

# Research Background

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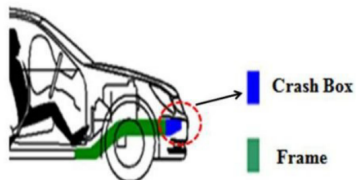
## 1 Education

## 2 Research Projects

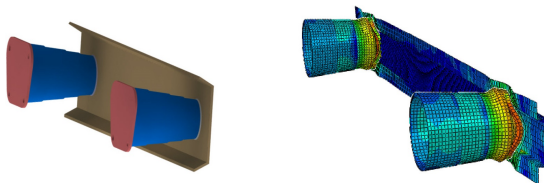
- 2016-2017      Crash-Box selection (Master's thesis)
- 2017            Foam-Filled Crash-Box with Initiator
- 2018            Inversion of Crash-Box Process
- 2018-2019      Crash-Box Multi-Objective Optimization
- 2018-2021      Crash-Box Manufacturing Process Selection

- MSc in Mechanical Engineering - Applied Design
- BSc in Mechanical Engineering - Solid Design

# Crash-boxes



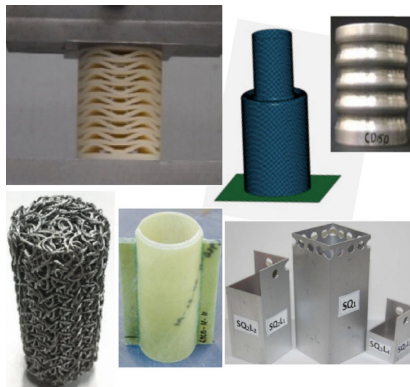
The front structural components in a vehicle (Hussain et al. 2017)



The model of front structural with a pair of crash boxes

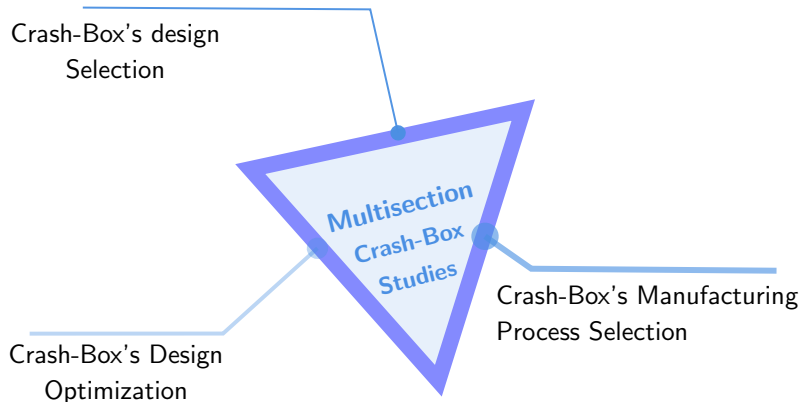
# Crash-boxes

The Crash-boxes' design indicators<sup>↗</sup> based on the force-displacement curve<sup>↗</sup>: Absorbed energy ( $E_{absorbed}$ ), Initial peak load ( $F_i$ ), Maximum peak load ( $F_i$ ), CFE, SEA and Mass of the structure (M).

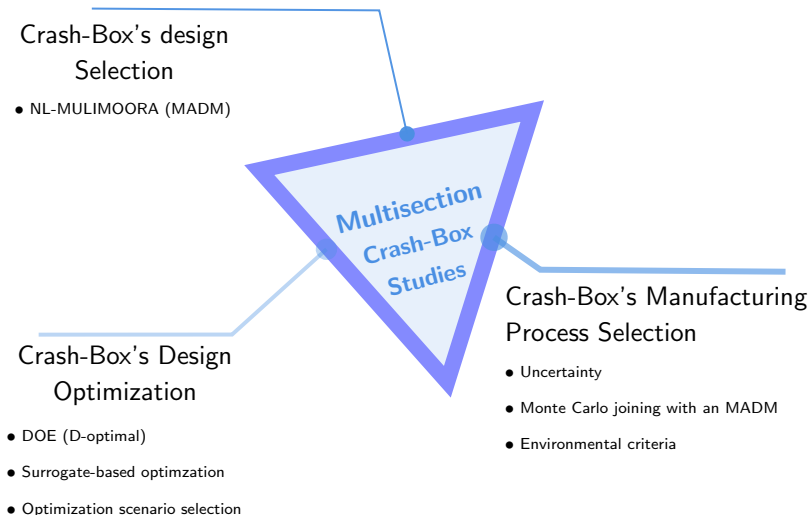


Different design of crash-boxes (Abdullah et al. 2020, Yusof et al. 2017)

