Evaluation of the efficiency of COVID-19 propagation control in Colombia through DEA optimization models

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Abstract

This paper presents the evaluation of COVID-19 propagation control in Colombia using Data Envelopment Analysis in certain departments for different time periods. The first model implemented seeks to evaluation of the control in the spreading of the virus, taking into account the population density, the tests carried out and the confirmed cases. At the same time, a second model considers the recovered cases, the deaths from the virus, the confirmed cases and the beds in ICU to evaluate the quality in the medical treatment. The departments are classified into four categories with respect to their efficiencies in both models, where different analyses of the observed behaviors and the strategies implemented in each territory are presented.

Keywords: COVID-19, Efficiency Evaluation, Data Envelopment Analysis, Control Propagation, Medical Treatment.

1 Introduction

The COVID-19 pandemic has become a major problem for the entire world, testing the health systems in all the countries it has reached and claiming thousands of lives. Colombia has not been the exception in terms of the affectations that this virus causes, because to date, as it is registered by El Mundo (2020), this country is located in the Top 10 in countries with the highest number of registered infections and also, is one of the twenty countries with the most deaths because of the virus, Statista (2020). Now, the numbers of cases per day continue to increase, so, as a prevention measure since March of this year, the National Government implemented different types of measures to preserve the economic and social level of the country, Presidencia de la República (2020). In addition, each city has adopted some plans to reduce the spread of the virus, such as quarantine periods, constant concern for personal care and disinfection of public spaces.

A recent report shows that the main problems this country has controlling the virus are related to sociodemographic factors such as the population density and the governments prioritization of the number of deaths over the new infections, Rodriguez & Loaiza (2020). These strategies have generated multiple impacts in each one of the communities involved, where in many cases they have not been sufficient to stop the virus from affecting them. That is why there are cities where the number of infected people is alarming, as in the case of Bogotá, Barranquilla, Cali and Medellín, Ministerio de Salud (2020).

Unfortunately, according to the literature reviewed, the country does not currently have a mechanism to compare the efficiency of the strategies implemented in the different cities, which would determine which of these measures guarantee greater effectiveness in controlling the pandemic according to the economic, social and health conditions of each territory. For this reason, the question has arisen of how to estimate the efficiency of the control of the spread of the virus in certain departments in Colombia, applying different models of Data Envelopment Analysis (DEA), a non-parametric mathematical programming approach that have been widely used in multiple sectors, such as the health, education, industrial and financial sectors. These models manage to integrate problems that link different relationships between inputs and outputs by analyzing the effectiveness of Decision Making Units. Serrano & Blasco (2006). Thus, it is possible to represent non-linear problems in other linear equivalents, which when implemented, generate optimal results with extremely useful information for the identification of possible action plans that could benefit the Colombia's territories most affected in these times, where the numbers of infections and deaths are still increasing.

The present investigation is focused on the analysis of different control strategies that certain departments in Colombia have implemented to mitigate the virus in their territories. Despite the efforts that have been made by different health and government organizations, Ministerio de Salud (2020) states that the number of registered infections exceeds 1'200.000 and the number of deaths is above 35.000 for the entire country. This situation has some cities, such as Bogotá, that has present in some periods an occupancy in Intensive Care Units (ICU) superior than 80% , one of the highest and alarming in the country, SALUDATA (2020).

Taking this into account, the study in this paper will show the evaluation process of eleven selected departments in Colombia over a period of ten weeks; with each territory classified according to its population density, whether it is high or intermediate, as well as some territories that are considered to be remotely located in the country. The main objective in this project is to analyze, according to the information obtained from the past and through DEA models, those contingency plans that have proven effective in some places to improve the spread and management of the COVID-19 in each territory. This will allow using results to propose possible new action plans within the framework of certain strategies that can have a positive impact in different municipalities with similar characteristics to those departments that present greater effectiveness. In addition, it serves as a starting point for analyzing the behavior of the virus in different time periods in the country.

The exposition of the paper is as follows. In the next section, the bibliographic review is presented. Section 3 contains the research methodology. In Section 4, the results obtained for the different models implemented are reported and different analyses are discussed. For Section 5, the conclusions and future implementations are illustrated and finally, this paper is concluded with the acknowledgements and references.

2 State of the art

Data Envelope Analysis (DEA) is a non-parametric optimization methodology where, according to Serrano & Blasco (2006), linear programming models are used in the analysis of the efficiency of a set of elements called Decision Making Units (DMUs), which integrate different inputs and outputs comparable to each other. Some representative publications on the subject can be found in Cooper *et al.* (2000) and Thanassoulis (2001), who have proposed different methods to solve problems using DEA. The first developments in this area were inspired by the definition of productivity proposed by Farrell (1957), which speaks of effectiveness as the profitability produced by the weight of the products (outputs) with respect to the resources used (inputs). According to this research, Charnes *et al.* (1978) develop an optimization model known as RATIO, a non-linear model that seeks to define the optimal values of the weights that maximize the efficiency of the units evaluated, and where its authors propose different methods of assigning efficiencies to enhance the planning and control of activities in public programs. This programming model is designed for Constant Returns to Scale (CRS) problems, which refer to those that keep a constant proportion between the increase or decrease in the values of inputs and outputs. Based on this first model, as it is explained by Ibargüen (2003), new linear models has been developed. Thus, in the CCR-Input model, the input are minimized, while keeping the outputs constant, and the CCR-Output model, keeping the inputs constant, the aim is to maximize the outputs. Later, Banker *et al.* (1984) propose certain linear models focused on Variable Returns to Scale (VRS) problems, called BCC. Those refer to the situations where there is not a proportional relationship between inputs and outputs. CCR model

measures the overall technical efficiency, while BCC model measures the pure technical efficiency.

