Discovering the underlying topological space of extracted features

Hamid Karim[1] Harrison LeFrois[2]

[1]Department of Computer Science and Engineering
Michigan State University

[2]Department of Mathematics
Michigan State University

April 2017

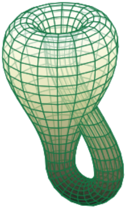
Table of Contents

The Big Problem

What does our data look like?

Source:
www.gnuplotting.org/klein-bottle/

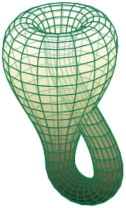
The Big Problem



What does our data look like?

- Data is maybe from a manifold?

Source: www.gnuplotting.org/klein-bottle/

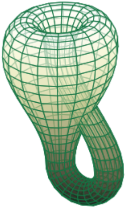


What does our data look like?

- Data is maybe from a manifold?
- Manifold learning makes this assumption

Source: www.gnuplotting.org/klein-bottle/

The Big Problem

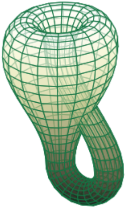


What does our data look like?

- Data is maybe from a manifold?
- Manifold learning makes this assumption
- Question 1: What if our data isn't from a manifold? Could the features be parametrized by a manifold?

Source: www.gnuplotting.org/klein-bottle/

The Big Problem



What does our data look like?

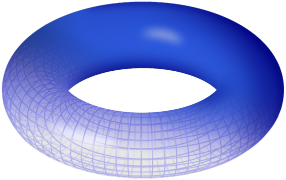
- Data is maybe from a manifold?
- Manifold learning makes this assumption
- Question 1: What if our data isn't from a manifold? Could the features be parametrized by a manifold?
- Question 2: Neural networks often become a black box. What is happening inside?

Source: www.gnuplotting.org/klein-bottle/

The Big Problem

Why do we care?

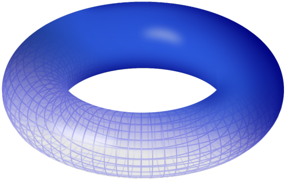
Source: https://en.wikipedia.org/wiki/Torus



Why do we care?

- Manifolds have nice mathematical structure (locally Euclidean, lots of structure, etc)

Source: https://en.wikipedia.org/wiki/Torus

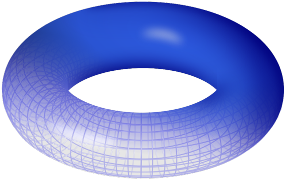


Why do we care?

- Manifolds have nice mathematical structure (locally Euclidean, lots of structure, etc)
- Can assign coordinates on the manifold rather than in $\mathbb{R}^n$

Source: https://en.wikipedia.org/wiki/Torus

The Big Problem



Source: https://en.wikipedia.org/wiki/Torus

Why do we care?

- Manifolds have nice mathematical structure (locally Euclidean, lots of structure, etc)
- Can assign coordinates on the manifold rather than in $\mathbb{R}^n$
- This might help with prediction!

Tools

Topology and Machine Learning

- Persistent homology: used to deduce topological structure from data points. Comes from Topological Data Analysis (TDA)
- Convolutional Neural Network (CNN): Great for feature extraction from images

Quick Topology Intro

Concerned with properties that don't change under continuous deformations

- Like stretching, shrinking, and bending
- But NOT cutting, gluing, etc.

Example: Circle = Square = Triangle


Source:
https://www.shapeways.com/blog/archives/
a-3d-printed-topology-joke.html

Quick Topology Intro

Homology is a sequence of groups (an algebraic structure) that keeps track of "holes" and their higher dimensional analogues. So if X is a topological space, then:

- $H_0(X)$ counts the number of connected pieces
- $H_1(X)$ counts the number of loops
- $H_2(X)$ counts the number of "voids" (like the inside of a beach ball)