# Summary of the core research on the Crash-boxes

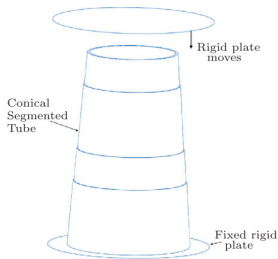


# Summary of the core research on the Crash-boxes



# Master's Thesis: Crash-Box Selection employing MADM & Simulation

- Proposing a hybrid method  $\text{MADM, MULTIMOORA}^{\zeta} + \text{NL}^{\zeta} \Rightarrow \text{NL-MULTIMOORA}$
- Simulating the crashing process $^{\zeta}$
- Generating, Sorting and Selecting the best design in two steps:
  - The best design selection through 39 designs based on Length & Thickness  $^{\zeta}$
  - The best design selection through 16 designs based on Taper's Angle  $^{\zeta}$
- The best prototype design selected



Simulation of the conical segmented tube under axial loading (Souzangarzadeh et al. 2017)



## Proposed prototype selection process:

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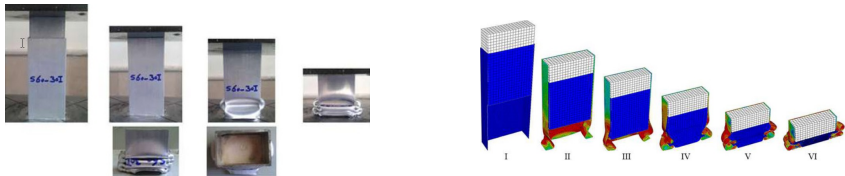
▽ **Step two**  $\Rightarrow$  Generating 39 points  $\Rightarrow$  NL-MULTIMOORA selected the best design base on thicknesses and lengths  $\Rightarrow$  Specifying the best thicknesses and lengths for the next step

## Proposed prototype selection process:

- ▽ **Step one**  $\Rightarrow$  Modeling the crushing process of Aluminum segmented tubes with 3D Solid elements.
- ▽ **Step two**  $\Rightarrow$  Generating 39 points  $\Rightarrow$  NL-MULTIMOORA selected the best design base on thicknesses and lengths  $\Rightarrow$  Specifying the best thicknesses and lengths for the next step
- ▽ **Step three**  $\Rightarrow$  Considering different taper angles  $\Rightarrow$  Generating other 14 points + the two best designs from second step  $\Rightarrow$  Selecting the best design base on taper angle through NL-MULTIMOORA

# Foam-Filled Crash-Box with Initiator: Experimental & Numerical

- Evaluating the effect of Initiator & foam on the behavior of the Crash-box
  - Numerically ☞
  - Experimentally ☞
- Detecting the effect of the Initiator ☞
- Detecting the most efficient Initiator's length

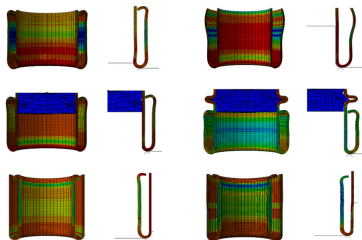


Different stages of crushing for partially foam-filled with an initiator (Razazan et al. 2018)

# 3

## Inversion of Crash-Box Process: Experimental & Numerical

- Evaluating the effect of triggering, foam & inversion process on the behavior of the Crash-box
  - Numerically ☞
  - Experimentally ☞
- Detecting the effect of the inversion mechanism, ☞

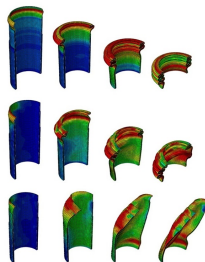
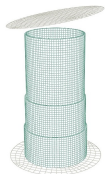
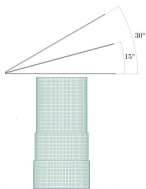


Crushed shapes of empty and foam-filled circular specimens derived from experimental and numerical simulation (Rezvani and

Souzangarzadeh 2020)

