancels in period l $(< t)$ is given by $f_{ij}(t)\gamma_{ij}(l)$. If a passenger who does not show up for a service (i, j), the refund d_{ij} is paid. Note that $f_{ij}(t)\gamma_{ij}(l) \geqslant d_{ij}$, for all $l < t$.

Given x reservations at the departure time, the number of passengers who show up for a service (i, j) is denoted by $S_{ij}(x)$.

Let $\pi_{ij}(x)$ denote the overbooking penalty when x passengers show up on the departure time. In general, $\pi_{ij}(x)$ is a nonnegative, nondecreasing and convex function of x for $x > c_{ij}$, with $\pi_{ij}(x) = 0$, $x \leqslant c_{ij}$. Then, the expected refund $G_{ij}(t)$ from the time period t to the departure time is given by the following recursive equation:

$$\begin{cases} G_{ij}(t) = \alpha_{ij}(t) f_{ij}(t)\, \gamma_{ij}(t) \\ \qquad\qquad + (1 - \alpha_{ij}(t)) G_{ij}(t-1), \quad \text{for } t_j^i < t, \\ G_{ij}(t_j^i) = \eta_{ij} d_{ij}. \end{cases} \qquad (3)$$

This equation can be rewritten as follows;

$$\begin{cases} G_{ij}(t) = f_{ij}(t) H_{ij}(t) + L_{ij}(t), \quad \text{for } t_j^i < \text{t}, \\ L_{ij}(t) = \eta_{ij} d_{ij} \prod_{u=t_j^i+1}^{t} \Big(1 - \alpha_{ij}(u)\Big), \\ H_{ij}(t) = \alpha_{ij}(t)\gamma_{ij}(t) + (1 - \alpha_{ij}(t)) H_{ij}(t-1), \\ H_{ij}(t_j^i) = 0. \end{cases} \qquad (4)$$

In the equation above, $L_{ij}(t)$ represents the expected no-show refund in period t, and $H_{ij}(t)$ represents the expected refund rate of cancellation in period t. Then, the reduced revenue, for service (i, j) after refund, is defined as $\hat{f}_{ij}(t) = f_{ij}(t) - G_{ij}(t)$.

Lemma 1: For any i and j, we have $H_{ij}(t) < 1$ for $t \geqslant t_j^i$.

Proof. The proof is by induction on t. First, this is trivially true for $t = t_j^i$ by the boundary condition $H_{ij}(t_j^i) = 0$ for all i and j. Assume it is true for period $t-1$, and consider period t. Then, we have

$$\begin{aligned} H_{ij}(t) &< \alpha_{ij}(t)\gamma_{ij}(t) + 1 - \alpha_{ij}(t) \\ &= 1 - (1 - \gamma_{ij}(t))\alpha_{ij}(t) \leqslant 1. \quad \square \end{aligned}$$

Remark 1: If cancel rate does not depend on time, that is, $\alpha_{ij}(t) = \alpha_{ij}$, then $H_{ij}(t)$ can be expressed as

$$H_{ij}(t) = \alpha_{ij} \sum_{u=t_j^i+1}^{t} (1 - \alpha_{ij})^{t-u} \gamma_{ij}(u). \qquad (5)$$

Moreover, if the cancel refund rate also does not depend on time, $\gamma_{ij}(t) = \gamma_{ij}$, then we have $H_{ij}(t) = \gamma_{ij} \alpha_{ij}^{t-t_j^i}$, $(t \geqslant t_j^i)$.

The objective for each transportation is dynamically to set each price of schedules in order to maximize the expected total revenue over the horizon from period T to 0. At the beginning of period t, the manager of each transport observes the pair of the states $\boldsymbol{f}_t$ and $\boldsymbol{x}_t$; let $v_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t)$ denote mode i's expected total revenue from period t to period 0 if all modes use equilibrium strategies from period t to period 0. Moreover, given state $\boldsymbol{x}_t$, let $\Phi_i(\boldsymbol{x}_t, t)$ denote the optimal expected revenue from period t to period 0. Then, we formulate the problem as the following dynamic programming formulation:

$$\Phi_i(\boldsymbol{x}_t, t) = \max_{f_t^i} \phi_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t) \qquad (6)$$

where

$$\begin{aligned}\phi_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t) &= \lambda v_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t) \\ &+ \sum_{k=1}^{n}\sum_{j=1}^{m_k} \alpha_{kj}\Phi_i(\boldsymbol{x}_t - \boldsymbol{e}_{kj}, t-1) \\ &+ q_0(t)\Phi_i(\boldsymbol{x}_t, t-1) \\ &+ E[-\pi(S_{ij}(x_{ij}))]1_{\{t=t_j^i\}} \qquad (7)\end{aligned}$$

and $\boldsymbol{e}_{ij}$ is a $n \times m_{\bar{n}}$ matrix with the 1 is in the (i,j)-th position and there are zeros everywhere else.

