



Clustering Algorithms and their Application to Facial Image Analysis

Hamid Sadeghi

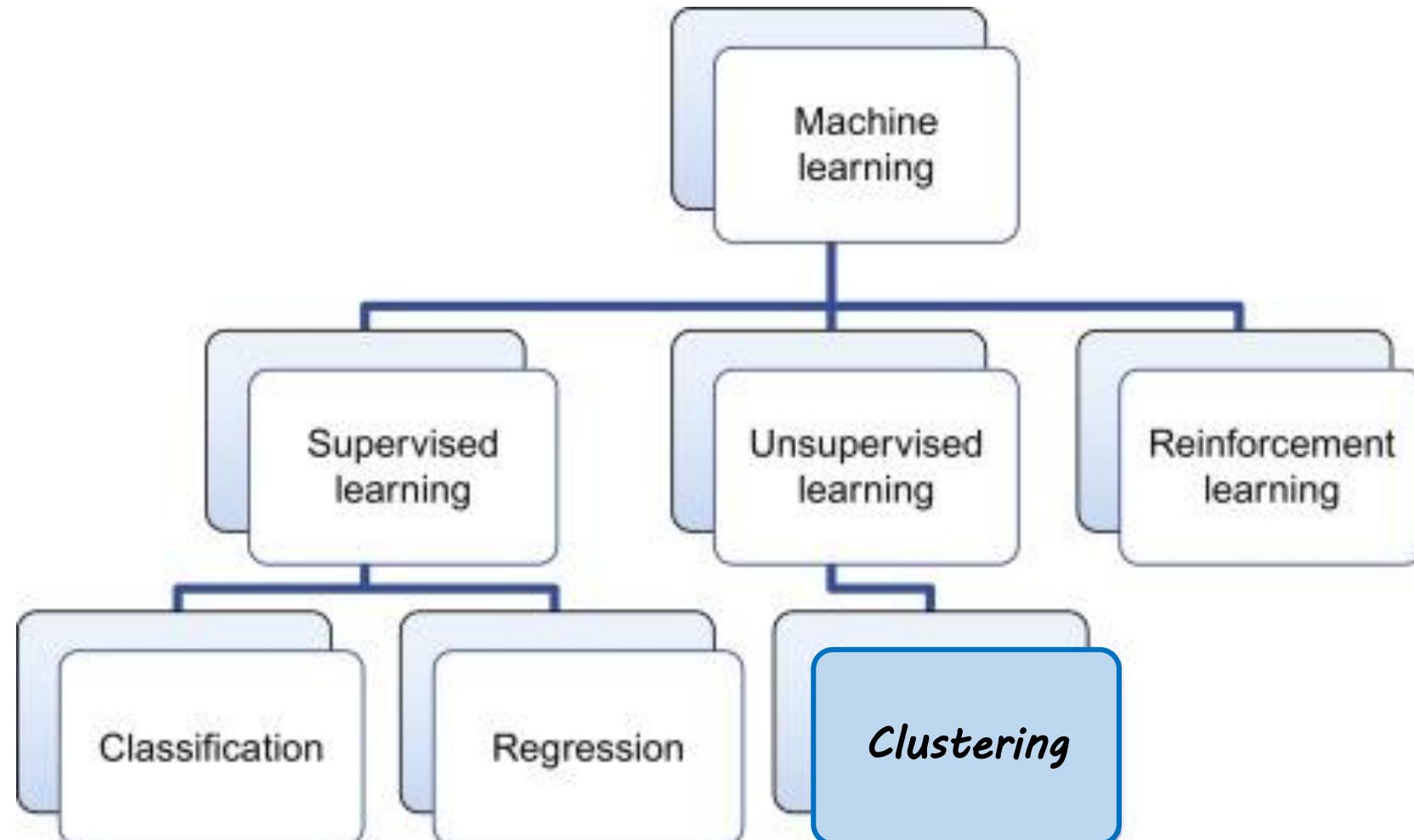
nextera, FaceCup 1400 (2022)



Contents

- *Introduction*
- *Clustering Algorithms*
- *Evaluation*
- *Face Analysis*

Introduction

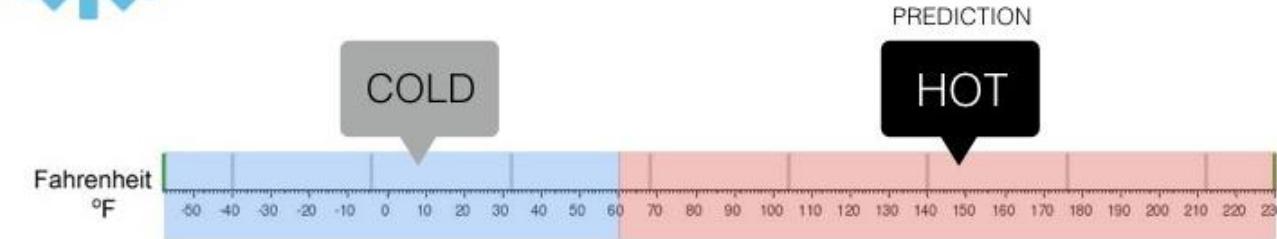


Introduction (Classification vs Regression)



Classification

Will it be Cold or Hot tomorrow?

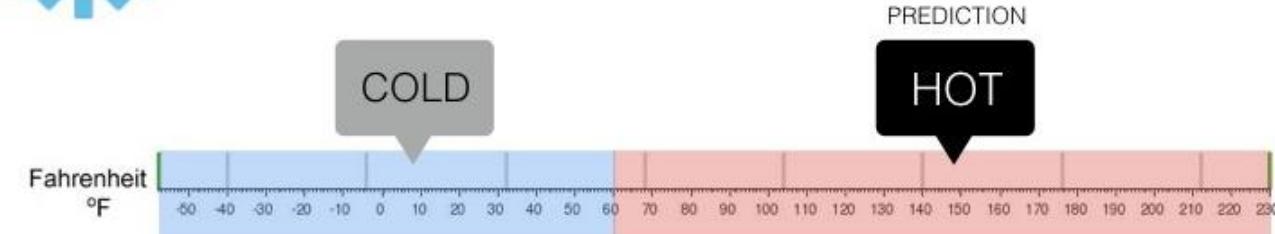


Introduction (Classification vs Regression)



Classification

Will it be Cold or Hot tomorrow?

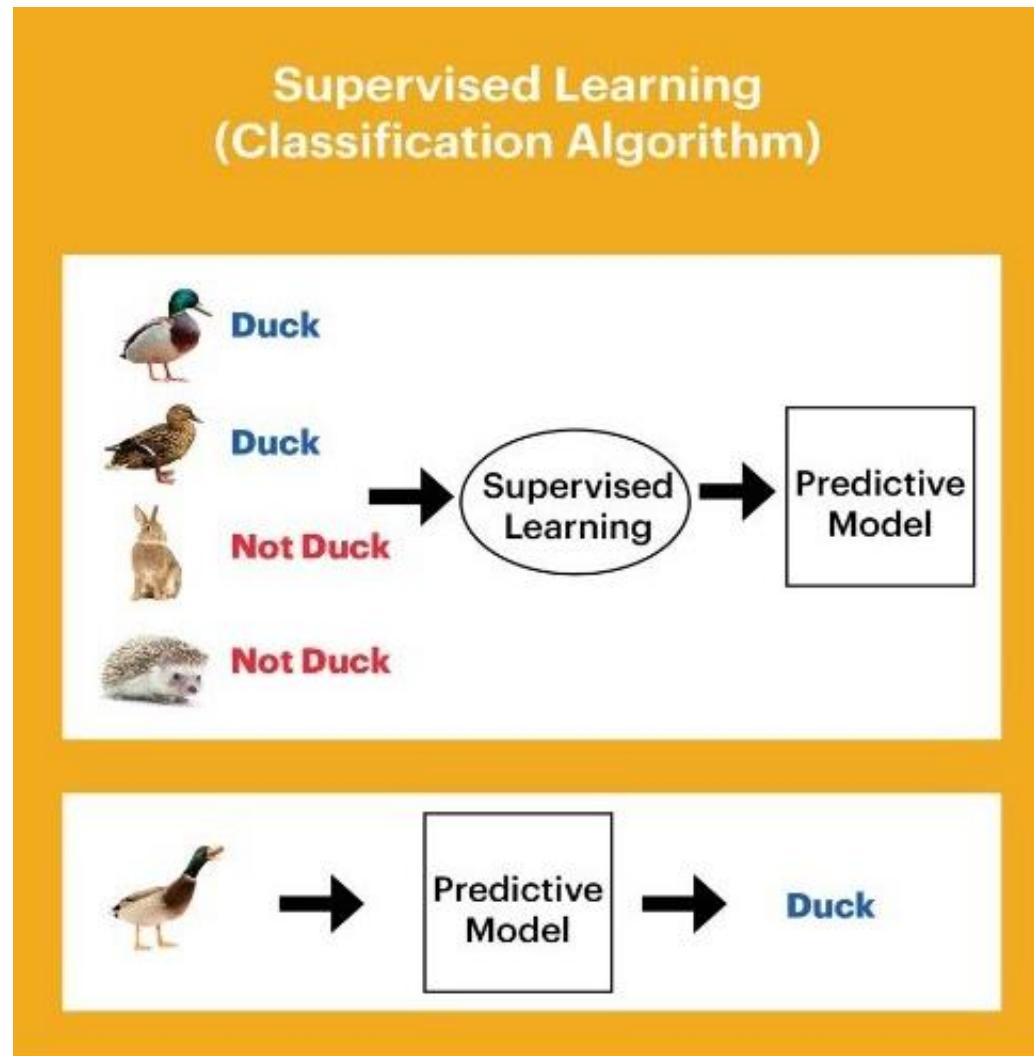


Regression

What is the temperature going to be tomorrow?

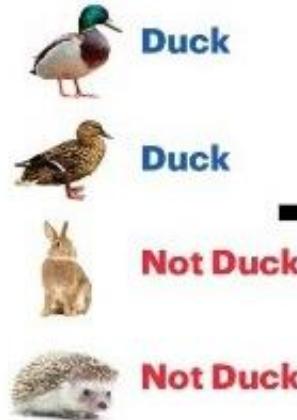


Introduction (Classification vs Clustering)

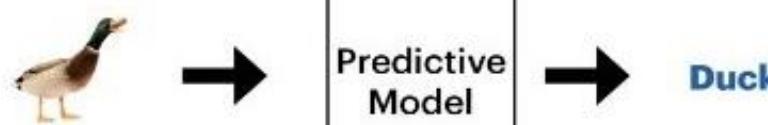


Introduction (Classification vs Clustering)

Supervised Learning (Classification Algorithm)



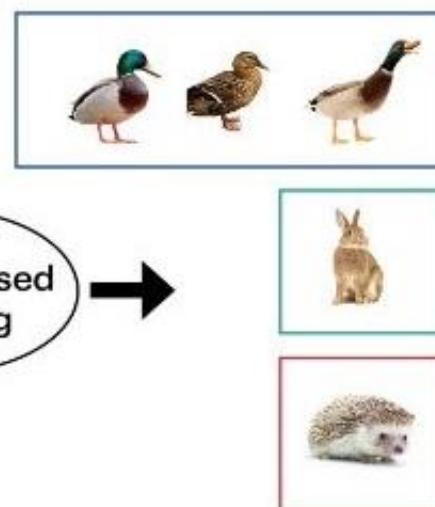
Supervised Learning → Predictive Model



Unsupervised Learning (Clustering Algorithm)



Unsupervised Learning



Introduction



Clustering





Clustering Algorithms

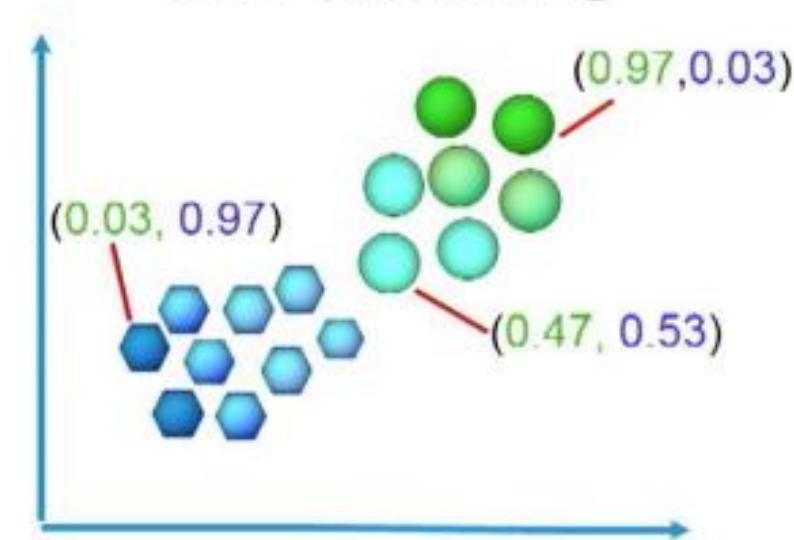
- *Introduction*
- *K-means Clustering*
- *DBSCAN Clustering*
- *Agglomerative Clustering*

Clustering Algorithms: Introduction

Hard Clustering

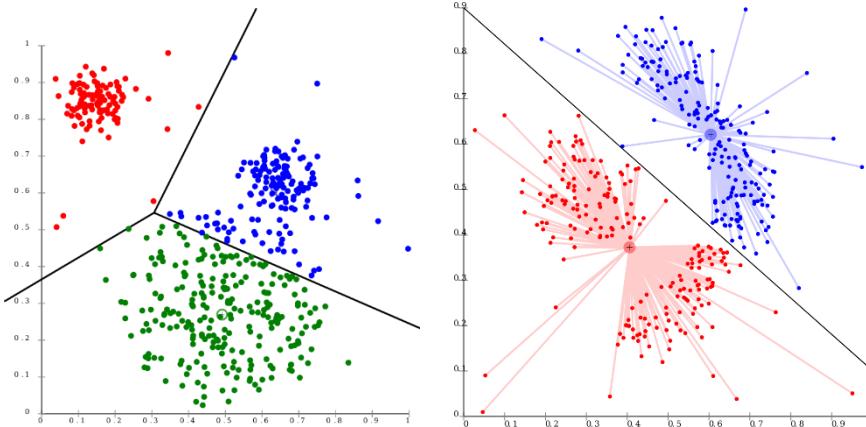


Soft Clustering

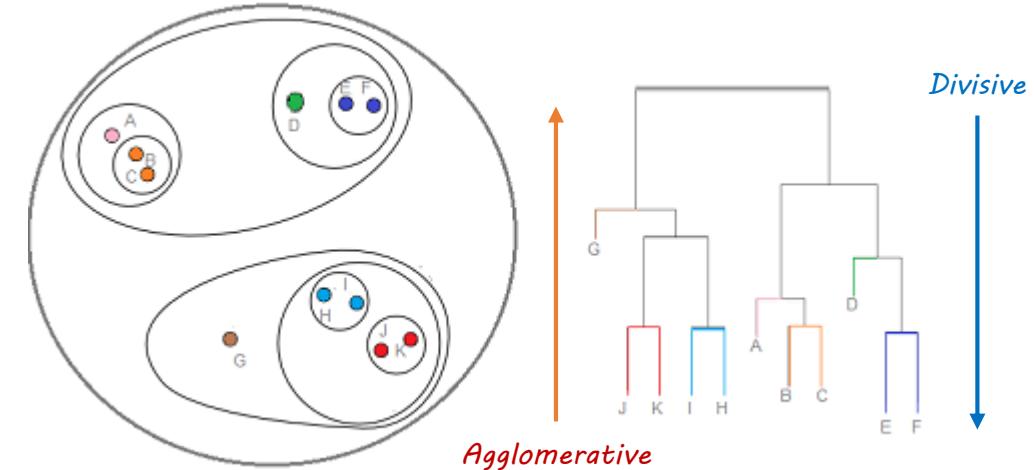


Clustering Algorithms: Introduction

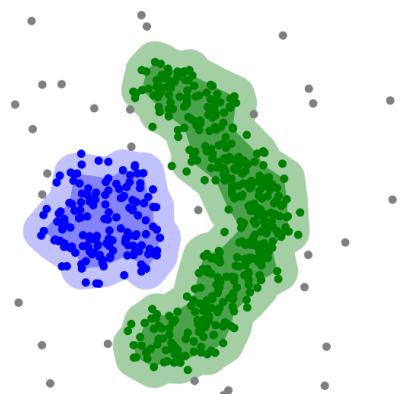
Centroid-based (Partitional) clustering



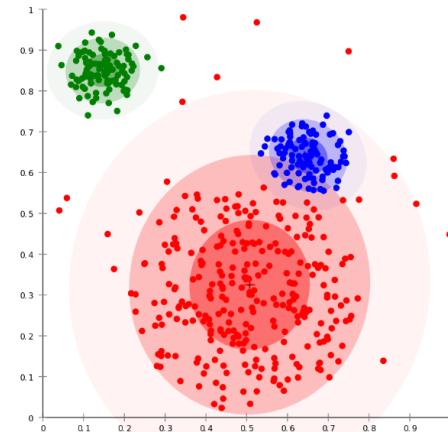
Connectivity-based (hierarchical) clustering