## Optimization the design of Crash-box employing DOE, MADM, & Simulation

▷ Five design variables & Ten Objective Functions [↗](#)



Deforming process of the selected tube under axial, 15°, and 30° loads (Souzangarzadeh et al. 2020)

## Proposed optimization and selection process:



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- ▽ DoE (RSM D-optimal)<sup>Ⓢ</sup>  $\Rightarrow$  34 points  $\Rightarrow$  Ten regression models
  - $\Rightarrow$  Comparing the regression model with FEM
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- ▽ 16 optimization scenarios  $\Rightarrow$  16 optimized designs <sup>↗</sup>
- ▽ Using combinative weighting (CW) method to combine three different methods <sup>↗</sup>
- ▽ Selecting the best candidate/scenarios through CW-MULTIMOORA<sup>↗</sup>

## ⑤ Automobile Crash-Box Manufacturing Selection

### Assumptions and Conditions:

- Cylindrical-Steel-segmented tube (Souzangarzadeh et al. 2020) and Conical-aluminum-segmented tube (Souzangarzadeh et al. 2017)
- Uncertain data in the case study location, Iran
- Extracted data of time and cost were interval not exact

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### Methods:

- ELECTRE-IDAT as MADM method deals with interval and exact data
- Using Monte Carlo to simulate the selection process for 5000 times
- The most promising manufacturing processes and material-design selected

# References



N.N, Hussain, S.P, Regalla, Y.V.D., Rao, 2017

Comparative study of trigger configuration for enhancement of Crashworthiness of automobile Crash box subjected to axial impact loading.

*Procedia Engineering, 173, pp.1390-1398.*



H, Souzangarzadeh, M, Rezvani, A, A, Jahan, 2017

Selection of optimum design for conical segmented aluminum tubes as energy absorbers: Application of MULTIMOORA method

*Applied Mathematical Modelling*



M, Razazan, M.J, Rezvani, H, Souzangarzadeh, 2018

Evaluation of the Performance of Initiator on Energy Absorption of Foam-Filled Rectangular Tubes: Experimental and Numerical Assessment

*Experimental Techniques*



M.J, Rezvani, H, Souzangarzadeh, 2020

Effects of triggering and polyurethane foam on energy absorption of thin-walled circular tubes under the inversion process

*Journal of Energy Storage*



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Multi-objective optimization of cylindrical segmented tubes as energy absorbers under oblique crushes: D-optimal design and integration of MULTIMOORA with combinative weighting

*Structural and Multidisciplinary Optimization*



N.A.Z. Abdullah, M.S.M. Sani, M.S. Salwani, N.A. Husain 2020

A review on crashworthiness studies of crash box structure

*Thin-Walled Structures journal*



N S B, Yusof, S M, Sapuan, M T H, Sultan, M, Jawaidd, M A, Maleque, 2017

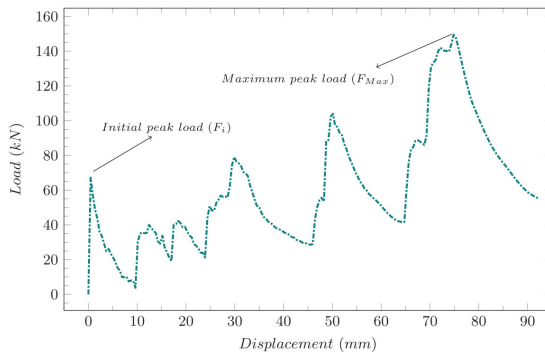
Design and materials development of automotive crash box : a review

*Ciência & Tecnologia dos Materiais*

# The End



# Appendix 1



Typical load-displacement curve of the conical segmented tube

# Appendix 2

## ① Absorbed energy

The total absorbed energy:  $\Rightarrow E_{absorbed} = \int F d\delta$

Where  $F$  and  $\delta$  are the crush force and crush distance, respectively.