In equation (7), $v_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t)$ is the expected total revenue of the mode i when a customer arrives, and denoted by

$$\begin{aligned}v_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t) &= \sum_{j=1}^{m_i} p_{ij}(\boldsymbol{f}_t)\left(\hat{f}_{ij}(t) + \Phi_i(\boldsymbol{x}_t + \boldsymbol{e}_{ij}, t-1)\right) \\ &+ \sum_{k\neq i}^{n}\sum_{j=1}^{m_k} p_{kj}(\boldsymbol{f}_t)\Phi_i(\boldsymbol{x}_t + \boldsymbol{e}_{kj}, t-1) \\ &+ p_0(\boldsymbol{f}_t)\Phi_i(\boldsymbol{x}_t, t-1). \qquad (8)\end{aligned}$$

The boundary condition is

$$\Phi_i(\boldsymbol{x}_t, 0) = \begin{cases} E[-\pi(S_{im_i}(x_{im_i}))] & \text{for } t = t_{m_i} = 0, \\ 0, & \text{otherwise.}\end{cases} \qquad (9)$$

From equation (2), equation (7) can be rewritten as follows:

$$\phi_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t) = \lambda\tilde{\phi}_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t) + \psi_i(\boldsymbol{x}_t, t) \qquad (10)$$

where

$$\begin{aligned}\tilde{\phi}_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t) &= \sum_{j=1}^{m_i} p_{ij}(\boldsymbol{f}_t)\left(\hat{f}_{ij}(t) + \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1)\right) \\ &+ \sum_{k\neq i}^{n}\sum_{j=1}^{m_k} p_{kj}(\boldsymbol{f}_t)\Delta_{kj}\Phi_i(\boldsymbol{x}_t, t-1), \qquad (11)\end{aligned}$$

$$\begin{aligned}\psi_i(\boldsymbol{x}_t, t) &= \Phi_i(\boldsymbol{x}_t, t-1) \\ &- \sum_{i=1}^{n}\sum_{j=1}^{m_i} \alpha_{ij}(t)\Delta_{ij}\Phi_i(\boldsymbol{x}_t - \boldsymbol{e}_{ij}, t-1) \\ &- E\left[\pi\left(S_{ij}(x_{im_i})\right)\right]1_{\{t=t_j^i\}}, \qquad (12)\end{aligned}$$

and $\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t) = \Phi_i(\boldsymbol{x}_t + \boldsymbol{e}_{ij}, t) - \Phi_i(\boldsymbol{x}_t, t)$.

Here, $-\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t)$ interpreted the marginal expected revenue for service (i,j). The value function (6) can be modified as

$$\Phi_i(\boldsymbol{x}_t, t) = \lambda \max_{f_t^i}\{\tilde{\phi}_i(\boldsymbol{f}_t, \boldsymbol{x}_t, t)\} + \psi_i(\boldsymbol{x}_t, t). \qquad (13)$$

Theorem 1: (Levin *et al*, 2009) Under Assumptions 2 and 3, there exists a Nash equilibrium in mixed strategies.

Remark 2: The equation (13) can be reduced to the monopolistic and multi-product model (Dong *et al*, 2009, Zhang and Cooper, 2009) when we set $n = 1$ and $\alpha_{ij}(t) = 0$ for all i, j and t. It can be reduced to the oligopoly and single-product model (Lin and Sibdari, 2009) when we set $m_i = 1$ for all i and $\alpha_{ij}(t) = 0$ for all i, j and t.

OPTIMAL POLICY OF THE COMPETITION MODEL

In this section, we specify the function of choice probabilities $P_{ij}(\boldsymbol{f}_t)$ and $P_0(\boldsymbol{f}_t)$ by using the MNL model and derive an optimal pricing policy. The utility of customer taking schedule j

with mode i at price f_{ij} is given by

$$U_{ij} = w_{ij} - \beta f_{ij} + \varepsilon_i, \qquad (14)$$

and the utility of no purchase is

$$U_0 = u_0 + \varepsilon_0, \qquad (15)$$

where w_{ij} is a constant value that does not depend on price f_{ij}, and ε_i is independent and identically distributed Gumbel random variables with mean 0 and variance $(1/6)\mu^2\pi^2$. If the customer's preference can be identified to be a constant value per mode b_{ij} (for example accessibility of terminals, reliability, comfort), the total trip time T_i, the fare from point of departure to the gate of mode i, $\bar{f}$, and the frequency m_i on all modes, then w_{ij} is expressed as follows;

$$w_{ij} = b_{ij} - \beta_1 \bar{f}_i + \beta_2 m_i + \beta_3 T_i, \qquad (16)$$

where β_k, $k = 1, 2, 3$, is the weight in logit model setting the importance of parameters per operator category. These parameters are estimated by using maximum likelihood estimation (see Cramer, 2003) or statistical software (for example 'mlogit' package for R).

