

# اصول علم ربات – اسلاید شانزدهم

Fundamentals of Robotics – Slide 16

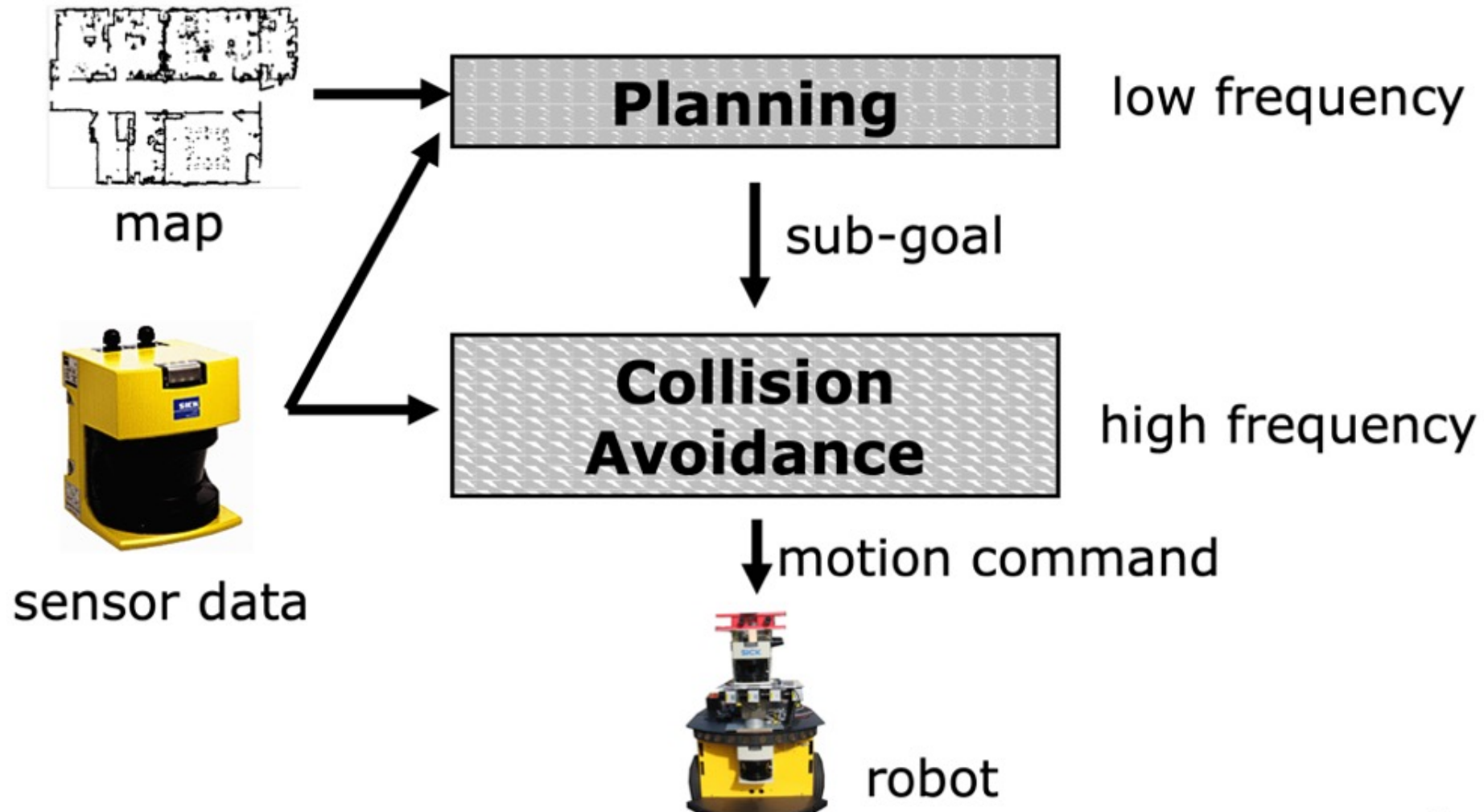
## Navigation in Presence of Obstacles Maps, Local Maps

دکتر مهدی جوانمردی

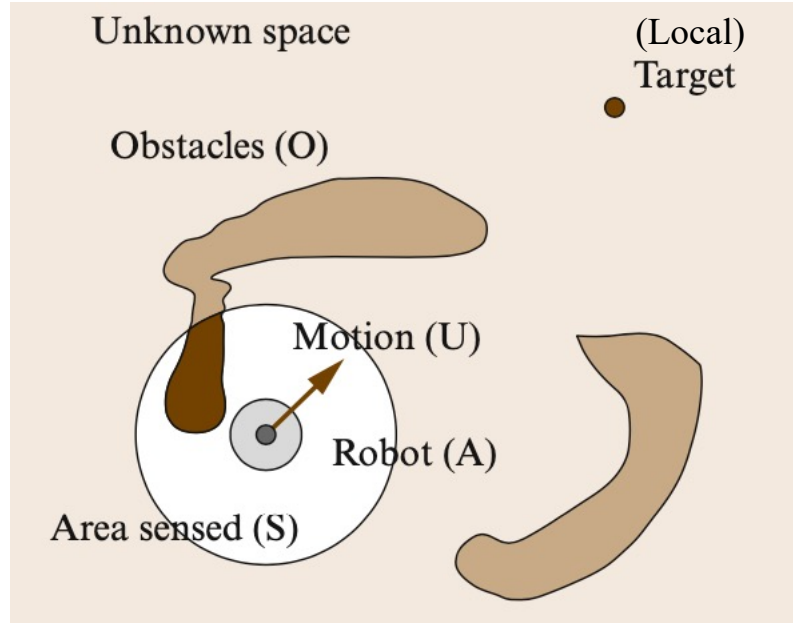
زمستان ۱۴۰۰ – بهار ۱۴۰۱

[slides adapted from Gianni Di Caro, @CMU with permission]

# Standard two-layered architecture for map-based planning & navigation



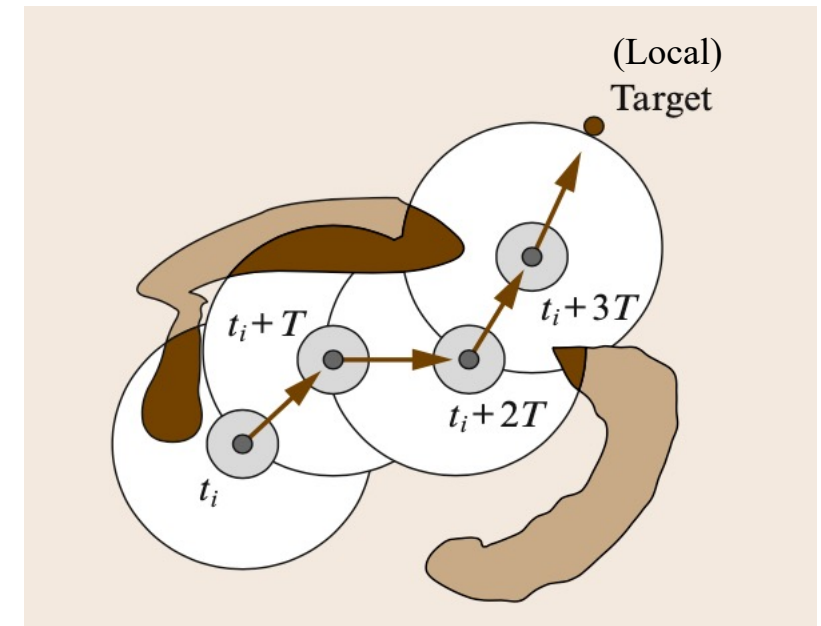
# Obstacle Avoidance / Local motion planning



- **Local target:** next waypoint, pose in a plan
- Use sensors to acquire information about surrounding environment
- Plan (or re-plan) the path in real-time avoiding obstacles
- Aim to reach target as fast and as reliably possible

❖ **Obstacle Avoidance / Local Planner problem:** computing a motion control that avoid collisions with the obstacles as observed by sensors, whilst driving the robot towards the target location.