## ② Mean crushing load

The ratio of total absorbed energy to total crushing distance.  $\Rightarrow F_m = \frac{1}{\delta_t} \int F d\delta$

## ③ Maximum crushing load

The initial peak load ( $F_i$ ) occurs at the onset of the curve. The maximum peak load ( $F_{Max}$ ) in conical segmented tubes usually occurs at the end of the crush range

## ④ Crush force efficiency (CFE)

This parameter is used to compare the efficiency of different energy absorbers.  $\Rightarrow CFE = \frac{F_m}{F_{Max}}$

## ⑤ The specific energy absorption

Energy Absorbed Per Unit Mass  $\Rightarrow SEA = \frac{E_{absorbed}}{M}$

NL  $\implies$  subjective weighting method  $\implies$  any numeric weight between 0 and 1 for each criteria:

$$W_{j,NL} = \frac{\sum_{k=1}^n C_{jk}}{\sum_{j=1}^n \sum_{k=1}^n C_{jk}}, j \text{ and } k = \{1, \dots, n\} \text{ and } j \neq k$$

NL weighting.

Attributes	Attributes type	Relative numeric weights ( $C_k$ )					Weights
$E_{absorbed}$ (kJ)	Maximize $\uparrow$	0.55	0.6	0.75			0.3167
CFE ( $\frac{F_{in}}{F_{max}}$ )	Maximize $\uparrow$	0.45			0.5	0.65	0.2667
$F_i$ (kN)	Minimize $\downarrow$		0.4		0.5	0.65	0.2583
M (kg)	Minimize $\downarrow$			0.25		0.35	0.1583

## The ratio system

Normalized matrix multiplied by the deviation of the subjective weight of beneficial attributes from the non-beneficial ones, where  $g$  is the number of beneficial attributes, and  $n$  is the number of all attributes:

$$y_i^w = \sum_{j=1}^g w_j x_{ij}^* - \sum_{j=g+1}^n w_j x_{ij}^*$$

Based on the ratio system, the optimum alternative has the maximum assessment value of  $y_i^w$ .

## The reference point approach

the maximal attribute reference point vector ( $r$ ) and the deviation of normalized rating from the reference point ( $d$ ).

$$r_j = \begin{cases} \max_i x_{ij}^*, & j \leq g, \\ \min_i x_{ij}^*, & j > g. \end{cases} \quad \Rightarrow \quad d_{ij} = (r_j - x_{ij}^*) \quad \Rightarrow \quad z_i^w = \max_j w_j d_{ij}$$

Based on the reference point approach, the optimum alternative has the minimum assessment value of  $z_i^w$ .

## The full multiplicative form

The value of the full multiplicative form, which was weighted by considering the subjective weight as the exponent:

$$U_i^w = \frac{\prod_{j=1}^g (x_{ij}^*)^{w_j}}{\prod_{j=g+1}^n (x_{ij}^*)^{w_j}}$$

the optimum alternative has the maximum assessment value of  $U_i^w$ .

## Final rank

The subordinate ranks calculated above can be conjoined to the final rank, which is known as the MULTIMOORA rank, by utilizing the dominance theory.

# Appendix 5



The collapse pattern of SGMT2-2 under axial compression.

The numerical outcomes and rankings of the NL-weighted MULTIMOORA method.

