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A seat assignment model for high-speed railway ticket booking system with customer preference consideration1

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Abstract

This paper addresses the railway revenue management problem of homogeneous seats. Within a certain period of time, multiple trains characterized by different train stop plans are alternatives to each other. For homogeneous seats, multiple fare classes are designed based on the customer preference orders of different customer types. A seat inventory control method is developed based on characteristics of the China high-speed railway (HSR). In addition, an efficient heuristic approach is proposed to solve the nonlinear integer programming model of the HSR revenue management problem. The Beijing-Shanghai HSR is taken as a case study. Numerical examples show that revenue can be increased by reasonably controlling the deterministic booking limits of different fare classes according to customer preference orders. Furthermore, our method can increase the total revenue by optimizing the resources of multiple trains with different train stop plans.

Keywords: High-speed railway; Homogeneous seats; Multiple trains; Train stop plan; Customer preference order

1 Introduction

The high-speed railway (HSR) has many advantages, such as a large transportation capacity, less land occupation, low pollution, all-weather service and so on, which promotes HSR development in many countries. However, the construction of an HSR requires a large investment. For instance, China has experienced a rapid development of HSR due to rapid urbanization and economic growth in the past few years (Li and Sheng, 2016), but previous massive investments led to significant debt for the China Railway Corporation (CRC). Therefore, the HSR revenue management problem is important for the sustainable development and operation of HSRs.

Our research concerns the revenue management problem of the current largest HSR network in the world, the China HSR. The ticket sales method used by the China Railway Customer Service Center (CRCSC) is a deterministic booking-limit control strategy, namely, ticket allocation, which determines the number of seats allocated to each origin-destination (O-D) itinerary for each train during the entire

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sales horizon (China Railway Corporation, 2014). In our view, there are two major flaws that prevent the China HSR from increasing revenue.

The first point is that no matter how the passenger demand changes, the CRCSC only provides a single ticket price for homogeneous seats during the entire sales horizon (Wang et al., 2016). This type of pricing form limits the revenue potential from ticket sales in terms of the loss of higher yields and also the loss of stimulated passenger demand. According to the passenger demand changes, designing different fare classes for homogenous seats can not only reasonably match passenger demand and seat capacity but also improve seat utilization and ticket revenue. Hence, in our method, there will be different fare classes for homogenous seats, and customer choice behaviour is described by a choice model based on the concept of preference order. For each arriving customer, the customer preference order describes a list of options to be followed in case the customer's preferred order is not available. That is, if the customer's first choice is not available, she either tries her second choice or decides not to purchase anything. If her second choice is not available either, again she moves to her next choice or decides not to purchase anything, and so on.

The second point is that each train's control strategy is determined by the predicted passenger demand for a single train (Bao et al., 2015), which ignores the substitutional relationship of passenger transport products of different trains. Within a certain period of time, there could be multiple trains providing transportation services for the same O-D itinerary. For an O-D itinerary, when the departure times, arrival times and travel times of different trains are all approximate, the transport products of the different trains can be alternative to each other. Fig. 1 shows the train stop plans of the trains that originate at Beijing and stop at Jinan from 7:00 to 7:50 in the morning. We hold the opinion that transport products of G105 are usually able to meet the customers' needs who originally want to take G57 from Beijing to Jinan. If different train transport products can replace each other, considering the seat allocation of multiple trains as a whole could be more beneficial with regard to the reasonable allocation of resources.

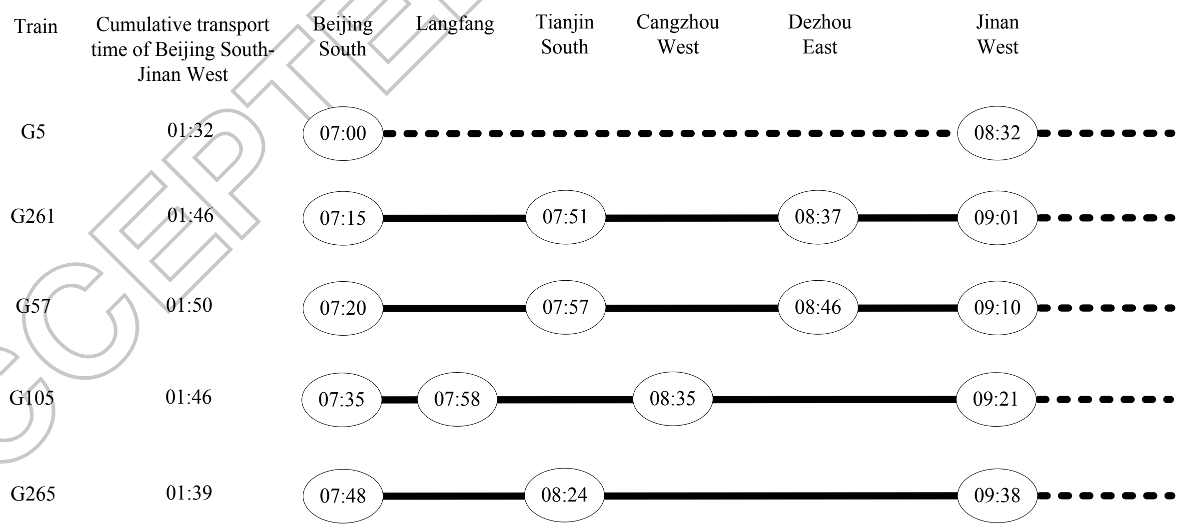

Fig. 1. Train stop plans from Beijing South to Jinan West and the departure time (h:min)

To better illustrate the second point, a simple example is shown in Fig. 2. It is assumed that train 1 and train 2 can be substituted for each other in the same O-D itinerary, and the service capacity of each train is 560. According to the predictive passenger demand, Train 1 has 280 customers for the $A-D$ itinerary, 280 customers for the $A-B$ itinerary and 280 customers for the $C-D$ itinerary, and Train 2 has 280 customers for the $A-C$ itinerary and 560 customers for the $B-D$ itinerary. Previous strategies control each train based only on each train's demand, for which the demand of the 280 customers for the $B-D$ itinerary cannot be satisfied because there are no extra seats. In contrast, our control strategy considers the two trains as a whole and adjusts the booking limits based on the different train stop plans; thus, this scheme satisfies all the demands for each O-D itinerary. Hence, multiple trains with different train stop plans need to be comprehensively considered. In an actual HSR network, the train stop plans are more complex.

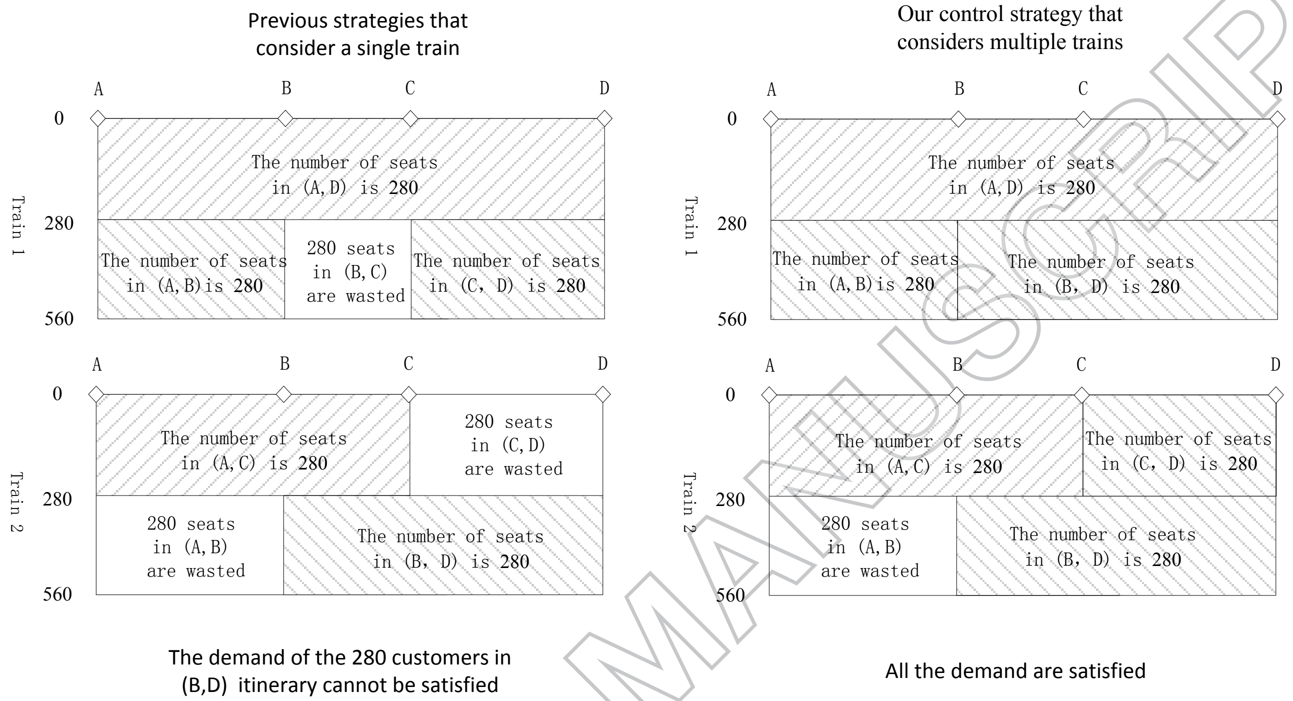

Fig. 2. Seat control scheme comparison

The objective of this study is to develop an HSR revenue management method suitable for the China HSR characteristics. Our strategy is a deterministic booking-limit seat inventory control method, which is consistent with the current control method adopted by the CRCSC, so it is more easily embedded in the existing system.

In practice, the seats are classified into three levels: business, first-class and second-class. According to the comfort level, they are heterogeneous seats. It should be pointed out that the proportion of second-class is more than 80% in each of the China high-speed trains, such as train mode CRH2C and CRH380CL shown in Fig. 3. Hence, the second-class seats are our focus that can be regarded as homogeneous seats. We will design different fare classes for homogeneous seats based on different customer types, which can increase ticket revenue by better meeting various needs of different types of customers. The customer choice behaviour of each customer type is described by a choice model based on the notion of customer preference order. In addition, multiple trains with different train stop plans are regarded as a whole for optimization. Furthermore, the actual passenger demand is characterized by randomness, and a control strategy without considering the randomness of passenger demand is difficult to obtain better control results. Therefore, our control strategy is provided based on stochastic passenger demand and the passenger demand function can follow any random distribution. And we use random passenger demand density function to generate simulated demands to assess the effectiveness of our control strategy. Finally, to demonstrate the superiority of our strategy, the current control method of the CRCSC is regarded as a comparison scheme in a real-life case.