➤ Result of applying this technique online, at each sample time, is a sequence of motions that drive the vehicle free of collisions to the target



# Taxonomy of obstacle avoidance algorithms

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- ❖ Methods that **compute the motion in one step** and that **do it in more than one**
  - *Sensors → Motion*: One-step methods directly reduce the sensor information to motion control
    - Various **heuristics**, e.g., **Bug algorithms** (reactive algorithms)
    - Use **physical analogies** assimilate the obstacle avoidance to a known physical problem, e.g., **Potential field method**
  - *Sensors → Intermediate Information Building → Motion*: Methods with more than one step compute some intermediate information, which is processed next to obtain the motion.
- ✓ The methods of **subset of controls** compute an intermediate **set of motion controls**, and next choose one of them (the *best*) as a solution.
  - Subset of **motion directions**, e.g., **Vector Field Histogram**
  - Subset of **velocity controls**, e.g., **Dynamic Window Adaptation**

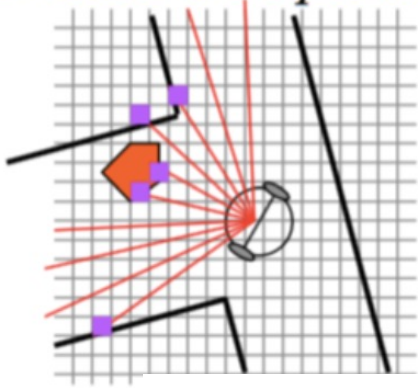


# Vector Field Histogram

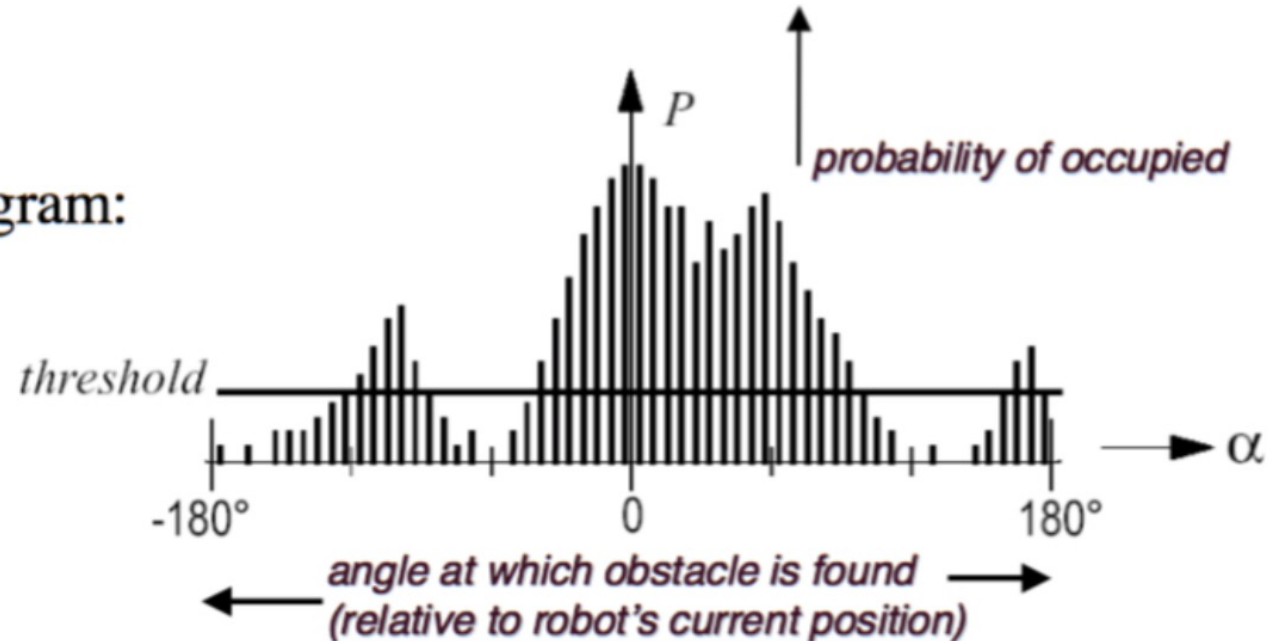
- Environment represented in a grid (2 DOF)

*Koren & Borenstein, ICRA 1990*

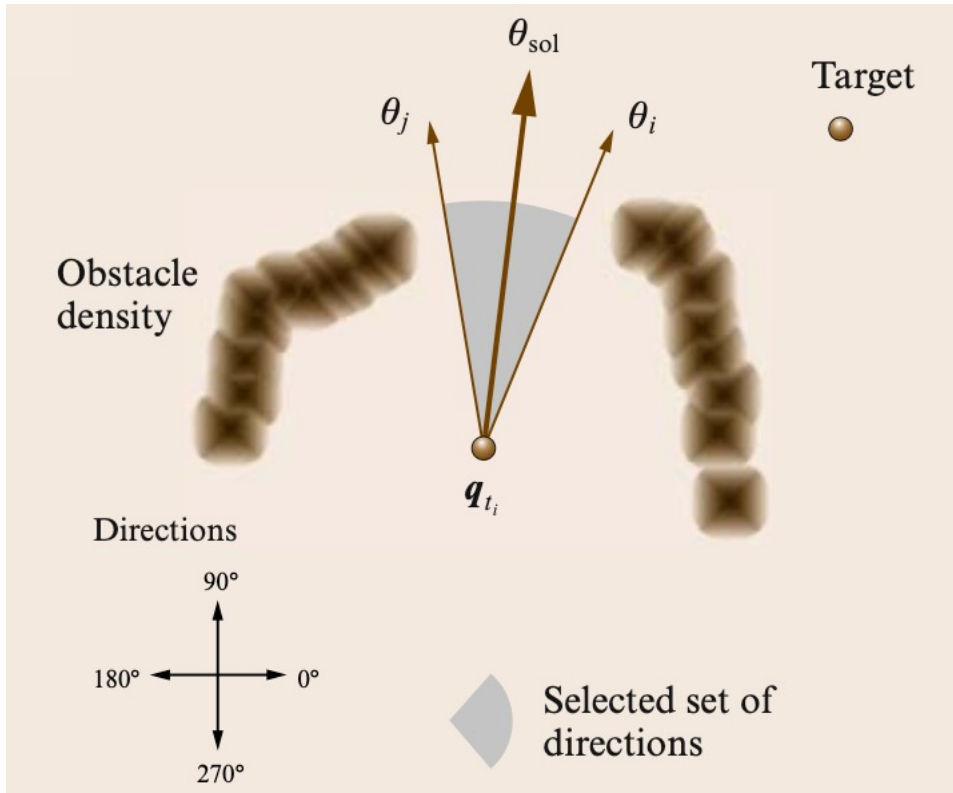
➤ *cell values are equivalent to the probability that there is an obstacle*



- Generate polar histogram:



# Vector Field Histogram



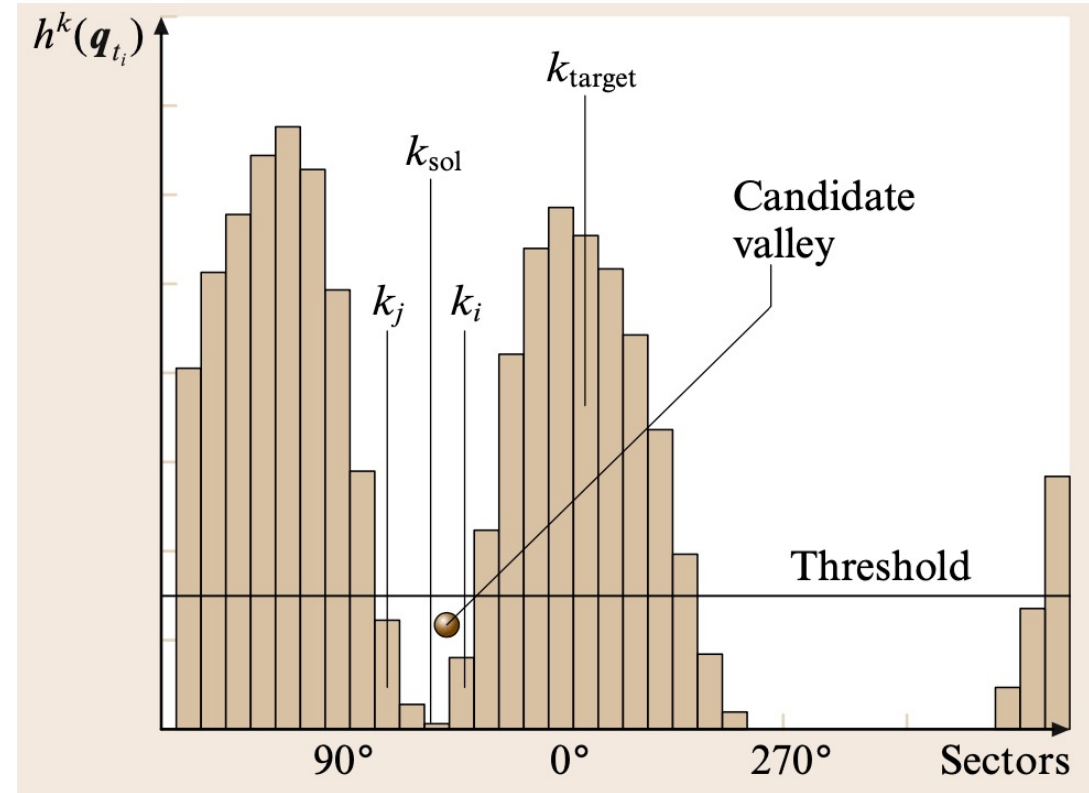
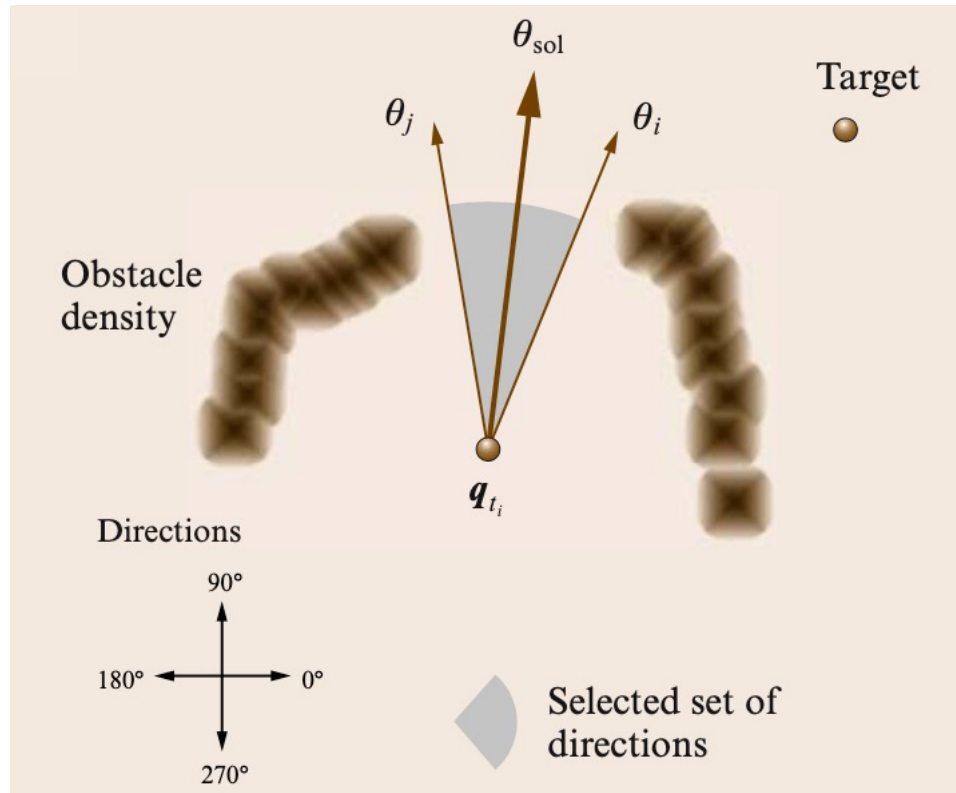
- Space is divided into sectors  $k = 1, \dots, N$  from robot location.
- Using sensor data, a polar histogram  $H$  is constructed around the robot, where each component represents the obstacle polar density in the corresponding sector.
- Function mapping observed obstacle distribution in sector  $k$  to a density value  $h^k(q_{t_i})$  in the histogram representation:

$$h^k(q_{t_i}) = \int_{\Omega_k} P(\mathbf{p})^n \left( 1 - \frac{d(q_{t_i}, \mathbf{p})}{d_{\max}} \right)^r d\mathbf{p}$$

where  $\Omega_k$  is the set of points  $\mathbf{p}$  falling within a certain maximal distance from the robot

$h^k(q_{t_i}) \propto$  probability that a point is occupied by an obstacle  $\times$  factor that increases as distance to point decreases

# Vector Field Histogram: Step 1, select candidate directions

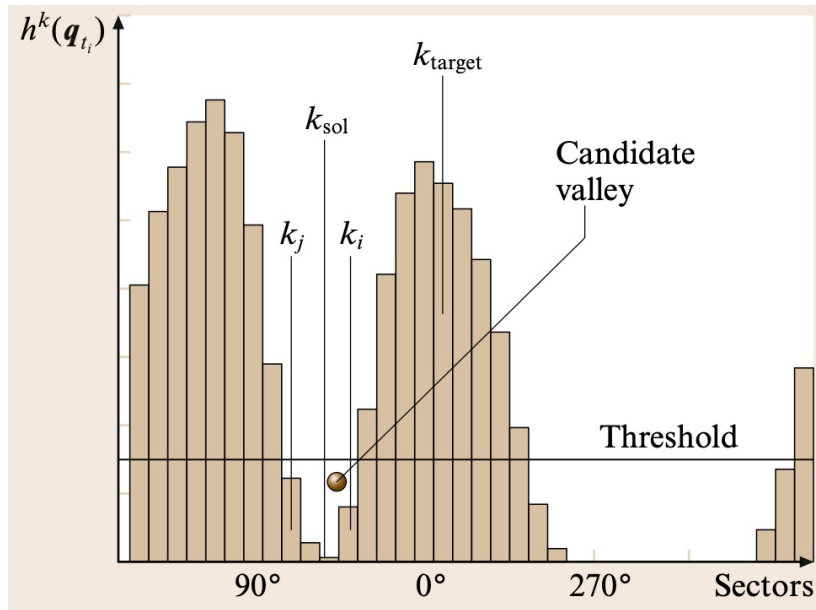
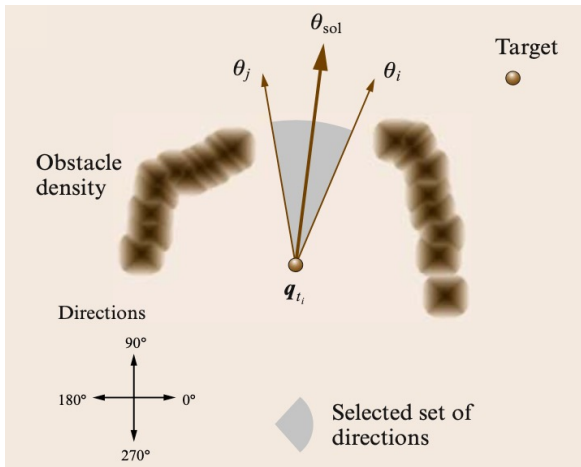


$$h^k(q_{t_i}) = \int_{\Omega_k} P(p)^n \left( 1 - \frac{d(q_{t_i}, p)}{d_{\max}} \right)^r dp$$