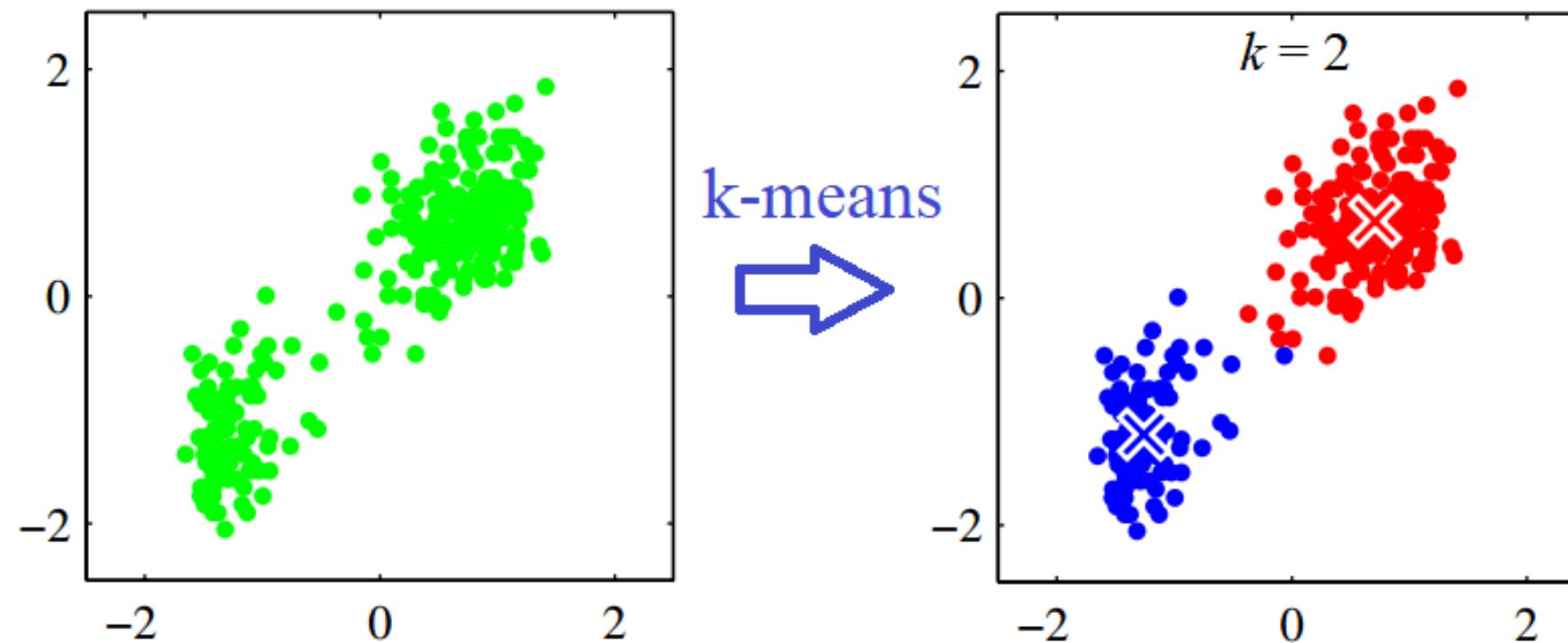
Density-based clustering



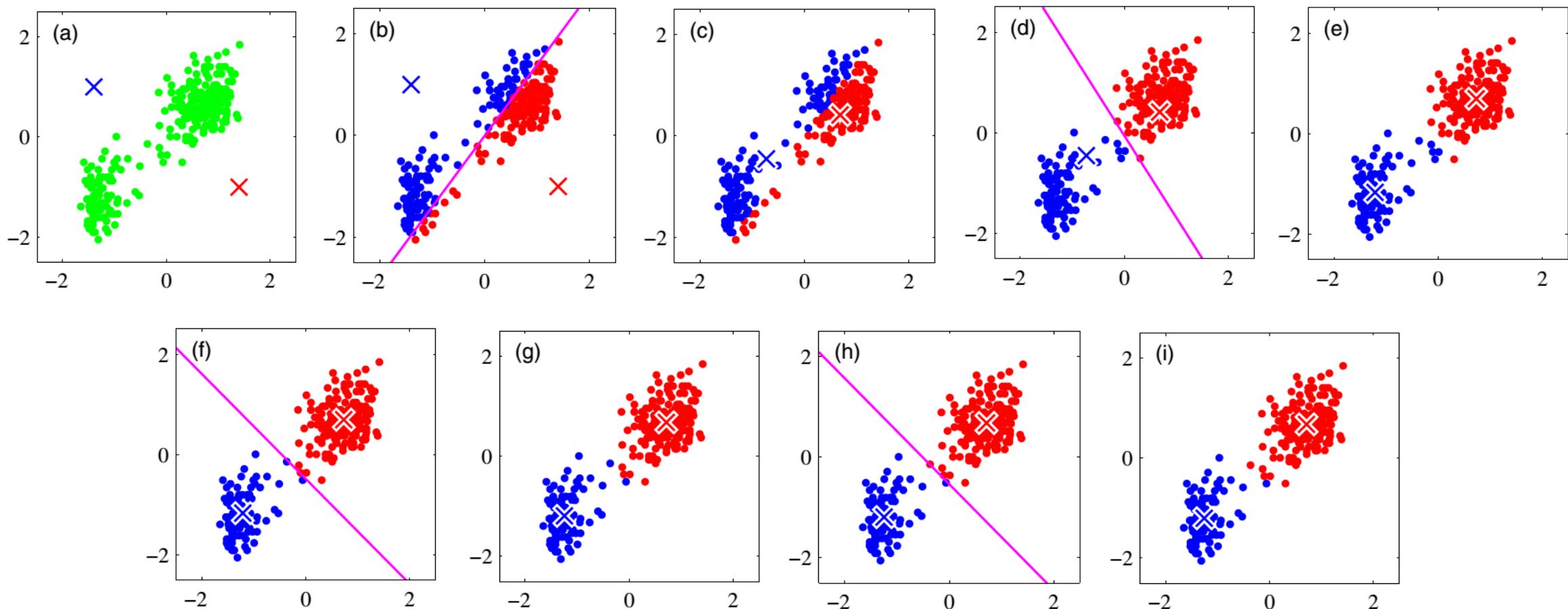
Distribution-based clustering



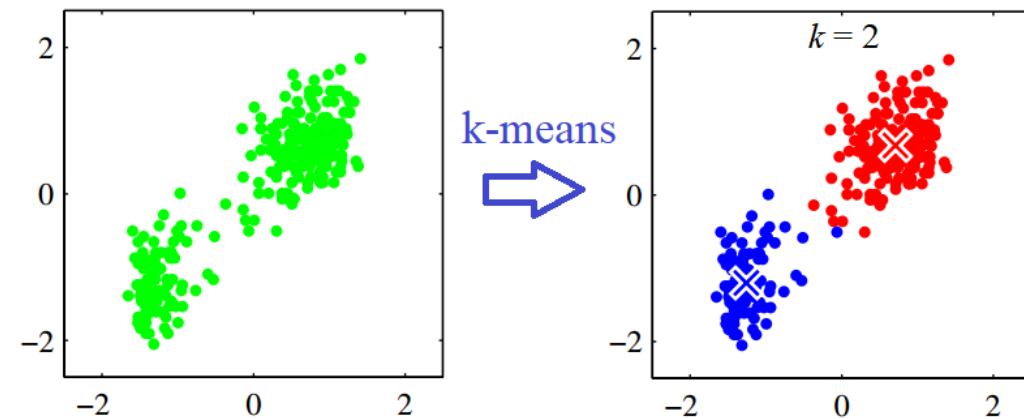
Clustering Algorithms: K-means



Clustering Algorithms: K-means



Clustering Algorithms: K-means



Algorithm 1 k -means algorithm

- 1: Specify the number k of clusters to assign.
 - 2: Randomly initialize k centroids.
 - 3: **repeat**
 - 4: **expectation:** Assign each point to its closest centroid.
 - 5: **maximization:** Compute the new centroid (mean) of each cluster.
 - 6: **until** The centroid positions do not change.
-

Clustering Algorithms: K-means

image segmentation and image compression

$K = 2$

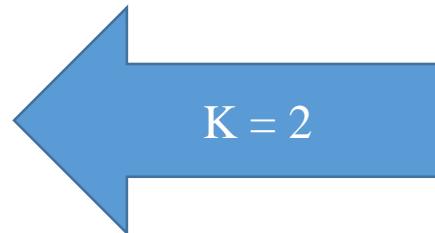


Original image



each pixel: {R, G, B}

$K = 2$



Clustering Algorithms: K-means

image segmentation and image compression

$K = 2$



$K = 3$



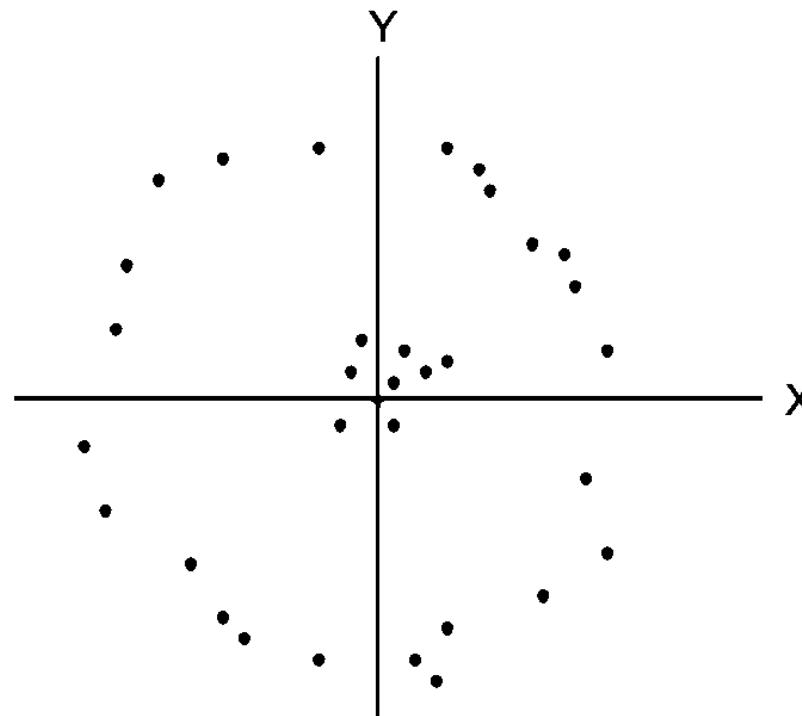
$K = 10$



Original image

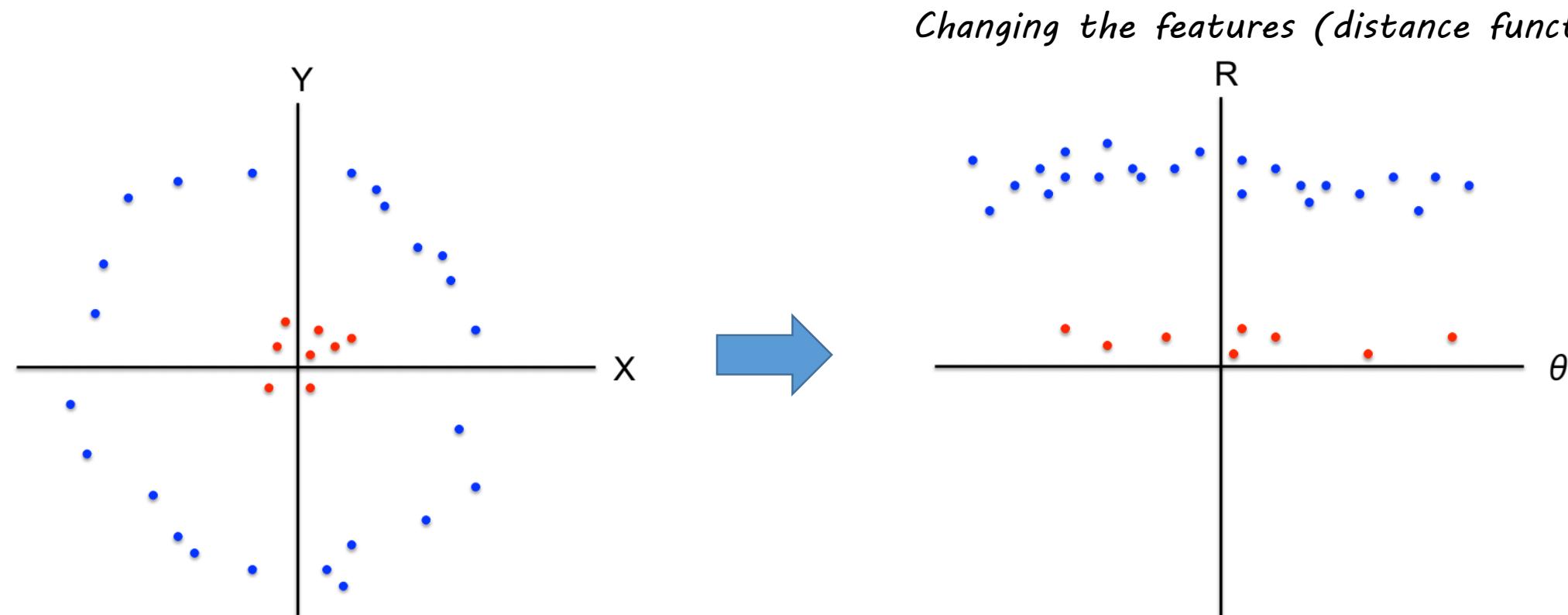


Clustering Algorithms: K-means

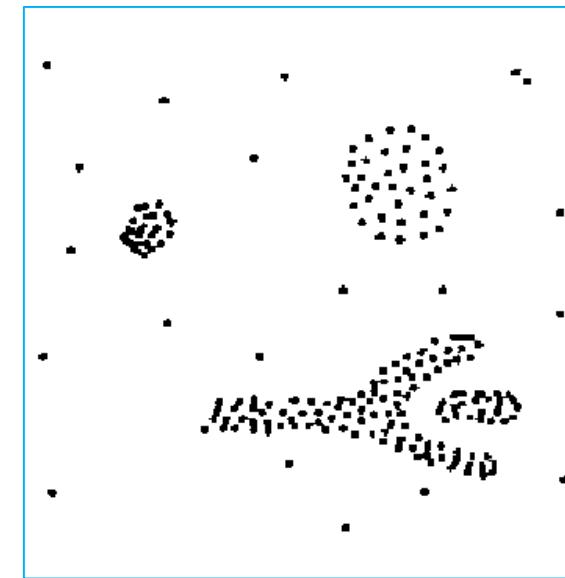
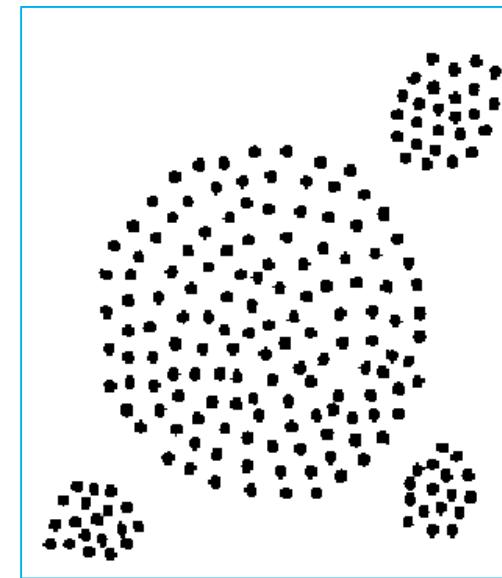
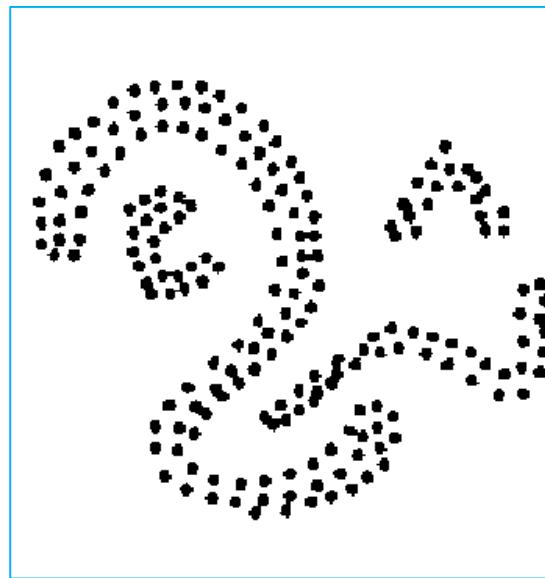


K-means not able to properly cluster

Clustering Algorithms: K-means

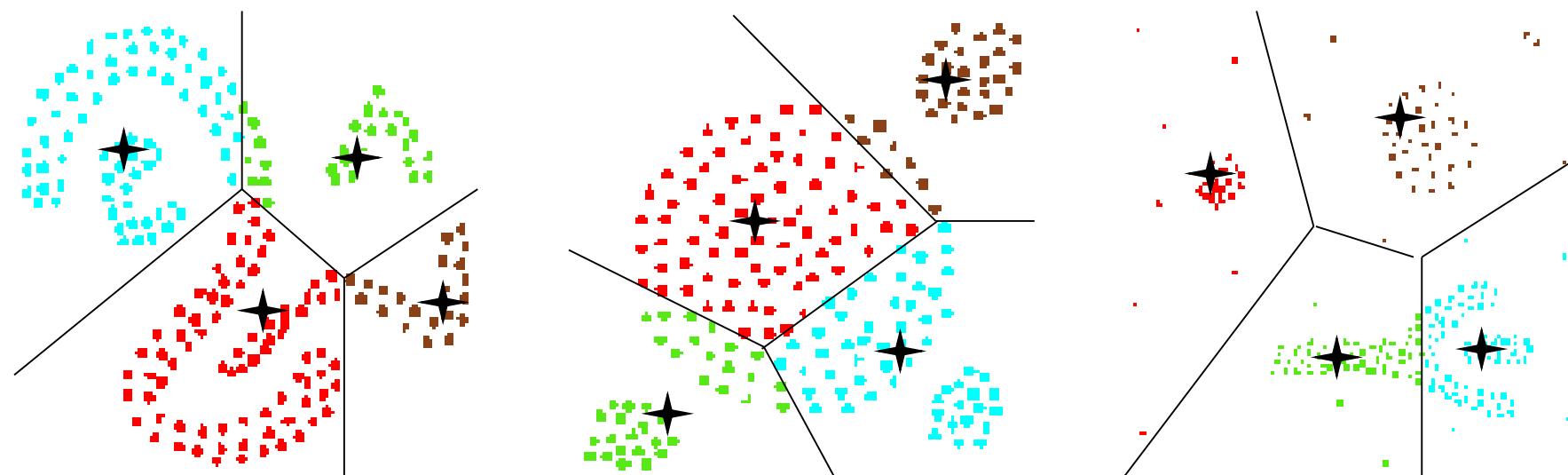


Clustering Algorithms: K-means



Clustering Algorithms: K-means

Results of k-medoids algorithm for k=4



(In contrast to the k-means algorithm, k-medoids chooses actual data points as centers.)

Clustering Algorithms: K-means

- *Implementations:*



GitHub <https://github.com/hamidsadeghi68/face-clustering>

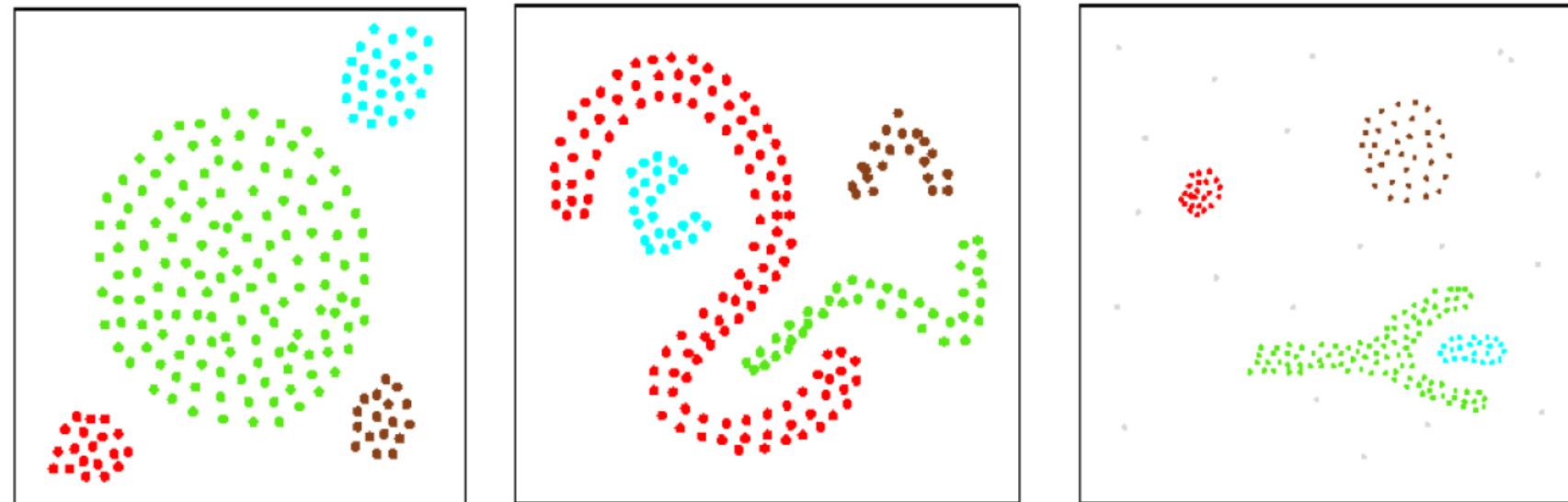
- *K-means:*

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_kmeans.ipynb



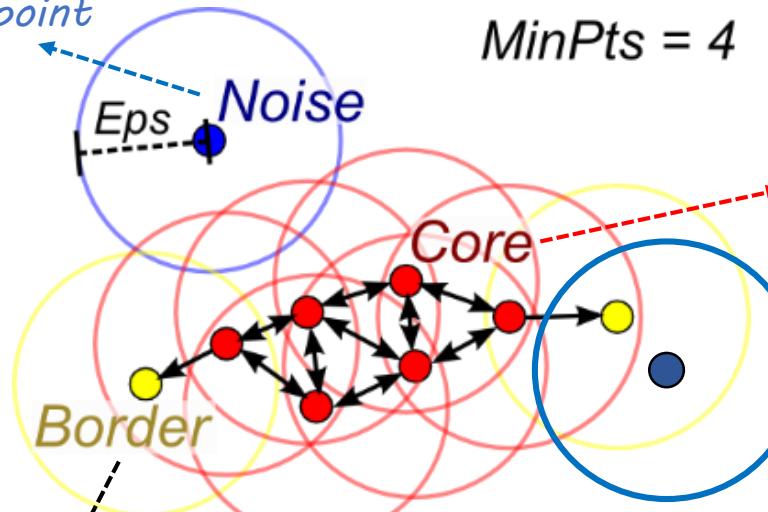
Clustering Algorithms: DBSCAN

Results of a DBSCAN clustering



Clustering Algorithms: DBSCAN

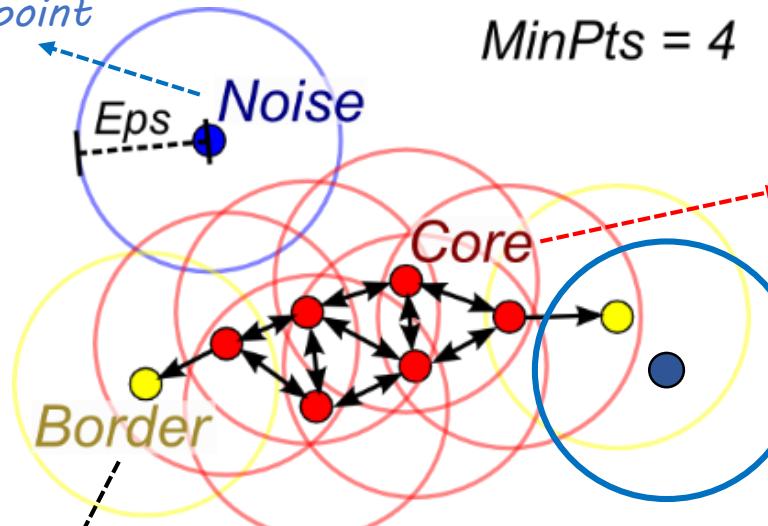
not reachable from any core point



The point is within distance ϵ from core point p

Clustering Algorithms: DBSCAN

not reachable from any core point



ALGORITHM 2: Abstract DBSCAN Algorithm

- 1 Compute neighbors of each point and identify core points // Identify core points
- 2 Join neighboring core points into clusters // Assign core points
- 3 **foreach** non-core point **do**
- 4 Add to a neighboring core point if possible // Assign border points
- 5 Otherwise, add to noise // Assign noise points