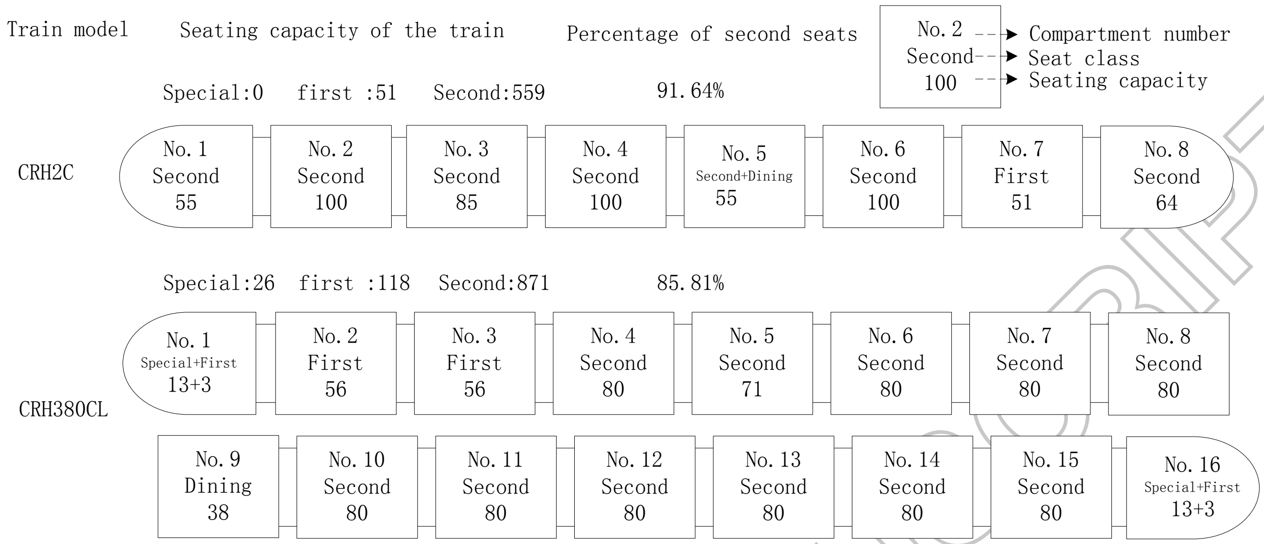

Fig. 3. Seating capacity of trains

The literature is reviewed in the next section. We present a revenue management model for homogeneous seats based on the characteristics of the China HSR in Section 3. The hybrid heuristic algorithm used to solve the model and the numerical experiments used to test the algorithm are described in Section 4. The Beijing-Shanghai HSR is used as a case study and the developed control method is compared with the current control method adopted by the CRCSC in Section 5. We present our conclusions and further research in Section 6.

2 Literature review

Revenue management has been studied in many areas, such as airlines (Talluri and van Ryzin, 1998; Lu et al., 2017), media (Popescu and Crama, 2016), hotel (Zhang and Weatherford, 2016) and shipping (Wang et al., 2015), in recent decades. Among all these studies, airline revenue management draws a considerable amount of attention, which contributes to railway learning, such as bid-price control (McGill and van Ryzin, 1999; Adelman, 2007; Tong and Topaloglu, 2014; Vossen and Zhang, 2015a) and virtual nested booking-limit control (Bertsimas and De Boer, 2005; Van Ryzin and Vulcano, 2008a; Van Ryzin and Vulcano, 2008b). However, the objective of our study is to develop a revenue management method that is suitable for the China HSR. Hence, according to the characteristics of the China HSR, we will focus on the deterministic booking-limit control that is consistent with the control method used by the CRCSC and has been adopted by Ciancimino et al. (1999), You (2008) and Wang et al. (2016).

In the past decade, with the rapid development of railway in Asia, the railway passenger transport revenue management problem has received increased attentions, especially in China. Armstrong and Meissner (2010) provided a summary of the available models, including possible extensions.

The ticket allocation problem of the China railway has been widely studied, such as Shan et al. (2011), Qian et al. (2013), Bao et al. (2014a), Bao et al. (2014b) and Bao et al. (2015). Among them, Bao et al. (2014b) noted that the deterministic booking-limit control method, which incorporates the dynamic aspect, is better than the bid-price control and virtual nested booking-limit control methods for the China railway. In addition, Bharill and Rangaraj (2008) studied the overbooking problem for Indian railways. However, the above studies all focused on the railway revenue management of a single train. In practice, there are multiple trains with different train stop plans within a certain period of time, and they should be regarded as a whole for optimization.

According to deterministic passenger demand, some studies (Shi et al., 2008; Lan and Zhang, 2009; Wang et al., 2012; Jiang et al., 2015) attempted to optimize the seat resources of multiple trains. In practice, passenger demands are complicated and changeable, and it is difficult to accurately predict passenger demands. Hence, the study based on stochastic passenger demand is more consistent with the actual situation.

Customer choice behaviour has been the focus of some studies of railway revenue management. Hetrakul and Cirillo (2014) proposed a latent class model to explain the ticket purchase timing of passenger railway and jointly optimized pricing and seat allocation. However, in their study, the traffic demand of each day is considered as a deterministic value. Wang et al. (2016) developed a deterministic booking-limit control method with customer choice. The customer choice behaviour is described by a discrete choice model based on a variety of attributes, such as the rail line, cabin class, and travel time. The discrete choice model has been widely adopted by bid-price controls of airline revenue management (Talluri and van Ryzin, 2004; Liu and van Ryzin, 2008; Zhang and Adelman, 2009). In airline revenue management, the booking process is described by dynamic programming and the offer set of available products is also changing because the remaining resources are reducing. That is, the choice decisions of customers would be affected by the offer set of available products that are affected by the remaining resources. Although the dynamic programming successfully describes the dynamic customer choice behaviours, the problem becomes intractable because of the high-dimensional state space, which is known as Bellman’s curse of dimensionality. Therefore, subsequent studies began to focus on the solution efficiency (Tong and Topaloglu, 2014; Vossen and Zhang, 2015a; Vossen and Zhang, 2015b). However, Wang et al. (2016) simplistically considered that the offer set of available products would not affect the choice decisions of customers, which obviously deviates from the actual situation. Significant differences could occur between the actual demand and the control scheme, which may result in wasted seat resources and lost revenue. However, in our study, the choice behaviour is described by the customer preference order in which customers make their choices according to the offer set of available products.

The control strategy developed in our work is similar to that adopted by Ciancimino et al. (1999) and You (2008). Ciancimino et al. (1999) investigated a multi-leg single-fare railway seat allocation problem and provided a probabilistic nonlinear programming model under random demand. Based on Ciancimino et al. (1999), You (2008) employed a pricing discount to attract customers and developed a hybrid heuristic approach to calculate the booking limits of the railway seat inventory control problem. Compared with Ciancimino et al. (1999) and You (2008), our research presents two main differences: (1) Ciancimino et al. (1999) investigated only a single-fare problem, and You (2008) regarded the demand of each fare class as an independent irrelevant variable without considering customer choice behaviour. In our research, homogeneous seats are differentiated by multiple fare classes based on customer preference orders. (2) Their models both solve the problem for a single train. However, within a certain period of time, the passenger transport products from different trains can serve as alternatives for each other. Therefore, our research, which comprehensively considers multiple trains with different train stop plans, is more realistic and challenging.

The main contribution of our work is to develop an effective revenue management method for the characteristics of China HSR. First, we adopt a deterministic booking-limit control strategy that is consistent with the ticket allocation strategy used by the CRCSC, so our method can be more easily embedded in the existing ticket system. Additionally, since the China HSR mainly offers homogeneous seats, a differential pricing method for homogeneous seats is developed, and the customer choice behaviour of each customer type is described by a customer preference order. In addition, based on stochastic passenger demand, multiple trains characterized by different train stop plans are regarded as a whole for optimization. Furthermore, because the proposed revenue management problem is formulated as a nonlinear integer programming model, we design a hybrid heuristic algorithm to solve it and the effectiveness in solving a real-case size instance is numerically evaluated. Finally, to demonstrate the practicability and superiority of our strategy for solving the China HSR revenue management problem, the current control method adopted by the CRCSC is used as a contrasting solution. Table 1 summarizes the studies on railway revenue management and compares them with our proposed study.

Table 1

Summary of railway revenue management studies

Authors	MT	MTSP	RD	CCB	HMS
Ciancimino et al. (1999)	×	×	√	×	×
You (2008)	×	×	√	×	√
Bharill and Rangaraj (2008)	×	×	×	×	√
Shi et al. (2008)	√	√	×	×	×
Lan and Zhang (2009)	√	√	×	×	√
Shan et al. (2011)	×	×	×	×	×
Wang et al. (2012)	√	√	×	×	√
Qian et al. (2013)	×	×	√	√	×
Bao et al. (2014a)	×	×	√	×	×
Bao et al. (2014b)	×	×	×	×	×
Hetrakul and Cirillo (2014)	√	×	×	√	√
Jiang et al. (2015)	√	√	×	×	×
Bao et al. (2015)	×	×	×	×	×
Wang et al. (2016)	√	×	√	√	×
This research	√	√	√	√	√

MT: Considers multiple trains;

MTSP: Includes multiple train stop plans;

RD: Develops a control strategy under the condition of random demand;

CCB: Considers customer choice behaviour when developing a control strategy; and

HMS: Includes differential pricing of homogeneous seats.

3 Problem formulation

3.1 Symbol notations

O-D itinerary are divided into multiple classes. An individual customer has a personal preference in purchasing a ticket for an O-D itinerary. The following notations are used throughout this paper.

Notation

g	train, $g = 1, 2, ..., G$, where G is the number of trains.
i	rail leg, $i = 1, 2, ..., m$, where m is the number of rail legs.
j	O-D itinerary, $j = 1, 2, ..., n$, where n is the number of O-D itineraries.
u_{gj}	judgement entry, which is decided by the train stop plan. If train g can provide a product in the O-D itinerary j , $u_{gj} = 1$; otherwise, $u_{gj} = 0$.
a_{ij}	judgement entry, which is decided by the HSR network structure. If the O-D itinerary j contains rail leg i , $a_{ij} = 1$; otherwise, $a_{ij} = 0$.
C_{gi}	service capacity, for rail leg i of train g .

3.2 Multiple trains with different train stop plans

When a single train becomes the control object (namely, $G=1$), as studied by You (2008), the expected sales for O-D itinerary j are given by

$$S_j(b_j) = \int_0^{b_j} x f_j(x)\mathrm{d}x + b_j \int_{b_j}^{\infty} f_j(x)\mathrm{d}x \tag{1}$$

where b_j denotes the booking limit for O-D itinerary j , $f_j(x)$ is the density function of the predicted passenger demand for O-D itinerary j .

The weakness of regarding a single train as the control object has been illustrated in the introduction. Multiple trains characterized by different train stop plans will be regarded as a whole for optimization in this section. In practice, the departure time taste may significantly change the customer's choice decision, which has been studied by Hetrakul and Cirillo (2014). They used many departure times, including: early morning, a.m., p.m., evening and so on to reflect the departure time taste of customers. Their research illustrates that customers' taste are heterogeneous for different departure times of a day but homogenous within a certain period of time, such as early morning. Notably, multiple trains departing within a certain period of time are our focus. Additionally, for the same O-D itinerary, the travel times between different trains of the China HSR are often approximate. Certainly, some trains have travel times that are very different from others, and these trains are not included within the scope of our research. In our work, multiple trains are characterized by substitutability for each other.

Therefore, when multiple trains are controlled as a whole, the expected sales for G trains in O-D itinerary j are given by

$$S_j(b_{gj}) = \int_0^{\sum_{g=1}^{G} b_{gj}} x f_j(x)\mathrm{d}x + \sum_{g=1}^{G} b_{gj} \int_{\sum_{g=1}^{G} b_{gj}}^{\infty} f_j(x)\mathrm{d}x \tag{2}$$

where b_{gj} denotes the booking limit of train g for O-D itinerary j , and $\sum_{g=1}^{G} b_{gj}$ denotes the sum of the booking limits of G trains for O-D itinerary j .

3.3 Customer preference order

Customer choice behaviours have been widely described by discrete choice models via offer sets with associated probabilities (Wang et al., 2016; Zhang and Adelman, 2009; Liu and van Ryzin, 2008; Talluri and van Ryzin, 2004). In such a model, customers are divided into segments, each segment corresponds to a vector of 'preference weights' for each itinerary-fare class, and the probabilities are calculated based on the utility of various product attributes. For example, in a segment of customers, there are three products A, B and C. The preference values are $v_A = 4$, $v_B = 3$ and $v_C = 2$, and the no-purchase value is $v_O = 1$. Then, $P(A \mid A) = 4/(4+1) = 4/5$, $P(A \mid AB) = 4/(4+3+1) = 1/2$, $P(A \mid AC) = 4/(4+2+1) = 4/7$, $P(A \mid ABC) = 4/(4+3+2+1) = 2/5$, etc.