- ✓ **Set of candidate directions:** set of adjacent components with lower density than a given threshold, and close to the component that contains the target direction

- Candidate valleys

# Vector Field Histogram: Step 2, select motion

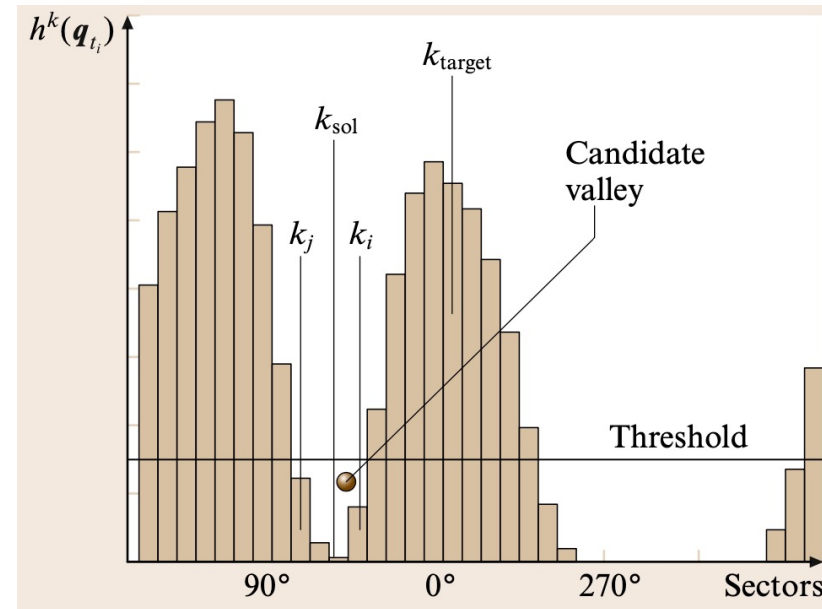
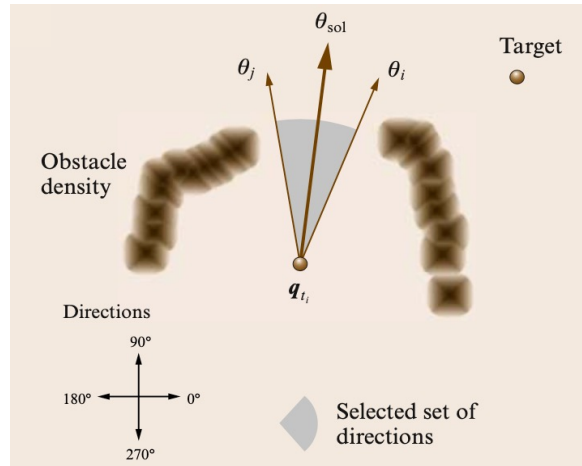


- ✓ **Set of candidate directions:** set of adjacent components with lower density than a given threshold, and close to the component that contains the target direction
- Select the *best* direction (i.e., sector)  $k_{sol}$ 
  - Heuristic based on three cases

- ✓ **Case 1:** goal sector in the selected valley  $\rightarrow k_{sol} = k_{target}$  where  $k_{target}$  is the sector that contains the goal location
- ✓ **Case 2:** goal sector not in the selected valley and the number of sectors in the valley is greater than a threshold  $m$  (e.g.,  $m = 8 \rightarrow$  valley of  $\approx 45^\circ \rightarrow$  large valley)  $\rightarrow k_{sol} = k_{closer} \pm \frac{m}{2}$  where  $k_{closer}$  is the sector of the valley closer to  $k_{target}$
- ✓ **Case 3:** goal sector not in the selected valley and number of sectors in the valley is lower than  $m$  (i.e., a narrow valley)  $\rightarrow k_{sol} = \frac{k_i + k_j}{2}$  where  $k_i$  and  $k_j$  are the extremal sectors of the valley



# Vector Field Histogram: Step 2, select motion



- ✓ **Case 3:** goal sector not in the selected valley and number of sectors in the valley is lower than  $m = 8$  (i.e., a narrow valley)  
→  $k_{sol} = \frac{k_i + k_j}{2}$  where  $k_i$  and  $k_j$  are the extreme sectors of the valley.
  - The result is a sector  $k_{sol}$  whose **bisector angle value** is the direction solution for the direction to move  $\theta_{sol}$
  - The linear velocity  $v$  is set inversely proportional to the distance to the closest obstacle.
- ✓ The **control** is  $u_t = (v_{sol}, \theta_{sol}) \rightarrow (v_{sol}, \omega_{sol} = \dot{\theta}_{sol})$

# Dynamic Window Adaptation (DWA): velocity space

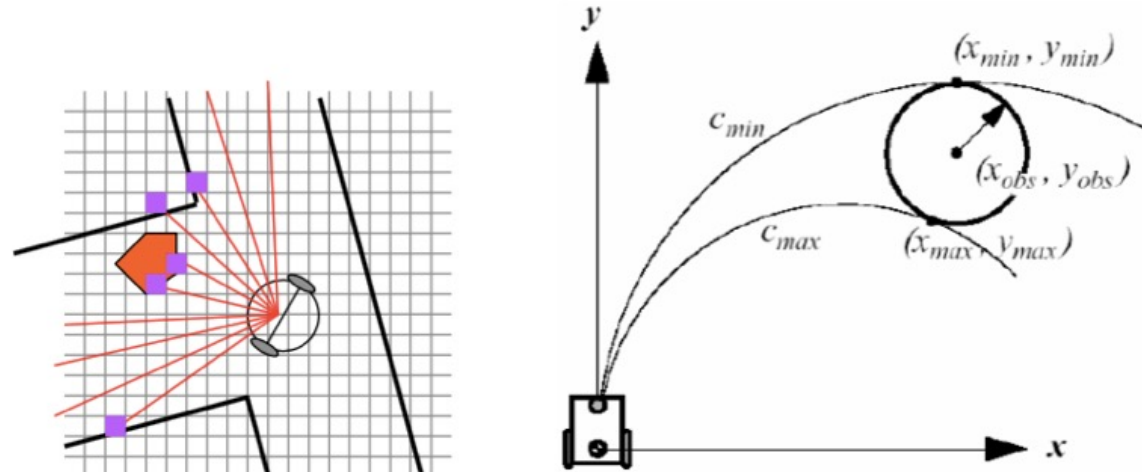
Basic ideas:

- Robot instantaneously moves over **circular trajectories**
- Radius is defined by  $c = \frac{\omega}{v}$
- What are the **velocities** that determine **obstacle-free** and **short** circular trajectories (toward target)?
- → **Work in velocity space!**

## Obstacle Avoidance: **Basic Curvature Velocity Methods** (CVM)

*Simmons et al.*

- Adding **physical constraints** from the robot and the environment on the **velocity space** ( $v, \omega$ ) of the robot
  - Assumption that robot is traveling on arcs ( $c = \omega / v$ )
  - Constraints:  $-v_{max} < v < v_{max}$      $-\omega_{max} < \omega < \omega_{max}$
  - Obstacle constraints: Obstacles are transformed in velocity space
  - Objective function used to select the optimal speed