Clustering Algorithms: DBSCAN

```

DBSCAN(database: DB, distFunc, eps, minPts) {
    C ← 0
    for each point P in DB {
        if label(P) ≠ undefined then continue
        Neighbors N ← RangeQuery(DB, distFunc, P, eps)
        if |N| < minPts then {
            label(P) ← Noise
            continue
        }
        C ← C + 1
        label(P) ← C
        SeedSet S ← N \ {P}
        for each point Q in S {
            if label(Q) = Noise then label(Q) ← C
            if label(Q) ≠ undefined then continue
            label(Q) ← C
            Neighbors N ← RangeQuery(DB, distFunc, Q, eps)
            if |N| ≥ minPts then {
                S ← S ∪ N
            }
        }
    }
}
    
```

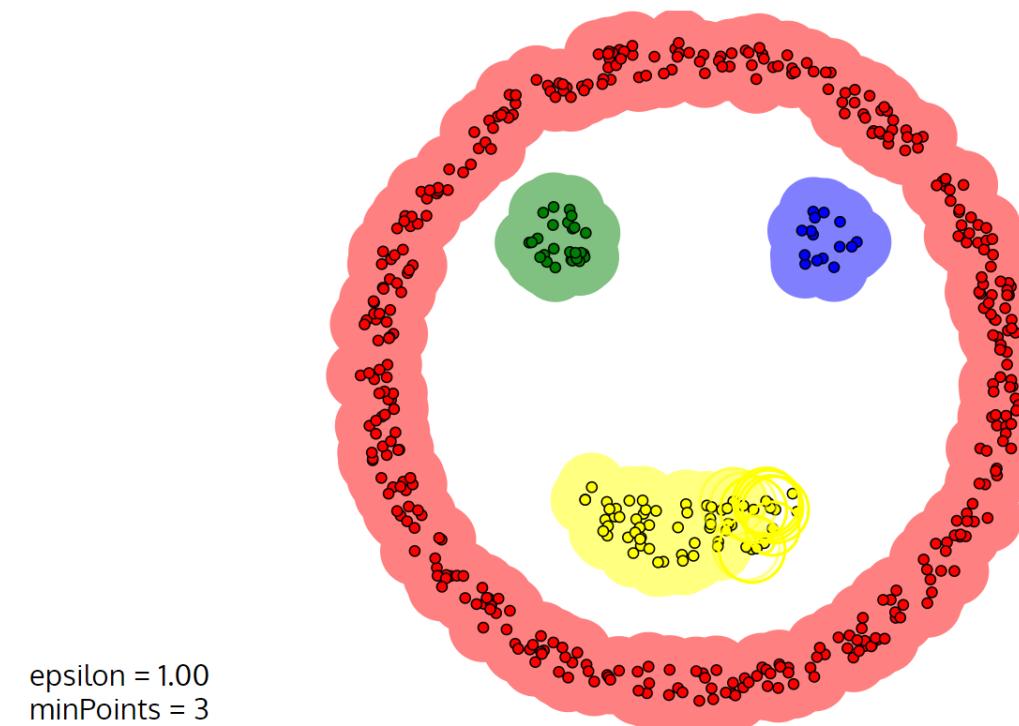
/ Cluster counter */*
/ Previously processed in inner loop */*
/ Find neighbors */*
/ Density check */*
/ Label as Noise */*

/ next cluster label */*
/ Label initial point */*
/ Neighbors to expand */*
/ Process every seed point Q */*
/ Change Noise to border point */*
/ Previously processed (e.g., border point) */*
/ Label neighbor */*
/ Find neighbors */*
/ Density check (if Q is a core point) */*
/ Add new neighbors to seed set */*

Clustering Algorithms: DBSCAN

- Online visualization:

<https://www.naftaliharris.com/blog/visualizing-dbscan-clustering/>



Clustering Algorithms: DBSCAN

- *implementation:*

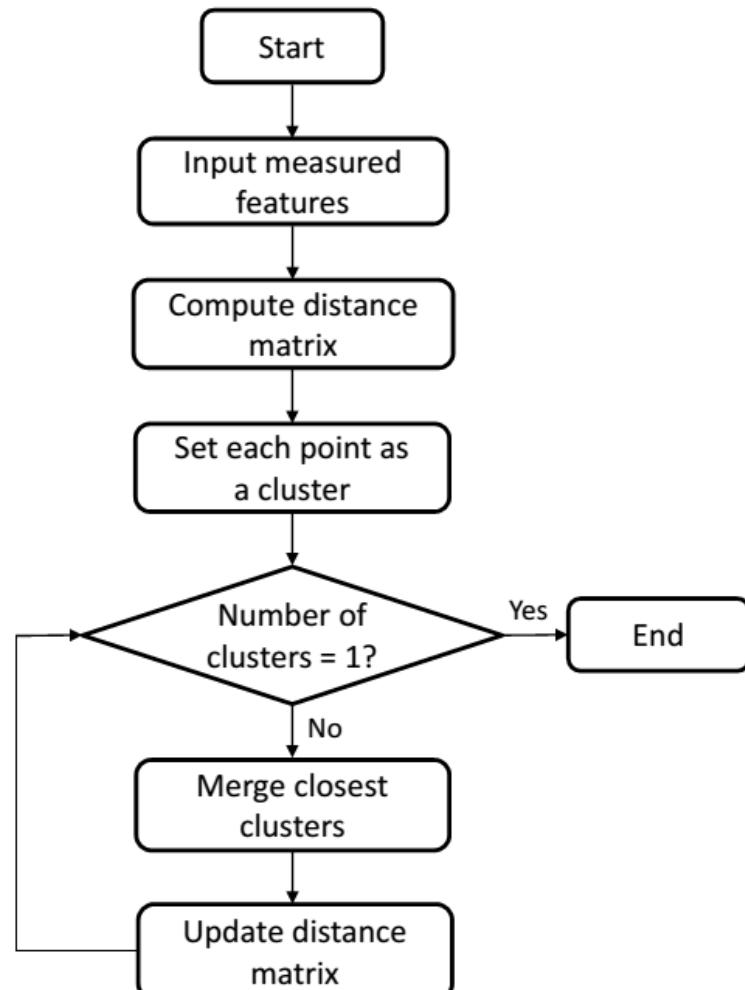
https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_dbSCAN.ipynb



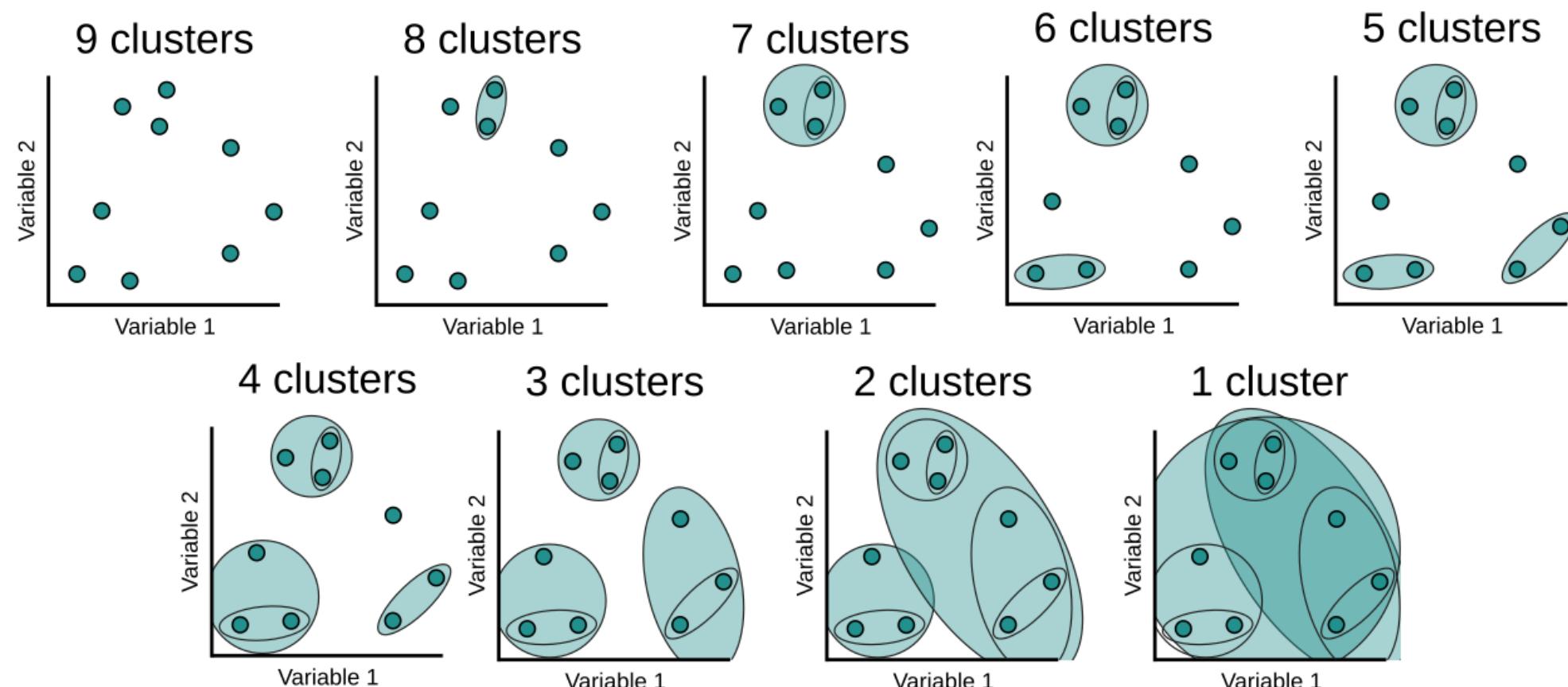
Clustering Algorithms: Agglomerative

- each observation starts in its own cluster
- pairs of clusters are merged

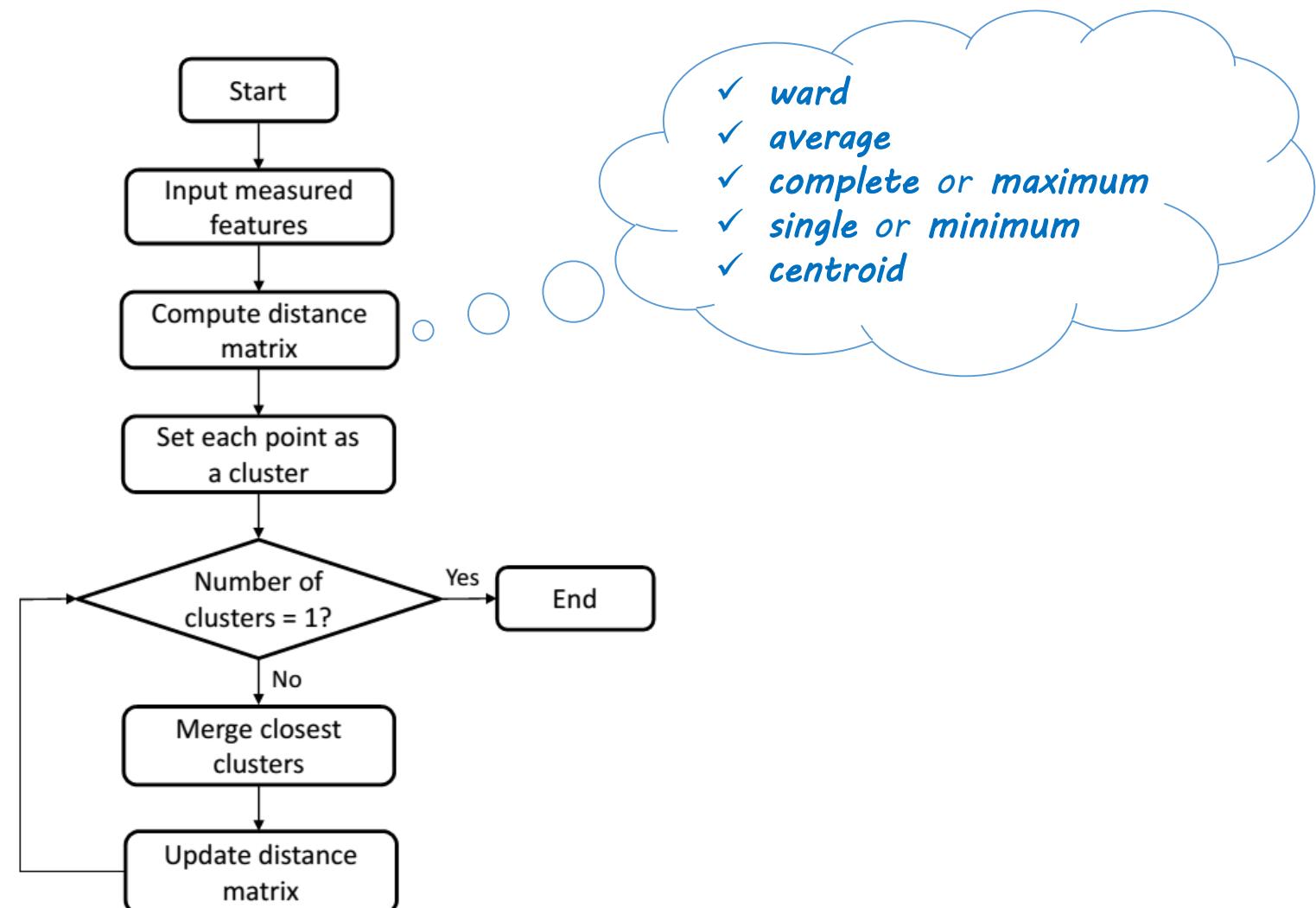
Clustering Algorithms: Agglomerative



Clustering Algorithms: Agglomerative



Clustering Algorithms: Agglomerative

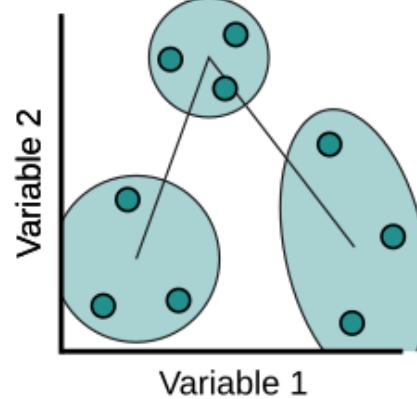


Clustering Algorithms: Agglomerative

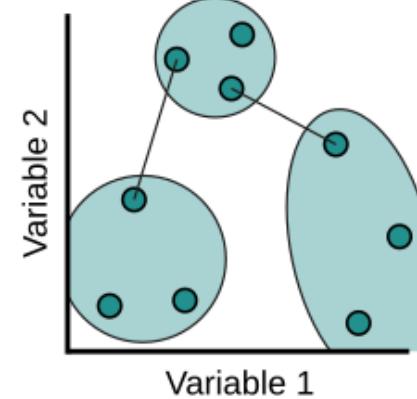
- ‘**ward**’ minimizes the variance of the clusters being merged.
- ‘**average**’ uses the average of the distances of each observation of the two sets.
- ‘**complete**’ or ‘**maximum**’ linkage uses the maximum distances between all observations of the two sets.
- ‘**single**’ or ‘**minimum**’ uses the minimum of the distances between all observations of the two sets.
- ‘**centroid**’: uses the distance between the centroid for cluster 1 (a mean vector of length p variables) and the centroid for cluster 2

Clustering Algorithms: Agglomerative

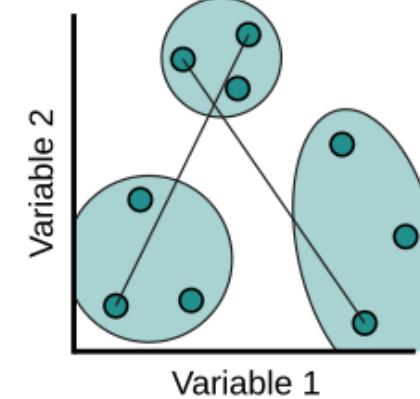
Centroid linkage



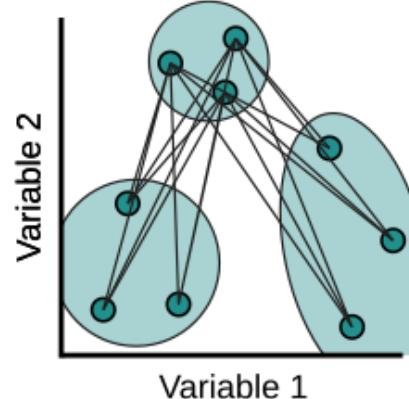
Single linkage



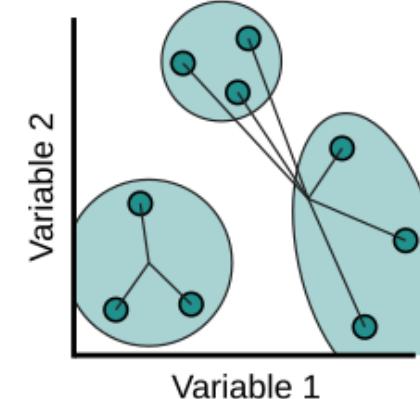
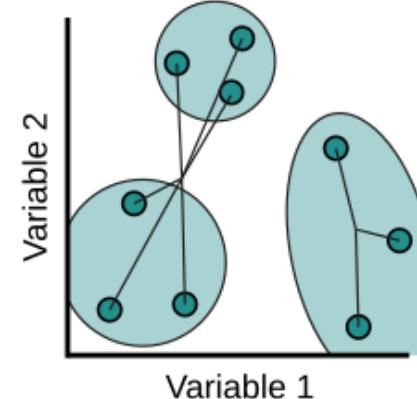
Complete linkage