The passenger demand probabilities for a service (i, j) can be expressed as

$$p_{ij}(\boldsymbol{f}_t) = \frac{e^{\frac{w_{ij}-\beta f_{ij}}{\mu}}}{\sum_{i=1}^{n}\sum_{j=1}^{m_i} e^{\frac{w_{ij}-\beta f_{ij}}{\mu}} + e^{\frac{u_0}{\mu}}}, \qquad (17)$$

$$p_0(\boldsymbol{f}_t) = \frac{e^{\frac{u_0}{\mu}}}{\sum_{i=1}^{n}\sum_{j=1}^{m_i} e^{\frac{w_{ij}-\beta f_{ij}}{\mu}} + e^{\frac{u_0}{\mu}}}, \qquad (18)$$

where u_0 denotes the utility of no-purchase choice (see Anderson *et al*, 1992).

As $\Phi_i(\boldsymbol{x}_t, t)$ is not quasi-concave in $\boldsymbol{f}_t^i$ (a counter-example is shown by Hanson and Martin, 1996), the optimal price vector cannot be obtained by concave optimization. Thus, we use the same way as Dong *et al* (2009), which establish a one-two-one mapping between the price space $\mathbb{R}^{m_1+\cdots+m_n} \cap \{+\infty\}$ and the probability space $\Theta = \{p_0, p_{ij}, \quad i = 1, \ldots, n, j = 1, \ldots, m_i\} \in [0, 1)^{m_1+\cdots+m_n} \times (0, 1]$. It follows from equations (17) and (18) that $p_{ij}(\boldsymbol{f}_t)/p_0(\boldsymbol{f}_t) = \exp\{(w_{ij} - \beta f_{ij} - u_0)/\mu\}$. Hence, we obtain

$$f_{ij} = \frac{1}{\beta}(w_{ij} - u_0 - \mu \log p_{ij} + \mu \log p_0). \qquad (19)$$

Putting equation (19) into equation (11), we have

$$\begin{aligned}\tilde{\phi}_i(\boldsymbol{p}_t, \boldsymbol{x}_t, t) = \sum_{j=1}^{m_i} p_{ij}\bigg\{\frac{1}{\beta}\bar{H}_{ij}(t)(w_{ij} - u_0 \\ -\mu \log p_{ij} + \mu \log p_0) - L_{ij}(t)\bigg\} \\ + \sum_{k=1}^{n}\sum_{j=1}^{m_k} p_{kj}\Delta_{kj}\Phi_i(\boldsymbol{x}_t, t-1), \qquad (20)\end{aligned}$$

where $\boldsymbol{p}_t = \{p_{ij}(\boldsymbol{f}_t)\}$ and $\bar{H}_{ij}(t) = 1 - H_{ij}(t)$.

Lemma 2: The expected total revenue $\tilde{\phi}_i(\boldsymbol{p}_t, \boldsymbol{x}_t, t)$ is concave in $\boldsymbol{p}_t^i$ for $\boldsymbol{p}_t^i \in [0, 1)^{m_i} \times (0, 1]$.

Proof. The Hessian of the function $-\tilde{\phi}_i$ $(\boldsymbol{p}_t, \boldsymbol{x}_t, t)$ is given by

$$\mathcal{H}_{-\tilde{\phi}_i} = \left(\frac{\mu}{\beta}\right)^{m_i+1} A_i,$$

$$A_i = \begin{pmatrix} \frac{\bar{H}_{i1}(t)}{p_{i1}} & \cdots & 0 & -\frac{\bar{H}_{i1}(t)}{p_0} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \frac{\bar{H}_{im_i}(t)}{p_{im_i}} & -\frac{\bar{H}_{im_i}(t)}{p_0} \\ -\frac{\bar{H}_{i1}(t)}{p_0} & \cdots & -\frac{\bar{H}_{im_i}(t)}{p_0} & \frac{\sum_{j=1}^{m_i} p_{ij}\bar{H}_{ij}(t)}{p_0^2} \end{pmatrix}. \qquad (21)$$

For a vector $\mathbf{y} = (y_1, y_2, \ldots, y_{m_i+1}\ \ )$ and Lemma 1, we have

$$\boldsymbol{y}\mathcal{H}\boldsymbol{y}^T = \sum_{j=1}^{m_i} \frac{\bar{H}_{ij}(t)}{p_{ij}}\left(y_j - \frac{1}{p_0}p_{ij}y_{m_i+1}\right)^2 \geqslant 0. \qquad (22)$$

As $\mathcal{H}_{-\tilde{\phi}_i}$ is positive definite, $-\tilde{\phi}_i(\boldsymbol{p}_t, \boldsymbol{x}_t, t)$ is strictly convex. Hence, $\tilde{\phi}_i$ $(\boldsymbol{p}_t, \boldsymbol{x}_t, t)$ is concave in p_{ij}. □

Theorem 2: Given $\boldsymbol{x}_t$ in period t, an optimal price for a service (i, j) is given by

$$f_{ij}^*(\boldsymbol{x}_t, t) = \frac{\mu}{\beta} + \frac{1}{\bar{H}_{ij}(t)}(L_{ij}(t) - \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1) + \tau_i(t)), \quad (23)$$

where $\tau_i(t)$ is the unique solution of the following equation;

$$\frac{\beta\tau_i(t)}{\mu}\exp\left(\frac{u_0+\mu}{\mu}\right) - \sum_{j=1}^{m_i}\bar{H}_{ij}(t) \times \exp\left\{w_{ij} - \frac{\beta}{\bar{H}_{ij}(t)}(L_{ij}(t) - \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1) + \tau_i(t))\right\} = 0 \quad (24)$$

where $\bar{H}_{ij}(t) = 1 - H_{ij}(t)$.