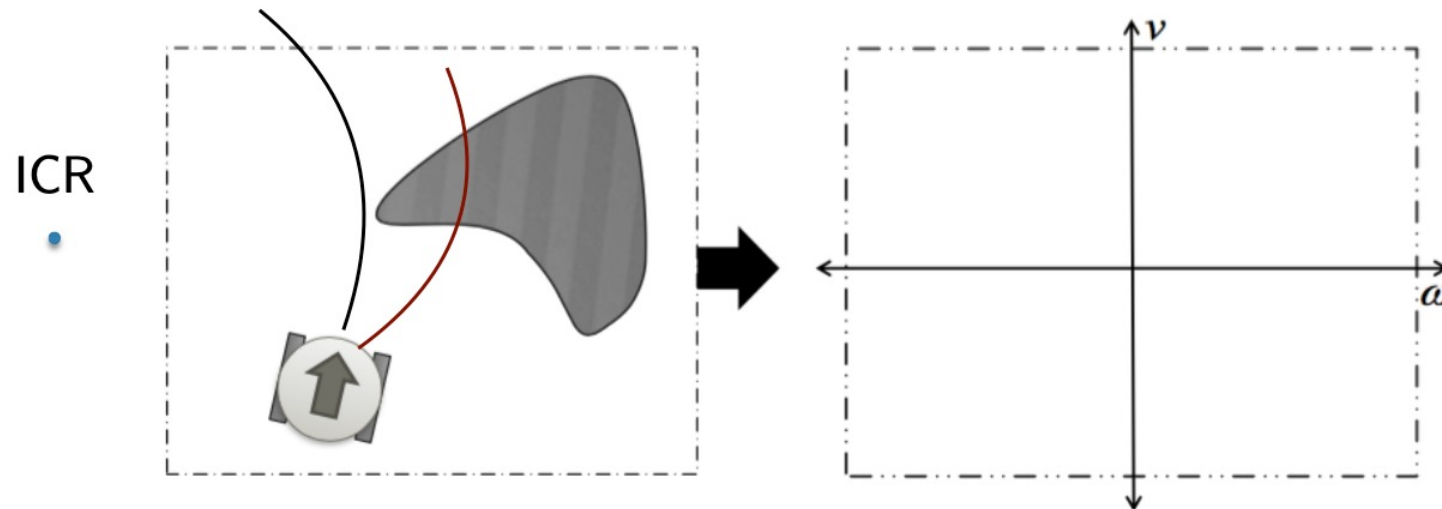


# Dynamic Window Adaptation (DWA): velocity space

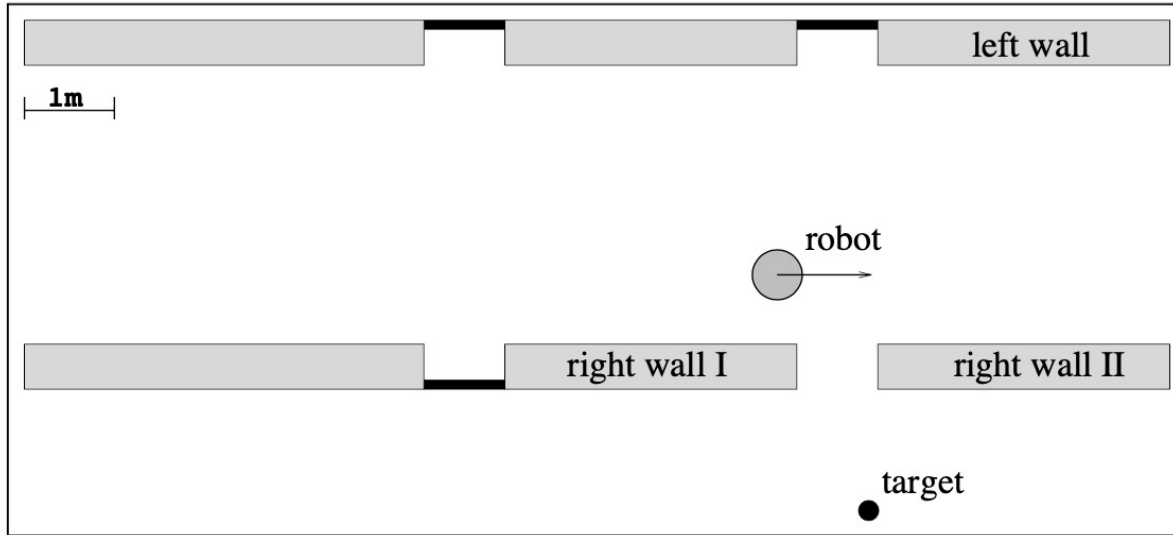
## Dynamic Window Approach (DWA, 1987)

- Robot is assumed to instantaneously move on circular arcs  $(v, \omega)$
- 2D evidence grid is transformed into  $(v, \omega)$  input-space based on robot deceleration capabilities / kino-dynamics, leading to  $V_o$
- Static window  $V_s$  constrains velocities
- Dynamic window  $V_d$  accounts for vehicle dynamics
- Selection of  $(v, \omega)$ -pair within  $V_r = V_o \cap V_s \cap V_d$  maximizing objective containing heading, distance to goal and velocity terms

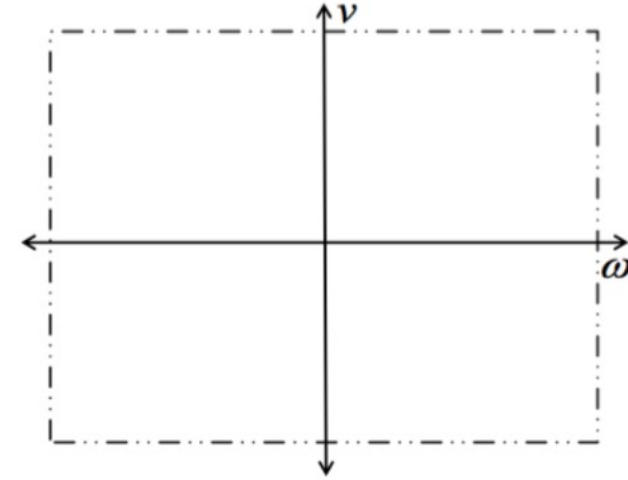
D. Fox, W. Burgard, S. Thrun, The Dynamic Window Approach to Collision Avoidance, IEEE Robotics & Automation Magazine 4(1):23 - 33 · April 1997



# Dynamic Window Adaptation (DWA): Admissible velocities



$V_s$  = Space of possible velocities for the robot



- Robot move with a curvature defined by  $(v, \omega)$
- $d(v, \omega)$  = closest distance to an obstacle on the corresponding curvature
- **Admissible velocity**  $(v, \omega)$ : the robot can stop before hitting the obstacle
- $\dot{v}_b, \dot{\omega}_b$  maximum accelerations ( $\pm$ ) available for **breakage**

Admissible velocities  $V_a$ :

$$V_a = \left\{ (v, \omega) \mid v \leq \sqrt{2 \cdot \text{dist}(v, \omega) \cdot \dot{v}_b} \wedge \omega \leq \sqrt{2 \cdot \text{dist}(v, \omega) \cdot \dot{\omega}_b} \right\}$$