Our focus is to develop a differential pricing method for homogeneous seats and we will adopt a customer preference order to describe the leave-or-stay process.

There have been some studies related to customer preference order (Van Ryzin and Vulcano, 2014; Park and Seo, 2011; Chen and Homem, 2010; Van Ryzin and Vulcano, 2008b). Customers consume homogeneous products in their preference order until their desired quantity is met. That is, a customer purchases her most preferred fare class first. If this fare class is not available or runs out, the customer transfers to purchase the second most preferred fare class with a transition probability or decides not to purchase anything. For example, in a segment of customers, consider a preference order with three fare classes $A \to B \to C$ and transition probabilities P_{AB}, P_{BC}. Particularly, P_A denotes the probability of customers for purchasing fare class A and $P_O = 1 - P_A$ denotes the no-purchase probability. Then $P(A \mid A) = P_A$, $P(A \mid AB) = P_A$, $P(A \mid AC) = P_A$, $P(A \mid ABC) = P_A$, $P(B \mid B) = P_A P_{AB}$, $P(B \mid BC) = P_A P_{AB}$, $P(C \mid C) = P_A P_{AB} P_{BC}$. When the number of total customers is λ, the number of customers who request fare class A is

$$\lambda_A = \lambda P_A$$

Denote the booking-limits for fare classes A, B, C by b_A, b_B, b_C, respectively. Then, the number of customers, who obtain their first-choice fare class A is

$$S_A = \min(\lambda_A, b_A) = \int_0^{\frac{b_A}{p_A}} p_A x f(x)\mathrm{d}x + b_A \int_{\frac{b_A}{p_A}}^{\infty} f(x)\mathrm{d}x$$

where $f(x)$ denotes the density function of passenger demands λ. Therefore, there will be $\lambda_A - \min(\lambda_A, b_A)$ customers who face the choice between staying or leaving. Hence, the number of customers who actually request fare class B is

$$\lambda_B = [\lambda_A - \min(\lambda_A, b_A)]P_{AB} = \max\{0, \lambda_A - b_A\}P_{AB}$$

Then, the number of customers who get fare class B is

$$S_B = \min(\lambda_B, b_B) = \int_{\frac{b_A}{p_A}}^{\frac{b_A}{p_A}+\frac{b_B}{p_A p_{AB}}} p_A p_{AB}\left(x - \frac{b_A}{p_A}\right) f(x)\mathrm{d}x + b_B \int_{\frac{b_A}{p_A}+\frac{b_B}{p_A p_{AB}}}^{\infty} f(x)\mathrm{d}x$$

Similarly, the number of customers who choose to stay and request fare class C is

$$\lambda_C = \max\{0, \lambda_B - b_B\}P_{BC}$$

and the number of customers who get fare class C is

$$S_C = \min(\lambda_C, b_C) = \int_{\frac{b_A}{p_A}+\frac{b_B}{p_A p_{AB}}}^{\frac{b_A}{p_A}+\frac{b_B}{p_A p_{AB}}+\frac{b_C}{p_A p_{AB} p_{BC}}} p_A p_{AB} p_{BC}\left[x - \left(\frac{b_A}{p_A} + \frac{b_B}{p_A p_{AB}}\right)\right] f(x)\mathrm{d}x + b_C \int_{\frac{b_A}{p_A}+\frac{b_B}{p_A p_{AB}}+\frac{b_C}{p_A p_{AB} p_{BC}}}^{\infty} f(x)\mathrm{d}x$$

3.4 Multiple classes for different types of customers

Let k denote the customer type, $k = 1, 2, \ldots, K$, where K is the number of customer types. Each customer type has a unique customer preference order. λ_j^k denotes the predicted number of customers for customer type k and O-D itinerary j, which is a random variable. The density function of λ_j^k is represented by $f_j^k(x)$. Then, the mean value of λ_j^k can be calculated by

$$\mu_j^k = \int_0^{\infty} x f_j^k(x)\mathrm{d}x \quad (3)$$

The variance value of λ_j^k can be calculated by

$$\sigma_j^k = \sqrt{\int_0^{\infty} (x - \mu_j^k)^2 f_j^k(x)\mathrm{d}x} \tag{4}$$

For O-D itinerary j and customer type k, the preference order is $1 \rightarrow 2 \rightarrow, \ldots, \rightarrow r \rightarrow, \ldots, \rightarrow R_j^k$, namely, class $r = 1, 2, \ldots, R_j^k$. Let p_{jr}^k denotes the transition probability from class $r-1$ to class r for customer type k and O-D itinerary j (Particularly, $p_{j,1}^k$ denotes the purchase probability of class 1 for customer type k and O-D itinerary j). Then, for customer type k and O-D itinerary j, the number of customers who request class 1 is

$$\lambda_{j,1}^k = \lambda_j^k \cdot p_{j,1}^k \tag{5}$$

and the number of customers who actually request class r is

$$\begin{aligned} \lambda_{jr}^k &= [\lambda_{j,(r-1)}^k - \min(\lambda_{j,(r-1)}^k, \sum_{g=1}^{G} b_{g,j,(r-1)}^k)] p_{jr}^k \\ &= \max\{0, \lambda_{j,(r-1)}^k - \sum_{g=1}^{G} b_{g,j,(r-1)}^k\} p_{jr}^k \end{aligned} \tag{6}$$

Therefore, for customer type k and O-D itinerary j, the number of customers who obtain class r is

$$\begin{aligned} S_{jr}^k(b_{gjr}^k) &= \min(\lambda_{jr}^k, \sum_{g=1}^{G} b_{gjr}^k) \\ &= \int_{\sum_{\gamma=1}^{r-1} \frac{\sum_{g=1}^{G} b_{gj\gamma}^k}{\prod_{\Upsilon=1}^{\gamma} p_{j\Upsilon}^k}}^{\sum_{\gamma=1}^{r} \frac{\sum_{g=1}^{G} b_{gj\gamma}^k}{\prod_{\Upsilon=1}^{\gamma} p_{j\Upsilon}^k}} \prod_{\gamma=1}^{r} p_{j\gamma}^k \left(x - \sum_{\gamma=1}^{r-1} \frac{\sum_{g=1}^{G} b_{gj\gamma}^k}{\prod_{\Upsilon=1}^{\gamma} p_{j\Upsilon}^k}\right) f_j^k(x)\mathrm{d}x + \sum_{g=1}^{G} b_{gjr}^k \int_{\sum_{\gamma=1}^{r} \frac{\sum_{g=1}^{G} b_{gj\gamma}^k}{\prod_{\Upsilon=1}^{\gamma} p_{j\Upsilon}^k}}^{\infty} f_j^k(x)\mathrm{d}x \end{aligned} \tag{7}$$

where b_{gjr}^k denotes the booking limit for train g, O-D itinerary j, customer type k, and class r; this is a decision variable.

Let h_{jr}^k denote the ticket price of class r for customer type k and O-D itinerary j. Then, the expected revenue of class r for customer type k and O-D itinerary j is

$$Q_{jr}^k(b_{gjr}^k) = h_{jr}^k \cdot S_{jr}^k(b_{gjr}^k) \tag{8}$$

The total expected revenue of all classes for customer type k and O-D itinerary j is

$$Q_j^k(b_{gjr}^k) = \sum_{r=1}^{R_j^k} Q_{jr}^k(b_{gjr}^k) \tag{9}$$

3.5 Optimization model

3.5.1 Objective function

The total expected revenue of all the customers for all O-D itineraries is

$$Q(b_{gjr}^k) = \sum_{j=1}^{n} \sum_{k}^{K} Q_j^k(b_{gjr}^k) \tag{10}$$

3.5.2 Constraints

The CRCSC simultaneously sells a ticket and determines the seat. Thus, if a ticket for train g in O-D itinerary j has been sold, the corresponding seat of train g in O-D itinerary j cannot be utilized again. We consider only the tickets with seats and do not allow for overbooking; therefore, decision variable b_{gjr}^{k} should satisfy the service capacity constraint in each rail leg of each train

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{r=1}^{R_j^k} a_{ij} b_{gjr}^{k} \le C_{gi} \qquad \forall g, i \tag{11}$$

In addition, because different trains have different train stop plans, certain trains cannot provide a service product in O-D itinerary j. Therefore, the decision variable b_{gjr}^{k} should satisfy the constraints of the train stop plan

$$\begin{array}{ll} if \quad u_{gj} = 0, \quad then \quad b_{gjr}^{k} = 0 & \forall g, j, k, r \\ if \quad u_{gj} = 1, \quad then \quad b_{gjr}^{k} \ge 0 & \forall g, j, k, r \end{array} \tag{12}$$

Constraint (12) can be simplified as

$$(u_{gj} - 1) b_{gjr}^{k} = 0 \qquad \forall g, j, k, r \tag{13}$$

Decision variable b_{gjr}^{k} should satisfy the actual integer constraints as

$$b_{gjr}^{k} \ge 0 \qquad \forall g, j, k, r \tag{14}$$

$$b_{gjr}^{k} \in \mathbb{Z} \qquad \forall g, j, k, r \tag{15}$$

3.5.3 Nonlinear integer programming

Based on the predicted passenger demands of different types of customers in each O-D itinerary, to maximize the HSR ticket revenue according to the service capacity in each rail leg of each train, the HSR revenue management model for multiple fare classes with customer choice and multiple trains with different train stop plans is given by

$$(\mathbf{NLIP}) \qquad \max Q(b_{gjr}^{k}) \tag{16}$$

$$s.t.$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{r=1}^{R_j^k} a_{ij} b_{gjr}^{k} \le C_{gi} \qquad \forall g, i \tag{17}$$

$$(u_{gj} - 1) b_{gjr}^{k} = 0 \qquad \forall g, j, k, r \tag{18}$$

$$b_{gjr}^{k} \ge 0 \qquad \forall g, j, k, r \tag{19}$$

$$b_{gjr}^{k} \in \mathbb{Z} \qquad \forall g, j, k, r \tag{20}$$

4 Solution algorithm and numerical experiment

To address the model proposed in the previous section, an efficient and practical solution procedure is developed, and numerical experiments are presented to test this procedure in this section. The model is a nonlinear integer programming problem, and many software products, such as Lingo, provide solution procedures for these problems; however, efficient algorithms that can optimally solve these problems are still in the exploratory stage. Therefore, we have attempted to develop a hybrid heuristic approach to solving the model based on its characteristics and the artificial bee colony (ABC) optimization framework.

Although $(\mathbf{NLIP})$ derived under the condition of random demand is difficult to solve, the model with the condition of deterministic demand is a linear programming problem that can generate a better initial solution for $(\mathbf{NLIP})$.

4.1 Initial solution

When the integer constraints of $(\mathbf{NLIP})$ are relaxed, $(\mathbf{NLIP})$ will become a nonlinear programming problem:

$$(\mathbf{NLP}) \qquad \max Q(b_{gjr}^k) \tag{21}$$

$s.t.$

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{r=1}^{R_j^k} a_{ij} b_{gjr}^k \le C_{gi} \qquad \forall g,i \tag{22}$$

$$(u_{gj}-1) b_{gjr}^k = 0 \qquad \forall g,j,k,r \tag{23}$$

$$b_{gjr}^k \ge 0 \qquad \forall g,j,k,r \tag{24}$$

Ciancimino et al. (1999) has been demonstrated that the nonlinear programming problem can be transformed into a deterministic linear programming problem when the random demand is replaced by its mean value. Similar to their ideas, we randomly generated a deterministic demand, namely, a $(k \times n)$-matrix $\lambda(\beta) \equiv (\lambda_j^k(\beta))$ with the probability $\mathcal{P}(\beta)$ based on the density function of demand $f_j^k(x)$. $\lambda_j^k(\beta)$ denotes the deterministic demand for customer type k and O-D itinerary j. (Relax the integer constraints and allow $\lambda_j^k(\beta) \in \mathbb{R}$; β denotes any passenger demand that might occur, and $\beta = 1, 2, \ldots \omega$. Because of $\lambda_j^k(\beta) \in \mathbb{R}$, the possible values of $\lambda(\beta)$ are infinite; therefore, $\omega \to \infty$ and $\sum_{\beta=1}^{\omega} \mathcal{P}(\beta) = 1$.)