Proof. Putting $p_0 = 1 - \sum_{i=1}^{n}\sum_{j=1}^{m_i} p_{ij}$ into equation (20), and taking derivative with respect to p_{ij}, we have

$$\frac{\partial\tilde{\phi}_i}{\partial p_{ij}} = \frac{1}{\beta}\bar{H}_{ij}(t)(w_{ij} - u_0 - \mu\log p_{ij} + \mu\log p_0 - \mu) - L_{ij}(t) + \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1) - \frac{\mu}{\beta p_0}\sum_{j=1}^{m_i} p_{ij}\bar{H}_{ij}(t). \quad (25)$$

Let p_{ij}^* be the value that satisfies the equation $(\partial\tilde{\phi}_i/\partial p_{ij}) = 0$. Then we obtain

$$\bar{H}_{ij}(t)(w_{ij} - u_0 - \mu\log p_{ij}^* + \mu\log p_0 * - \mu) - \beta L_{ij}(t) + \beta\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1) - \frac{\mu}{p_0 *}\sum_{j=1}^{m_i} p_{ij}\bar{H}_{ij}(t) = 0. \quad (26)$$

Here, we assume that $\tau_i(t) = (\mu/\beta p_0^*)\xi_i(t)$ where $\xi_i(t) = \sum_{j=1}^{m_i} p_{ij}\bar{H}_{ij}(t)$ which implies $p_0^* = (\mu/\beta\tau_i(t))\xi_i(t)$. Thus, equation (26) can be rewritten as follows:

$$p_{ij}^* = \frac{\mu}{\beta\tau_i(t)}\xi_i(t)\exp\left\{\frac{1}{\mu}(w_{ij} - u_0 - \mu) - \frac{\beta}{\mu\bar{H}_{ij}(t)} \times (L_{ij}(t) - \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1) + \tau_i(t))\right\}. \quad (27)$$

Multiplying $\bar{H}_{ij}(t)$ by both sides and then taking summation, we have

$$\sum_{j=1}^{m_i} p_{ij}^*\bar{H}_{ij}(t) = \frac{\mu}{\beta\tau_i(t)}\xi_{(}t)\exp\left(-\frac{u_0+\mu}{\mu}\right) \times \sum_{j=1}^{m_i}\bar{H}_{ij}(t)\exp\left\{w_{ij} - \frac{\beta}{\bar{H}_{ij}(t)}(L_{ij}(t) - \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1) + \tau_i(t))\right\}. \quad (28)$$

From $\xi_i(t) = \sum_{j=1}^{m_i} p_{ij}\bar{H}_{ij}(t)$, we obtain equation (24). Let $g(\tau_i)$ be the left hand side of equation (24). We have $g(0) < 0$, $g(\infty) = \infty$ and

$$\frac{\partial g(\tau_i)}{\partial\tau_i} = \frac{\beta}{\mu}\exp\left(\frac{u_0+\mu}{\mu}\right) + \beta\sum_{j=1}^{m_i}\exp\left\{w_{ij} - \frac{\beta}{\bar{H}_{ij}(t)}(L_{ij}(t) - \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1) + \tau_i(t))\right\} > 0. \quad (29)$$

Hence, there exists a unique $\tau_i \in (0, \infty)$ satisfying $g(\tau_i) = 0$. By putting equation (27) into equation (19), we obtain equation (23). □

By using equation (23) and $\tau_i(t) = (\mu/\beta p_0^*)\xi_i(t)$, the reduced revenue $\hat{f}_{ij}(t) = f_{ij}(t) - G_{ij}(t)$ can be modified as

$$\hat{f}_{ij}^*(\boldsymbol{x}_t, t) = \left(1 + \frac{p_0(\boldsymbol{f}_t^*)}{\sum_{l\neq j}^{m_i} p_{il}(\boldsymbol{f}_t^*)\bar{H}_{il}(t)}\right)\tau_i(t) - \Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1). \quad (30)$$

Moreover, it follows from equations (20) and (27) and $\tau_i(t) = (\mu/\beta p_0^*)\xi_i(t)$ that

$$\Phi_i(\boldsymbol{x}_t, t) = \tau_i(t) - \sum_{k\neq i}^{n}\sum_{j=1}^{m_k} p_{kj}(\boldsymbol{f}_t^*)(\tau_i(t) - \Delta_{kj}\Phi_i(\boldsymbol{x}_t, t-1)) + \psi_i(\boldsymbol{x}_t, t). \quad (31)$$

In equation (31), $-\Delta_{kj}\Phi_I(\boldsymbol{x}_t, t-1)$, $k \neq i$ can be interpreted as the future value of the mode i when a customer purchases other modes. If $\tau_i(t)$ refers to the contribution of selling one unit of seat for mode i to the overall revenue, the second term can be considered as the loss of the contribution by losing the sale to the competitor. Furthermore, from equation (30), the reduced revenue is proportional to the rate of the contribution $\tau_i(t)$.