Average linkage

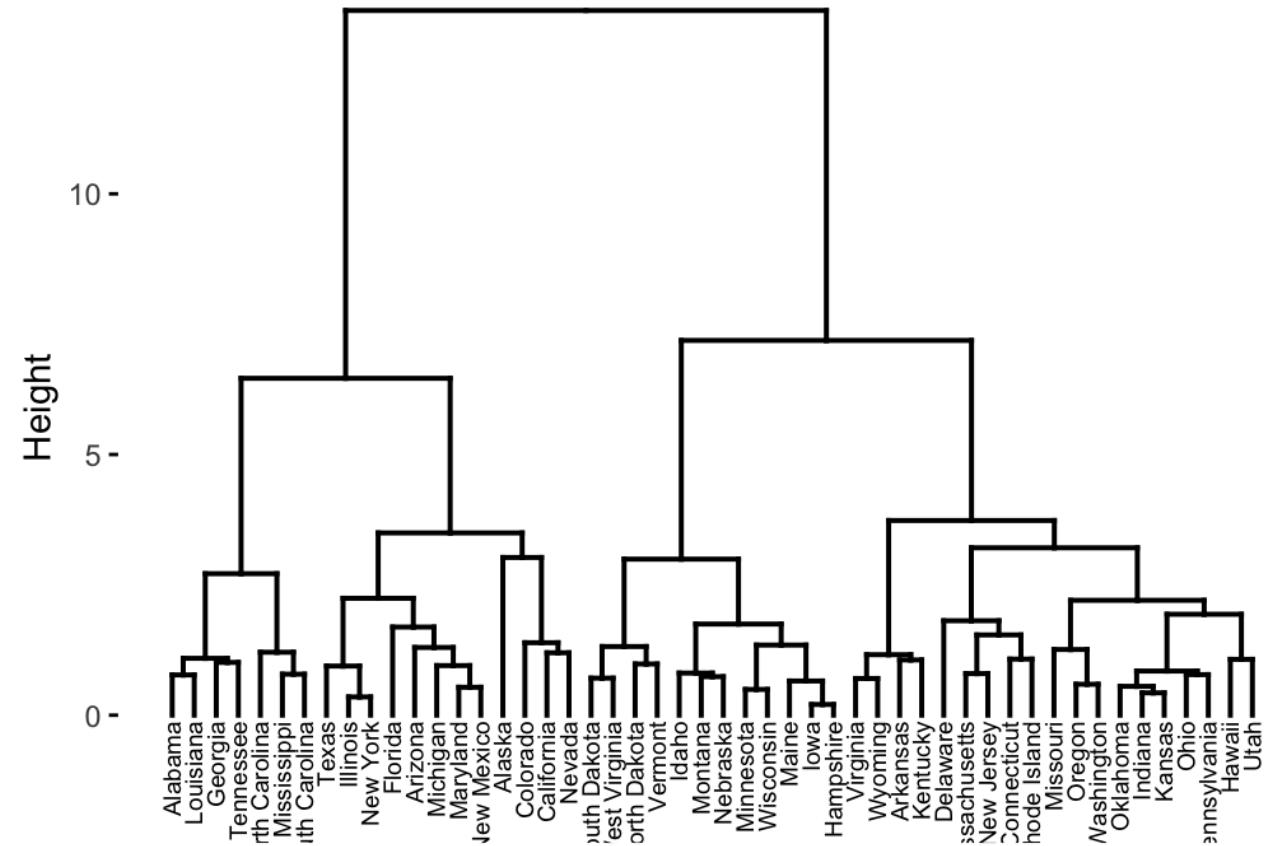


Ward's method

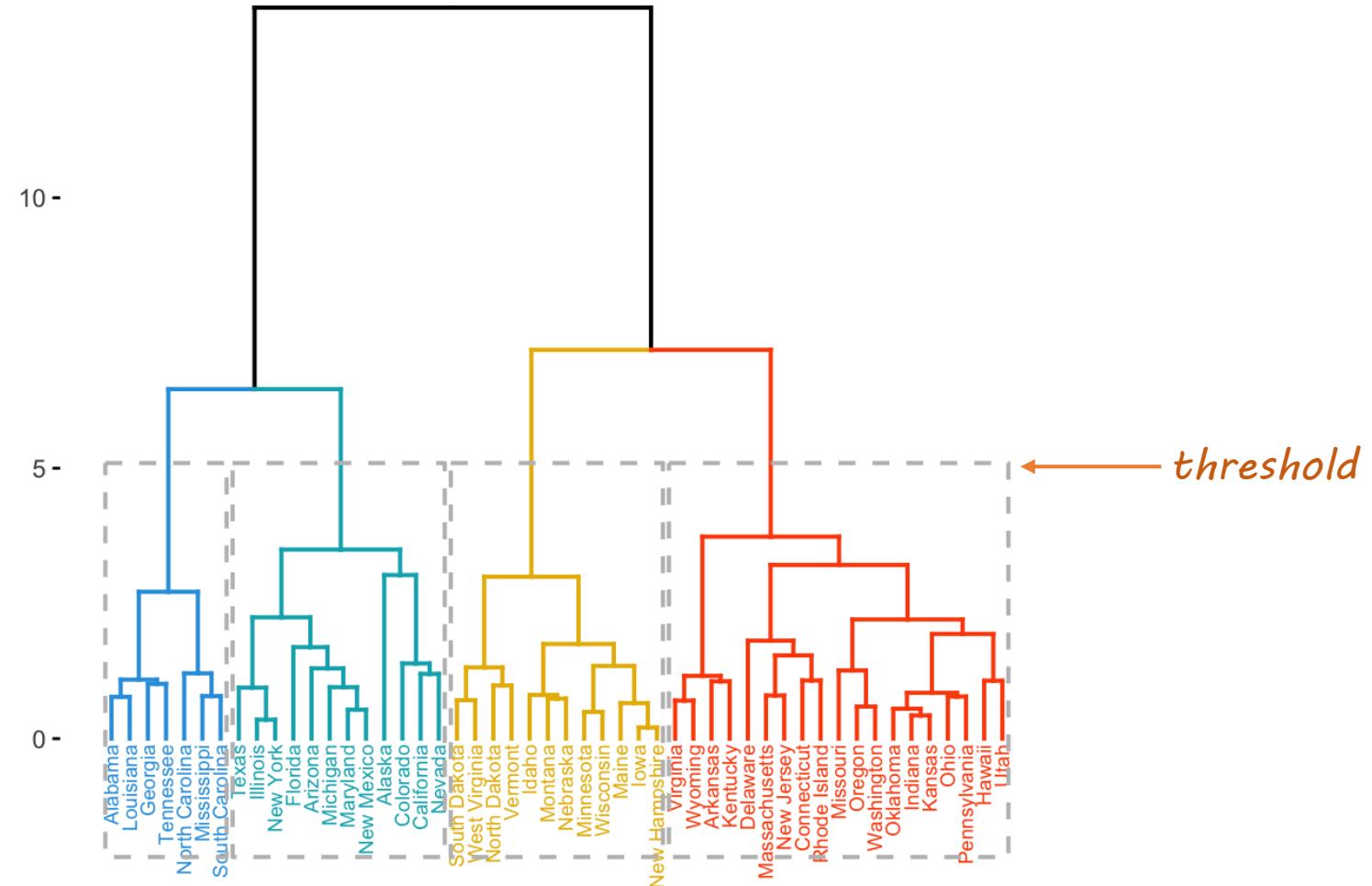


Clustering Algorithms: Agglomerative

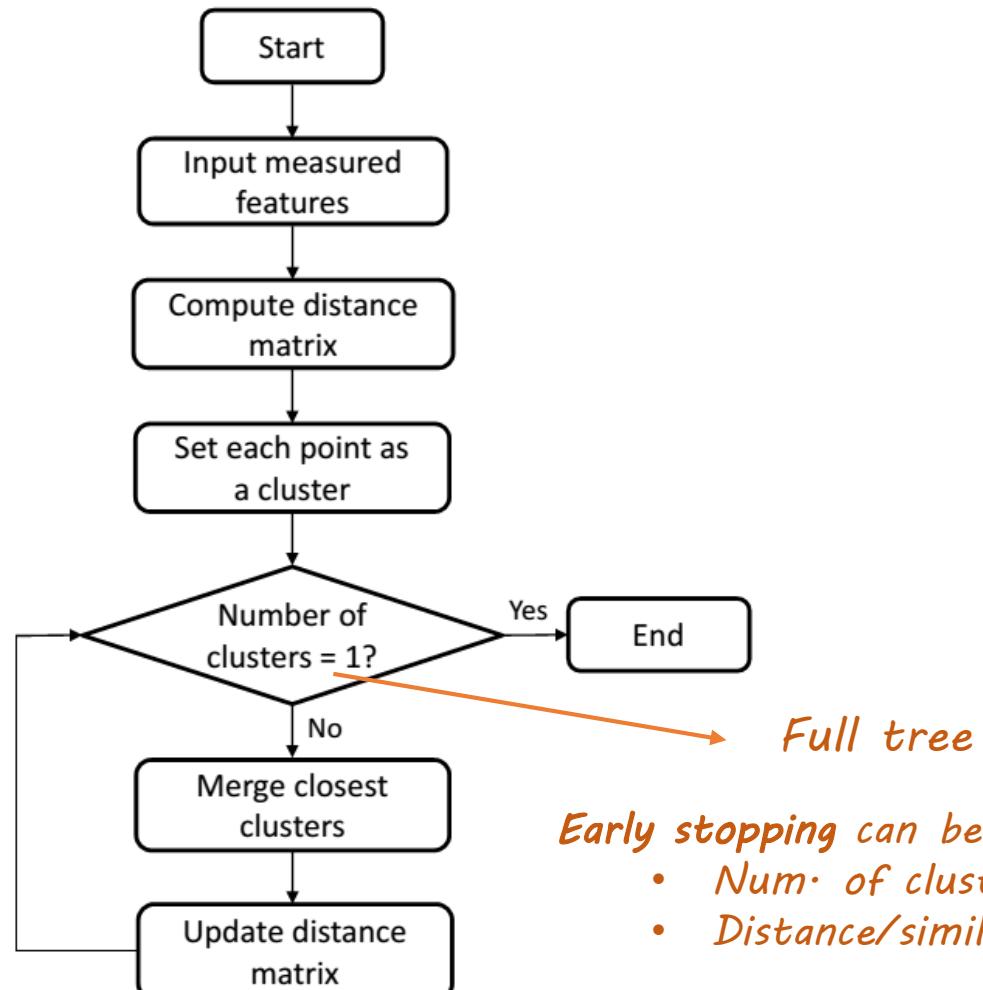
Cluster Dendrogram



Clustering Algorithms: Agglomerative



Clustering Algorithms: Agglomerative



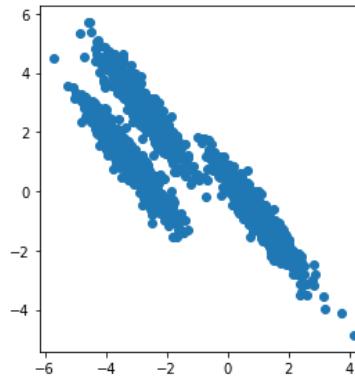
Clustering Algorithms: Agglomerative

- *implementation:*

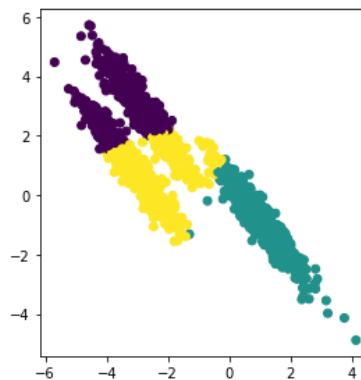
https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_agglomerative.ipynb



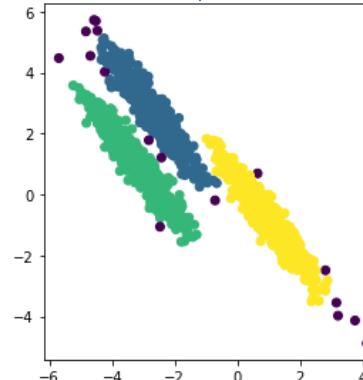
Clustering summary



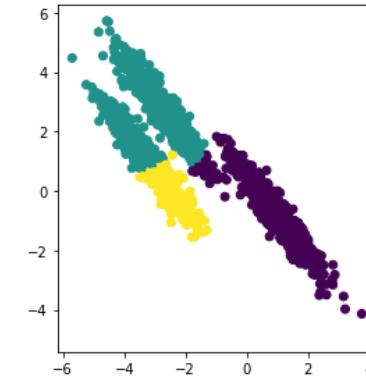
K-means



DBSCAN



Agglomerative



Evaluation

How to evaluate the quality of clusters?

Internal Evaluation (unsupervised)

Using clustered data itself

- Silhouette coefficient
- Davies-Bouldin index
- Dunn index

External Evaluation (supervised)

Using ground truth or gold standard

- Purity
- Rand Index
- Normalized Mutual Information (NMI)
- F-measure

Evaluation

- *Purity*

$$purity(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$

Evaluation

- Purity

$$\text{purity}(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$

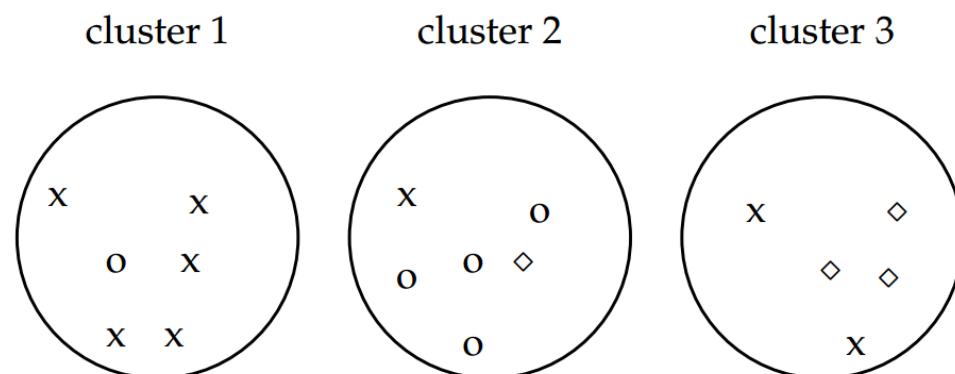
clusters
classes

Evaluation

- *Purity*

$$\text{purity}(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$

clusters
classes



► **Figure 16.4** Purity as an external evaluation criterion for cluster quality. Majority class and number of members of the majority class for the three clusters are: x, 5 (cluster 1); o, 4 (cluster 2); and ◇, 3 (cluster 3). Purity is $(1/17) \times (5 + 4 + 3) \approx 0.71$.

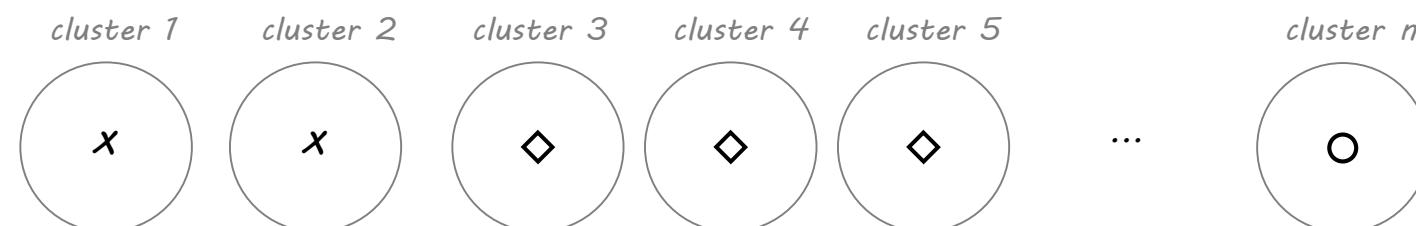
Evaluation

- *Purity*

$$\text{purity}(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cup c_j|$$

clusters
classes

If each data gets 1 cluster → purity = 1 (!)



Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

True Positive pairs → $TP + TN$

True Negative pairs → TN

False Positive pairs ← $TP + TN$

False Negative pairs → $FN + TN$

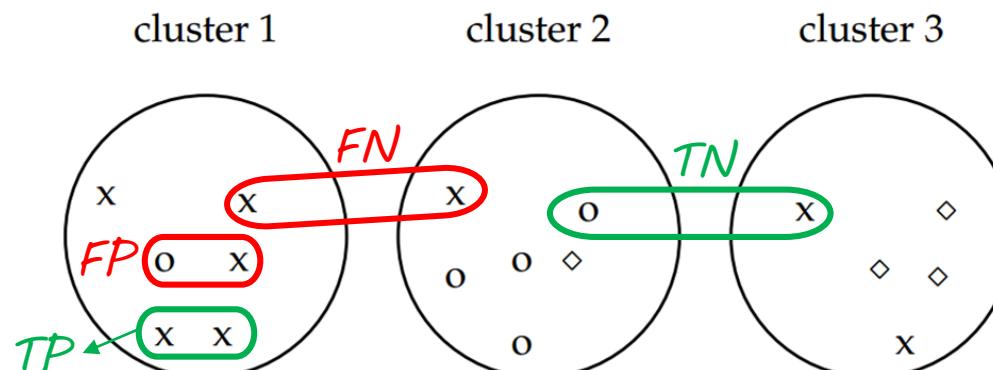
Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

True Positive pairs →
True Negative pairs →
False Positive pairs ←
False Negative pairs →

Sample pairs:



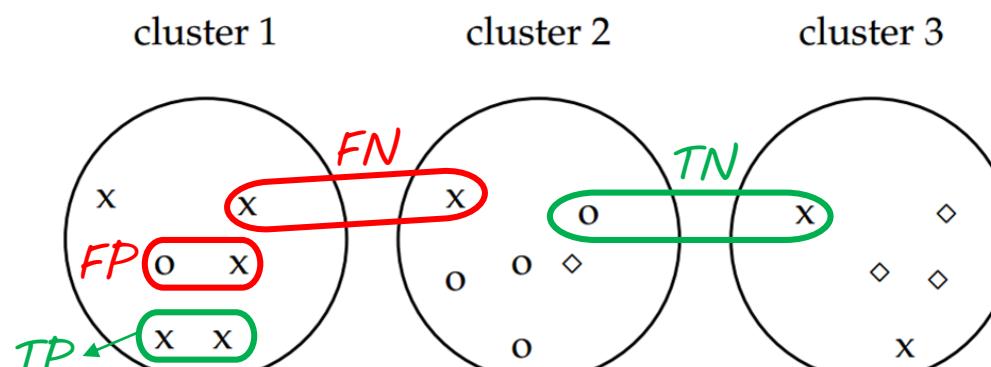
Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

TP + TN → True Positive pairs
TP + TN → True Negative pairs
FP + FN ← False Positive pairs
FP + FN → False Negative pairs

Sample pairs:



$$TP + FP = \binom{6}{2} + \binom{6}{2} + \binom{5}{2} = 40 \text{ Positive pairs}$$

$$TP = \binom{5}{2} + \binom{4}{2} + \binom{3}{2} + \binom{2}{2} = 20$$

$$\rightarrow FP = 40 - 20 = 20$$

	Same cluster	Different clusters
Same class	TP = 20	FN = 24
Different classes	FP = 20	TN = 72

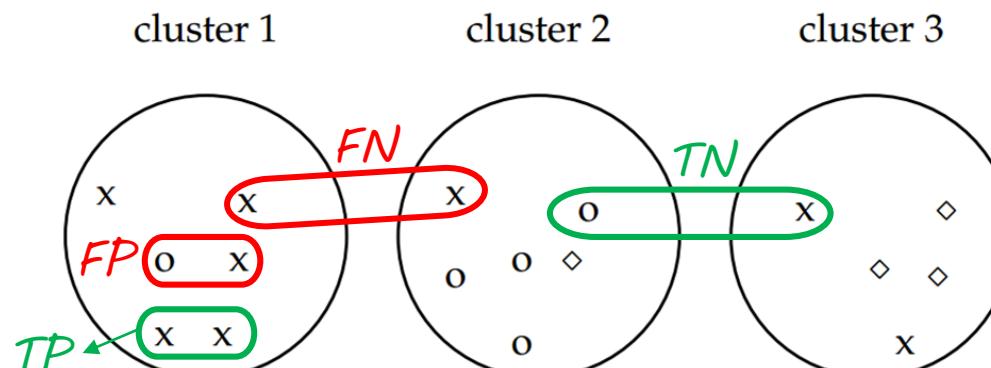
Evaluation

- Rand Index

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

→ True Positive pairs
→ True Negative pairs
← False Positive pairs
→ False Negative pairs

Sample pairs:



	Same cluster	Different clusters
Same class	TP = 20	FN = 24
Different classes	FP = 20	TN = 72

$$RI = (20 + 72) / (20 + 20 + 24 + 72) \approx 0.68$$

Evaluation

- *Normalized Mutual Information (NMI)*

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$$

Evaluation

- Normalized Mutual Information (*NMI*)

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$$

Diagram illustrating the components of NMI:

- Mutual Information* (orange text) is represented by a double-headed arrow between Ω and C .
- clusters* (orange text) is associated with C .
- classes* (orange text) is associated with Ω .
- entropy* (orange text) is represented by a double-headed arrow between $H(\Omega)$ and $H(C)$.

Evaluation

- Normalized Mutual Information (NMI)

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$$

Mutual Information \longleftrightarrow clusters

entropy \longleftrightarrow classes

$I(\Omega, C) = \sum_k \sum_j P(\omega_k \cap c_j) \log \frac{P(\omega_k \cap c_j)}{P(\omega_k) P(c_j)} = \sum_k \sum_j \frac{|\omega_k \cap c_j|}{N} \log \frac{N |\omega_k \cap c_j|}{|\omega_k| |c_j|}$

$$I(\Omega, C) = \sum_k \sum_j P(\omega_k \cap c_j) \log \frac{P(\omega_k \cap c_j)}{P(\omega_k) P(c_j)} = \sum_k \sum_j \frac{|\omega_k \cap c_j|}{N} \log \frac{N |\omega_k \cap c_j|}{|\omega_k| |c_j|}$$

Evaluation

- Normalized Mutual Information (NMI)

$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2}$$

Mutual Information \longleftrightarrow clusters
 \downarrow
 $I(\Omega, C)$ \longleftrightarrow classes
 \uparrow
 entropy \longleftrightarrow

probabilities of a data being in the intersection of ω_k and c_j

$$I(\Omega, C) = \sum_k \sum_j P(\omega_k \cap c_j) \log \frac{P(\omega_k \cap c_j)}{P(\omega_k) P(c_j)} = \sum_k \sum_j \frac{|\omega_k \cap c_j|}{N} \log \frac{N |\omega_k \cap c_j|}{|\omega_k| |c_j|}$$

