

CONTEXT FREE GRAMMAR:

A context free grammar (**CFG**) can be define in 4-tuple (S, T, N, P) where -

- ❖ **S** is a special variable called the **Start symbol**, $S \in N$ (*S is a representation of start symbol as we have q0 in FA, it name may be change*)
- ❖ **T** or Σ is a finite non-empty set of **Terminal symbols**.
 - Represented by Small letters (a to z) E.g. $\Sigma = \{a, b\}$
 - **OR** Represented by digits (0 to 1) E.g. $\Sigma = \{0, 1\}$
 - **OR** special symbols E.g. \$, #, @ etc.
 - *It is always on right side of production.*
- ❖ **N** or **VN** is a finite non-empty set of variables or **non-terminal symbols**.
 - *Represented by Capital letters (A to Z) E.g. {S, A}*
 - *It is always on left side of production, but also possible on right side as well.*
- ❖ **P** is Production rules for Terminals and Non-terminals.
 - A production rule can be in the form of $\alpha \rightarrow \beta$ where
$$\alpha \rightarrow \beta \text{ (}\alpha \text{ determine } \beta\text{)}$$
$$\alpha \in N, \alpha \text{ must be only 1}$$
$$\beta \in (\Sigma \cup N)^*$$

For Example:

- $S \rightarrow Aa$
- $A \rightarrow a$
- $A \rightarrow b$
- $A \rightarrow \epsilon$
- $A \rightarrow 0/1/2/3$

The above is the example of CFG, each line is called **rule of grammar**, due to that we define grammar as; “a set of rules and regulations”.

Backus-Nour Form / Backus Normal Form:

It states that if left side of production is same then we can write it together instead of separate.

For Example:

- $S \rightarrow A$
- $A \rightarrow a$
- $A \rightarrow b$

➤ $A \rightarrow \varepsilon$

We notice that on left side of production letter are same, then we can write it together, like

➤ $A \rightarrow a / b / \varepsilon$

Why we use Context Free Grammar?

- ❖ Before going to main question we must know that what is regular languages and non-regular languages?
- ❖ **Regular languages** are those languages for which we can draw Finite automata (NFA/DFA)
- ❖ **Non-Regular languages** are those languages for which we cannot draw FA.
- ❖ Now the question is that how to handle Non-Regular languages, answers is simple to handle Non-Regular languages context-free grammar is use.

Examples of Context-Free Grammar

How to generate a string from CFG?

1. Let's we have given a grammar.

➤ $S \rightarrow A$

➤ $A \rightarrow A+A$

➤ $A \rightarrow A*A$

➤ $A \rightarrow 0/1/2/3/4/5/6/7/8/9$

- ❖ And tell us that generate the string “2+3*5” from above grammar.

Let's solve it...

- $S \rightarrow A$ // Replace the A with A+A, follow above grammar)
- $A \rightarrow A+A$ // Now replace right A with A*A, follow grammar)
- $A \rightarrow A+A*A$ // Now replace left A with 2, follow grammar)
- $A \rightarrow 2+A*A$ // Now replace middle A with 3, follow grammar)
- $A \rightarrow 2+3*A$ // Now replace right A with 5, follow grammar)
- $A \rightarrow 2+3*5$ // finally, we got the desire string)

2. Let's we take another example, suppose we have given a grammar.

- $S \rightarrow A$
- $A \rightarrow \epsilon$
- $A \rightarrow 0 / 1$
- $A \rightarrow 0 A 0$
- $A \rightarrow 1 A 1$

❖ And tell us that generate the string “0110” from above grammar.

Let's solve it...

- $S \rightarrow A$ // Replace the A with 0A0, follow above grammar)
- $A \rightarrow 0A0$ // Now replace right A with 1A1, follow grammar)
- $A \rightarrow 01A10$ // Now to remove the A, use ϵ)
- $A \rightarrow 01\epsilon10$ // ϵ not effect any string)
- $A \rightarrow 0110$ // finally, we got the desire string)

How to convert Regular Expression to CFG?

1. Let's we have R.E.

R.E = a // Grammar start from S, it generate only “a”

CFG = $S \rightarrow a$

2. Let's we have R.E.

$$\text{R.E} = 1$$

$$\text{CFG} = S \rightarrow 1$$

// Grammar start from S, it generate only "1"

3. Let's we have R.E.

$$\text{R.E} = a + b$$

$$\text{CFG} = S \rightarrow X / Y$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

OR we can write it as well,

$$S \rightarrow a / b$$

OR we can write it as well,

$$S \rightarrow a$$

$$S \rightarrow b$$

We know that, $a+b$ = union and $a.b / ab$ = concatenation in R.E, now in CFG, for union we use this symbol " / " and concatenation will be same.

4. Let's we have R.E.

$$\text{R.E} = (a+b) (a+b)$$

$$\text{CFG} = S \rightarrow X X$$

$$X \rightarrow A / B$$

$$A \rightarrow a$$

$$B \rightarrow b$$