When the deterministic demand is $\lambda(\beta) \equiv (\lambda_j^k(\beta))$, the revenue management problem under random demand is transformed into a revenue management problem under deterministic demand. That is, $(\mathbf{NLP})$ is transformed into

$$(\mathbf{LP}) \qquad \max \sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{r=1}^{R_j^k}\sum_{g=1}^{G} h_{jr}^k b_{gjr}^k \tag{25}$$

$s.t.$

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\sum_{r=1}^{R_j^k} a_{ij} b_{gjr}^k \le C_{gi} \qquad \forall g,i \tag{26}$$

$$\sum_{r=1}^{R_j^k}\sum_{g=1}^{G} \frac{b_{gjr}^k}{p_{jr}^k} \le \lambda_j^k(\beta) \qquad \forall j,k \tag{27}$$

$$(u_{gj}-1) b_{gjr}^k = 0 \qquad \forall g,j,k,r \tag{28}$$

$$b_{gjr}^k \ge 0 \qquad \forall g,j,k,r \tag{29}$$

where formula (25) is the objective function, formula (26) is the service capacity constraint, formula (27) is the demand constraint that states that the booking limit should not exceed the number of customers who purchase tickets, formula (28) is the constraint of the train stop plan and formula (29) is the nonnegative constraint. $(\mathbf{LP})$ is a linear programming problem, and the solution of $(\mathbf{LP})$ can be an initial solution for $(\mathbf{NLIP})$.

4.2 Heuristic approach

Most deterministic optimization algorithms are based on numerical linear and nonlinear programming methods that require substantial gradient information and usually seek to improve the solution in the neighbourhood of a starting point. However, these algorithms reveal a limited approach to complicated real-world optimization problems. Therefore, the computational drawbacks of deterministic algorithms have forced researchers to rely on stochastic meta-heuristic algorithms to solve real-world engineering optimization problems. Compared with earlier meta-heuristic optimizations, the ABC algorithm imposes fewer mathematical requirements and is easier to implement with few parameters. Furthermore, the ABC algorithm is good at solving multivariable optimization problems.

The ABC algorithm is a population-based computational method developed by Karaboga (2005). This approach is derived from simulations of the behaviour of a bee colony for seeking food sources. Compared with other population-based approaches, such as genetic algorithms and particle swarm algorithms, the ABC algorithm offers the key advantage of completing a global search and local search in each cycle. Thus, the probability of finding the optimal solution is greatly increased, while avoiding becoming stuck on a local optimal solution.

4.2.1 ABC algorithm for the HSR revenue management problem

In the ABC algorithm, the position of a food source represents a possible solution of the HSR revenue management problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. The position of food source α in the designed solution space is represented by a D-dimensional vector $x_\alpha = \{x_{\alpha 1}, x_{\alpha 2}, \ldots, x_{\alpha D}\}$, where $\alpha = 1, 2, \ldots, SN$, with SN denoting the number of food source positions. There are three species of bees: employed bee, onlooker bee and scout bee. The number of employed or onlooker bees is equal to the number of food sources SN. The number of bees, namely, the population size, is BN. The maximum cycle number is MCN, and there is a maximum of one scout bee that will seek a new food source in each cycle.

We use the ABC algorithm to generate the booking limits for our problem. First, we design the mapping rule between the variables of the booking limits and the particles in the ABC system.

Because there are G trains, n O-D itineraries and k customer types and each customer type in each O-D itinerary has R_j^k classes, we setup a searching space with $D = G\sum_{j=1}^{n}\sum_{k=1}^{K} R_j^k$ dimensions for the HSR revenue management problem. The position of a food source x_α is used to represent the booking limit decision. Element $x_{\alpha d}$ is used to represent the booking limit decision of the αth food source for train g, O-D itinerary j, customer type k and class r, where

$$d = (g-1)\sum_{j=1}^{n}\sum_{k=1}^{K} R_j^k + \sum_{j=1}^{j-1}\sum_{k=1}^{K} R_j^k + \sum_{K=1}^{k-1} R_j^k$$

For example, for a booking system with $G = 2$, $n = 10$, $K = 2$ and $R_j^k = 3, \forall j, k$, the booking limit for train $g = 2$, O-D itinerary $j = 7$, customer type $k = 2$ and class $r = 2$ is represented by $x_{\alpha,101}$.

4.2.2 Generating the food source position and calculating fitness

First, according to $f_j^k(x)$, $\lambda(\beta) \equiv (\lambda_j^k(\beta))$ is randomly generated. Then, $(\mathbf{LP})$ is calculated using $\lambda(\beta)$, and the solution of $(\mathbf{LP})$ is an initial food source position. SN food source positions are generated by $(\mathbf{LP})$ in the initial stage, and BN artificial bees are employed bees, onlooker bees and scout bees. The number of employed bees or onlooker bees is equal to $BN/2 = SN$.

Note that the food source position generated by $(\mathbf{LP})$ is a real number. Because the values of booking limits are required to be integers, the food source position is not a feasible solution and cannot be viewed as a candidate solution. To evaluate the fitness of each candidate solution, $x_{\alpha d}$ is rounded to the nearest integer $x_{\alpha d}$ and the values of booking limits $x_\alpha = \{x_{\alpha 1}, x_{\alpha 2}, \ldots, x_{\alpha D}\}$ are obtained. The fitness of each food source position is calculated by substituting x_α into the objective function of $(\mathbf{NLIP})$.

4.2.3 Artificial bees

4.2.3.1 Modified method for finding a new food source

Each employed bee or onlooker bee probabilistically modifies the food source position in her memory to find a new food source and tests the nectar amount of the new source. The standard ABC algorithm (Karaboga and Basturk 2007; Karaboga and Akay 2009) uses the following expression

$$v_{\alpha d} = x_{\alpha d} + \phi_{\alpha d}(x_{\alpha d} - x_{\kappa d}) \tag{30}$$

where $\kappa \in \{1, 2, ..., BN/2\}$ ($\kappa \neq \alpha$) and $d \in \{1, 2, ..., D\}$ are randomly chosen indexes, $\phi_{\alpha d}$ is a random number from [-1,1], v_α is a candidate food position from the old position x_α, and $v_{\alpha d}$ is the modification.

The candidate food position v_α changes only one component $v_{\alpha d}$ compared with x_α. Because there are multiple components of x_α under conditions of multiple trains with different train stop plans and multiple fare classes with different customer types, it is inefficient to have one component $v_{\alpha d}$ changed in each iteration. In a real network, D, the number of components of x_α is usually greater than 10^3.

The other important deficiency of formula (30) is that its optimization ability is limited for the HSR revenue management problem. The booking limits for different O-D itineraries and different fare classes are related. In other words, the components of x_α are affected by each other. Therefore, the increase in fitness from changing only one of the booking limits is negligible, and the increased fitness is derived from a reasonable combination of changes under certain constraints of service capacity.

For the characteristic HSR revenue management problem, in our ABC algorithm, a candidate food position is obtained from

$$v_\alpha = x_\alpha + \phi_\alpha(x_\alpha - x_\kappa) \tag{31}$$

where $\kappa \in \{1, 2, ..., BN/2\}$ ($\kappa \neq \alpha$) and ϕ_α is a random number from [-1,1]. In each iteration, the candidate food position v_α changes multiple components of x_α in our modified formula (31) which is more efficient than formula (30).

4.2.3.2 Onlooker bees

An onlooker bee chooses a food source depending on the probability value associated with that food source, p_α, which is calculated by the following expression

$$p_\alpha = \frac{fit_\alpha}{\sum_{\kappa=1}^{SN} fit_\kappa} \tag{32}$$

where fit_α is the fitness value of the solution α evaluated by its employed bee, which is proportional to the nectar amount of the food source in position α and SN is the number of food sources which is equal to the number of employed bees ($BN/2$).

4.2.3.3 Scout bee

In the ABC algorithm, if a position cannot be further improved through a predetermined number of cycles called $limit$, then that food source is assumed to be abandoned. A scout bee finds a new food source position using $(\mathbf{LP})$.

4.2.4 Solution procedure

Step 1: Initialization

1.1 Set the initial ABC parameters: SN, BN, $limit$, Maximum cycle number MCN and iterative number $l = 0$.

1.2 Randomly generate SN food source positions x_α using $(\mathbf{LP})$ and calculate the fitness of x_α.

Step 2: Creating and selecting new food source positions

2.1 Each employed bee finds a new food source position according to formula (31) with the greedy selection criterion. Record the unvaried parameter of each employed bee.

2.2 Each onlooker bee chooses a food source using formula (32) and finds a new food source position according to formula (31) with the greedy selection criterion. Record the unchanged parameter of each onlooker bee.

2.3 An employed bee or onlooker bee whose unvaried parameter exceeds $limit$ becomes a scout bee that will find a new food source using $(\mathbf{LP})$.

Step 3: Determining whether to stop

3.1 Record the food source position with maximum fitness.

3.2 Update $l = l + 1$.

3.3 If $l > MCN$, stop; otherwise return to Step 2.1.

4.3 Control policy

In practice, if we cannot determine the type of arriving customer, we can only adopt the first-come-first-served (FCFS) control strategy. When the booking limits of a certain customer type are exhausted, the FCFS control strategy may lead to that the customer does not transfer to the next selection but occupy the booking limits of other customer types. This situation may result in some loss of revenue. Therefore, it is necessary to judge the type of arriving customer and adopt the corresponding control strategy. Van Ryzin and Vulcano (2014) proposed an approach for estimating the arrival rate and the customer preferences. Chen and Homem (2010) assumed that the type of arriving customer was known in their control. Van Ryzin and Vulcano (2008b) assumed that the arrival time was different for different customer types. In our work, we present a method based on ticket purchase behaviour to judge the type of arriving customer.

In China, there are two different types of customers according to the source of funds: customers who pay their own fare (customer type A) and customers whose fare is paid by their company (customer type B). The preferences of the two customer types are different. Customer type A mainly considers the travel cost when faced with multiple fare classes. They might decide not to travel if the fare is beyond their reservation price. In contrast, customer type B cares more about the service of rescheduling and cancellation (SRC), because the ticket cost is usually paid by her company and her trips may be changed frequently according to her adjustment of work schedule.

In the ticketing service system, we can set an option that represents whether to purchase the SRC. If an arriving customer chooses the SRC, we judge this customer belongs to customer type B; if a customer does not choose the SRC, we judge this customer belongs to customer type A.

Based on the above method for judging customer type, we solve the $(\mathbf{NLIP})$ model to obtain the optimal solution $\mathrm{B} = (b_{gjr}^{k\ *})$, where $b_{gjr}^{k\ *}$ is the optimal booking limit for train g, O-D itinerary j, customer type k and class r. Although $(\mathbf{NLIP})$ is a static model, we can incorporate the dynamic aspect by re-setting the booking limits by re-optimizing the model decisions according to the information on the demand already occurred (You, 2008). That is, the dynamic aspect can be built by re-optimizing the model decisions several times in the sub-sequential selling periods.

If the actual demand is consistent with the predictive demand, our control policy can ensure its effectiveness without re-optimizing and adjustment. However, the actual demand is complex and changeable. Therefore, CRCSC operators can adjust the booking limits from 11 p.m. to 6 a.m. according to the prior day's demand obtained (the CRCSC is maintaining from 11 p.m. to 6 a.m.). In our case study, the dynamic change in the passenger demand is not our research focus.