Alternatives		$E_{\text{absorbed}}$	CFE	$F_i$	M	Ranks			Final
		(kJ)	( $\frac{F_{\text{max}}}{F_{\text{min}}}$ )	(kN)	(kg)	$U_i^w$	$y_i^w$	$z_i^w$	
1	BSC 1	1.50	0.415	39.30	0.064	39	32	39	39
2	BSC 2	2.10	0.466	49.00	0.076	37	29	37	36
3	BSC 3	2.74	0.505	58.78	0.089	34	28	35	32
4	BSC 4	2.80	0.468	65.01	0.096	38	35	34	36
5	BSC 5	3.68	0.533	75.49	0.109	33	31	31	30
6	BSC 6	4.20	0.523	87.30	0.122	36	36	30	34
7	BSC 7	4.57	0.528	94.00	0.129	35	37	22	33
8	BSC 8	5.50	0.565	105.75	0.142	32	34	29	30
9	BSC 9	6.58	0.604	118.40	0.156	27	33	32	30
10	BSC 10	7.07	0.619	124.00	0.163	24	30	33	28
11	BSC 11	7.54	0.595	137.70	0.176	31	38	36	36
12	BSC 12	8.34	0.609	148.76	0.190	30	39	38	38
13	SGMT 1-1	5.02	0.398	54.09	0.126	22	22	19	22
14	SGMT 1-2	5.75	0.450	63.15	0.140	19	19	7	19
15	SGMT 1-3	7.28	0.484	70.25	0.153	9	6	4	6
16	SGMT 2-1	6.03	0.556	48.85	0.121	3	3	3	3
17	<b>SGMT 2-2</b>	<b>7.13</b>	<b>0.767</b>	<b>56.94</b>	<b>0.134</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
18	SGMT 2-3	7.01	0.434	66.41	0.148	11	9	9	9
19	SGMT 3-1	4.45	0.440	46.29	0.115	15	18	25	18
20	SGMT 3-2	5.10	0.431	55.38	0.129	20	21	13	21
21	SGMT 3-3	6.78	0.470	63.73	0.142	6	5	5	5
22	SGMT 4-1	4.72	0.441	50.71	0.110	14	17	20	17
23	SGMT 4-2	5.16	0.473	57.98	0.123	16	16	11	14
24	SGMT 4-3	5.56	0.404	67.20	0.136	25	24	18	24
25	SGMT 5-1	4.36	0.664	45.81	0.104	4	4	26	4
26	SGMT 5-2	4.91	0.340	55.75	0.117	28	26	27	26
27	SGMT 5-3	5.43	0.415	64.49	0.131	23	23	15	23
28	SGMT 6-1	4.64	0.492	50.80	0.113	10	11	21	11
29	SGMT 6-2	5.30	0.489	59.37	0.126	12	13	8	12
30	SGMT 6-3	6.69	0.418	68.15	0.139	18	15	14	16
31	SGMT 7-1	4.52	0.493	48.00	0.110	7	8	24	8
32	SGMT 7-2	5.02	0.510	58.91	0.123	13	14	16	14
33	SGMT 7-3	5.68	0.465	66.27	0.136	21	20	6	20
34	SGMT 8-1	5.34	0.411	46.75	0.118	8	10	17	10
35	SGMT 8-2	6.26	0.431	54.06	0.131	5	7	10	7
36	SGMT 8-3	7.32	0.654	64.82	0.145	2	2	2	2
37	SGMT 9-1	4.61	0.338	51.06	0.121	29	27	28	27
38	SGMT 9-2	5.25	0.361	56.98	0.134	26	25	23	25
39	SGMT 9-3	6.65	0.426	64.07	0.147	17	12	12	14

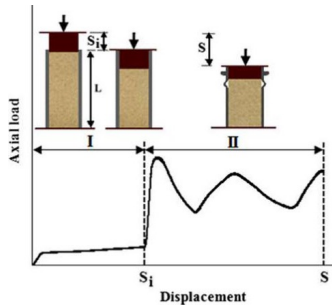
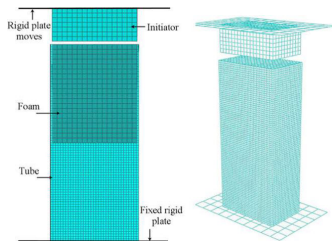
# Appendix 7

The dimensions of the conical segmented tubes of the second group.

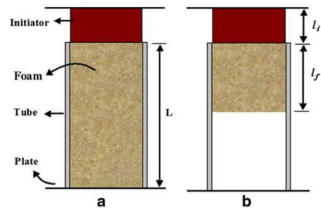
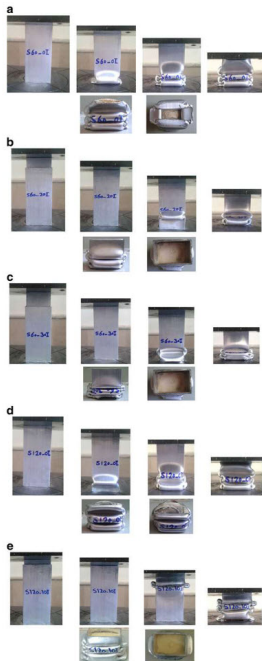
		$L_1$	$L_2$	$L_3$	$L_4$	$\alpha^\circ$			$L_1$	$L_2$	$L_3$	$L_4$	$\alpha^\circ$
1	SGMT 2-2	25	25	35	35	4°	9	SGMT 46	25	25	35	35	6°
2	SGMT 8-2	25	35	25	35	4°	10	SGMT 47	25	35	25	35	6°
3	SGMT 40	25	25	25	45	4°	11	SGMT 48	25	25	25	45	6°
4	SGMT 41	25	35	35	25	4°	12	SGMT 49	25	35	35	25	6°
5	SGMT 42	25	25	35	35	2°	13	SGMT 50	25	25	35	35	8°
6	SGMT 43	25	35	25	35	2°	14	SGMT 51	25	35	25	35	8°
7	SGMT 44	25	25	25	45	2°	15	SGMT 52	25	25	25	45	8°
8	SGMT 45	25	35	35	25	2°	16	SGMT 53	25	35	35	25	8°