Next, we show some properties for an optimal price.

Proposition 1:

(i) If $\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1)$ is decreasing in x_{ij} for any i, j, then $\tau_i(t)$ satisfying the equation (24) is decreasing in x_{ij}.

(ii) An optimal price $f_{ij}^*(\boldsymbol{x}_t, t)$ is increasing in μ.

Proof. (i) If $\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1)$ decreases in x_{ij}, then $g(\tau_i; x_{ij})$ defined as the left hand side of equation (24) is increasing in x_{ij} for fixed $\tau_i(t)$. As $g(\tau_i)$ increases in τ_i, $\tau_i(t; x_{ij})$ satisfying the equation (24) is decreasing in x_{ij}. (ii) It follows from equation (24) that $\tau_i(t;\ x_{ij})$ is increasing in μ. Hence, $f_{ij}^*(\boldsymbol{x}_t, t)$ is increasing in μ. □

From Proposition 1 (i) and equation (23), an optimal price f_{ij}^* is not always decreasing in x_{ij}. It implies that it is not always optimal for a company to cut a price for service (i, j) even if the marginal revenue $-\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1)$ increases when the number of unsold seat increases. As the variance of random variable ε is $(1/6)\mu^2\pi^2$, Proposition 1 (ii) implies that optimal price should be increased when the variance of idiosyncratic taste differences for customer is large enough.

Proposition 2: Suppose a service (i, j) has surplus seat, $t \leqslant c_{ij} - x_{ij}(t)$ for all t, and overbooking is not allowed for all service, that is, $\theta_{ij} = 0$ for all i, j.

(i) If $\alpha_{ij}(t) = \alpha_{ij}$ and $\gamma_{ij}(t) = \gamma_{ij}$, then the optimal reduced revenue $\hat{f}_{ij}^*\ (\boldsymbol{x}_t, t)$ is decreasing in α_{ij} and γ_{ij} for all i and j.

(ii) If $\alpha_{ij}(t) = 0$ for all i, j and t, then we have the fixed optimal price as follows:

$$f_{ij}^*(\boldsymbol{x}_T, T) = \frac{\mu}{\beta} + \eta_{ij} d_{ij} + \tau_i \quad (32)$$

where τ_i is a unique value satisfying the equation

$$\frac{\beta\tau_i}{\mu}\exp\left(\frac{u_0 + \mu}{\mu}\right) = \sum_{j=1}^{m_i} \exp\{w_{ij} - \beta(\eta_{ij} d_{ij} + \tau_i)\}. \quad (33)$$

Proof. When $t \leqslant x_{ij}$ and $\theta_{ij} = 0$ for all i and j, we have $\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1)$ by induction on t. (i) From Remark 1, $H_{ij}(t)$ is increasing in α_{ij} and γ_{ij} when $\alpha_{ij}(t) = \alpha_{ij}$ and $\gamma_{ij}(t) = \gamma_{ij}$, respectively. Thus, $\tau_i(t;\ \alpha_{ij}, \gamma_{ij})$ decreases for α_{ij} and γ_{ij}, and $\hat{f}_{ij}^*(\boldsymbol{x}_t, t)$ is decreasing in α_{ij} and γ_{ij} from equation (30). (ii) By putting $\alpha_{ij}(t) = 0$ and $\Delta_{ij}\Phi_i(\boldsymbol{x}_t, t-1)$ into equations (23) and (24), we obtain equations (32) and (33). □

NUMERICAL EXAMPLE

In this section, we demonstrate how optimal prices change during the selling period and show the effect of parameters on the prices numerically. We consider the competition between an HSR ($i = 1$) and an air ($i = 2$), and assume that each mode provides two schedules ($m_1 = m_2 = 2$). The capacities for each mode are $c_{11} = c_{12} = 6$ (HSR) and $c_{21} = c_{22} = 5$ (air). We suppose that the time period is $T = 54$ and arrival rate is $\lambda = 0.5$. Departure times of each mode's schedule are $t_1^1 = t_1^2 = 1$ and

$t_2^1 = t_2^2 = 0$. The demand utility function consists of constant value, price and trip time: $u_{ij} = b_{ij} - \beta f_{ij} + \beta_1 T_i + \varepsilon_i$. The weights for logit model are $\beta = 0.4$ and $\beta_1 = 0.35$. We assume that the demand of later schedules for HSR and air are larger than of early schedules, so we set constant values $b_{11} = 6.0$, $b_{12} = 9.0$, $b_{21} = 5.5$ and $b_{22} = 8.5$, and trip times $T_1 = 2.6$, $T_2 = 1.5$. We do not consider the no-purchase alternative $u_0 = 0$, that is, an arriving customer purchase either an HSR or an air ticket.