Inspiration

Persistent homology of rotations of everyday objects

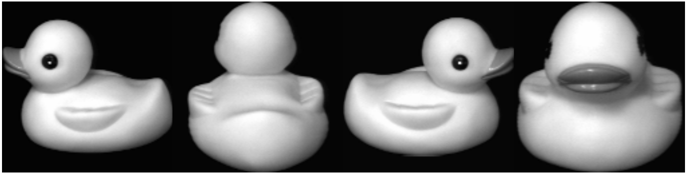
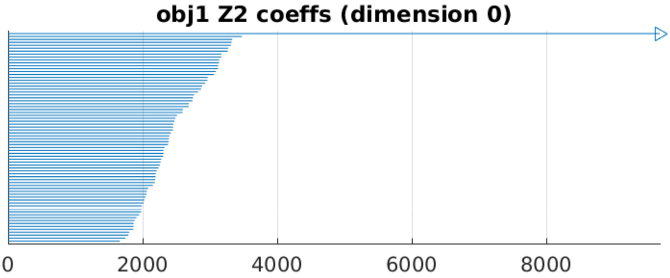
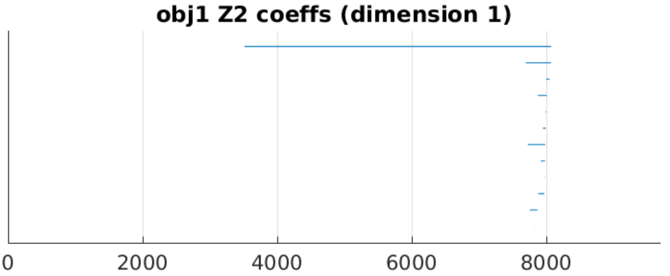


Figure: Rotation of rubber duck

Inspiration

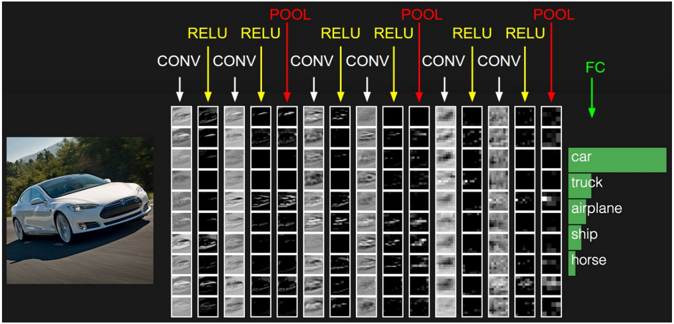




Our Data

Needed thousands of photos to train a neural network - so we used the CASIA-Webface dataset. It is a public dataset consisting of of 494,414 images [2].

Convolutional neural network

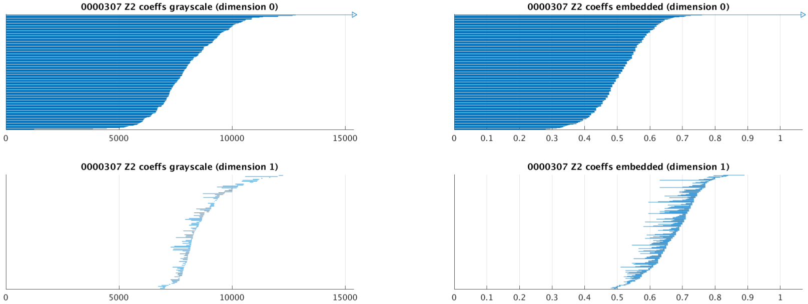


Architecture

layer	size-in	size-out	kernel	param
conv1	220×220×3	110×110×64	7×7×3, 2	9K
pool1	110×110×64	55×55×64	3×3×64, 2	0
rnorm1	55×55×64	55×55×64		0
conv2a	55×55×64	55×55×64	1×1×64, 1	4K
conv2	55×55×64	55×55×192	3×3×64, 1	111K
rnorm2	55×55×192	55×55×192		0
pool2	55×55×192	28×28×192	3×3×192, 2	0
conv3a	28×28×192	28×28×192	1×1×192, 1	37K
conv3	28×28×192	28×28×384	3×3×192, 1	664K
pool3	28×28×384	14×14×384	3×3×384, 2	0
conv4a	14×14×384	14×14×384	1×1×384, 1	148K
conv4	14×14×384	14×14×256	3×3×384, 1	885K
conv5a	14×14×256	14×14×256	1×1×256, 1	66K
conv5	14×14×256	14×14×256	3×3×256, 1	590K
conv6a	14×14×256	14×14×256	1×1×256, 1	66K
conv6	14×14×256	14×14×256	3×3×256, 1	590K
pool4	14×14×256	7×7×256	3×3×256, 2	0
concat	7×7×256	7×7×256		0
fc1	7×7×256	1×32×128	maxout p=2	103M
fc2	1×32×128	1×32×128	maxout p=2	34M
fc7128	1×32×128	1×1×128		524K
L2	1×1×128	1×1×128		0
total				140M