For all tubes:  $t_1 = 1.2$  mm,  $t_2 = 1.7$  mm,  $t_3 = 2.2$  mm,  $t_4 = 2.7$  mm.

# Appendix 8

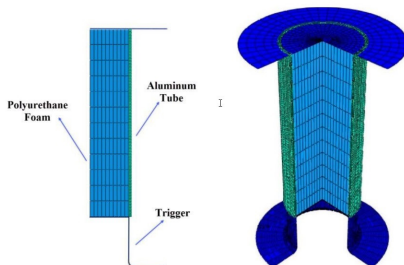




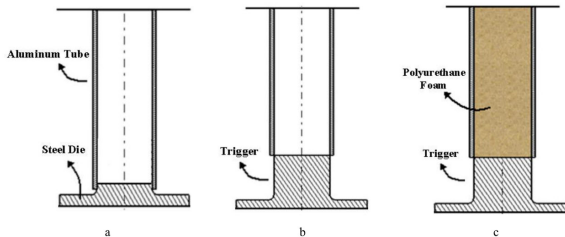


A schematic of two groups of the thin-walled rectangular tubes with the aluminum initiator: (a) foam-filled; (b) partially foam-filled

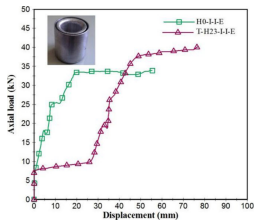
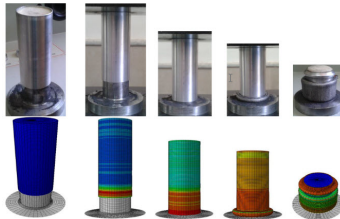
# Appendix 10

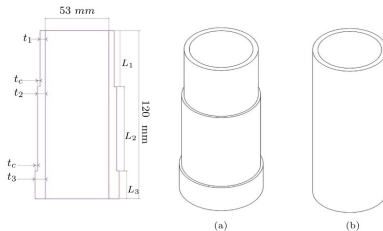


Finite element mesh and 3D view for the partially foam-filled rectangular tube with trigger.



Details of the position of the specimens on the die: a) H0-E-I-E, b) T-H23-E-I-E, c) T-H23-E-I-F.





Variables	-1	0	+1
Length of sections ( $L_1$ , $L_2$ , and $L_3$ )	20	40	60
Thickness of first section ( $t_1$ )	1	1.4	1.8
Thickness factor ( $t_c$ )	0.3	0.5	0.8

$$\left\{ \begin{array}{l} L_1 : 15 \leq x \leq 75 \text{ mm} \\ L_2 : 15 \leq x \leq 75 \text{ mm} \\ L_3 : 15 \leq x \leq 75 \text{ mm} \\ t_1 : 1 \leq x \leq 1.9 \text{ mm} \\ t_c : 0.25 \leq x \leq 0.95 \text{ mm} \\ \text{where } L_1 + L_2 + L_3 = 120 \text{ mm} \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Minimize} \left\{ \begin{array}{l} f_1 = F_i^0(x) \\ f_2 = F_i^{15}(x) \\ f_3 = F_i^{30}(x) \end{array} \right. \\ \text{Minimize} \left\{ \begin{array}{l} f_4 = F_{max}^0(x) \\ f_5 = F_{max}^{15}(x) \\ f_6 = F_{max}^{30}(x) \end{array} \right. \\ \text{Maximize} \left\{ \begin{array}{l} f_7 = E^0(x) \\ f_8 = E^{15}(x) \\ f_9 = E^{30}(x) \end{array} \right. \\ \text{Less is better } f_{10} = M(x) \\ S.t \ x^l \leq x \leq x^u \end{array} \right.$$