Moreover, we assume that the cancel rates does not depend on time, that is, $\alpha_{11} = \alpha_{12} = 0.01$ and $\alpha_{21} = \alpha_{22} = 0.008$. The refund rates for cancellation are set to be $\gamma_{ij}(l) = 0.5$ if $l < 5$, $\gamma_{ij}(l) = 0.9$ if $l \geqslant 5$ for $i = 1, 2$, $j = 1, 2$. No-show rate and the refund are $\eta_{ij} = 0.04$ and $d_{ij} = 0$ for $i = 1, 2$, $j = 1, 2$. The number of overbookings is $\theta_{ij} = 1$ for $i = 1, 2$, $j = 1, 2$. We suppose that the overbooking penalty function is defined as $\pi_{ij}(x) = \sigma_i (S_{ij}(x) - c_{ij})^+$, where σ_i is overbooking cost and $\sigma_1 = \sigma_2 = 5$. We also suppose that $S_{ij}(x_{ij})$ is a binomial-$(x_{ij}, 1 - \eta_{ij})$ random variable.

Figures 1 and 2 show a sample booking path for each mode and their corresponding optimal prices during the selling period. We can see that the higher demand schedules (HSR2 and Air2) are more expensive than the lower demand schedules (HSR1 and Air1).

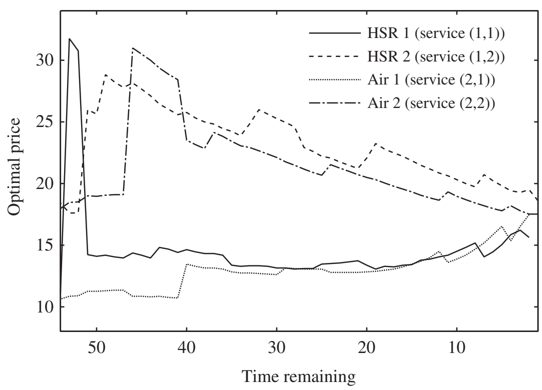

Figure 1: Optimal prices correspond to the booking path.

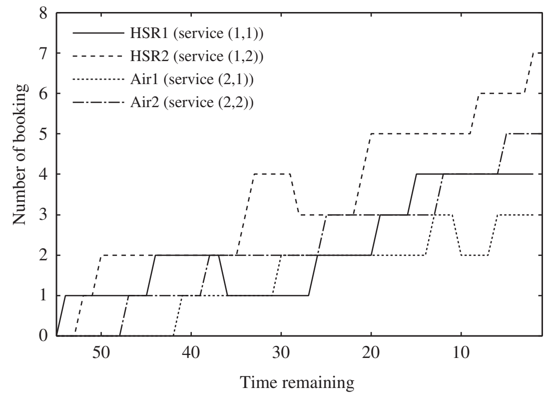

Figure 2: Sample booking paths.

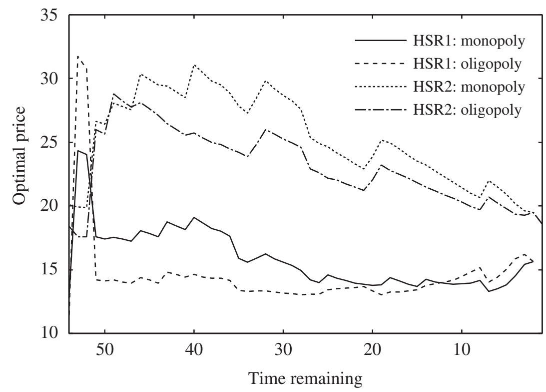

Figure 3: Comparison between the optimal prices for HSR in monopoly and oligopoly.

We can also see that the price goes up when a seat is booked, and the price goes down as the booked reservation is canceled. Moreover, the competitor's price changes slightly affects the own optimal prices.

Figure 3 shows the optimal prices when HSR is monopoly in a market, and the prices correspond to the booking path shown in Figure 2. In the monopoly case, the optimal prices for each schedule of HSR exceed the ones for oligopoly case. It implies that the existence of competitor reduces the prices of HSR.

Next, we investigate some properties of the optimal price. Figures 4–7 show how the optimal prices are affected by the number of

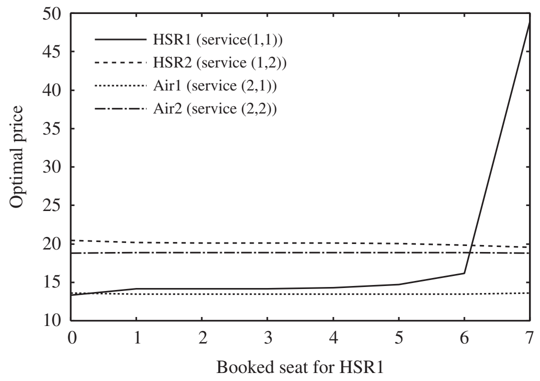

Figure 4: Optimal prices for the fixed number of booked seat for HSR1.