# Dynamic Window Adaptation (DWA): Admissible velocities

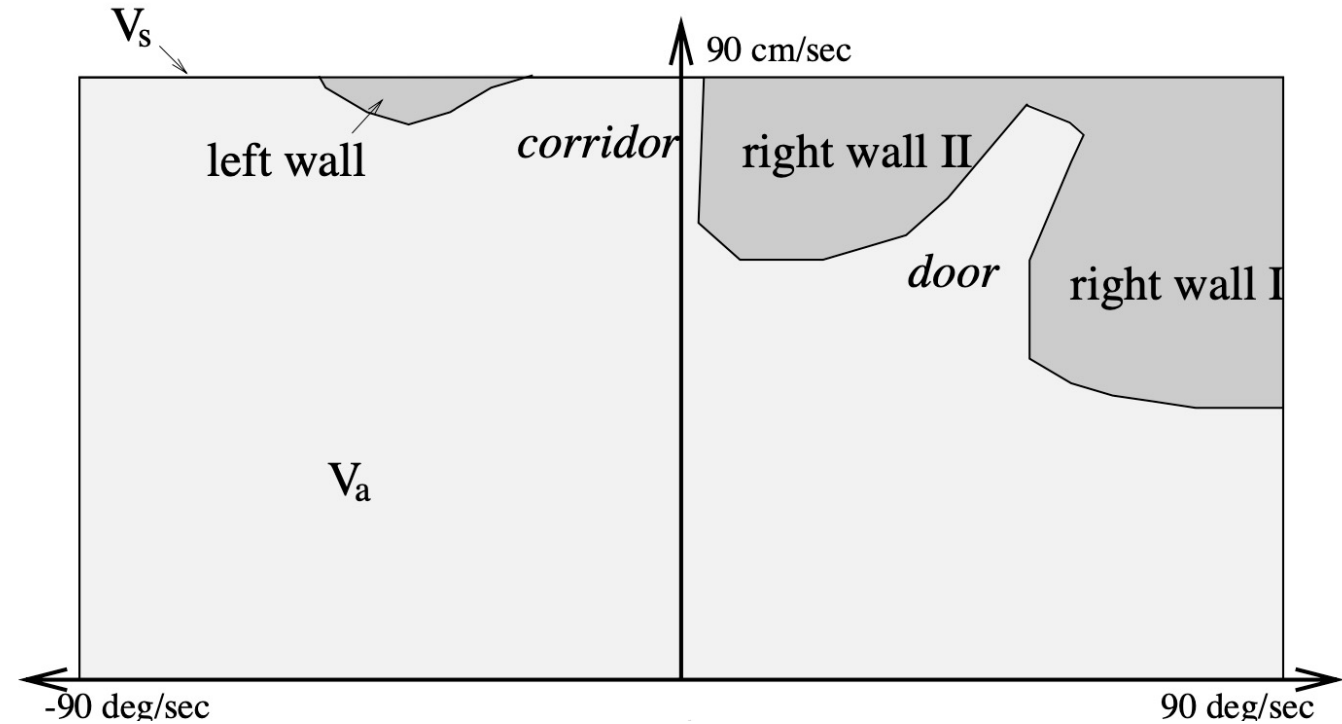
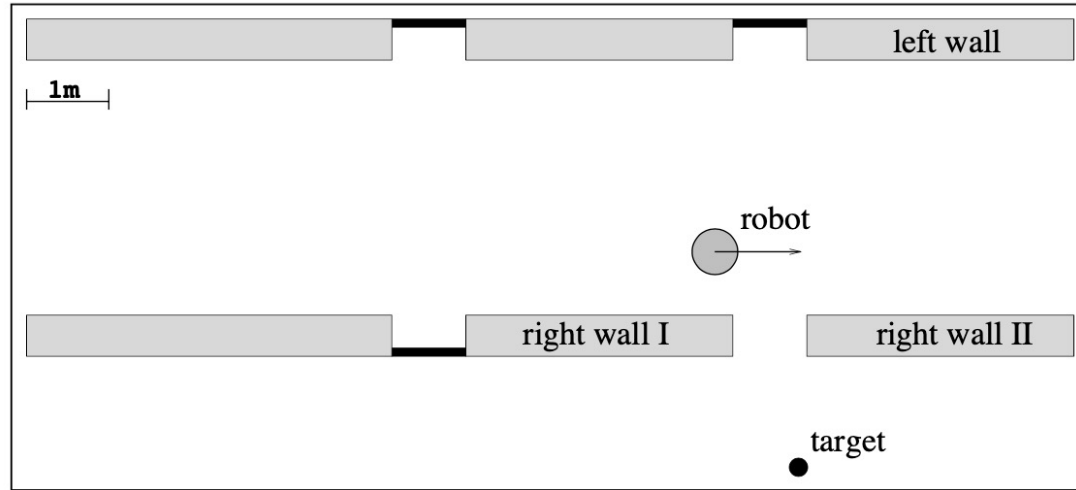
Admissible velocities  $V_a$ : 
$$V_a = \left\{ (v, \omega) \mid v \leq \sqrt{2 \cdot \text{dist}(v, \omega) \cdot \dot{v}_b} \wedge \omega \leq \sqrt{2 \cdot \text{dist}(v, \omega) \cdot \dot{\omega}_b} \right\}$$

From kinematics:

- Assuming a constant acceleration, distance  $d$  traveled in time interval  $t$  (from  $t = 0$ ) is  $d = \frac{v_f - v_i}{2} t$  where  $v_i$  is the initial velocity,  $v_f$  is the final velocity, and  $\frac{v_f - v_i}{2}$  is the average velocity
- It is also true that  $v_f = v_i + at$ , where  $a$  is the (constant) acceleration in the interval
- Substituting  $v_f$  in  $d = \frac{v_f - v_i}{2} t$  and making a few additional operations:
$$v_f = v_i^2 + 2ad$$
- In our case,  $v_f$  must be 0
- $0 = v_i^2 + 2ad$
- $v_a = \sqrt{2da}$



# DWA: Admissible velocities

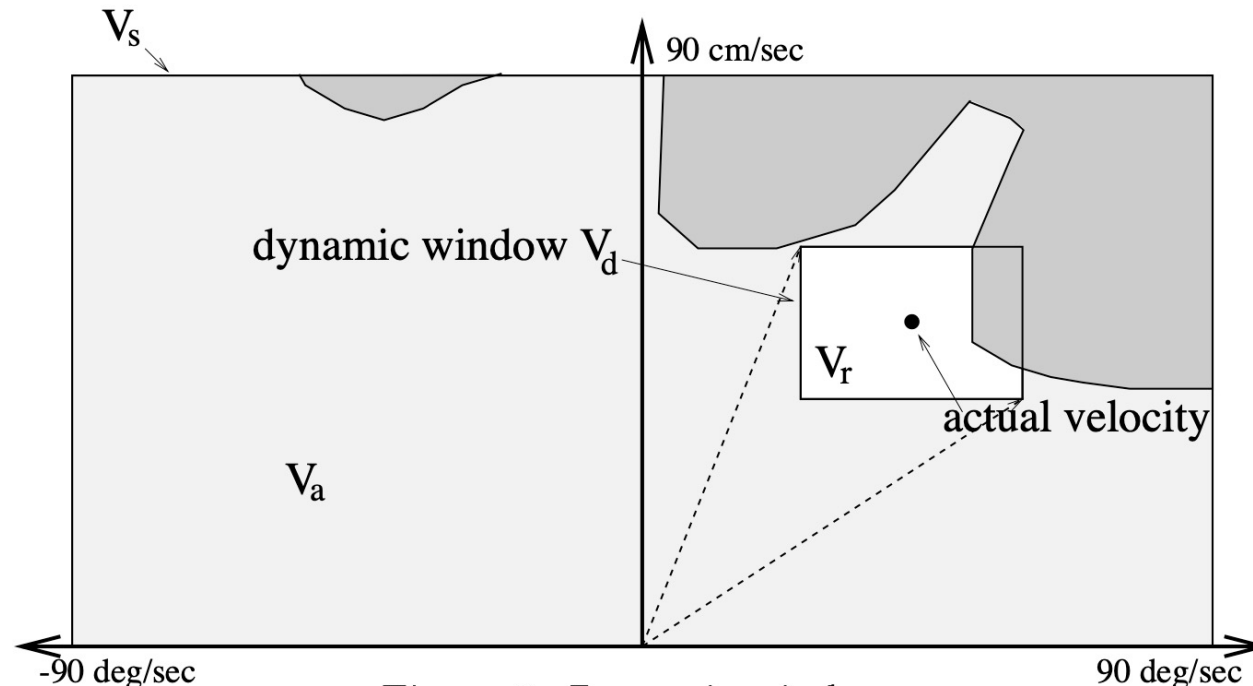


**Example 1** Again consider the example given in Figure 2. Figure 4 shows the velocities admissible in this situation given the accelerations  $\dot{v}_b = 50 \text{ cm/sec}^2$  and  $\dot{\omega}_b = 60 \text{ deg/sec}^2$ . The non-admissible velocities are denoted by the dark shaded areas. For example all velocities in area right wall II would cause a sharp turn to the right and thus cause the robot to collide with the right wall in the example situation. The non-admissible areas are extracted from real world proximity information; in this special case this information was obtained from sonar sensors (see Section 5).