# Appendix 13

Table The ANOVA of design factors for each response (objective function)				
Responses	Factors	F value	P value	Final equation
$f_1$	$t_1$	594.629	< 0.0001	$10^{(4.16996+0.57713 \times t_1 + 0.22260 \times t_c - 0.25329 \times t_1 \times t_c)}$
	$t_c$	20.446	< 0.0001	
	$t_1 t_c$	10.213	0.0024	
	$L_1$	17.030	0.0001	
	$L_2$	19.530	< 0.0001	
$f_2$	$t_1$	170.530	< 0.0001	$10^{(4.49370-2.26674E-3 \times L_1 - 2.21568E-3 \times L_2 + 0.42023 \times t_1 + 0.78968 \times t_c - 0.27190 \times t_1 \times t_c)}$
	$t_c$	141.450	< 0.0001	
	$t_1 t_c$	8.690	0.005	
	$L_1$	65.894	< 0.0001	
	$L_2$	31.996	< 0.0001	
$f_3$	$t_1$	468.696	< 0.0001	$10^{(3.10313-3.69279E-3 \times L_1 - 2.34881E-3 \times L_2 + 0.51899 \times t_1 + 0.78189 \times t_c - 0.26789 \times t_1 \times t_c)} \times 0.5$
	$t_c$	204.036	< 0.0001	
	$t_1 t_c$	12.303	0.0010	

# Appendix 14

**Table** Scenarios chosen for pareto front generation

	Design variables					Output responses									
	$L_1$ mm	$L_2$ mm	$L_3$ mm	$t_1$ mm	$t_c$	$f_1$ $F_i^0$	$f_2$ $F_{max}^0$	$f_3$ $E^0$	$f_4$ $F_i^{15}$	$f_5$ $F_{max}^{15}$	$f_6$ $E^{15}$	$f_7$ $F_i^{30}$	$f_8$ $F_{max}^{30}$	$f_9$ $E^{30}$	$f_{10}$ $M$
1*	23.59	34.34	62.07	1	0.51	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
2	23.91	25.81	70.28	1.05	0.9	–	–	✓	–	–	✓	–	–	✓	✓
3	38.91	37.82	43.27	1	0.9	✓	–	✓	✓	–	✓	✓	–	✓	✓
4	15	30.12	74.88	1	0.81	✓	–	✓	✓	–	✓	✓	–	✓	–
5	26.89	74.99	18.12	1.88	0.25	–	✓	✓	–	✓	✓	–	✓	✓	–
6	14.21	40.82	64.97	1.9	0.8	–	–	✓	–	–	✓	–	–	✓	–
7	26.88	33.29	59.83	1	0.25	✓	–	–	✓	–	–	✓	–	–	–
8	39.07	63.12	17.81	1	0.25	✓	–	–	✓	–	–	✓	–	–	✓
9	26.04	56.06	37.9	1	0.25	–	✓	–	–	✓	–	–	✓	–	✓
10	56.74	48.23	15.03	1.14	0.25	–	✓	–	–	✓	–	–	✓	–	–
11	24.05	33.21	62.74	1	0.34	✓	✓	–	✓	✓	–	✓	✓	–	–
12	60.82	43.48	15.7	1	0.25	✓	✓	–	✓	✓	–	✓	✓	–	✓
13	15	31.9	73.1	1.01	0.36	✓	✓	✓	–	–	I	–	–	–	✓
14	48.03	16.34	55.63	1	0.55	–	–	–	✓	✓	✓	–	–	–	✓
15	41.27	41.82	36.91	1.32	0.25	–	–	–	–	–	–	✓	✓	✓	✓
16**	–	–	–	1.98	–	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

\*Segmented tubes

\*\*Basic tube

# Appendix 15

**Table** Summarized table for the steps of CW method

Methods	Weighting strategies	Formula		
Entropy	Objective weighting	$w_j^o$	$\frac{1 - E_j}{\sum_{k=1}^n (1 - E_k)}$	(8)
Numeric logic	Subjective weighting	$w_j^s$	$\frac{\sum_{k=1}^n C_{jk}}{\sum_{k=1}^n \sum_{j=1}^n C_{jk}}$	(9)
Correlation's effect		$w_j^c$	$\frac{\sum_{j=1}^n (\sum_{k=1}^n (1 - R_{jk}))}{(\sum_{j=1}^n w_j^o w_j^s)^{1/3}}$	(11)
Combinative weighting		$w_j \uparrow$	$\sum_{j=1}^n (w_j^o w_j^s w_j^c)^{1/3}$	(12)