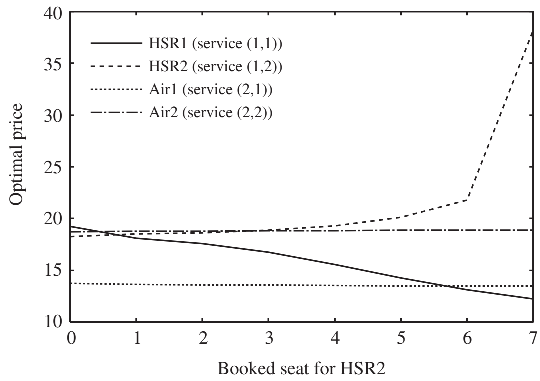

Figure 6: Optimal prices for the fixed number of booked seat for HSR2.

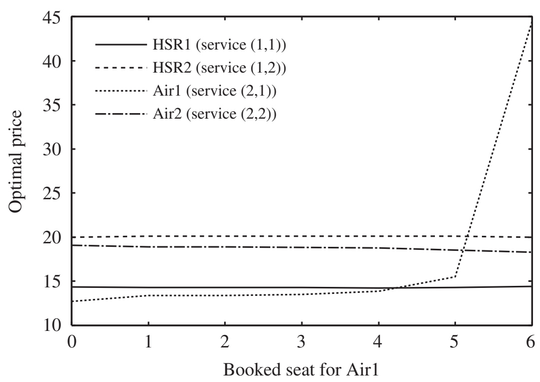

Figure 5: Optimal prices for the fixed number of booked seat for Air1.

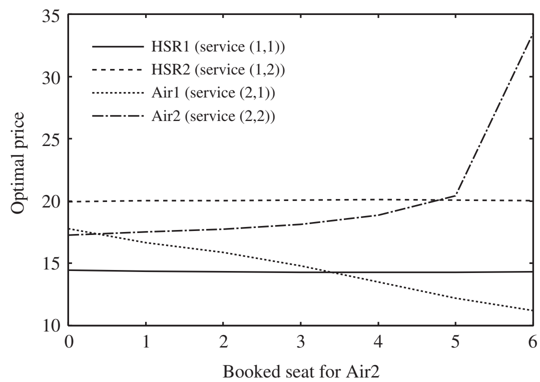

Figure 7: Optimal prices for the fixed number of booked seat for Air2.

booked seats for each services when the time period is fixed $t = 10$. When the number of booking is high for the lower demand schedule (HSR1 or Air1), the optimal price corresponds to the schedule increase. The price of the other schedule (HSR2 or Air2) does not change (see Figures 4 and 5). On the other hand, when the number of booking is high for the higher demand schedule (HSR2 or Air2), the optimal price increases, as well as the lower demand case. However, the price of the other schedule (HSR1 or Air1) decreases (see Figures 6 and 7). Figure 8 represents the optimal prices as the function of remaining time, when the number of booking is fixed $x_{ij} = 3$ for $i = 1, 2$ and $j = 1, 2$. The prices of higher demand schedules for each mode (HSR2 and Air2) are decreasing in time. As the selling period drew to close, the price of the schedules converges to the same price.

Finally, Figure 9 depicts the optimal prices for the cancellation rate $\alpha_{ij} = \alpha$ for $i = 1, 2$, $j = 1, 2$. The prices of high demand schedules (HSR2 and Air2) increase as the cancel rate becomes larger, and the difference between the prices for each schedule becomes larger.

CONCLUSION

This article studied a dynamic pricing model under the competition for different transportation

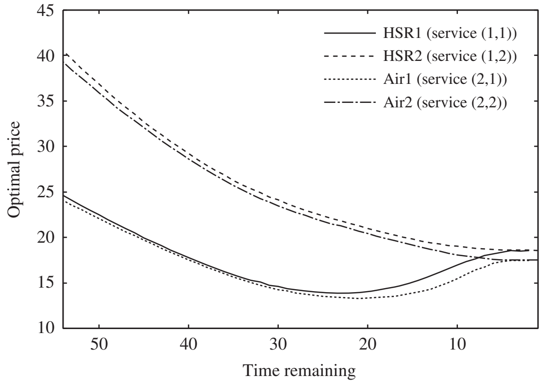

Figure 8: Optimal prices with respect to the time period for the fixed number of bookings.