4.4 Numerical test

To evaluate the performance of the proposed approach for the HSR revenue management problem, 6 test problems are solved and compared with the solution found using the Lingo 12.0 software. In addition, we randomly generate a simulated passenger flow based on the given predictive passenger flow to illustrate our control method and the dynamic change process of the fares.

4.4.1 Sample network parameters

There are three trains ($G=3$) in an HSR. To demonstrate the computational efficiency of the algorithm in different scales, the number of rail legs is $m=2,3$ and the number of stations is 3 or 4. The train stop plans of different trains for different numbers of rail legs are shown in Fig. 4.

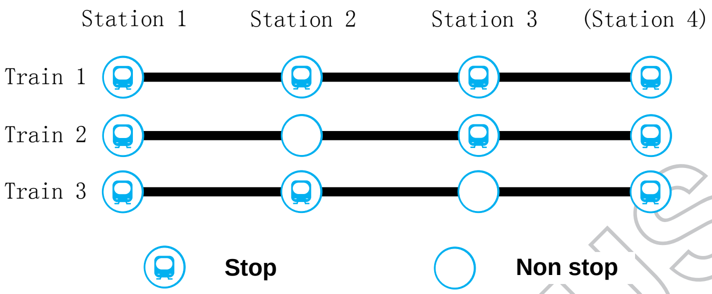

Fig. 4. The train stop plans for different numbers of rail legs

It is assumed that the demand of customer type k in OD itinerary j obeys a normal distribution $N(\mu_j^k, \sigma_j^{k^2})$. Customer type A is represented by $k=1$, and customer type B is represented by $k=2$. The values of parameters μ_j^k and σ_j^k are shown in Table 2.

Table 2

Parameters of the mean value μ_j^k and the variance value σ_j^k

μ_j^k (μ_j^1, μ_j^2)	Destination		
Origin	Station 2	Station 3	(Station 4)
Station 1	165 27	175 32	205 38
Station 2		160 27	180 36
(Station 3)			155 28
σ_j^k (σ_j^1, σ_j^2)	Destination		
Origin	Station 2	Station 3	(Station 4)
Station 1	32 6	34 5	36 6
Station 2		17 7	17 6
(Station 3)			16 5

For each O-D itinerary, we design three types of passenger transport products, which are Product I, Product II and Product III. The ticket prices of Product I and Product II are h_j^{I} and h_j^{II} respectively, and neither product provides the SRC. The ticket price of Product III, which provides the SRC, is h_j^{III}. h_j^{III} is shown in Table 3, where $h_j^{\mathrm{I}} = 0.8 \times h_j^{\mathrm{III}}$ and $h_j^{\mathrm{II}} = 0.9 \times h_j^{\mathrm{III}}$. The preference order of customer type A is $\mathrm{I} \to \mathrm{II} \to \mathrm{III}$, and that of customer type B is III. The transition probability p_{jr}^k is shown in Table 4.

Table 3

Ticket price h_j^{III} of Product III in each O-D itinerary (RMB)

O-D	Destination		
Origin	Station 2	Station 3	(Station 4)
Station 1	80	164	261
Station 2		90	184
(Station 3)			100

Note: RMB is the Chinese currency "Renminbi". US$1 approximates RMB6.6 as of November 1, 2017.

Table 4

Transition probability p_{jr}^k for different customer types

Customer type A ($k = 1$)	$p_{j,1}^1$ =95%	$p_{j,2}^1$ =80%	$p_{j,3}^1$ =80%
Customer type B ($k = 2$)	$p_{j,1}^2$ =90%	-	-

The parameters σ_j^k , h_{jr}^k and p_{jr}^k are constant. We design 6 cases in which the number of rail legs m , mean value μ_j^k and service capacity C_{gi} are changed. Table 5 lists the associated parameters m , μ_j^k and C_{gi} for all categories.

Table 5

Parameters m , μ_{kgj} and C_{gi} in different cases

Category	Number of rail legs	Mean value	Service capacity	Stop		
				Train 1	Train 2	Train 3
Case 1	$m = 2$	$0.9\mu_j^k$	$C_{gi} = 150$	Stations 1.2.3	Stations 1.3	Stations 1.2
Case 2	$m = 2$	μ_j^k	$C_{gi} = 150$	Stations 1.2.3	Stations 1.3	Stations 1.2
Case 3	$m = 2$	$1.1\mu_j^k$	$C_{gi} = 150$	Stations 1.2.3	Stations 1.3	Stations 1.2
Case 4	$m = 3$	$0.9\mu_j^k$	$C_{gi} = 225$	Stations 1.2.3.4	Stations 1.3.4	Stations 1.2.4
Case 5	$m = 3$	μ_j^k	$C_{gi} = 225$	Stations 1.2.3.4	Stations 1.3.4	Stations 1.2.4
Case 6	$m = 3$	$1.1\mu_j^k$	$C_{gi} = 225$	Stations 1.2.3.4	Stations 1.3.4	Stations 1.2.4

The parameters of the ABC algorithm are as follows: $SN = 30$, $BN = 60$, $limit = 50$ and $MCN = 500$.

4.4.2 Comparison results

Table 6 compares the results for the solution quality and computation efficiency between our method and Lingo software for different cases.

Table 6

Computational results for different cases

Category	Lingo		Heuristic		Gap	
	Sol (10^4 RMB)	Time (h:min:s)	Sol (10^4 RMB)	Time (h:min:s)	SG	TG
Case1	4.2	5:40:24	4.2	0:00:25	0%	-100%
Case2	4.5	0:01:40	4.5	0:00:22	0%	-78%
Case3	4.8	0:01:40	4.8	0:00:23	0%	-77%
Case4	12.3	0:14:26	12.3	0:00:40	0%	-95%
Case5	N/A	>7:00:00	13.1	0:00:41	-	$\leq -100\%$
Case6	N/A	>7:00:00	14.0	0:00:40	-	$\leq -100\%$

SG=(Heuristic Sol-Lingo Sol)/ Lingo Sol.
TG=(Heuristic Time-Lingo Time)/ Lingo Time.

The experiments were conducted in MATLAB 2012 on a PC with a 2×2.6 GHz CPU and 8GB of RAM. The gaps between the solutions obtained by the Lingo solver and the solutions found using our method are used to judge the quality and efficiency of our solution procedure; these gaps are expressed as a percentage. The solution quality gap (SG) between the Lingo solver and our method is 0%, but the computational efficiency gap (TG) between the Lingo solver and our method is greater than 77%. These results indicate that our method can obtain the similar solution quality more efficiently. Table 6 indicates that the computing times are quite stable for the proposed heuristic approach. However, the computing times for the Lingo software are sensitive to parameter m ,and the CPU time clearly increases as the value of m increases (in some cases, no satisfactory solution was found by the Lingo software within 7 hours).

4.4.3 Simulation results

4.4.3.1 Simulation runs of one case

To verify the effectiveness of our method in an actual seat inventory control process, we simulate the actual process of booking tickets based on the parameters in Case 5. Table 7 shows the booking limits for Case 5 calculated using our method.

Table 7

Results of the optimal booking limits $b_{gjr}^{k\,*}$ (Train 1, Train 2 and Train 3)

Fare class	O-D	Destination								
	Origin	Station 2			Station 3			Station 4		
$b_{g,j,1}^{1\,*}$	Station 1	89	0	99	2	7	0	35	98	72
	Station 2				2	0	0	11	0	55
	Station 3							100	65	0
$b_{g,j,2}^{1\,*}$	Station 1	0	0	0	11	47	0	0	0	0
	Station 2				2	0	0	18	0	65
	Station 3							0	0	0
$b_{g,j,3}^{1\,*}$	Station 1	0	0	0	7	44	0	0	0	0

	Station 2				84	0	0	0	0	0
	Station 3							0	0	0
	Station 1	12	0	15	11	17	0	10	12	14
$b_{g,j,1}^{2}{}^{*}$	Station 2				19	0	0	13	0	19
	Station 3							15	11	0

Based on the parameters μ_j^k and σ_j^k in Table 2, we randomly generate a simulated passenger flow. Then, we use the simulated passenger flow to simulate the booking process of passengers using the seat inventory control strategy shown in Table 7 and record the ticket revenue of each passenger. Table 8 demonstrates the results of 20 simulation runs.

Because the passenger flow is characterized by randomness, the ticket revenues range between 11.9 and 13.9 in the 20 simulation runs. However, the average value of 13.2 is approximately equal to the expected revenue given in Table 6. Therefore, the expected revenue of our strategy is consistent with the revenue of the simulation runs.

Table 8

Ticket revenue of 20 simulation runs

Simulation run	Ticket revenue (10^4 RMB)	Simulation run	Ticket revenue (10^4 RMB)
1	13.3	11	13.3
2	12.2	12	13.9
3	13.3	13	13.4
4	13.5	14	13.4
5	11.9	15	13.3
6	13.2	16	13.9
7	13.7	17	12.5
8	13.1	18	12.5
9	13.4	19	12.8
10	12.8	20	13.7
Average value		13.2	

4.4.3.2 Offer set of products

When passengers of different types of customers in different O-D itineraries arrive randomly, the available products in each O-D itinerary change dynamically throughout the entire booking process. Taking simulation run 1 of Table 8 as an example, there are 1290 customers in the simulated passenger flow of simulation run 1. Fig. 5 shows the offer set of available products of certain O-D itineraries during differet booking periods.

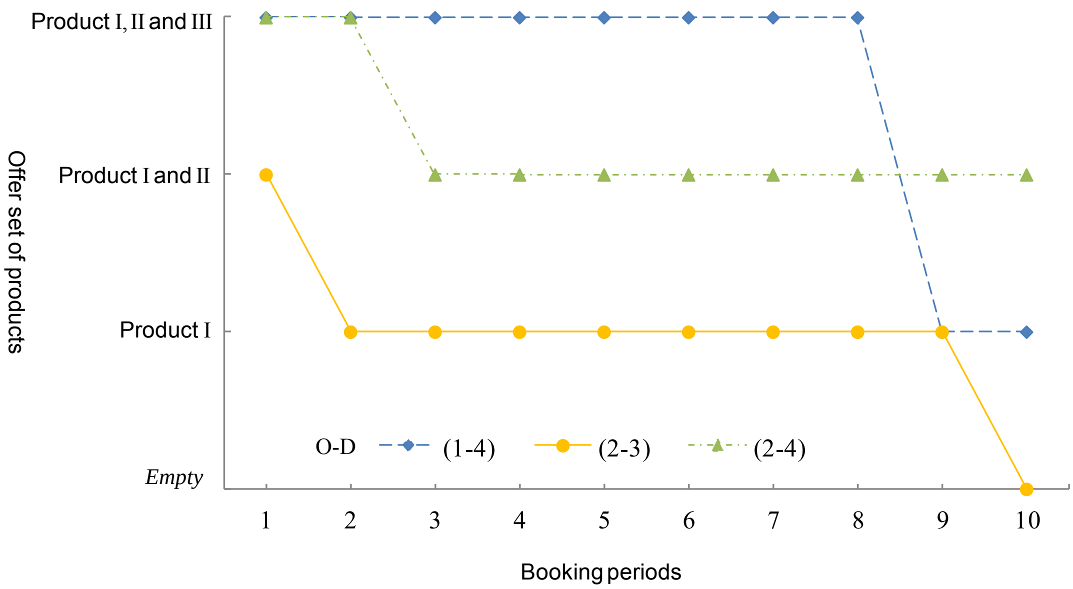

Fig. 5. Offer set of products in certain O-D itineraries

In Fig. 5, booking period 1 denotes the first period for booking a ticket, and booking period 10 denotes the last period. The available fare classes of each O-D itinerary are decreasing because of changes in the remaining resources during the booking process. The offer set for the 2-3 itinerary is empty in booking period 10 because all ticket for this itinerary have been sold out.