Shape of Data

Helena Bonham Carter - 254 images - Original features on left (pixels), 128 features on right (embedding from CNN)



New loss function

- We are basically interested in function f which embeds our data X in d-dimensional Euclidean space.
  $\mathcal{C}^i \leftarrow$ Data with same labels as i-th instance
  $C^i \leftarrow$ centroid of $\mathcal{C}^i$ in embedded space

$$
\begin{aligned}
\text{[1]Previous,} \quad L_{i,j,k} =& \lambda d(f^i, f^j) + (1-\lambda)[1 + d(f^i, f^j) - d(f^i, f^j)]_+ \\
\text{Current,} \quad L_{i,j,k} =& d(f^i, f^j) + [1 + d(f^i, f^j) - d(f^i, f^k)]_+ \\
& + d(f^i, C^i) + d(f^j, C^i) + d(f^k, C^k) \\
& + [1 - d(C^i, C^k)]_+ \\
\text{such that,} \quad C^i =& \frac{1}{|\mathcal{C}^i|} \sum_{s \in \mathcal{C}^i} f^s \quad \text{centroid in embedded space}
\end{aligned}
$$

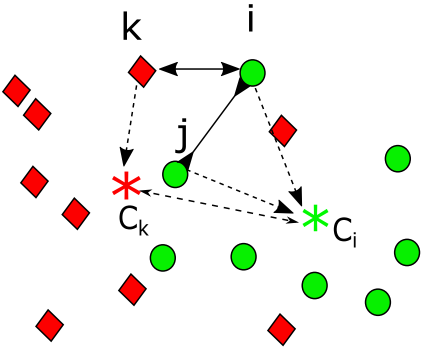
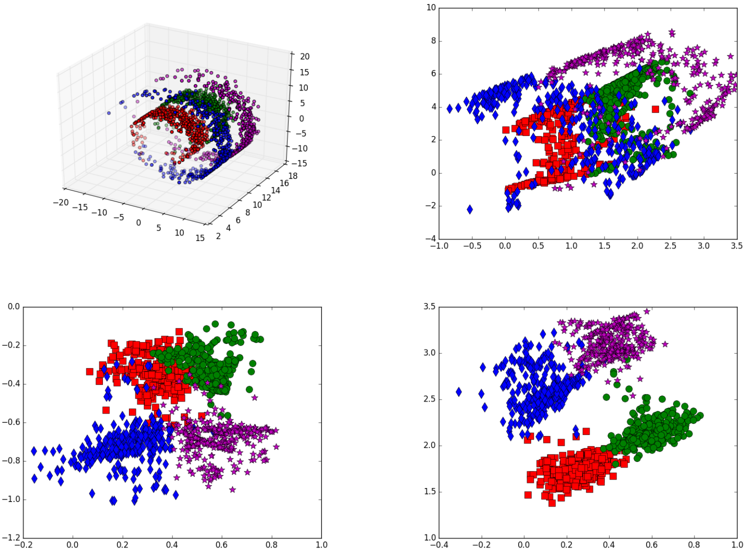


Figure: Graphical representation of loss function in the embedded space

Results on a toy data



Conclusions and Future Work

Conclusions thus far:

- From homology perspective, data appears to be connected with no loops
- Maybe other structure - like a dendrite? Clusters?

Future work:

- Apply more tools from TDA (mapper algorithm - check for dendrite structures)
- Other standard datasets (MNIST, natural images, spoken language) with appropriate NN architectures (CNN, RNN, etc)
- Scale space instead of pixel space for images

Bibliography

Florian Schroff, Dmitry Kalenichenko, and James Philbin. "Facenet: A unified embedding for face recognition and clustering". In: *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 2015, pp. 815–823.

Dong Yi et al. "Learning Face Representation from Scratch". In: *CoRR* abs/1411.7923 (2014). URL: http://arxiv.org/abs/1411.7923.