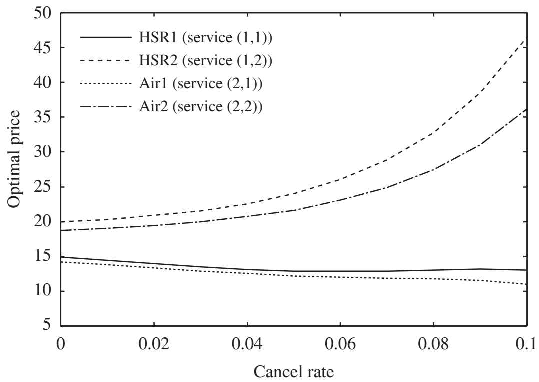

Figure 9: Optimal prices with respect to the cancellation rate, $\alpha_{ij} = \alpha$ for $i = 1, 2$ and $j = 1, 2$.

modes with multiple substitutable schedules. We allow a passenger to cancel and no-show, and also allow all of transport companies to overbook the reservation. Using the MNL model to describe a customer's discrete choice, we derive an optimal pricing policy and investigate some properties for the optimal policy. In numerical example, we provide the optimal prices under the HSR and air competition, based on the sample demand path. Furthermore, we illustrate in figures to demonstrate effects of parameters on the optimal price.

There remain many important research questions to explore within the framework of our model. First, we would like to relax the Assumption 2 in the second section. In practice, it is difficult to know the competitors' real-time information of remaining capacity. Thus, we need to consider the competitors' remaining capacity as a random variable and estimate the parameters of the distribution based on the information of the competitor's real-time price, the previous records of the price and the prices of the other schedules. Second, it may be worthwhile to compare the optimal expected revenue for the dynamic pricing policy with the one for fixed pricing policy. Currently, the fixed pricing policy is used by several HSR companies. Thus, we need to clarify competitive environment that is suitable to use the dynamic pricing. Finally, we wish to extend this model to the case incorporate situation where a customer can stop over or board between the departure and destination station.

ACKNOWLEDGEMENTS

This article was supported in part by the Grant-in-Aid for JSPS Fellows (No. 217470) of the Japan Society for the Promotion of Science in 2010.

REFERENCES

Adler, N., Pels, E. and Nash, C. (2010) High-speed rail & air transport competition: Game engineering as tool for cost-benefit analysis. *Transportation Research Part B: Methodological* 44(7): 812–833.

Anderson, S.P., Palma, A. and Thisse, J. (1992) *Discrete Choice Theory of Product Differentiation*. Cambridge, MA: MIT Press.

Armstrong, A. and Meissner, J. (2010) Railway revenue management: Overview and models. Lancaster University Management School. Working paper (available at http://www.meiss.com).

Clever, R. and Hansen, M.M. (2008) Interaction of air and high-speed rail in Japan. *Transportation Research Record: Journal of the Transportation Research Board* 2043: 1–12.

Cramer, J.S. (2003) *Logit Models from Economics and Other Fields*. Cambridge, UK: Cambridge University Press.

Currie, C.S.M., Chang, R.C.H. and Smith, H.K. (2008) Dynamic pricing of airline tickets with competition. *Journal of the Operational Research Society* 59(8): 1026–1037.

Dong, L., Kouvelis, P. and Tian, Z. (2009) Dynamic pricing and inventory control of substitute products. *Manufacturing & Service Operations Management* 11(2): 317–339.

Hanson, W. and Martin, K. (1996) Optimizing multinomial logit profit functions. *Management Science* 42(7): 992–1003.

Levin, Y., McGill, J. and Nediak, M. (2009) Dynamic pricing in the presence of strategic consumers and oligopolistic competition. *Management Science* 55(1): 32–46.

Li, J.-S. and Chen, S. (2009) *Real-time Dynamic Pricing for Multiproduct Models with Time-dependent Customer Arrival*

Rates. ACC'09 Proceedings of the 2009 Conference on American Control Conference. Piscataway, NJ, USA: IEEE Press, pp. 2196–2201.

Lin, K. and Sibdari, S. (2009) Dynamic price competition with discrete customer choices. *European Journal of Operational Research* 197(3): 969–980.

Park, Y. and Ha, H.-K. (2006) Analysis of the impact of high-speed railroad service on air transport demand. *Transportation Research Part E* 42(2): 95–104.

Román, C., Espino, R. and Martín, J.C. (2007) Competition of high-speed train with air transport: The case of Madrid-Barcelona. *Journal of Air Transport Management* 13(5): 277–284.

Sibdari, S., Lin, K.Y. and Chellappan, S. (2008) Multiproduct revenue management: An empirical study of auto train at Amtrak. *Journal of Revenue and Pricing Management* 7(2): 172–184.

Subramanian, J., Stidham, S. and Lautenbacher, C.J. (1999) Airline yield management with overbooking, cancellations, and no-shows. *Transportation Science* 33(2): 147–167.

Xu, X. and Hopp, W.J. (2006) A monopolistic and oligopolistic stochastic flow revenue management model. *Operations Research* 54(6): 1098–1109.

Zhang, D. and Cooper, W.L. (2009) Pricing substitutable flights in airline revenue management. *European Journal of Operational Research* 197(3): 848–861.