5 Case study

5.1 Network structure and passenger demand

According to the train operation scheme of the Beijing-Shanghai HSR in the CRCSC, five high-speed passenger trains, $\mathrm{G-I}$, $\mathrm{G-II}$, $\mathrm{G-III}$, $\mathrm{G-IV}$ and $\mathrm{G-V}$, run from 9:00 to 10:00 in the morning. $\mathrm{G-I}$ serves 3 stops and 3 O-D itineraries; $\mathrm{G-II}$ serves 8 stops and 28 O-D itineraries; $\mathrm{G-III}$ serves 10 stops and 45 O-D itineraries; $\mathrm{G-IV}$ serves 11 stops and 55 O-D itineraries; and $\mathrm{G-V}$ serves 4 stops and 6 O-D itineraries. Altogether, these five passenger trains have 13 railway legs ($m = 13$), 14 stops and 81 O-D itineraries. The train stop plans of the five passenger trains are shown in Fig. 6.

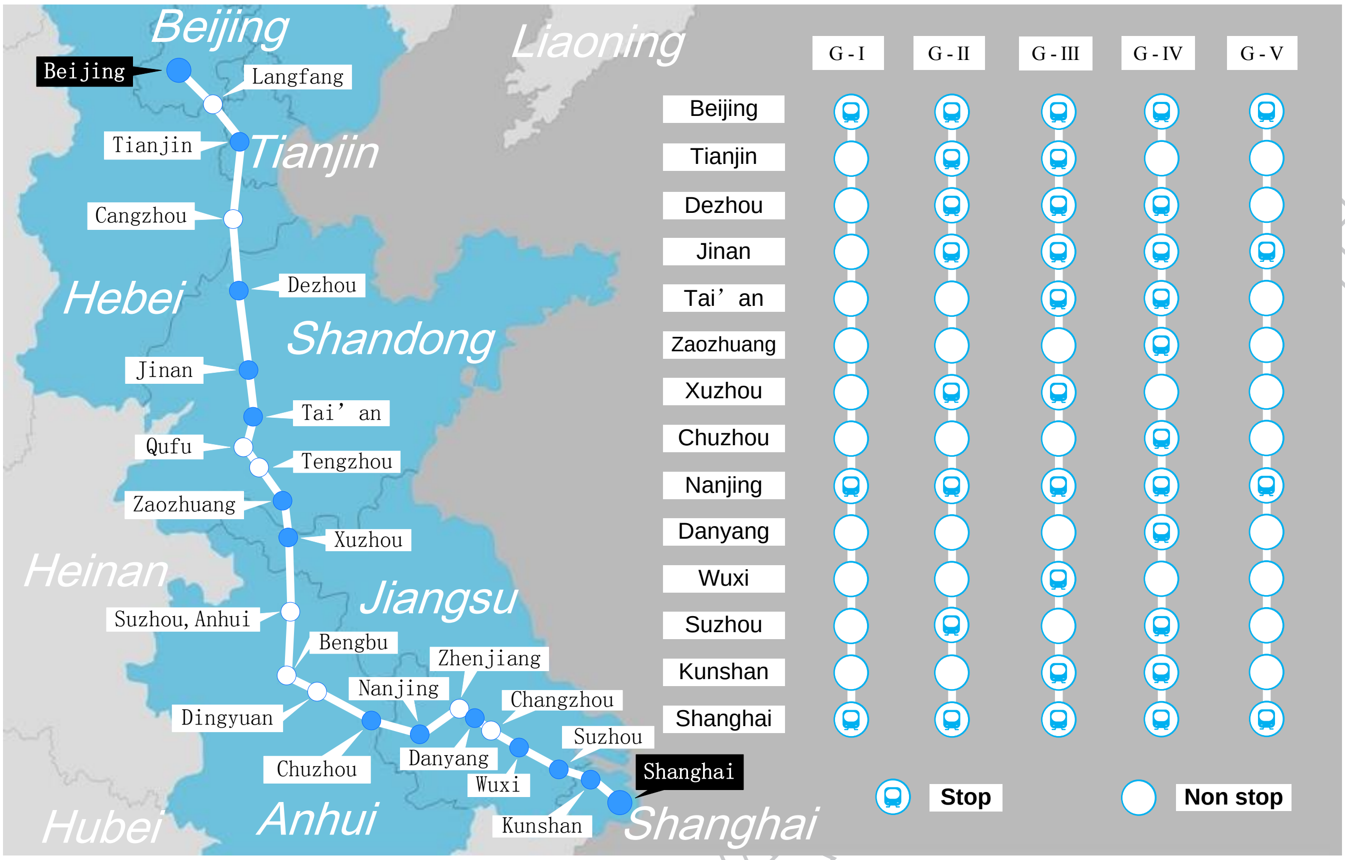

Fig. 6. Train stop plan in the Beijing-Shanghai HSR

Although the CRCSC can record each ticket that has been sold, because of limitations in the current seat inventory control strategy, the CRCSC data can only identify the passenger demand that has already occurred. In other words, the CRCSC data are not the actual prior passenger demand, and the future passenger demand is different from the prior passenger demand. Therefore, our data are predictive based on the prior passenger demand data in the CRCSC system recorded from July 20th, 2015 to July 26th, 2015. To demonstrate the superiority of the control for multiple trains, the control for multiple trains is compared with that for a single train. The predicted passenger demand of each train is assumed to obey the normal distribution $N\,(\mu_{gj}^{k}, {\sigma_{gj}^{k}}^{2})$. The values of parameters μ_{gj}^{k} and σ_{gj}^{k} are shown in Tables 19-23 in Appendix A.

For each O-D itinerary, there are three kinds of passenger transport products, which are Product Ⅰ, Product Ⅱ and Product Ⅲ. The ticket prices of Product Ⅰ and Product Ⅱ are h_j^{I} and h_j^{II} respectively, neither of which provides the SRC. The ticket price of Product Ⅲ, which provides the SRC, is h_j^{III}. h_j^{III} is shown in Table 9, where $h_j^{\mathrm{I}} = 0.8 \times h_j^{\mathrm{III}}$ and $h_j^{\mathrm{II}} = 0.9 \times h_j^{\mathrm{III}}$.

Table 9

Fare h_j^{III} in each O-D itinerary (RMB)

	Tian-jin	De-zhou	Jin-an	Tai'an	Zao-zhuang	Xu-zhou	Chu-zhou	Nan-jing	Dan-yang	Wu-xi	Su-zhou	Ku-shan	Shang-hai
Beijing	54.5	144.5	184.5	214	248	309	418.5	443.5	478.5	513.5	523.5	533.5	553
Tianjin		94.5	139.5	159.5	229	259	369	393.5	433.5	473.5	478.5	488.5	508.5
Dezhou			39.5	69.5	144.5	174.5	289	314	354	393.5	403.5	418.5	438.5
Jinan				24.5	99.5	129.5	254	279	319	354	364	378.5	398.5
Tai'an					74.5	104.5	229	254	294	329	344	354	374
Zao-zhuang						29.5	154.5	179.5	224.5	264	274	289	309
Xuzhou							124.5	149.5	194	239	249	259	279
Chuzhou								24.5	69.5	114.5	124.5	139.5	164.5
Nanjing									44.5	84.5	99.5	114.5	134.5
Danyang										39.5	59.5	74.5	94.5
Wuxi											24.5	39.5	64.5
Suzhou												14.5	34.5
Kushan													24.5

Table 10 shows the seat information for the five passenger trains, which is based on the data from July 20th, 2015, to July 26th, 2015 from the CRCSC system.

Table 10

Service capacity of each train in our case C_{gi}

	G - I	G - II	G - III	G - IV	G - V
Service capacity C_{gi}	989	535	977	1,092	1,092

The preference order of customer type A is $\mathrm{I} \rightarrow \mathrm{II} \rightarrow \mathrm{III}$, and the preference order of Customer type B is III. The parameter of the transition probability $p_{jr}^{k}(1)$ in our case study is based on a stated preference (SP) survey conducted at the Beijing south railway station in September 2016, and the statistical parameters are shown in Table 11.

Table 11

Transition probability $p_{jr}^{k}(1)$ of different types of customers

Customer type A ($k=1$)	$p_{j,1}^{1}(1)$ =95%	$p_{j,2}^{1}(1)$ =80.9%	$p_{j,3}^{1}(1)$ =81.3%
Customer type B ($k=2$)	$p_{j,1}^{2}(1)$ =90%	-	-

5.2 Analysis of algorithm parameters

In a real-size network, no satisfactory solution can be found using Lingo software within a reasonable time. To demonstrate the superiority of our ABC algorithm, we compare our algorithm with a particle swarm optimization algorithm (PSO), which is also a swarm intelligence algorithm and has been adopted by You (2008) to calculate booking limits. The initial solution of the PSO is calculated by $(\mathbf{LP})$. In the experiments, the population size (BN) is 20, 60 and 100 when maximum cycle number (MCN) is 1000. Other control parameters of the PSO are similar as those in You (2008). Each of the experiments is repeated 20 times.

The mean and the standard deviations of the expected revenue obtained by our ABC algorithm and the PSO are given in Table 12.

Table 12

Expected revenue obtained by the ABC and PSO algorithms

BN	ABC		PSO	
	Mean	Standard deviation	Mean	Standard deviation
20	2,192,946	939	2,165,786	3,394
60	2,195,092	603	2,168,583	4,122
100	2,195,602	932	2,172,894	3,440

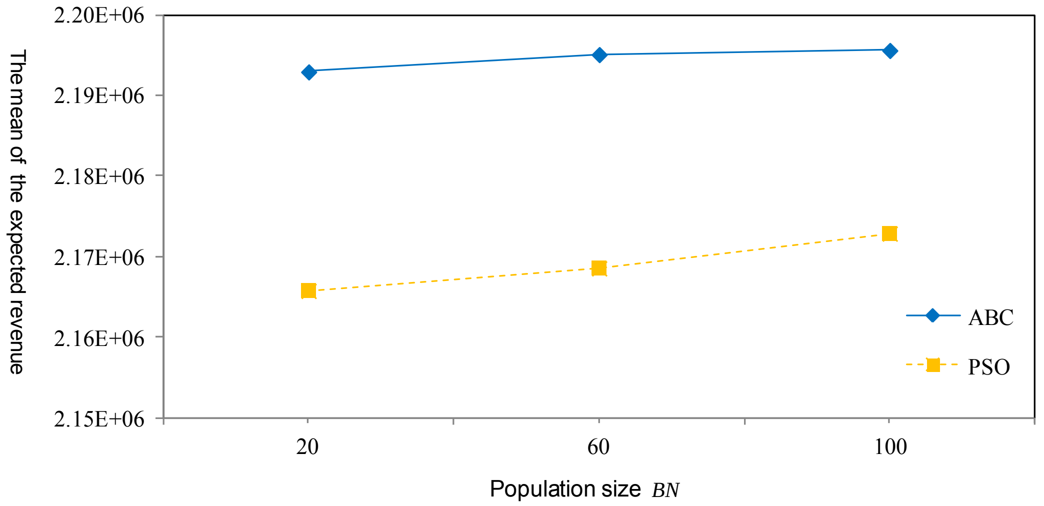

Fig. 7. Comparison of the means between the ABC and PSO algorithms

To show the performance of our ABC algorithm more clearly, the graphical representations of the results in Table 12 are reproduced in Fig. 7. The ABC shows a better performance than the PSO in each experiment. In our experiments, the PSO converges earlier than the ABC. In other words, the results demonstrate that the ABC can escape a local minimum in the search space and find the global minimum. In the ABC, while the exploration process conducted by artificial scouts is good for global optimization, the exploitation process managed by artificial employed and onlooker bees is efficient for local optimization. Hence, our ABC algorithm is quite successful in optimizing the objective of $(\mathbf{NLIP})$, which is a multivariable and multimodal function.