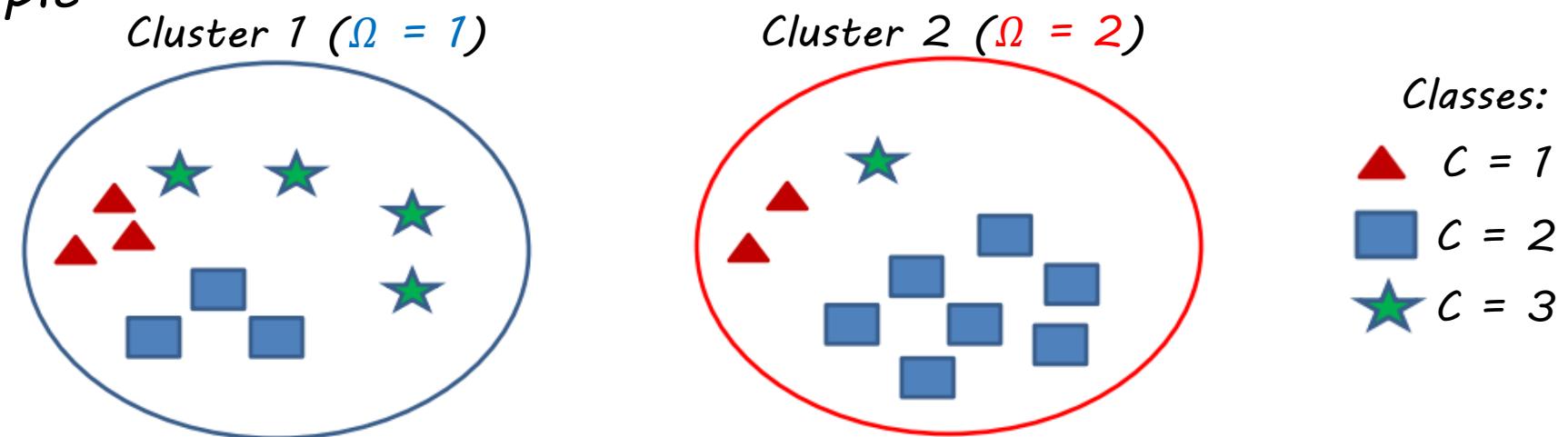
probabilities of a data being in cluster ω_k

$$H(\Omega) = \sum_k P(\omega_k) \log P(\omega_k) = \sum_k \frac{\omega_k}{N} \log \frac{\omega_k}{N}$$

probabilities of a data being in class c_j

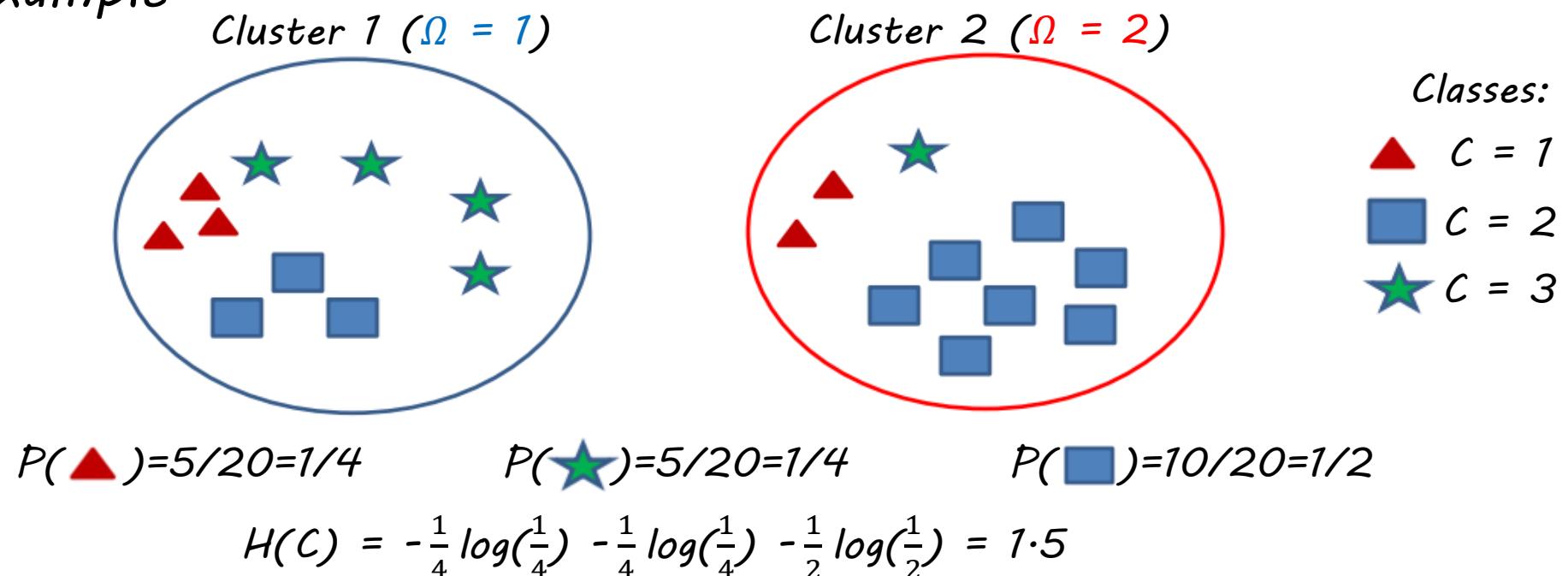
Evaluation

- NMI example



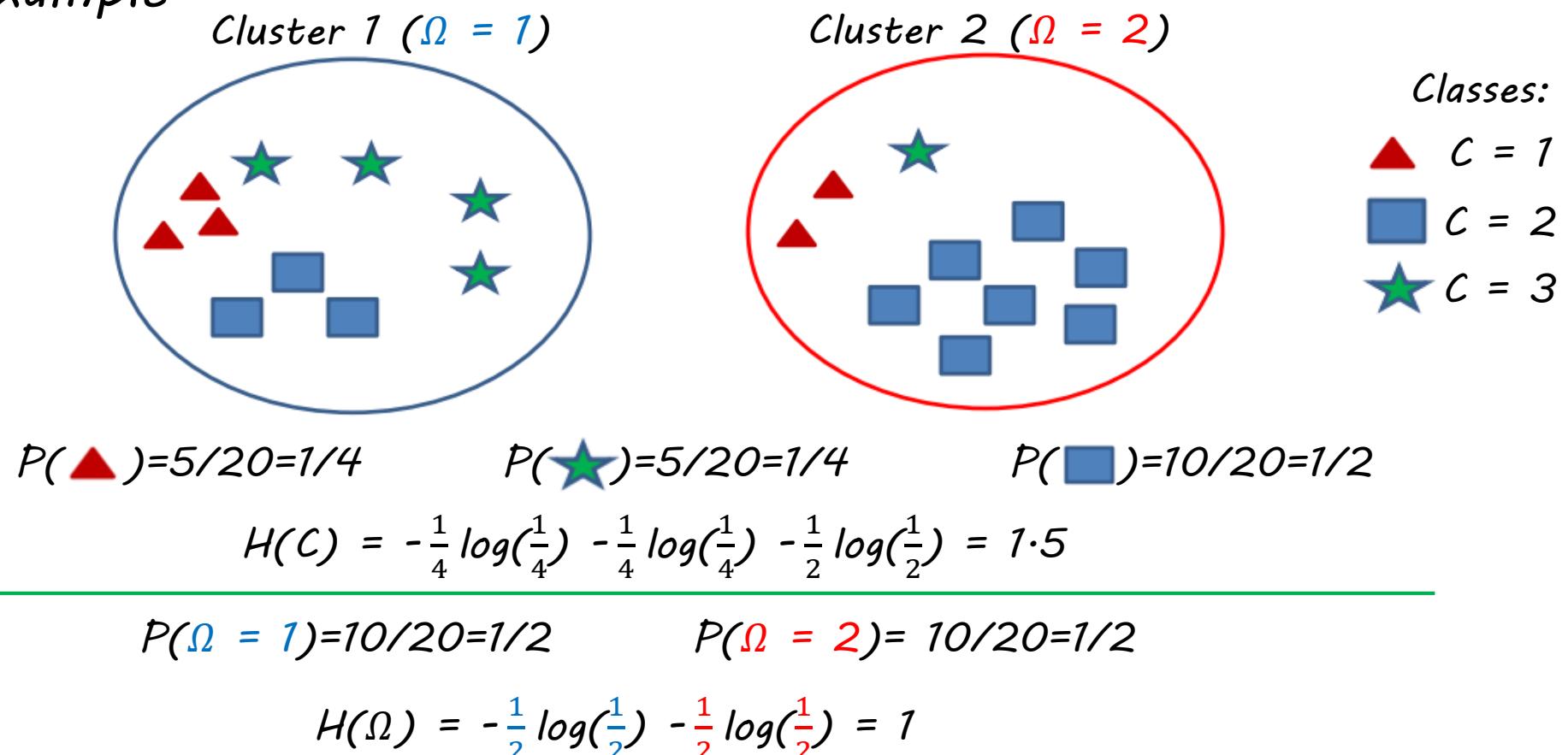
Evaluation

- NMI example



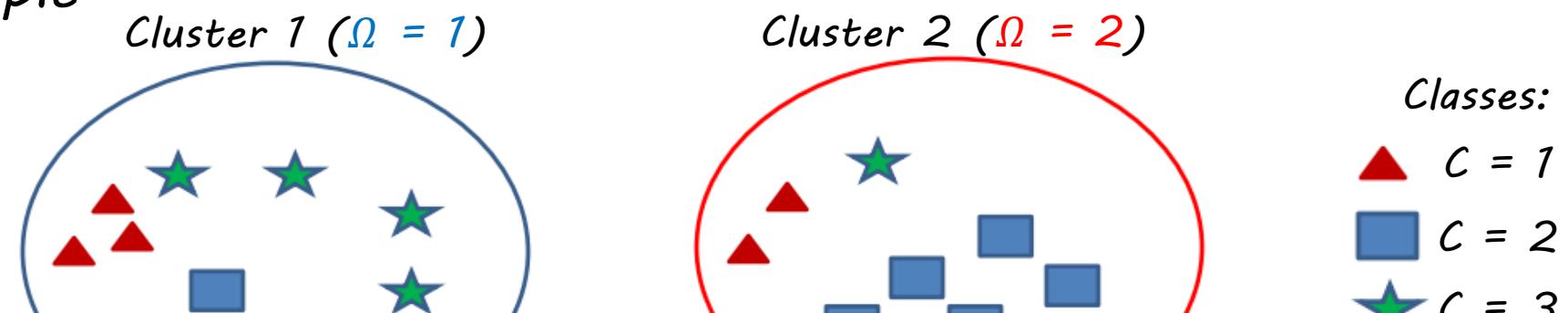
Evaluation

- NMI example



Evaluation

- NMI example

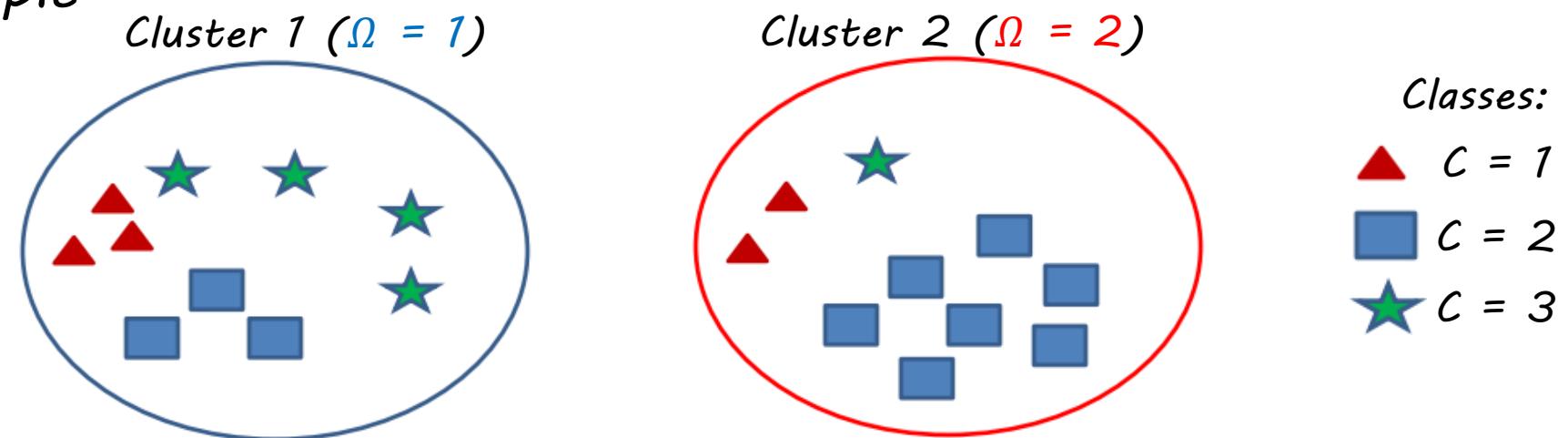


$$I(\Omega, C) = \sum_k \sum_j \frac{|\omega_k \cap c_j|}{N} \log \frac{N |\omega_k \cap c_j|}{|\omega_k| |c_j|}$$
$$= \frac{3}{20} \log \left(\frac{20 \times 3}{10 \times 5} \right) + \frac{3}{20} \log \left(\frac{20 \times 3}{10 \times 10} \right) + \frac{4}{20} \log \left(\frac{20 \times 4}{10 \times 5} \right) + \frac{2}{20} \log \left(\frac{20 \times 2}{10 \times 5} \right) + \frac{7}{20} \log \left(\frac{20 \times 7}{10 \times 10} \right) + \frac{1}{20} \log \left(\frac{20 \times 1}{10 \times 5} \right) = 0.1361$$



Evaluation

- NMI example



$$NMI(\Omega, C) = \frac{I(\Omega, C)}{[H(\Omega) + H(C)]/2} = \frac{0.1361}{[1 + 1.5]/2} = 0.1089$$

Evaluation

- *Precision*

(correct rate within clusters)
$$P = \frac{TP}{TP + FP}$$

- *Recall*

(correct rate within classes/ground truth)
$$R = \frac{TP}{TP + FN}$$

- *F-measure*

$$F_\beta = \frac{(\beta^2 + 1)PR}{\beta^2 P + R} \quad \begin{array}{l} \beta = 0: F = P \\ \beta \uparrow: R \end{array}$$

Evaluation

- *implementation:*

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_kmeans.ipynb

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_dbSCAN.ipynb

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/clustering_agglomerative.ipynb





Face Analysis

- *Introduction*
- *Face Detection & Preprocessing*
- *Face Recognition*
- *A Complete Face Clustering Algorithm*



Face Analysis: Introduction

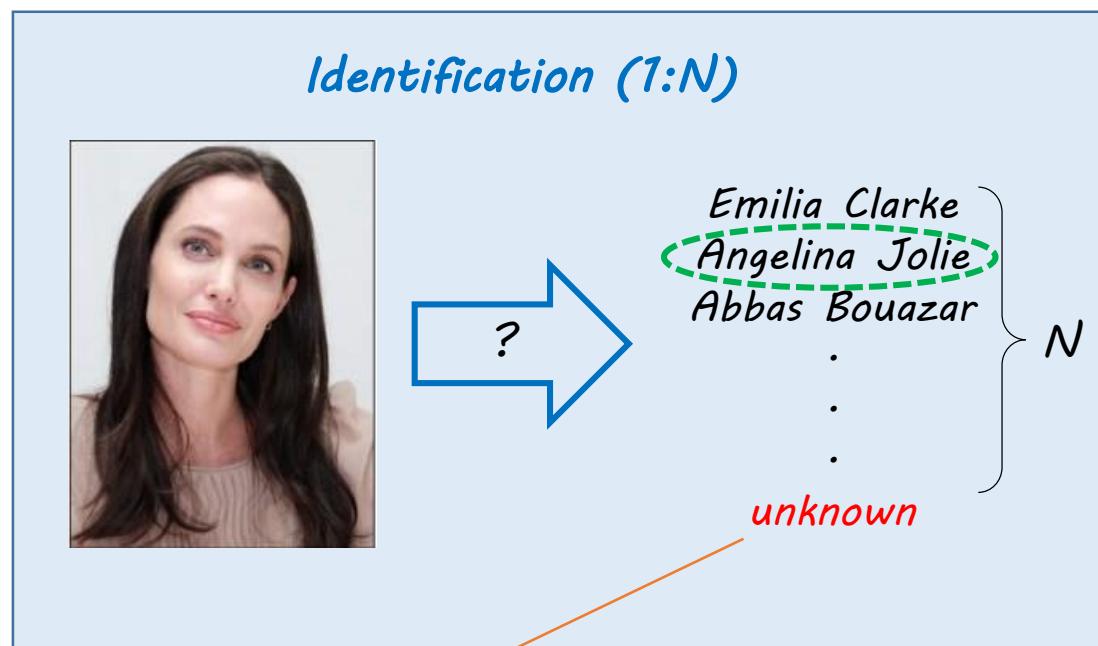
Face Recognition (It is different from face clustering!)

Identification (1:N)

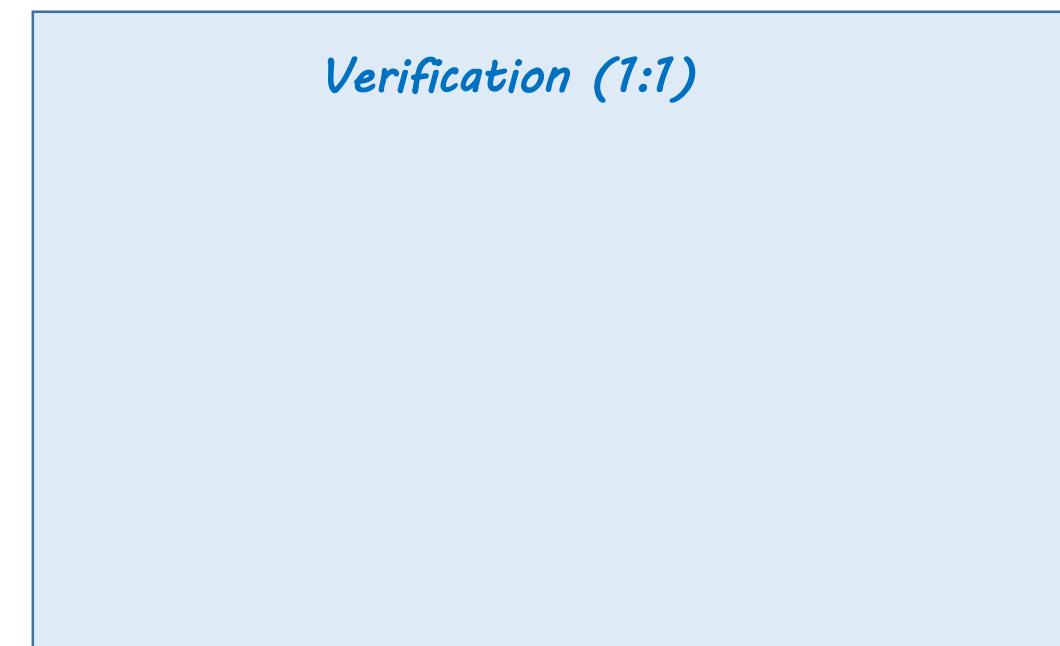
Verification (1:1)

Face Analysis: Introduction

Face Recognition (*It is different from face clustering!*)

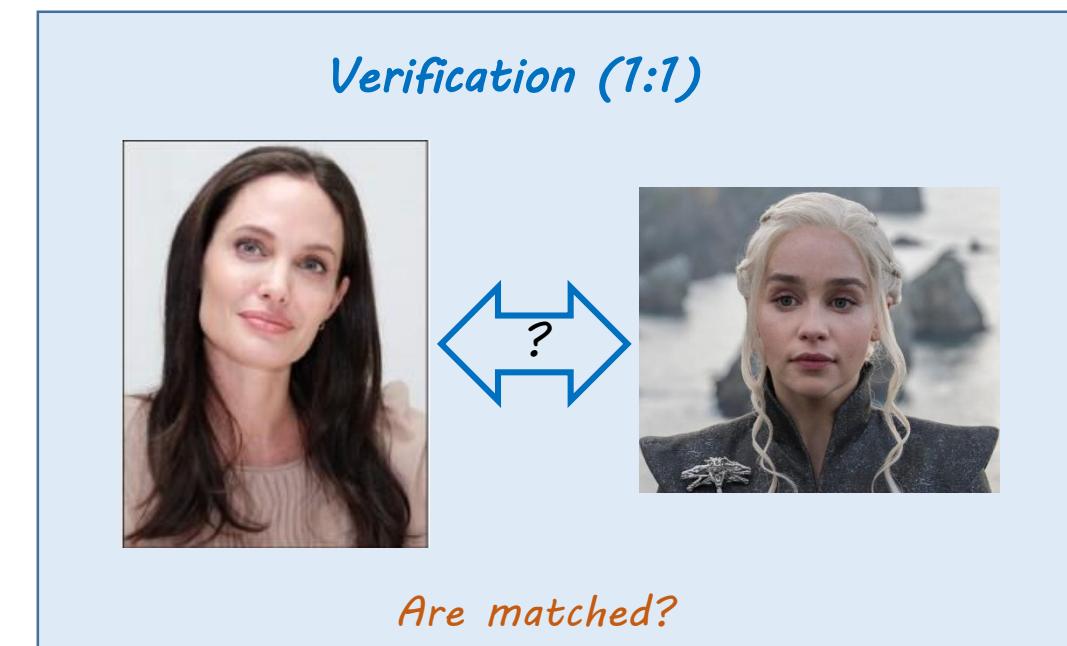
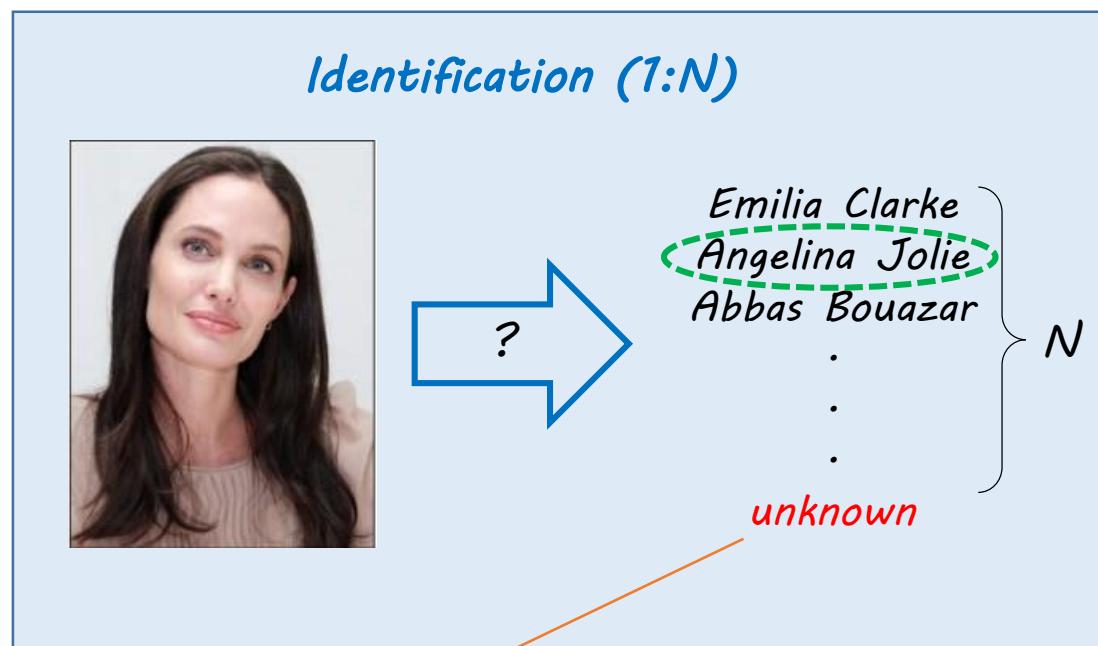