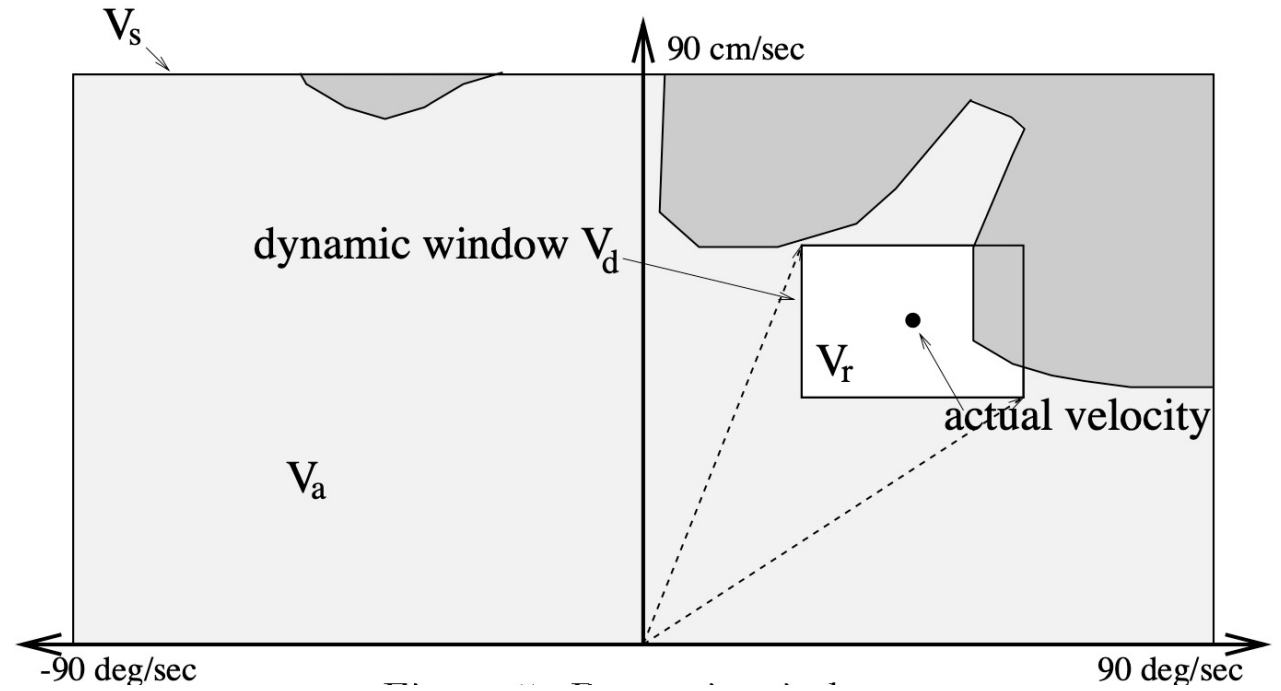
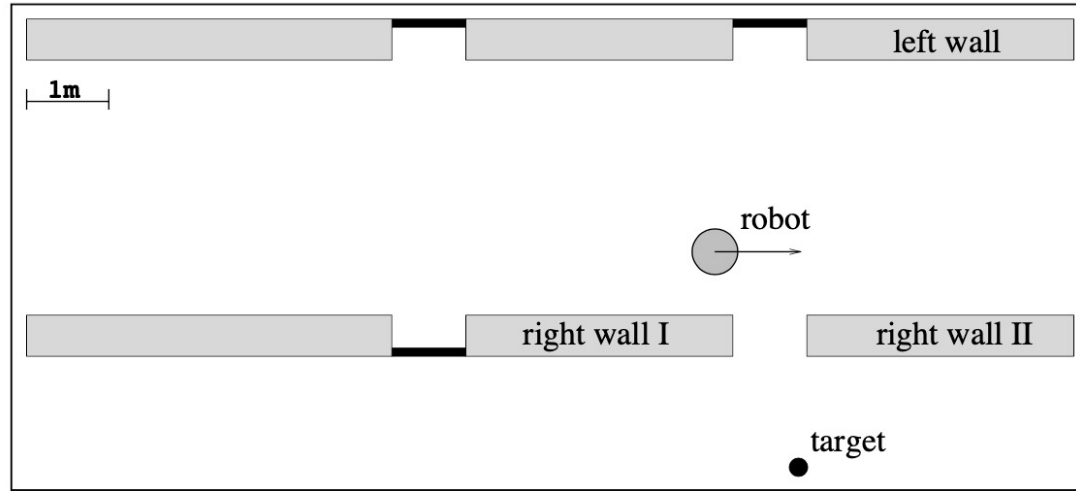
# DWA: Dynamic window

- Not all velocities can be reached, we can restrict to what velocities can be reached in the [next time window](#)
  - $t$  = Time interval during which the accelerations  $\dot{v}, \dot{\omega}$  will be applied
  - $(v_a, \omega_a)$  = actual velocity

Dynamic window velocities,  $V_d$ :  $V_d = \{(v, \omega) \mid v \in [v_a - \dot{v} \cdot t, v_a + \dot{v} \cdot t] \wedge \omega \in [\omega_a - \dot{\omega} \cdot t, \omega_a + \dot{\omega} \cdot t]\}$



# DWA: Dynamic window



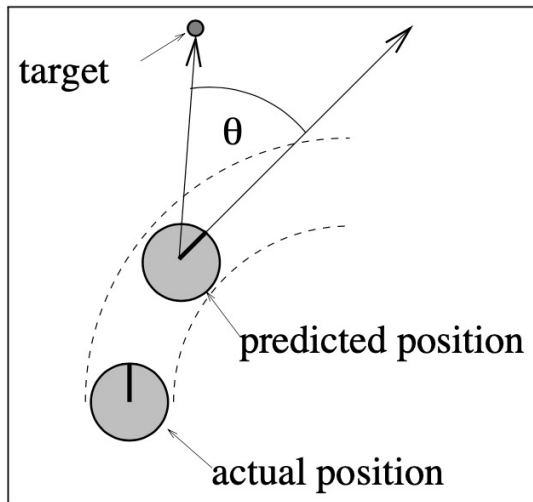
**Example 2** An exemplary dynamic window obtained in the situation shown in Figure 2 given accelerations of  $50 \text{ cm/sec}^2$  and  $60 \text{ deg/sec}^2$  and a time interval of  $0.25 \text{ sec}$  is shown in Figure 5. The two dotted arrows pointing to the corners of the rectangle denote the most extreme curvatures that can be reached.

# DWA: Set of velocities and Objective function

**Set of velocities:**  $V_r = V_s \cap V_a \cap V_d$

**Best velocities:** max over the cost function

$$G(v, \omega) = \sigma(\alpha \cdot \text{heading}(v, \omega) + \beta \cdot \text{dist}(v, \omega) + \gamma \cdot \text{velocity}(v, \omega))$$



Measure alignment  
with target:  
get to goal!

Measure Clearance:  
distance to closest  
obstacle on circular  
path → avoid obstacles

Robot velocity:  
move fast!