Table 13

Expected revenue obtained by the ABC algorithm

MCN	500			1000			2000		
BN	Mean	Standard deviation	Time	Mean	Standard deviation	Time	Mean	Standard deviation	Time
20	2,190,030	912	4 min4s	2,192,946	939	7 min25s	2,194,848	883	14 min25s
60	2,193,523	636	10 min59s	2,195,092	603	21 min39s	2,196,145	937	43 min12s
100	2,194,208	1010	17 min31s	2,195,602	932	36 min48s	2,196,570	730	76 min16s

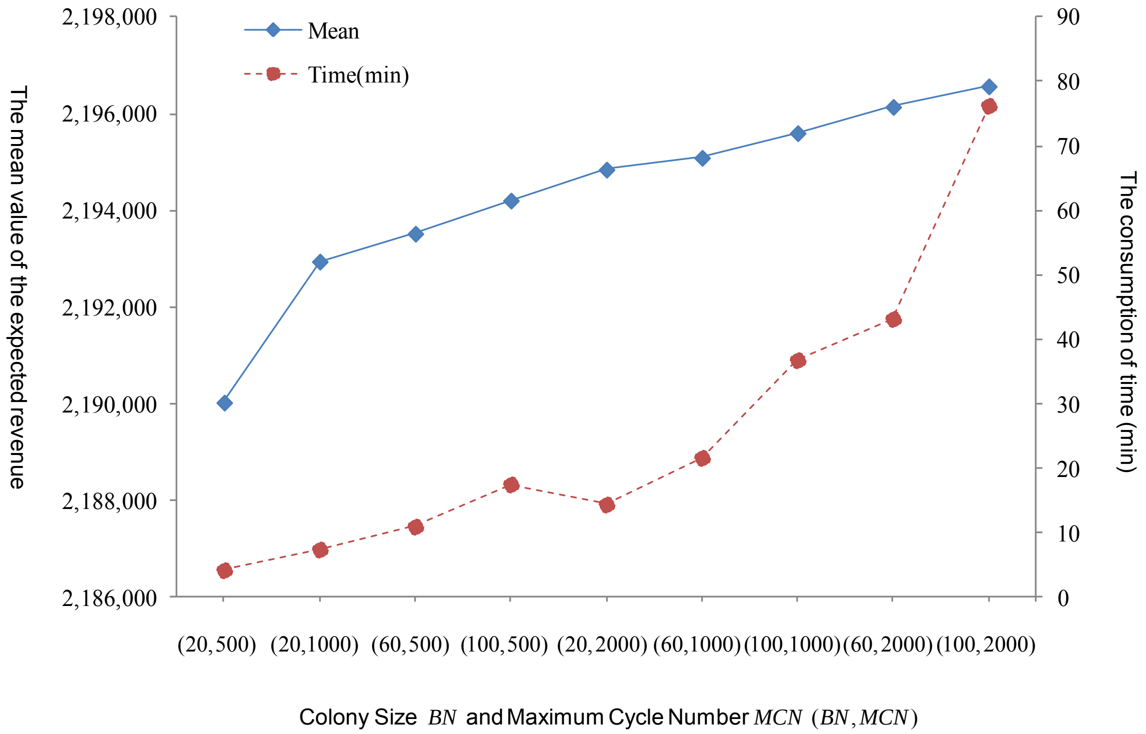

Fig. 8. Relationship between the mean and the time consumption obtained by the ABC

Table 13 shows the means and the standard deviations of the expected revenue obtained by our ABC algorithm when population size is 20, 60, 100 and maximum cycle number is 500, 1000, 2000. To demonstrate the relationship between the expected revenue and the time consumption more clearly, the graphical representations of the results in Table 13 are reproduced in Fig. 8. That the performance improves while the population size or maximum cycle number increases. However, a better result tends to consume more time. Based on the results above, in the following analysis, compared with the best result obtained by $(BN, MCN) = (100, 2000)$, we select the parameters $(BN, MCN) = (60, 1000)$ in the subsequent experiments because the time consumption is less than 30min and the gap is less than 0.1%.

5.3 Advantages of our method compared with the current control strategy

The current seat inventory control method in China HSR regards a single train as the control objective based on its own predicted passenger demand, and homogeneous seats have a single fixed price during the whole booking period. Compared with this approach, our method offers two main advantages: (i) controlling multiple trains with different train stop plans as a whole; and (ii) differentiating pricing for homogeneous seats based on customer preference order. Therefore, to demonstrate the superiority of our control strategy that considers multiple trains and multiple fare classes (MM), the other two strategies are used as contrasting schemes: (i) the strategy that considers a single train and a single fare class (SS); and (ii) the strategy that considers multiple trains and a single fare class (MS) (SS and MS both take h_j^{III} as their single fare class).

Table 14 shows the expected revenue of SS, MS and MM when the mean value is μ_{gj}^{k}, the variance value is σ_{gj}^{k}, the service capacity is C_{gi}, and the transition probability is $p_{jr}^{k} = p_{jr}^{k}(1)$.

Table 14

Expected revenue

Expected revenue (10^4 RMB)			Gap		
SS	MS	MM	GAP(A)	GAP(B)	GAP(AB)
201.55	210.52	219.51	4.45%	4.27%	8.91%

GAP(A)=(MS-SS)/SS.

GAP(B)=(MM-MS)/MS.

GAP(AB)=(MM-SS)/SS.

GAP(A) is the gap between the solution obtained by SS and the solution obtained by MS; GAP(B) is the gap between the solution obtained by MS and the solution obtained by MM; and GAP(AB) is the gap between the solution obtained by SS and the solution obtained by MM. These three gaps are used to judge the superiority of our method and expressed as percentages. GAP(A) reflects the advantage of controlling multiple trains with different train stop plans as a whole, and GAP(B) reflects the advantage of differentiated pricing for homogeneous seats according to the customer preference order. GAP(AB) reflects the overall improvements of our method compared with the current seat inventory control method for the China HSR.

Compared with SS, MM increases the expected revenue by 179.6 thousand RMB, i.e., by 8.91%. According to the annual report of CRC, the entire passenger revenue of the China railway in 2016 was 281.7 billion RMB. If the passenger revenue can be increased by 5%, the increase in revenue will be more than ten billion RMB which is considerable. Therefore, for Chinese railway passenger transportation, an improvement of 8.91% is significant.

To verify the effectiveness of MM, we simulate the actual process of booking tickets. The simulated passenger flow is randomly generated based on the parameters μ_{gj}^{k}, σ_{gj}^{k}, C_{gi} and $p_{jr}^{k} = p_{jr}^{k}(1)$. Table 15 shows the revenue results for 20 simulation runs.

Because the passenger flow is characterized by randomness, the revenue of each simulation differs from the expected revenue. However, the results of the average revenue of all the simulation runs in Table 15 are close to the results of the expected revenue in Table 14, which demonstrates the consistency between the control results and the expected results and the effectiveness of the proposed control policy.

Table 15

Revenue for 20 simulation runs

Simulation runs	Revenue (10^4 RMB)			Gap		
	SS	MS	MM	GAP(A)	GAP(B)	GAP(AB)
1	191.48	202.02	213.22	5.51%	5.54%	11.35%
2	203.09	215.63	220.74	6.18%	2.37%	8.69%
3	203.01	212.93	225.78	4.89%	6.03%	11.22%
4	203.87	213.08	224.30	4.52%	5.27%	10.02%

5	203.30	206.38	217.02	1.52%	5.15%	6.75%
6	200.39	211.13	223.68	5.36%	5.95%	11.63%
7	214.92	215.28	224.31	0.17%	4.19%	4.37%
8	204.21	211.82	220.32	3.72%	4.01%	7.89%
9	205.38	215.34	224.18	4.85%	4.10%	9.15%
10	206.01	214.06	222.07	3.90%	3.74%	7.79%
11	200.32	204.44	213.95	2.06%	4.65%	6.81%
12	200.01	207.43	215.74	3.71%	4.00%	7.86%
13	190.07	203.19	213.54	6.90%	5.09%	12.35%
14	192.35	203.17	213.65	5.62%	5.16%	11.07%
15	204.07	207.31	217.81	1.58%	5.07%	6.73%
16	212.98	219.98	229.29	3.29%	4.23%	7.66%
17	195.38	205.35	217.65	5.10%	5.99%	11.40%
18	203.45	212.61	215.06	4.50%	1.15%	5.71%
19	207.05	214.06	221.15	3.38%	3.31%	6.81%
20	195.38	202.81	214.14	3.81%	5.59%	9.60%
Average value	201.84	209.90	219.38	4.03%	4.53%	8.74%

5.4 Impact analysis of the mean value, the variance value, service capacity and transition probability

5.4.1 Mean value μ_{gj}^{k}, variance value σ_{gj}^{k} and service capacity C_{gi}

Table 16 shows the expected revenues of different strategies in different conditions when the transition probability $p_{jr}^{k} = p_{jr}^{k}(1)$ is invariant but the mean value μ_{gj}^{k}, the variance value σ_{gj}^{k} and the service capacity C_{gi} are variable. In Table 16, the percentages of mean value variations show the variation in mean value compared to the benchmark μ_{gj}^{k} we choose. For example, -20% means the mean value reduces by 20% of the current amount. The same format is used for the percentages of variance value variations and service capacity variations

Table 16

Expected revenues in different conditions

Varying parameter	μ_{gj}^{k}	σ_{gj}^{k}	C_{gi}	Expected revenue (10^4 RMB)			Gap		
				SS	MS	MM	GAP(A)	GAP(B)	GAP(AB)
Mean value variations	20%	0%	0%	227.89	241.61	243.61	6.02%	0.83%	6.90%
	10%	0%	0%	215.58	228.59	232.30	6.03%	1.62%	7.76%
	0%	0%	0%	201.55	210.52	219.51	4.45%	4.27%	8.91%
	-10%	0%	0%	185.53	191.26	205.70	3.09%	7.55%	10.87%

	-20%	0%	0%	167.39	169.75	190.06	1.41%	11.96%	13.54%
Variance value variations	0%	20%	0%	199.23	207.98	216.69	4.39%	4.19%	8.76%
	0%	10%	0%	199.97	209.39	218.36	4.71%	4.28%	9.20%
	0%	0%	0%	201.55	210.52	219.51	4.45%	4.27%	8.91%
	0%	-10%	0%	202.21	211.82	221.56	4.75%	4.60%	9.57%
	0%	-20%	0%	203.16	212.60	223.21	4.65%	4.99%	9.87%
Service capacity variations	0%	0%	20%	210.51	213.72	238.10	1.52%	11.41%	13.11%
	0%	0%	10%	206.31	213.43	229.48	3.45%	7.52%	11.23%
	0%	0%	0%	201.55	210.52	219.51	4.45%	4.27%	8.91%
	0%	0%	-10%	194.51	205.39	208.63	5.59%	1.58%	7.26%
	0%	0%	-20%	184.39	194.25	195.24	5.35%	0.51%	5.88%

Table 16 shows that GAP(A) and the mean value μ_{gj}^{k} have the same increasing and decreasing trends. When the mean value μ_{gj}^{k} is increasing, the possibility of travel demand locally exceeding ticket supply increases and becomes more serious. SS has difficultly solving this problem because it can only adjust the tickets of one train. However, MS is good at solving this problem because it can simultaneously adjust the tickets of the five passenger trains and address the overall unbalanced relationship between the travel demand and ticket supply. Thus, MS can utilize increased passenger demand to generate more ticket revenue than SS, when the mean value μ_{gj}^{k} increases.