For open-set problem



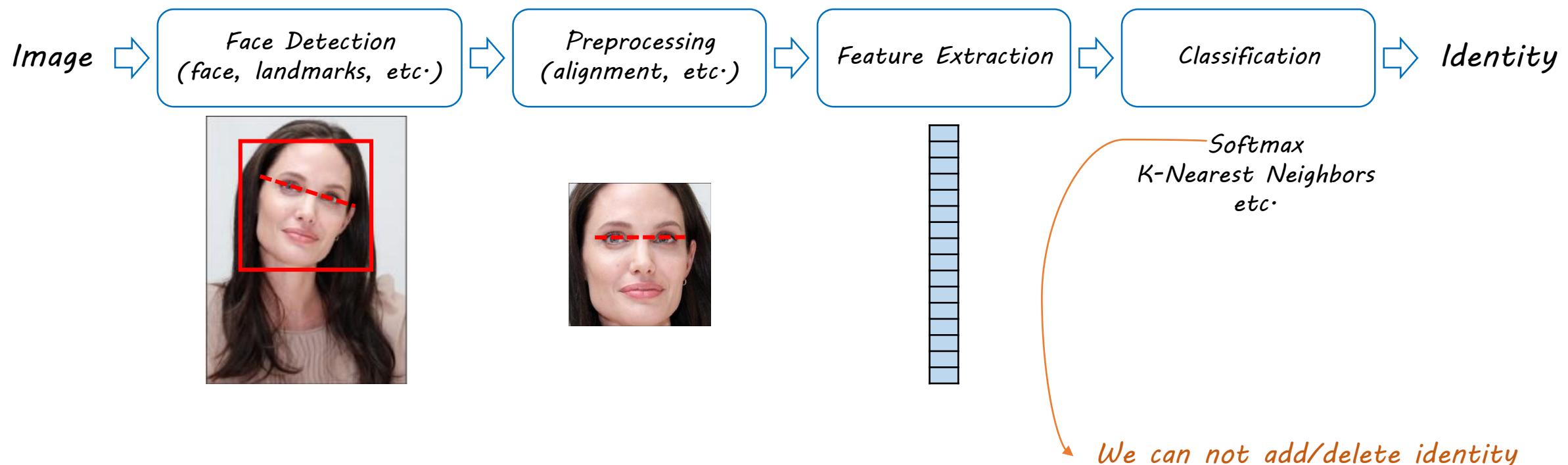
Face Analysis: Introduction

Face Recognition (*It is different from face clustering!*)

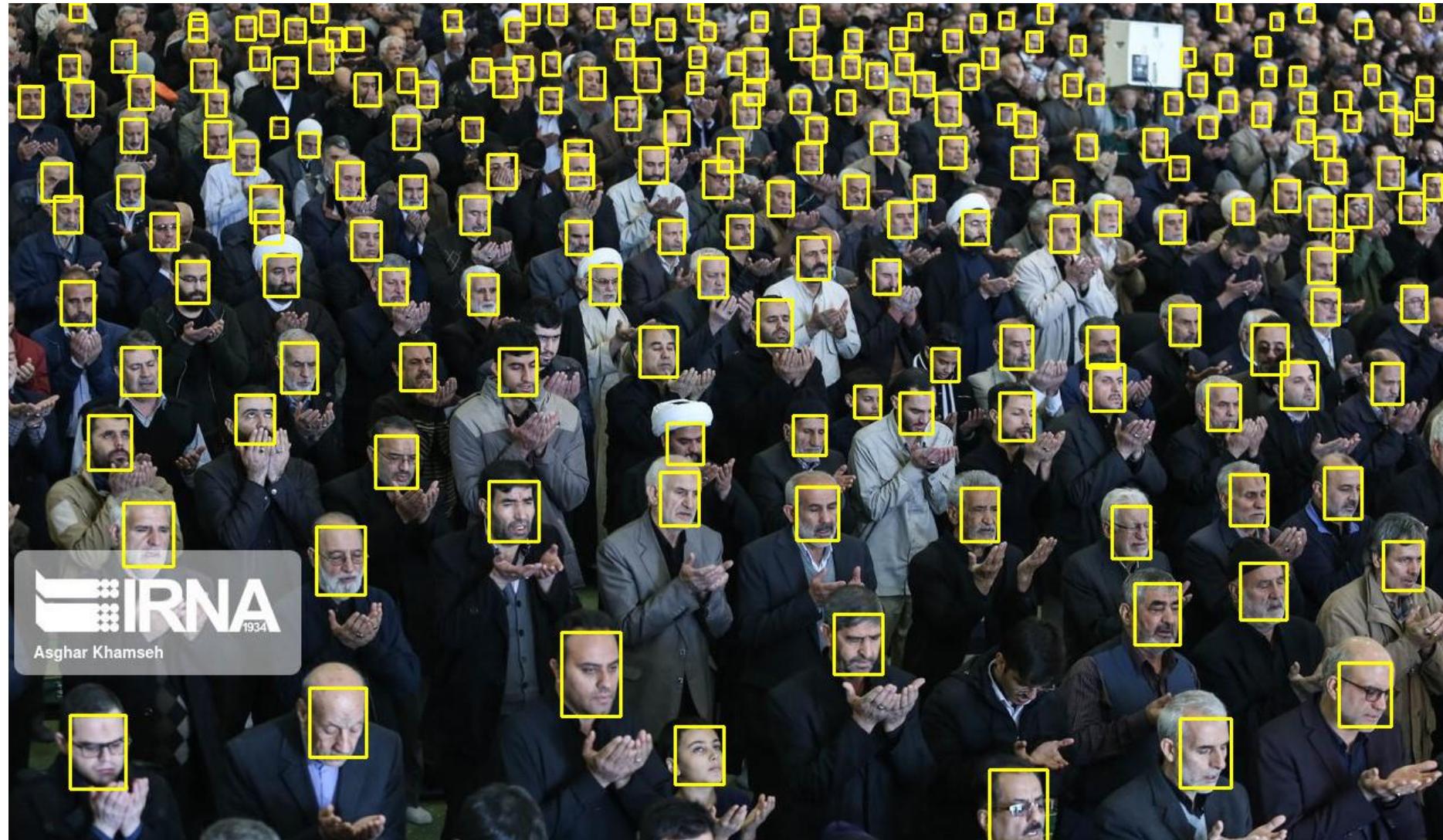


Face Analysis: Introduction

- Block diagram of a face recognition system

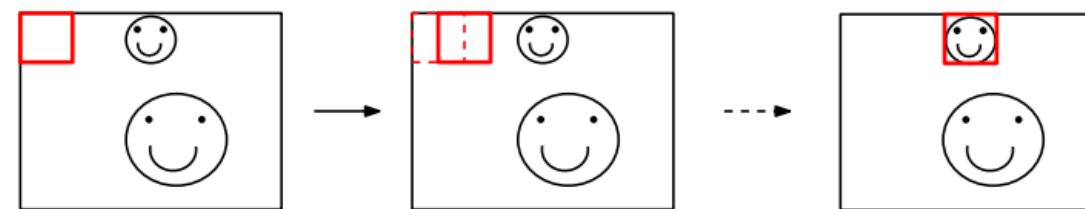


Face Analysis: Face Detection



Face Analysis: Face Detection

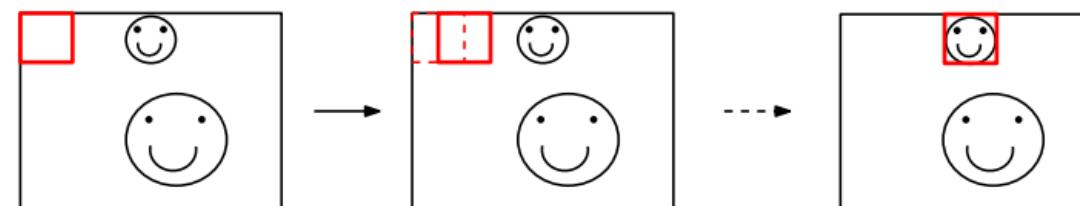
- *Haar Cascade*



F. Comaschi, et al., RASW: a Run-time Adaptive Sliding Window to Improve Viola-Jones Object Detection

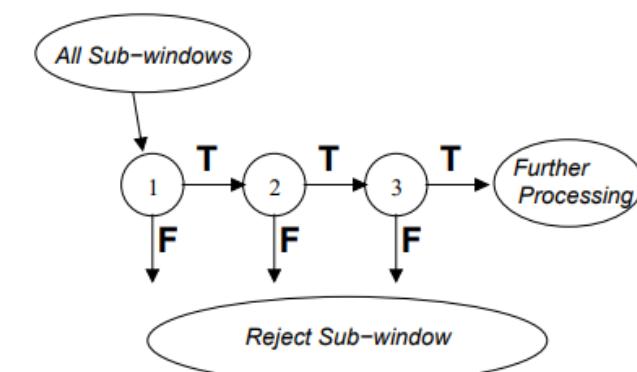
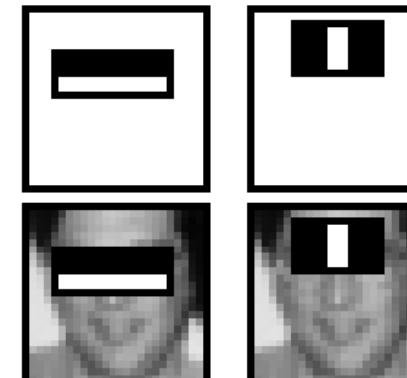
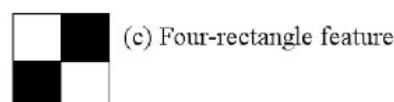
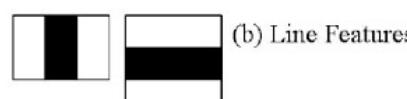
Face Analysis: Face Detection

- Haar Cascade



F. Comaschi, et al., RASW: a Run-time Adaptive Sliding Window to Improve Viola-Jones Object Detection

Each feature: $\sum(\text{pixels under black rectangle}) - \sum(\text{pixels under white rectangle})$



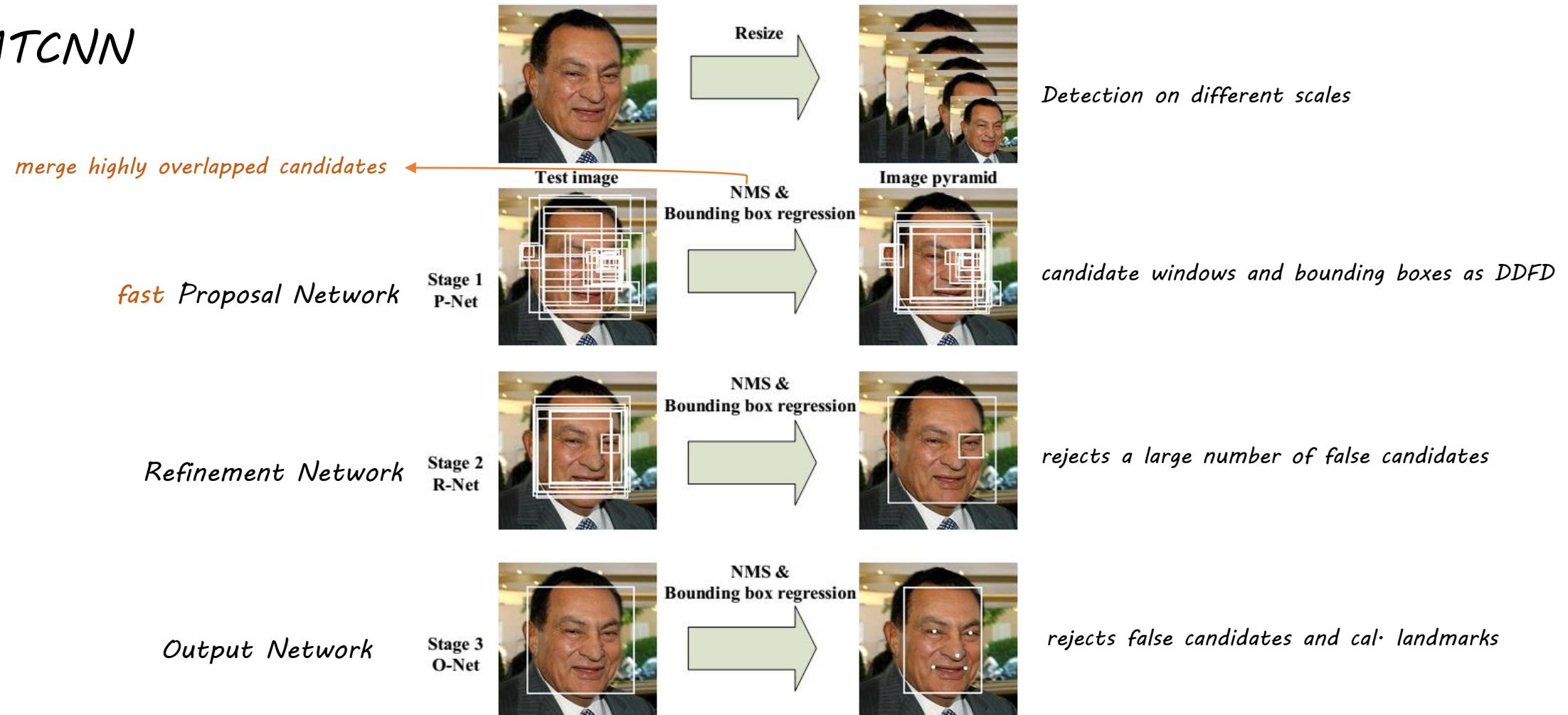
Code:

<https://towardsdatascience.com/face-detection-with-haar-cascade-727f68dafd08>

https://opencv24-python-tutorials.readthedocs.io/en/latest/py_tutorials/py_objdetect/py_face_detection/py_face_detection.html

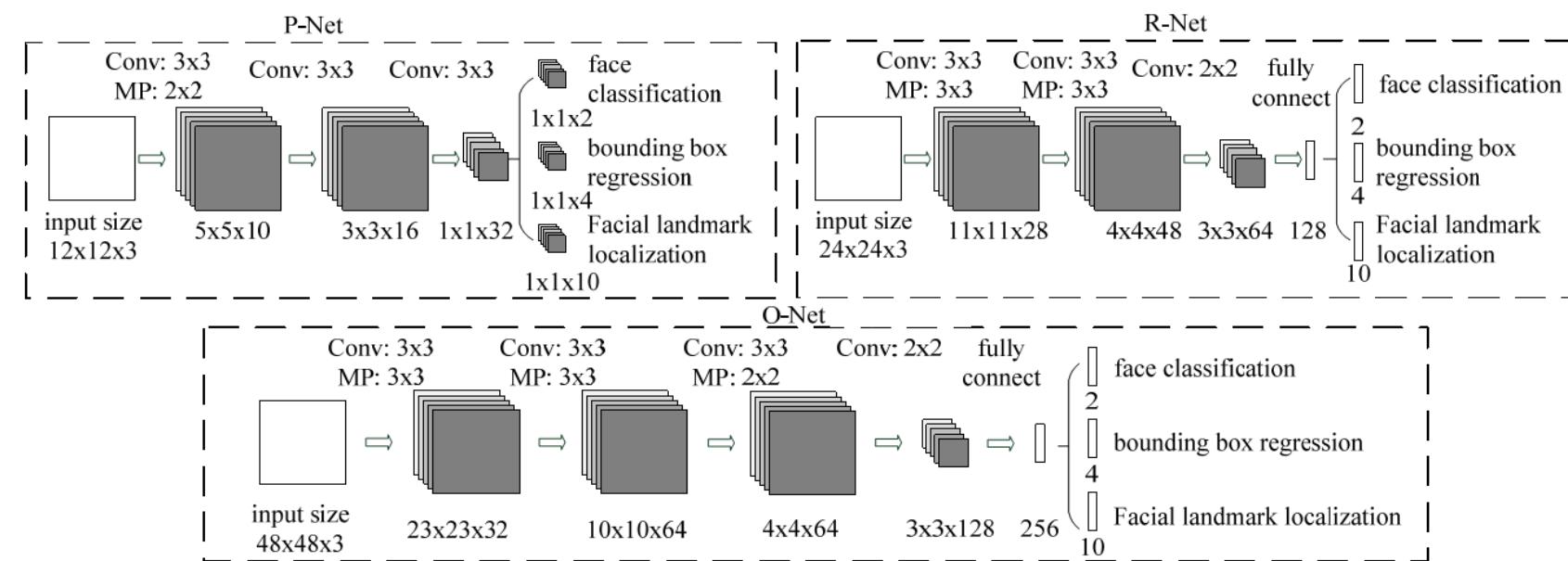
Face Analysis: Face Detection

- MTCNN



Face Analysis: Face Detection

- MTCNN



Code:

Python package: <https://github.com/ipazc/mtcnn>

<https://github.com/davidsandberg/facenet/tree/master/src/align>

[https://github.com/kpzhang93/MTCNN face detection alignment](https://github.com/kpzhang93/MTCNN_face_detection_alignment)

Face Analysis: Face Detection

- MTCNN

Face classification loss:

$$L_i^{det} = - \left(y_i^{det} \log(p_i) + (1 - y_i^{det})(\log(1 - p_i)) \right) \quad y_i^{det} \in \{0,1\}$$

Bounding box regression loss:

$$L_i^{box} = \|\hat{y}_i^{box} - y_i^{box}\|_2^2$$

landmark localization loss:

$$L_i^{Landmark} = \|\hat{y}_i^{Landmark} - y_i^{Landmark}\|_2^2$$

overall learning target:

$$\min \sum_{i=1}^N \sum_{j \in (det, box, landmark)} \alpha_j \beta_i^j L_i^j$$

$\beta_i^j \in \{0,1\}$ (sample type indicator)

task importance: $\begin{cases} PNet, RNet: & \alpha_{det} = 1, \alpha_{box} = 0.5, \alpha_{landmark} = 0.5 \\ ONet: & \alpha_{det} = 1, \alpha_{box} = 0.5, \alpha_{landmark} = 1 \end{cases}$