Use **kinematics equations!**

# DWA: In practice

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$$V_r = V_s \cap V_a \cap V_d$$

$$G(v, \omega) = \sigma(\alpha \cdot \text{heading}(v, \omega) + \beta \cdot \text{dist}(v, \omega) + \gamma \cdot \text{velocity}(v, \omega))$$

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**Algorithm 1** DWA pseudocode

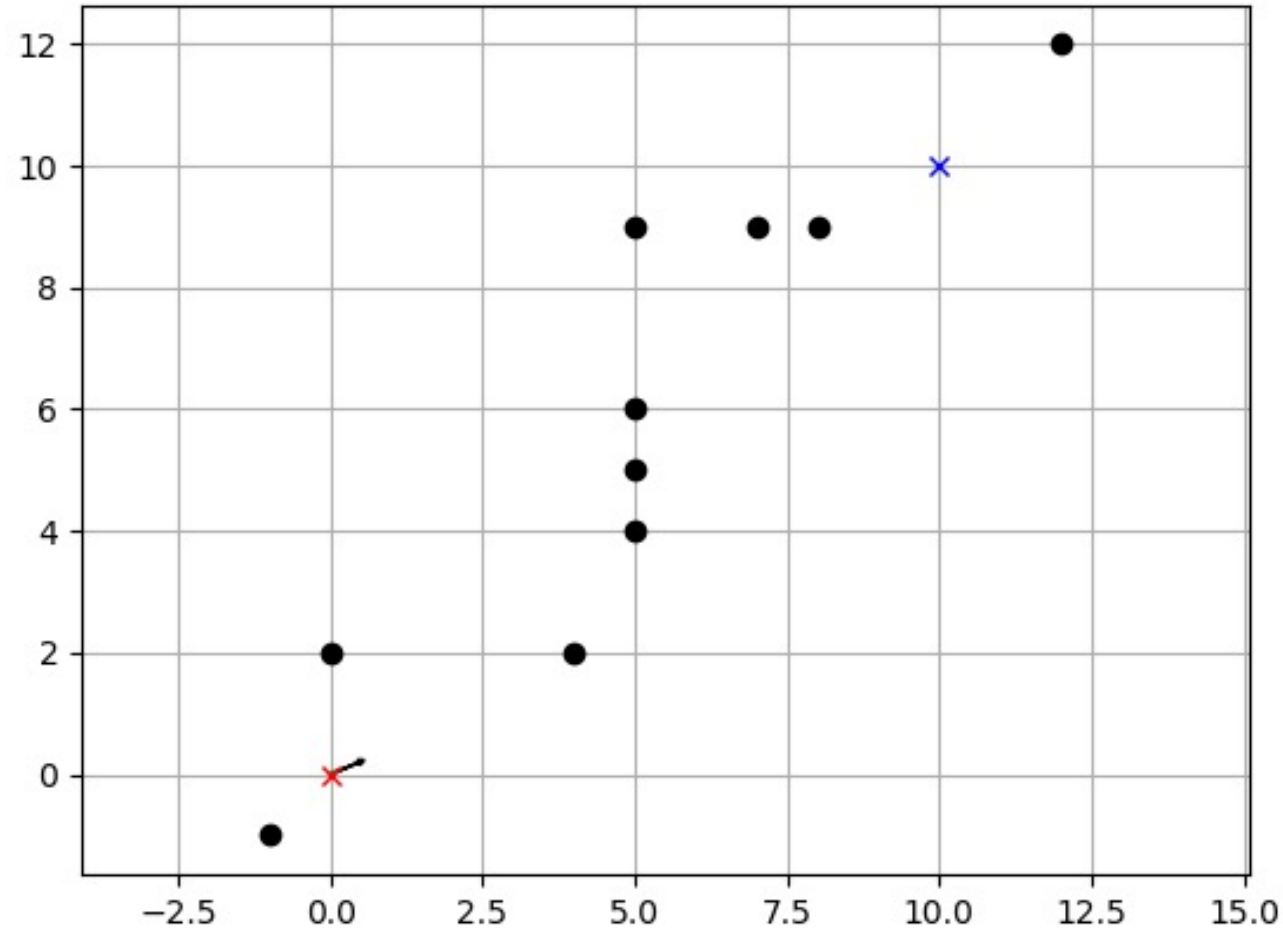
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```
1: function DWA(robotPose, robotGoal, robotModel)
2:   laserScan  $\leftarrow$  readScanner()
3:   (vallowable, wallowable)  $\leftarrow$  generateWindow(robotVW, robotModel)
4:   for (each v in vallowable) do
5:     for (each w in wallowable) do
6:       dist  $\leftarrow$  findDist(v, w, laserScan, robotModel)
7:       breakDist  $\leftarrow$  calculateBreakingDistance(v)
8:       if (dist > breakDist) then
9:         cost  $\leftarrow$  costFunction
10:        if (cost > optimal) then
11:          bestv  $\leftarrow$  v
12:          bestw  $\leftarrow$  w
13:          optimal  $\leftarrow$  cost
14:   return bestv, bestw
```

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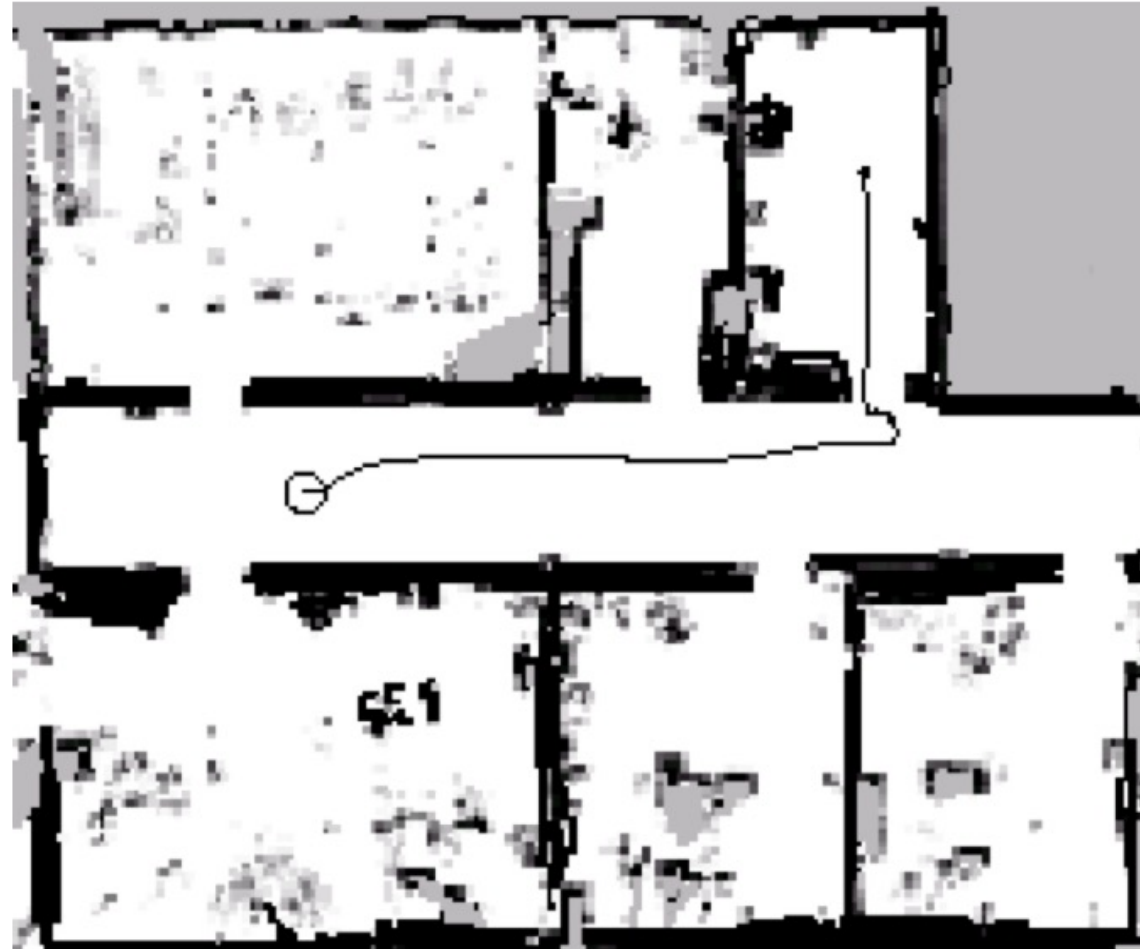


# DWA in action



# DWA in action: Problems with narrow passage, dependence on $V_d$

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Read more on the reference paper!

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance", *IEEE Robotics Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997.