# Appendix 16

The multi-objective design selection results

	$f_1$ $F_i^0$ kN	$f_2$ $F_{max}^0$ kN	$f_3$ $E^0$ kJ	$f_4$ $F_i^{15}$ kN	$f_5$ $F_{max}^{15}$ kN	$f_6$ $E^{15}$ kJ	$f_7$ $F_i^{30}$ kN	$f_8$ $F_{max}^{30}$ kN	$f_9$ $E^{30}$ kJ	$f_{10}$ $M$ kg		Ranks			Final
											$U_i^w$	$y_i^w$	$z_i^w$		
1*	<b>53.88</b>	<b>111.80</b>	<b>5.21</b>	<b>15.65</b>	<b>99.52</b>	<b>5.29</b>	<b>4.76</b>	<b>56.14</b>	<b>3.30</b>	<b>0.27</b>	3	6	7	4	
2	54.57	189.38	<b>8.91</b>	19.60	156.83	<b>8.75</b>	6.12	86.77	<b>5.40</b>	<b>0.38</b>	7	4	1	4	
3	<b>52.42</b>	161.41	<b>7.12</b>	<b>18.50</b>	116.88	<b>6.91</b>	<b>5.83</b>	69.09	<b>3.68</b>	<b>0.32</b>	10	11	5	10	
4	<u><b>52.75</b></u>	<u>170.84</u>	<u><b>8.17</b></u>	<u><b>17.63</b></u>	<u>151.54</u>	<u><b>7.78</b></u>	<u><b>5.58</b></u>	<u>80.99</u>	<u><b>5.27</b></u>	<u>0.36</u>	1	1	2	1	
5	155.44	<b>133.74</b>	<b>7.46</b>	39.05	<b>84.17</b>	<b>6.54</b>	9.29	<b>59.74</b>	<b>3.98</b>	0.34	14	15	4	15	
6	114.71	244.31	<b>14.43</b>	41.50	204.78	<b>10.53</b>	10.91	132.80	<b>7.43</b>	0.50	15	14	3	14	
7	<b>54.88</b>	81.04	3.74	<b>16.15</b>	72.92	3.95	<b>4.05</b>	43.98	2.51	0.21	8	5	12	8	
8	<b>54.88</b>	65.31	2.87	<b>16.04</b>	47.58	3.31	<b>4.05</b>	35.43	2.06	<b>0.19</b>	4	3	15	4	
9	54.88	<b>72.47</b>	3.33	16.16	<b>57.73</b>	3.76	4.05	<b>38.66</b>	2.26	<b>0.20</b>	2	2	13	2	
10	64.77	<b>71.98</b>	3.10	18.49	<b>54.32</b>	3.54	4.89	<b>39.56</b>	2.20	0.21	12	12	14	12	
11	<b>54.53</b>	<b>91.60</b>	4.27	<b>15.76</b>	<b>82.89</b>	4.37	<b>4.30</b>	<b>48.26</b>	2.80	0.23	5	7	11	6	
12	<b>54.88</b>	<b>64.44</b>	2.65	<b>15.83</b>	<b>49.90</b>	3.00	<b>4.05</b>	<b>36.73</b>	1.93	<b>0.19</b>	11	9	16	11	
13	<b>55.07</b>	<b>99.74</b>	<b>4.80</b>	15.98	92.79	4.47	4.41	51.85	3.13	<b>0.25</b>	6	8	9	7	
14	53.73	113.14	4.89	<b>15.51</b>	<b>95.02</b>	<b>5.17</b>	4.87	56.04	2.88	<b>0.26</b>	9	10	8	9	
15	80.14	93.30	4.41	22.67	74.13	4.93	5.96	<b>48.53</b>	<b>2.78</b>	<b>0.25</b>	13	13	10	13	
16**	<b>107.41</b>	<b>107.41</b>	<b>5.82</b>	<b>44.73</b>	<b>84.84</b>	<b>6.27</b>	<b>13.26</b>	<b>55.37</b>	<b>3.82</b>	<b>0.32</b>	16	16	6	16	

\*Regression models

\*\*Finite element method