Face Analysis: Face Detection

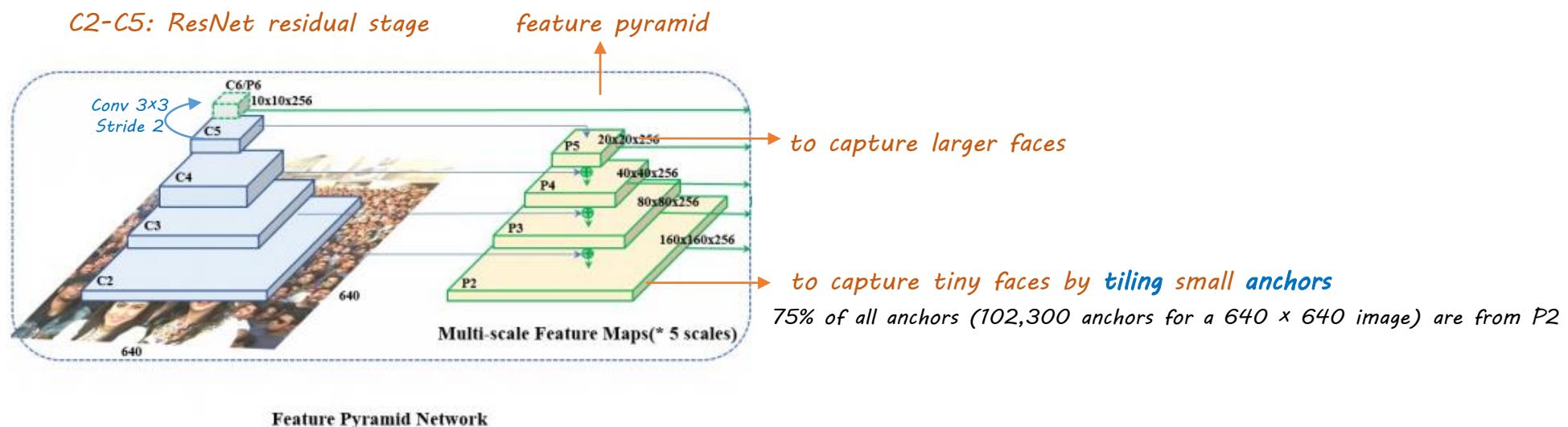
- *Implementation (MTCNN):*

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/face_detection_mtcnn.ipynb



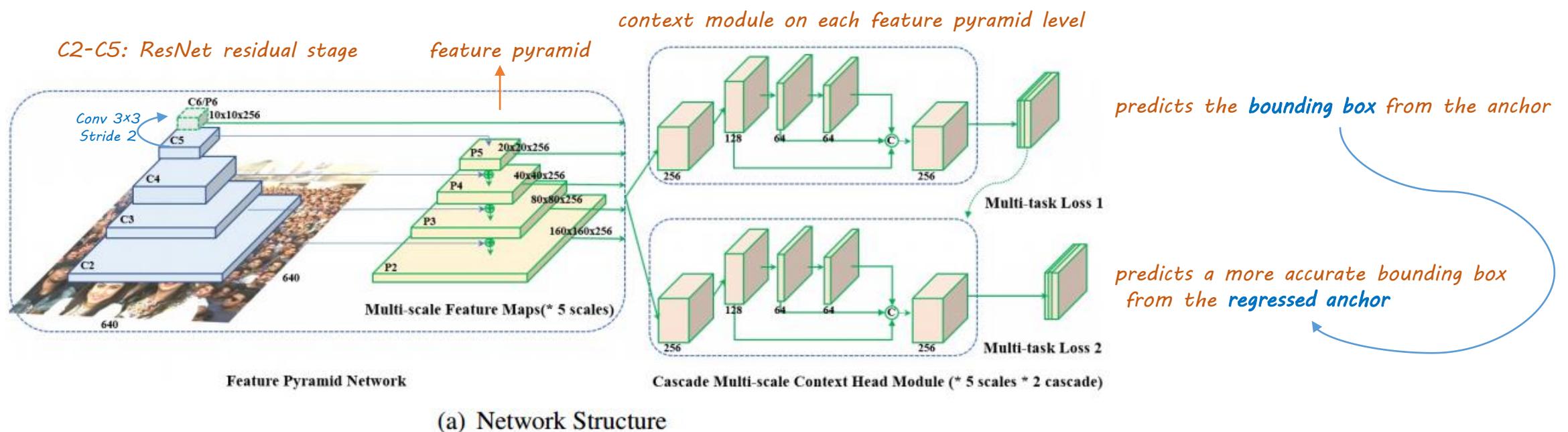
Face Analysis: Face Detection

- RetinaFace



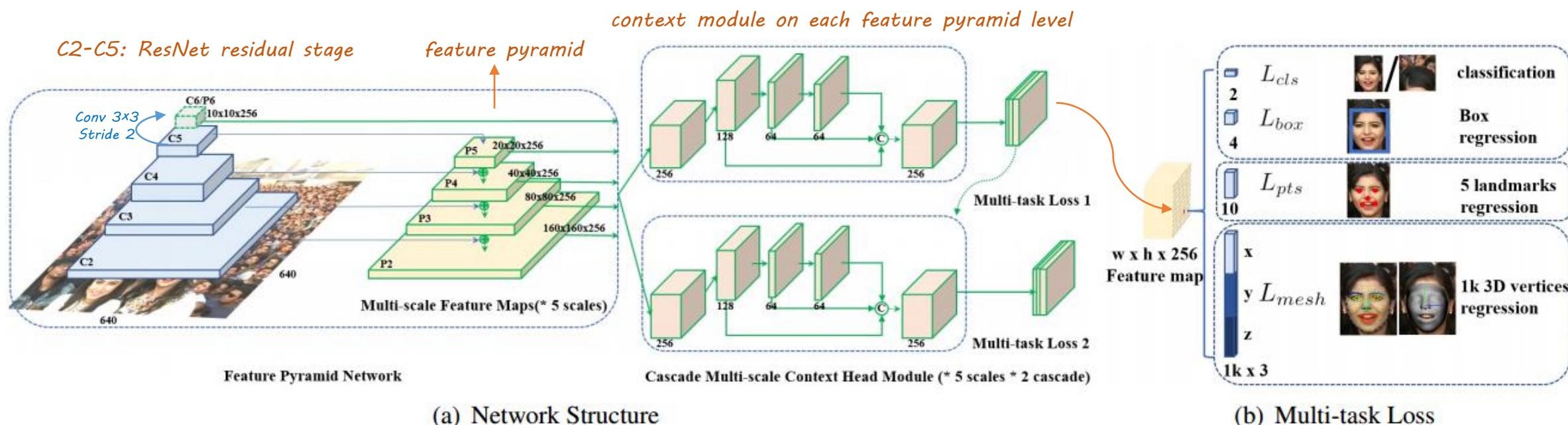
Face Analysis: Face Detection

- RetinaFace



Face Analysis: Face Detection

- RetinaFace

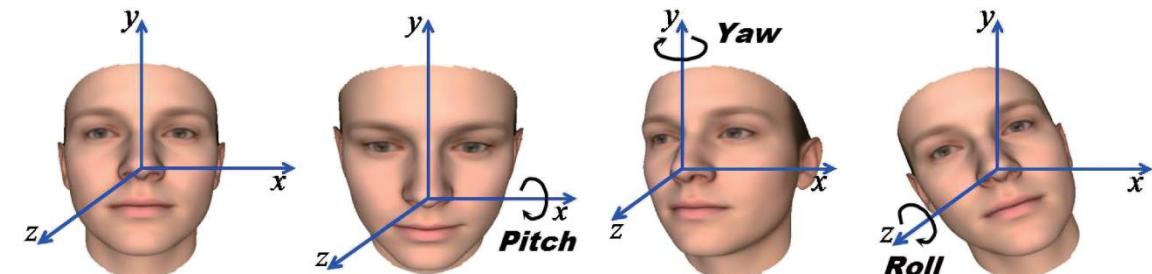


Code:

<https://github.com/deepinsight/insightface/tree/master/detection/retinafece>

Face Analysis: Preprocessing

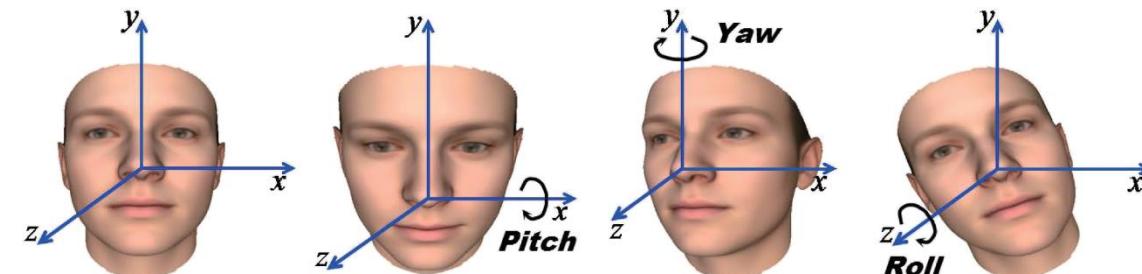
- Head pose



G. Sang, et al., Learning toward practical head pose estimation, SPIE, 2017

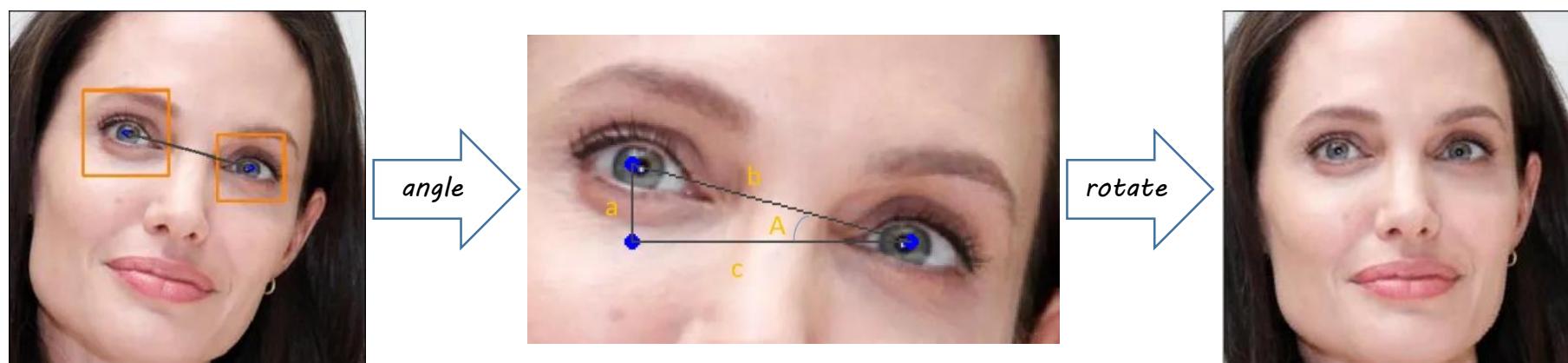
Face Analysis: Preprocessing

- Head pose



G. Sang, et al., Learning toward practical head pose estimation, SPIE, 2017

- Roll:

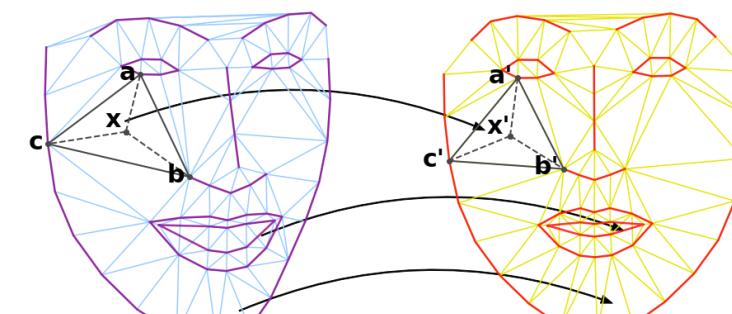


Code: <https://sefiks.com/2020/02/23/face-alignment-for-face-recognition-in-python-within-opencv/>

Face Analysis: Preprocessing

- Pitch and Yaw

piecewise affine warping



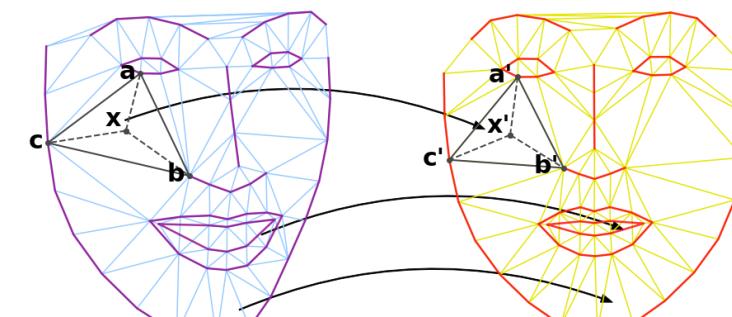
Delaunay triangulation

<http://jeankossaifi.com/pages/gagan.html>

Face Analysis: Preprocessing

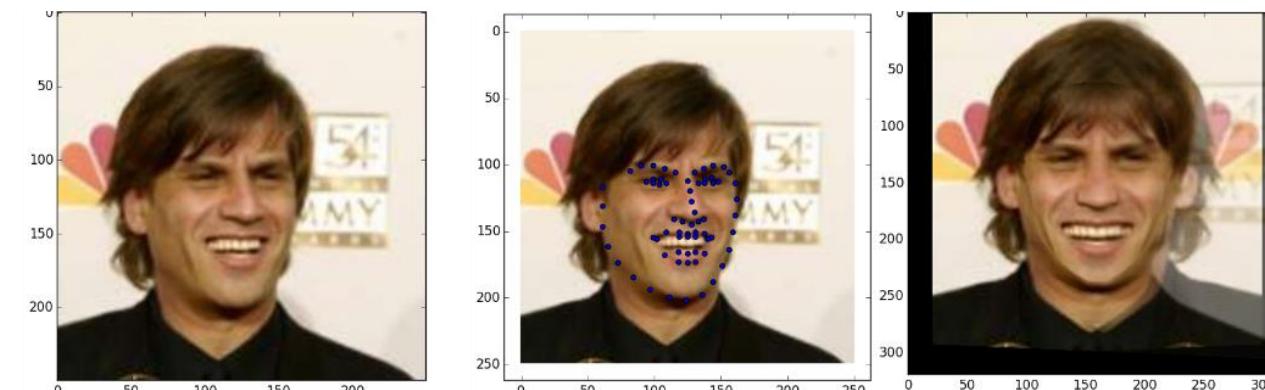
- Pitch and Yaw

piecewise affine warping



<http://jeankossaifi.com/pages/gagan.html>

Face Frontalization

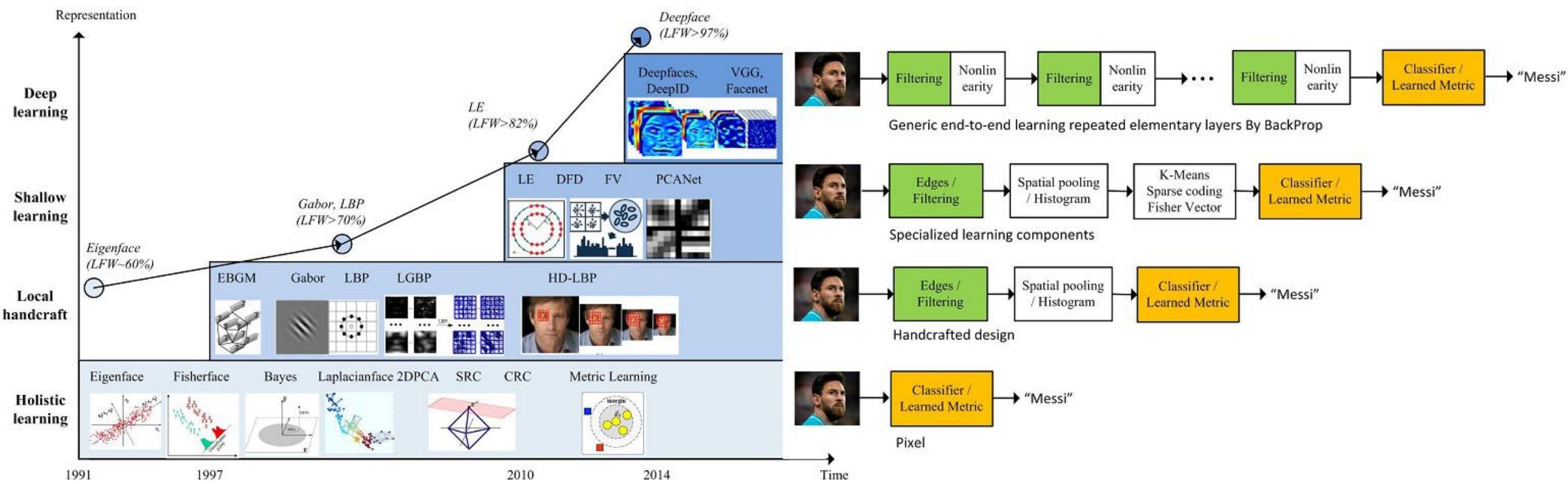


Code: <https://github.com/dougsouza/face-frontalization>

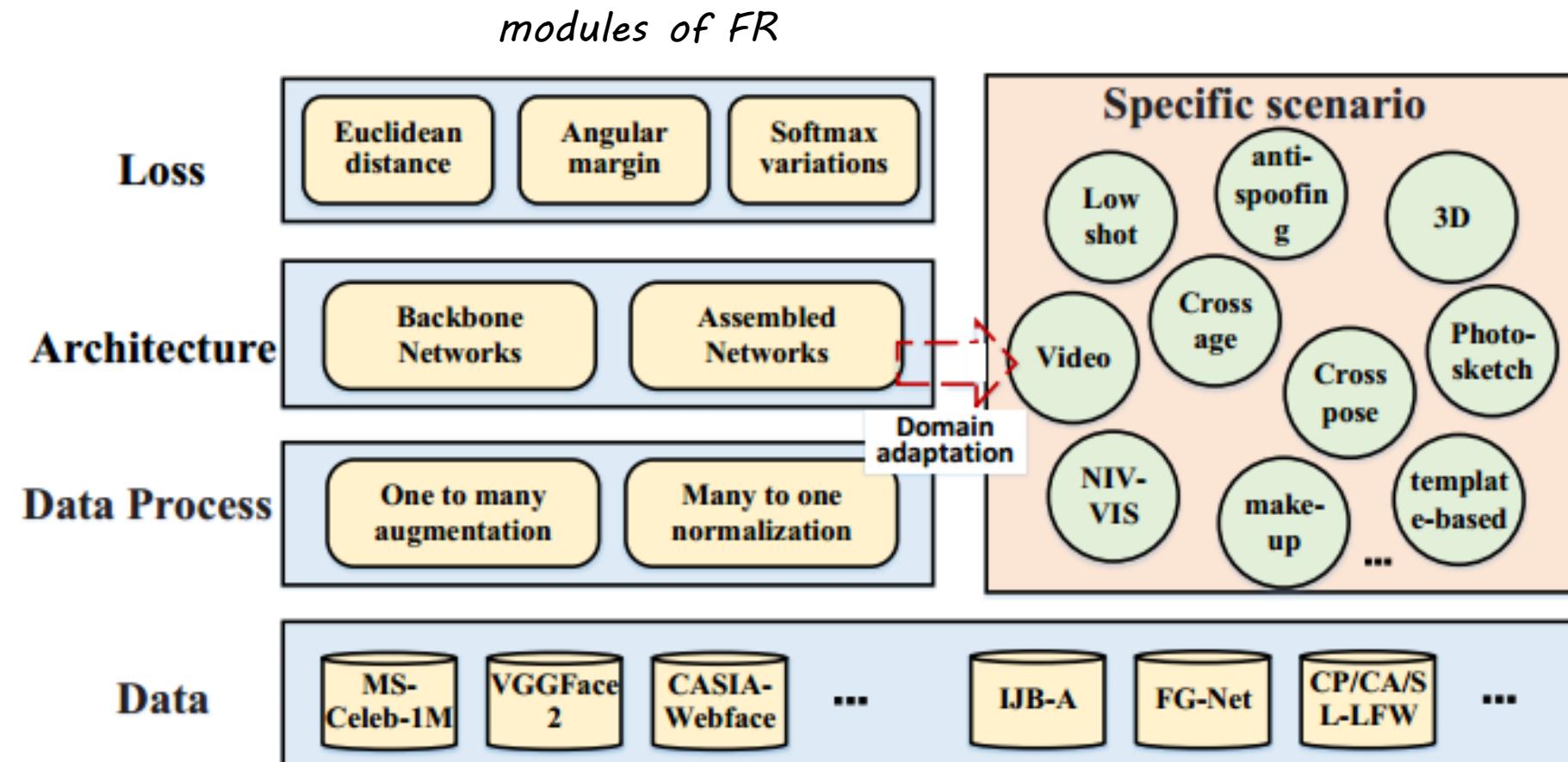
Insightface Code: <https://github.com/deepinsight/insightface/tree/48cc12e3d83c5e98122068eaf04f5d9829c8b399/alignment/coordinateReg>

Face Analysis: Face Recognition

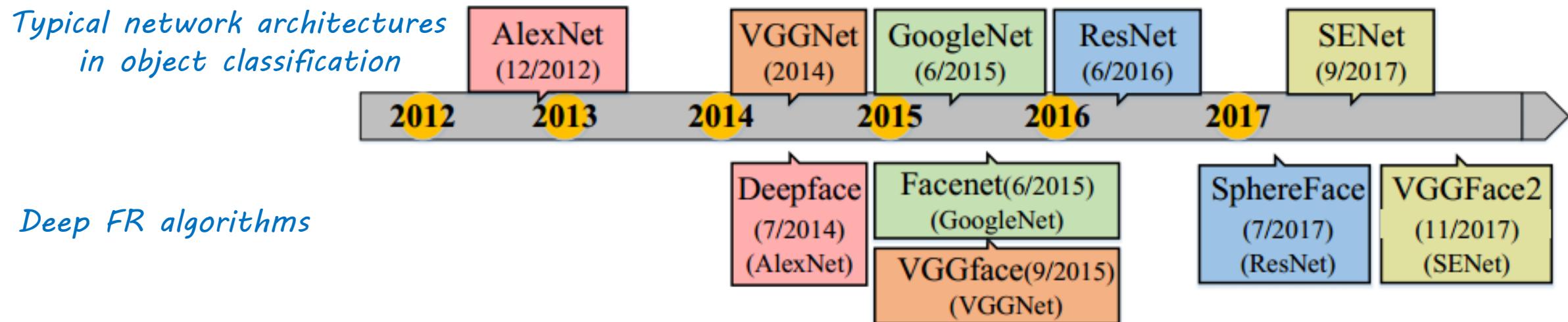
Milestones of face representation for recognition