Table 16 shows that GAP(B) and the mean value μ_{gj}^{k} have opposite increasing and decreasing trends. With decreasing the mean value μ_{gj}^{k}, there will be more redundant tickets that may be wasted without effective sales strategies. Because MS provides only one fare class for homogenous seats without considering customer choice behaviour, this model cannot prevent redundant tickets from being wasted. Compared with MS, MM has more flexible pricing and pays more attention to customer choice behaviour. MM can sell redundant tickets at a relatively low price. Although the revenue from each ticket is smaller, the total revenue is improved because the lower price increases the purchase intention of each customer and thus sells more redundant tickets. Thus, with a decreasing mean value μ_{gj}^{k}, the superiority of MM over MS is more obvious.

As the variance value σ_{gj}^{k} decreases, the expected revenues of SS, MS and MM all increase slightly because the variance value σ_{gj}^{k} affects the uncertainty of passenger demand. When the uncertainty of passenger demand decreases, the control problem becomes easier, thus slightly improving the expected revenue.

The increase in the service capacity C_{gi} is equivalent to the decrease in the mean value μ_{gj}^{k}, and the decrease in the service capacity C_{gi} is equivalent to the increase in the mean value μ_{gj}^{k}. In addition, Table 16 shows the corresponding increasing and decreasing trend of GAP(A) and GAP(B) when the service capacity C_{gi} is increasing or decreasing.

GAP(AB) is more than 5% for all conditions and even greater than 10% for certain conditions, which indicates the extensive applicability and the clear superiority of MM.

5.4.2 Transition probability p^k_{jr}

The transition probability $p^k_{jr}(1)$ is based on an SP survey. To verify the efficiency of our method for varying transition probabilities, we analyse the impact of changing the transition probability.

Table 17

Changes in the customer type A transition probability p^1_{jr}

$p^k_{jr}(2)$	Customer type A ($k=1$)	$p^1_{j,1}(2)$ =99%	$p^1_{j,2}(2)$ =77.7%	$p^1_{j,3}(2)$ =76.1%
	Customer type B ($k=2$)	$p^2_{j,1}(2)$ =90%	-	-
$p^k_{jr}(1)$	Customer type A ($k=1$)	$p^1_{j,1}(1)$ =95%	$p^1_{j,2}(1)$ =80.9%	$p^1_{j,3}(1)$ =81.3%
	Customer type B ($k=2$)	$p^2_{j,1}(1)$ =90%	-	-
$p^k_{jr}(3)$	Customer type A ($k=1$)	$p^1_{j,1}(3)$ =91%	$p^1_{j,2}(3)$ =84.5%	$p^1_{j,3}(3)$ =86.5%
	Customer type B ($k=2$)	$p^2_{j,1}(3)$ =90%	-	-

In Table 17, the purchase probability of customer type B for $p^k_{jr}(2)$ and $p^k_{jr}(3)$ is the same as that for $p^k_{jr}(1)$. However, the purchase probability of customer type A for $p^k_{jr}(2)$ is more sensitive to price than that for $p^k_{jr}(1)$, and the purchase probability of customer type A for $p^k_{jr}(3)$ is less sensitive to price than that for $p^k_{jr}(1)$. The other parameters are μ^k_{gj}, σ^k_{gj} and C_{gi}. The results of the expected revenue are shown in Table 18.

Table 18

Impact of changing the Customer type A purchase probability on the expected revenue

Varying parameter	p^k_{jr}	Expected revenue (10^4 RMB)			Gap		
		SS	MS	MM	GAP(A)	GAP(B)	GAP(AB)
Varying Customer type A purchase probability	$p^k_{jr}(2)$	193.33	200.91	219.92	3.92%	9.46%	13.75%
	$p^k_{jr}(1)$	201.55	210.52	219.51	4.45%	4.27%	8.91%
	$p^k_{jr}(3)$	207.93	219.4	221.79	5.52%	1.09%	6.67%

Compared with that for $p^k_{jr}(1)$, the GAP(B) of $p^k_{jr}(2)$ increases from 4.27% to 9.46% because the customer type A purchase probability of $p^k_{jr}(2)$ is more sensitive to price compared with $p^k_{jr}(1)$. In addition, the GAP(B) of $p^k_{jr}(3)$ decreases from 4.27% to 1.09% because the customer type A purchase probability of $p^k_{jr}(3)$ is less sensitive to price than that of $p^k_{jr}(1)$. This increase and decrease are both obvious, which indicates that the impact of the sensitivity of price on GAP(B) is significant because the important advantage of our method is increasing the revenue by utilizing the ticket price to stimulate the purchase intention of customers.

6 Conclusions and further research

This study has developed a seat inventory control method of homogeneous seats for the China HSR. The mathematical formula we proposed can consider multiple trains with different train stop plans as a whole to optimize and control inventory, and this formula also incorporates differential pricing of homogeneous seats and customer choice behaviour. An efficient approach that takes advantage of both the mathematic approach and the ABC optimization framework is provided to solve the nonlinear integer programming model. Numerical examples show the efficiency of the proposed algorithm. A case study of the Beijing-Shanghai HSR demonstrates the superiority of our method over the current control method of China HSR in a variety of scenarios.

The improvement of our method in revenue ranges from 5.88% to 13.75%, which is a significant improvement for China railway passenger transportation. In addition, the superiority of optimizing multiple trains as a whole becomes clear when the local travel demand exceeds the ticket supply, and the superiority of differential pricing of homogeneous seats becomes clear when the ticket supply exceeds the travel demand or customers are more sensitive to ticket price.

This paper focuses on the revenue management issue of homogenous seats. Although the proportion of homogenous seats is greater than 80% in each China high-speed train, there are still a certain number of heterogeneous seats. How to describe the customer preference order of customers for heterogeneous seats is a further research direction.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix A Passenger Demand Data

Table 19

Mean value μ_{gj}^{k} and variance value σ_{gj}^{k} of G - I in each O-D itinerary

$\mu_{gj}^{1}, \mu_{gj}^{2}$	Nanjing	Shanghai
Beijing	142 47	528 78
Nanjing		226 48
$\sigma_{gj}^{1}, \sigma_{gj}^{2}$	Nanjing	Shanghai
Beijing	32 13	45 45
Nanjing		34 7

Table 20

Mean value μ_{gj}^{k} and variance value σ_{gj}^{k} of G - II in each O-D itinerary

$\mu_{gj}^{1}, \mu_{gj}^{2}$	Tianjin	Dezhou	Jinan	Xuzhou	Nanjing	Suzhou	Shanghai
Beijing	61 15	78 4	47 15	48 5	43 4	177 14	190 23
Tianjin		8 1	6 4	17 1	79 3	10 4	38 4
Dezhou			22 1	5 1	6 1	14 2	20 1

	Tianjin	Dezhou	Jinan	Xuzhou	Nanjing	Suzhou	Shanghai
Jinan				41 3	14 1	21 1	17 16
Xuzhou					69 3	32 1	73 3
Nanjing						81 5	37 13
Suzhou							80 15
$\sigma_{gj}^{1},\sigma_{gj}^{2}$	Tianjin	Dezhou	Jinan	Xuzhou	Nanjing	Suzhou	Shanghai
Beijing	39 11	19 5	31 10	21 5	39 5	45 9	45 7
Tianjin		5 5	5 5	5 5	15 5	5 5	21 5
Dezhou			5 5	5 5	5 5	5 5	6 5
Jinan				10 5	5 5	5 5	16 15
Xuzhou					14 5	6 5	11 5
Nanjing						17 5	44 13
Suzhou							28 9

Table 21

Mean value μ_{gj}^{k} and variance value σ_{gj}^{k} of $\mathrm{G-III}$ in each O-D itinerary

$\mu_{gj}^{1},\mu_{gj}^{2}$	Tianjin	Dezhou	Jinan	Tai'an	Xuzhou	Nanjing	Wuxi	Kushan	Shanghai
Beijing	172 13	65 19	96 17	104 8	71 44	211 26	51 3	80 18	261 35
Tianjin		79 5	21 1	20 1	12 1	9 2	2 1	1 0	65 3
Dezhou			48 2	20 2	8 2	56 12	3 0	11 1	86 4
Jinan				99 21	23 2	36 3	5 1	40 2	23 5
Tai'an					24 3	67 3	3 1	16 1	82 15
Xuzhou						120 10	36 11	65 3	143 8
Nanjing							79 4	131 14	111 36
Wuxi								49 4	58 5
Kushan									98 8
$\sigma_{gj}^{1},\sigma_{gj}^{2}$	Tianjin	Dezhou	Jinan	Tai'an	Xuzhou	Nanjing	Wuxi	Kushan	Shanghai
Beijing	45 7	41 12	45 11	45 5	41 31	45 23	12 5	45 12	45 15
Tianjin		14 5	7 5	5 5	5 5	5 5	5 5	5 0	13 5
Dezhou			10 5	6 5	5 5	9 8	5 0	5 5	14 5
Jinan				32 12	9 5	7 5	5 5	8 5	17 5
Tai'an					8 5	18 5	5 5	5 5	34 10
Xuzhou						45 7	10 7	21 5	45 5
Nanjing							19 5	45 8	45 44
Wuxi								13 5	18 5
Kushan									41 5

Table 22

Mean value μ_{gj}^{k} and variance value σ_{gj}^{k} of $\mathrm{G-IV}$ in each O-D itinerary

$\mu_{gj}^{1},\mu_{gj}^{2}$	Dezhou	Jinan	Tai'an	Zao-zhuang	Chu-zhou	Nanjing	Dan-yang	Suzhou	Kushan	Shanghai
Beijing	154 17	156 48	144 23	88 8	122 14	120 22	76 8	160 17	134 9	309 31
Dezhou		39 4	35 5	3 1	18 5	46 5	6 1	51 14	70 3	65 7
Jinan			102 7	41 17	19 2	27 4	6 1	33 3	66 2	22 3

	Dezhou		Jinan		Tai'an		Zao-zhuang		Chu-zhou		Nanjing		Dan-yang		Suzhou		Kushan		Shanghai	
Tai'an							27	1	11	2	71	3	6	1	27	4	20	4	112	16
Zaozhuang									25	5	20	1	0	0	12	3	27	2	37	3
Chuzhou											138	9	11	1	59	8	62	17	65	6
Nanjing													11	3	78	10	96	15	64	9
Danyang															15	1	9	1	33	2
Suzhou																	10	1	163	10
Kushan																			111	12
$\sigma_{gj}^1, \sigma_{gj}^2$	Dezhou		Jinan		Tai'an		Zao-zhuang		Chu-zhou		Nanjing		Dan-yang		Suzhou		Kushan		Shanghai	
Beijing	45	12	45	31	45	14	27	5	30	8	45	14	16	5	45	12	39	6	45	11
Dezhou			13	5	12	5	5	5	5	5	21	5	5	5	18	9	16	5	23	5
Jinan					23	5	23	12	9	5	8	5	5	5	9	5	12	5	18	5
Tai'an							5	5	5	5	17	5	5	5	12	5	9	5	25	11
Zaozhuang									9	5	5	5	0	0	6	5	5	5	19	5
Chuzhou											45	6	5	5	31	5	26	12	35	5
Nanjing													6	5	45	6	29	9	45	15
Danyang															5	5	5	5	9	5
Suzhou																	5	5	45	6
Kushan																			29	7

Table 23

Mean value μ_{gj}^k and variance value σ_{gj}^k of G - V in each O-D itinerary

μ_{gj}^1, μ_{gj}^2	Jinan		Nanjing		Shanghai	
Beijing	112	11	247	39	389	39
Jinan			88	9	121	20
Nanjing					362	135
$\sigma_{gj}^1, \sigma_{gj}^2$	Jinan		Nanjing		Shanghai	
Beijing	45	7	45	23	45	31
Jinan			21	6	45	9
Nanjing					45	45

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