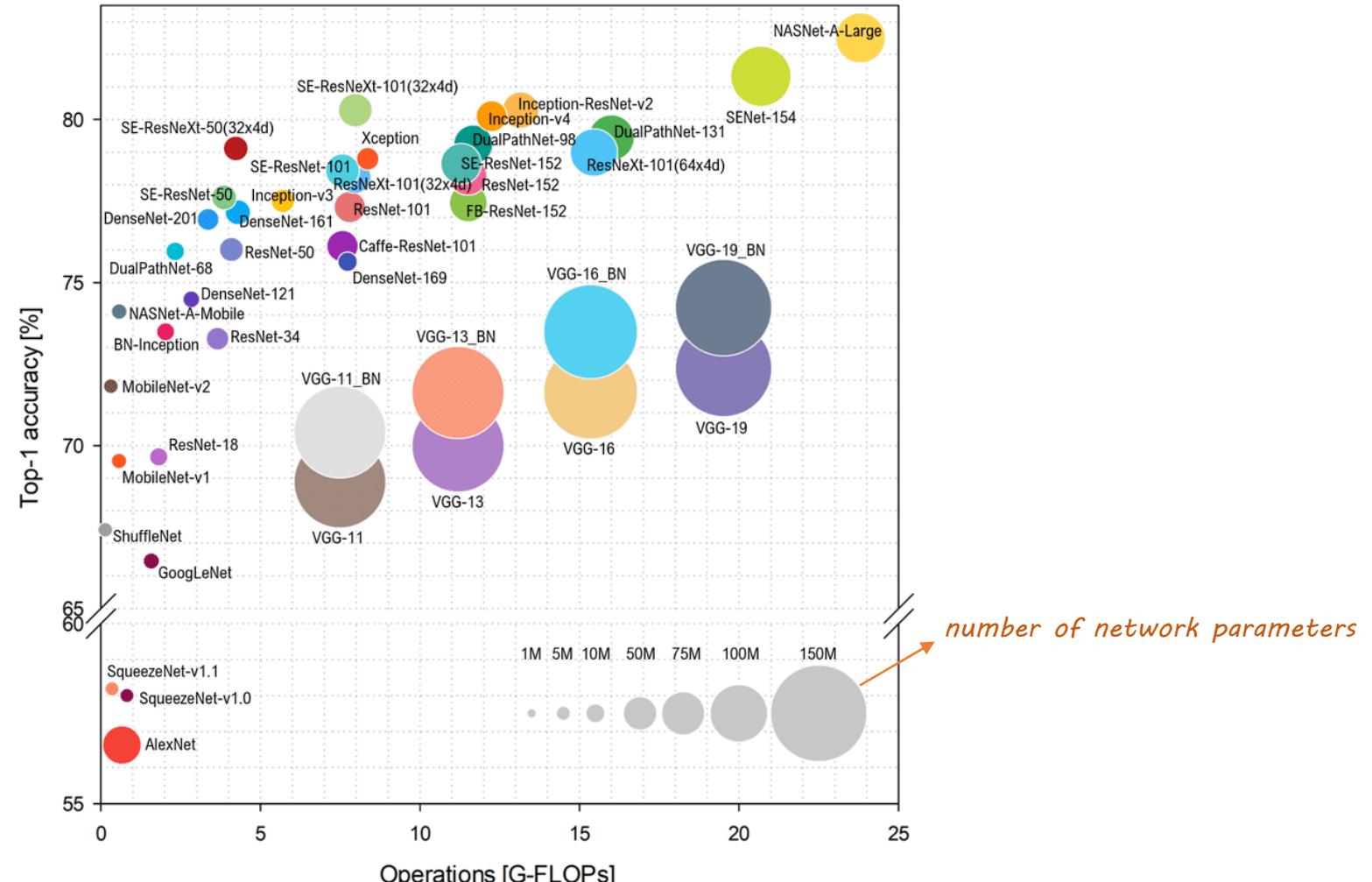
Face Analysis: Face Recognition



Face Analysis: Face Recognition



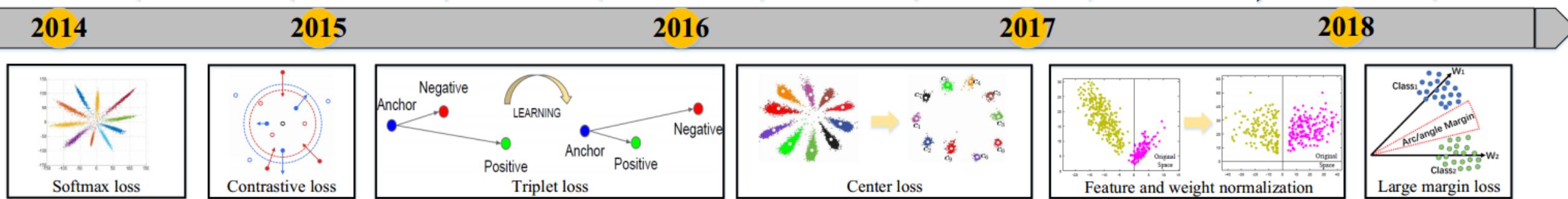
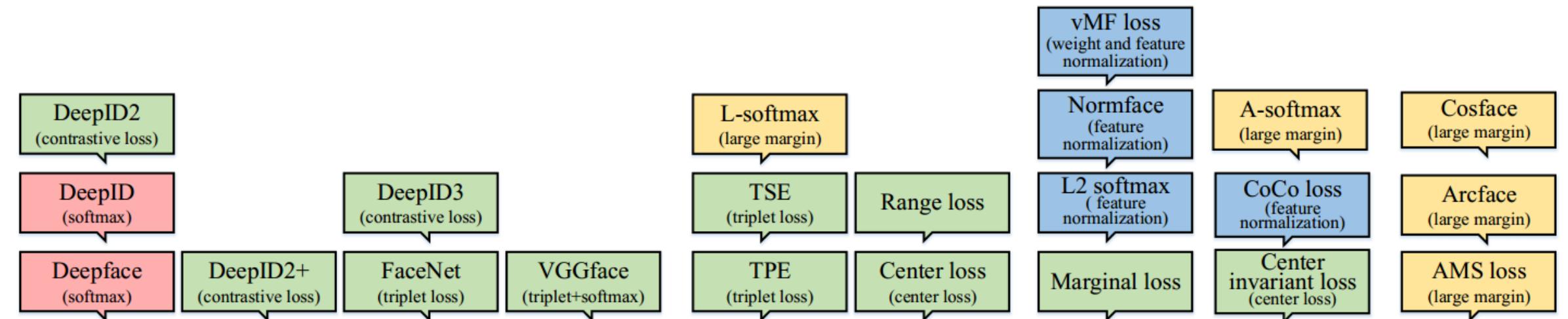
Face Analysis: Face Recognition



A. Canziani, An analysis of deep neural network models for practical applications, 2016
S. Bianco, Benchmark Analysis of Representative Deep Neural Network Architectures, 2018

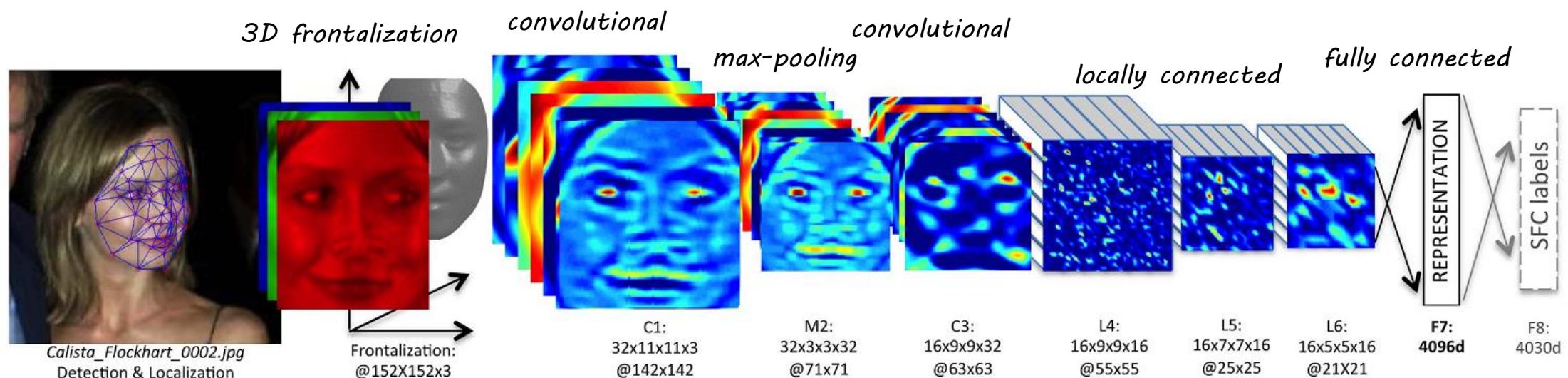
Face Analysis: Face Recognition

Loss function



Face Analysis: Face Recognition

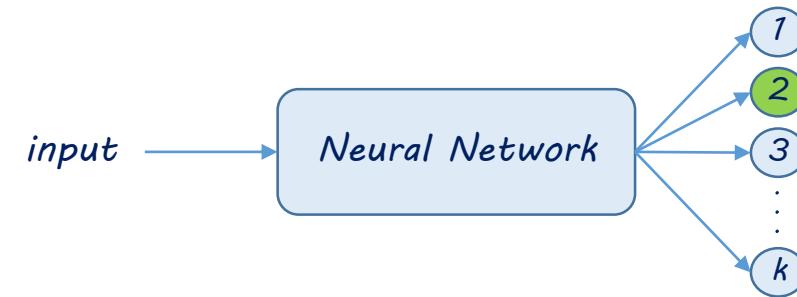
- DeepFace



- 1 - Train the NN using Softmax (for multi-class classification or identification)
- 2 - Train the two topmost layers using Siamese (for verification)

Face Analysis: Face Recognition

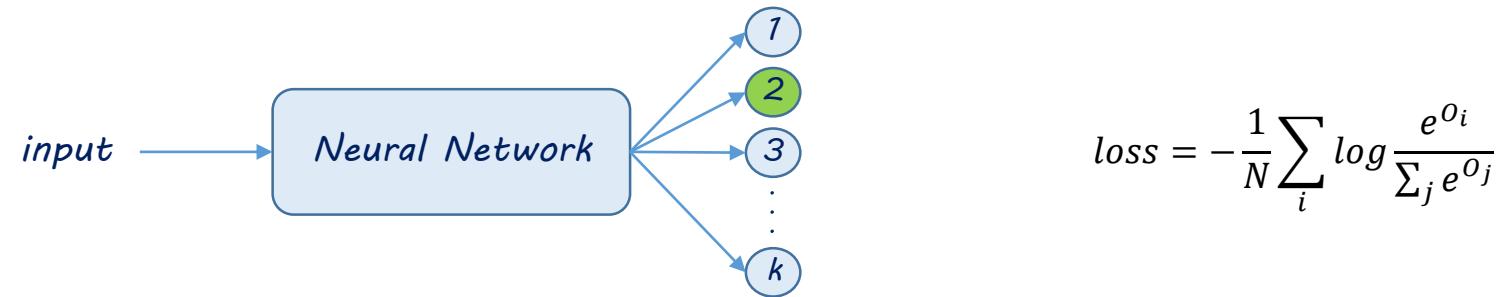
- Softmax



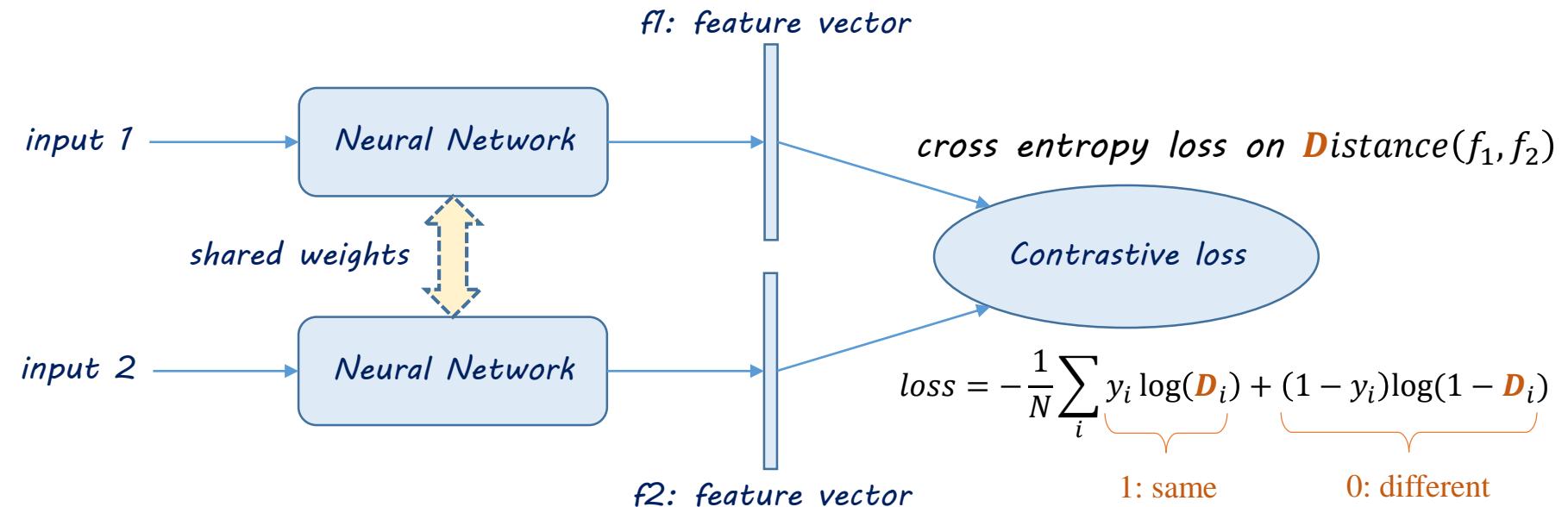
$$\text{loss} = -\frac{1}{N} \sum_i \log \frac{e^{o_i}}{\sum_j e^{o_j}}$$

Face Analysis: Face Recognition

- **Softmax**



- **Siamese**



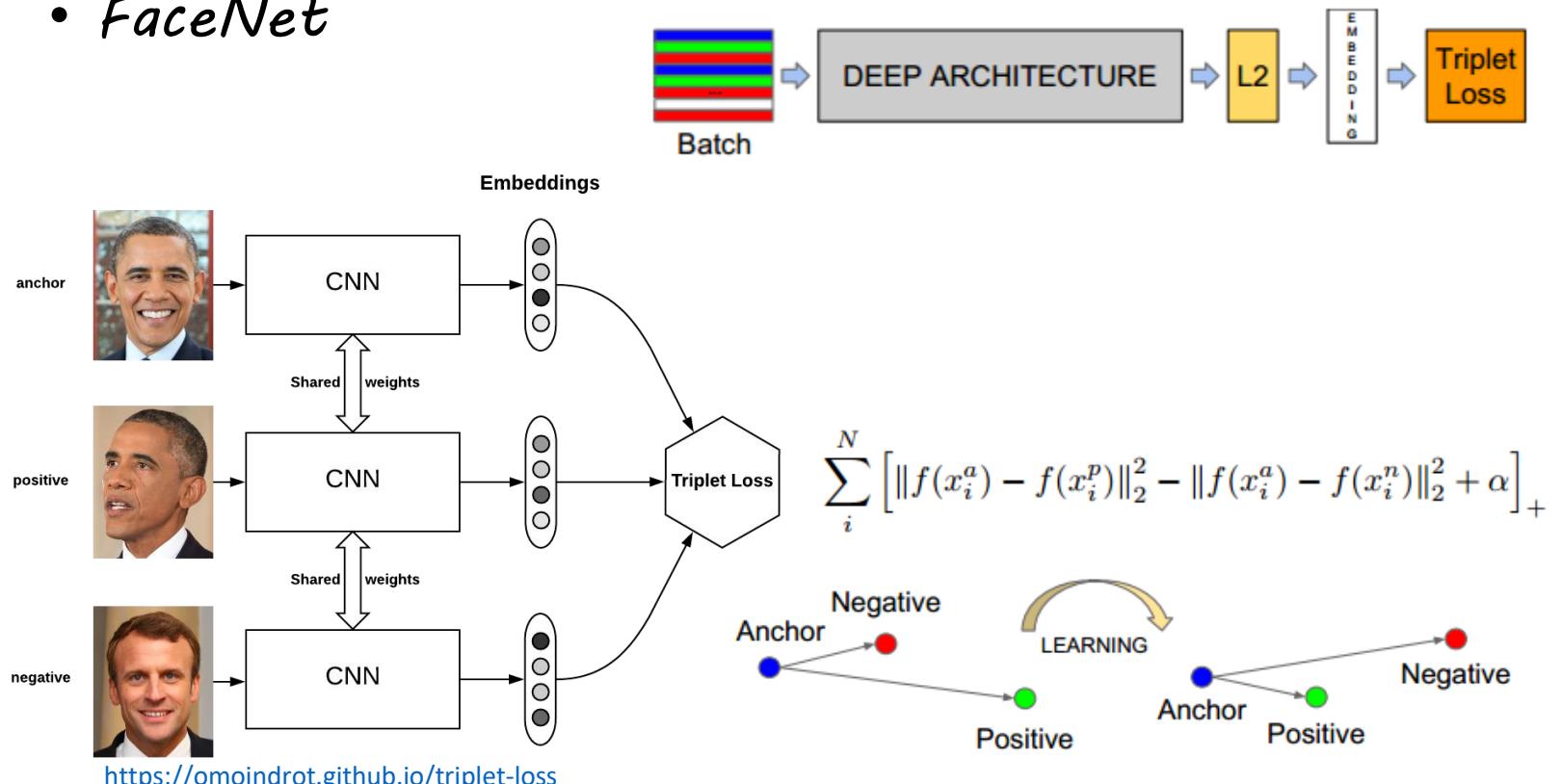
Face Analysis: Face Recognition

- FaceNet



Face Analysis: Face Recognition

- *FaceNet*



Code: <https://github.com/davidsandberg/facenet>

Face Analysis: Face Recognition

- ArcFace

Softmax:

$$L_1 = -\frac{1}{N} \sum_{i=1}^N \log \frac{e^{W_{y_i}^T x_i + b_{y_i}}}{\sum_{j=1}^n e^{W_j^T x_i + b_j}}$$

Face Analysis: Face Recognition

- ArcFace

Softmax:

$$L_1 = -\frac{1}{N} \sum_{i=1}^N \log \frac{e^{W_{y_i}^T x_i + b_{y_i}}}{\sum_{j=1}^n e^{W_j^T x_i + b_j}}$$

Annotations:

- #classes: points to the term n in the denominator.
- Batch size: points to the term N in the denominator.
- Label of x_i : points to the term y_i in the exponent.
- i^{th} data: points to the term x_i in the exponent.
- bias: points to the term b_{y_i} .

Deng, et al.: “Softmax loss function does not explicitly optimise the feature embedding to enforce higher similarity for intra-class samples and diversity for inter-class samples”

Pose, Age, etc.

Let $b = 0$, $\|W_j\| = 1$, $\|x_i\| = s$
 $W_j^T x_i = \|W_j\| \|x_i\| \cos \theta_j$

$$\rightarrow L_2 = -\frac{1}{N} \sum_{i=1}^N \log \frac{e^{s \cos \theta_{y_i}}}{e^{s \cos \theta_{y_i}} + \sum_{j=1, j \neq y_i}^n e^{s \cos \theta_j}}$$

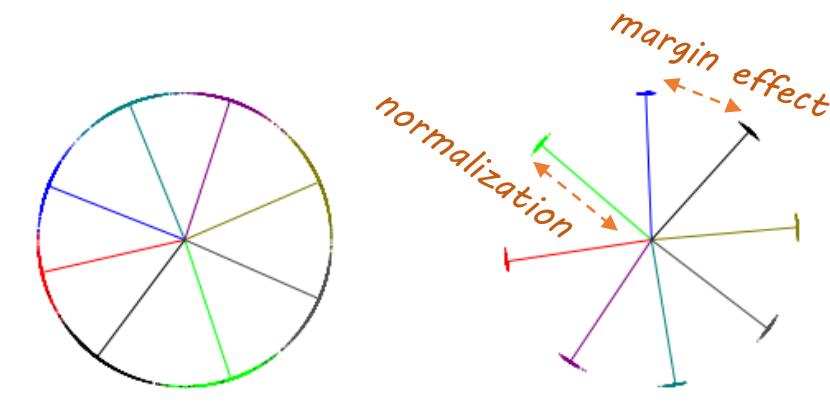
$$L_3 = -\frac{1}{N} \sum_{i=1}^N \log \frac{e^{s (\cos(\theta_{y_i} + m))}}{e^{s (\cos(\theta_{y_i} + m))} + \sum_{j=1, j \neq y_i}^n e^{s \cos \theta_j}}$$

margin

Face Analysis: Face Recognition

- ArcFace

8 identities with 2D normalized features:



(a) Softmax

(b) ArcFace

- Code:

<https://github.com/deepinsight/insightface>

python library, Originally MXNet, newly PyTorch

Face Analysis: Face Recognition

- *Implementations:*

- *face recognition using dlib:*

https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/face_recognition_using_dlib.ipynb

- *face recognition using ArcFace (insightface):*

<https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/arcface.ipynb>



A Complete Face Clustering Algorithm



Face Analysis: Face Clustering

- *Implementations:*
 - *Face clustering using ArcFace on a small set:*
https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/face_clustering_arcface.ipynb
 - *Face clustering using ArcFace on FaceCup sample dataset:*
https://colab.research.google.com/github/hamidsadeghi68/face-clustering/blob/main/face_clustering_arcface_facecup_samples.ipynb