Considering the wide applicability that the DEA methodology presents in different areas, it has been necessary to develop different methods that can be adapted to more particular problems since certain limitations such as unrestricted weights, changes in units over time, undesirable inputs and outputs, produce behaviors that cannot be handled by the already known models, Talluri (2000). One such development is the cross-efficiencies method implemented by Doyle & Green (1994). This method seeks to ensure that the efficiency of each DMU in relation to its assigned weight is as high as possible in comparison to the optimal weight of the other units. This process was applied in the different university accounting departments in the U.K., where it was evident that the results presented by this method are much better than those reported by the models described above with simple efficiency. In turn, Charnes *et al.* (1984) focus on systems that suffer different changes in efficiency over time, where for these a technique known as window analysis is proposed. This makes a special treatment of each DMU by handling it as a different entity in each of the time periods to be considered. The values of efficiency in each period are assigned simultaneously using the previous value as a benchmarks for the next, thus seeking to make this new value greater than the predecessors. These last mentioned advances arose thanks to the study of fourteen U.S. Air Force Wings to determine control systems that would allow establishing the necessary resources and capabilities that would improve the efficiency of their units, identifying limiting factors in the military operations being carried out. Later, Jahanshahloo *et al.* (2005) focus their developments on systems with undesirable inputs and outputs, that is, inputs that should be maximized and outputs that should be minimized. For this, they present a non-radial linear DEA model in which inputs and outputs are integrated both desirable and undesirable, which can be used in CRS or VRS problems.

As an example of the advances around the world that have used DEA programming models, in the academic sector BAL & GÖLCÜKCÜ (2016) present a development in the study of the efficiency of the educational system in 117 countries. For this analysis, CCR and BCC models were used, where for none of the scenarios evaluated, the number of efficient universities was greater than fifty. In a social approach and by means of slacks-based measure models with dynamic behavior in time, HUŽVÁR *et al.* (2016) present a work focused on the evaluation of the effectiveness in the administration of the municipal governments in Slovakia. In the results, the average efficiency values ranged from 43% for the year 2006 to 62% in 2013. The health sector has significant developments in the application of these methods, such as the one developed by DA CUNHA *et al.* (2016), where it is possible to observe the comparison of efficiencies in medical supply companies for 43 sales branches in Brazil. This study seeks to promote inefficient territories through benchmarks to achieve a homogenization in the efficiency of all coverage places. On the other hand, some countries have faced problems which present undesirable inputs or outputs. This is the case of China, since analyzing the behavior of the regional energy supplied to 30 regions of the country and the effects that its emissions have on the environment, Wang *et al.* (2012). They study the behavior of different DEA models to analyze whether it was more effective to use undesirable outputs or if it was better to consider them as inputs instead. For this process they used different performance indicators supported by window analysis techniques to evaluate dynamic behavior over time, which ultimately resulted in the undesirable outputs considered providing more information on the differences between energy and emissions behavior for the effectiveness evaluation.

This methodology is still a great tool for the evaluation and comparison of systems, as evidenced by Shirouyehzad *et al.* (2020), the recent use of this methodology in the current problem of

COVID-19 around the world. This research is carried out in 29 countries, where the main objective is to evaluate the control of infections and the medical treatments provided. For this purpose, a first stage is developed, integrating population density and the Average of 13 International Health Regulations Core Capacity Scores (IHRCCS) as inputs and confirmed cases as outputs. In a second stage, confirmed cases are used as inputs and deaths and recovered cases as outputs, where BCC models are implemented in both stages. The results are evaluated jointly and through a classification of countries it is possible to conclude that given the criteria considered and up to March 25 of this year, Singapore, Belgium and Vietnam had a higher than average efficiency in disease control and medical treatment offered, while China, Italy and Iraq had the most alarming performance.

In the process of searching for previous research, no record was found of this type of methodology for evaluating efficiency in Colombia, where the study of the different strategies adopted to combat the virus is of great importance for the current situation. Among the measures adopted to confront the pandemic are those implemented by the department of Antioquia. There, since the beginning of the pandemic, there has been constant monitoring of confirmed cases through the development of technologies for the collection and access to information, Consultor Salud (2020). In addition, the isolation of some communes in the capital of this department, and of certain municipalities such as Urabá, are part of the action plans provided by the government, El Tiempo (2020a) . Another of the territories of principal interest is Bogotá, which in the month of July and August, according to reports from Gobierno de Bogotá (2020), has implemented different sectorized quarantines for the most affected localities, a reduction in the capacity of the Transmilenio to 35% in order to avoid outbreaks of infection, Transmilenio (2020), and a significant investment in medical treatment. On the other hand, Amazonas is a department which presents different strategies implemented to avoid the growth of the virus in its territory. As reported by El Tiempo (2020b), the closure and control of the borders with Brazil and Peru, an obligatory quarantine in the city of Leticia for the month of July and different action plans to help the indigenous communities are some of the processes that have been developed there.

In the same way, there are some departments in the country where the restrictions have not been as strict, as is the case in Nariño, where they have been coupled with the isolation measures imposed by the government, without implementing any other type of restrictions. Plans to strengthen the medical service have been developed since July, Procuraduría (2020), and screening plans have been in place since August, Gobierno de Pasto (2020). In turn, another of the territories of great interest is San Andrés, given its remote location in the country. There, the least number of confirmed cases in the country have been presented, where it is also the territory that performs the least number of tests weekly. Furthermore, as reported by El Tiempo (2020c), few measures are presented for the adaptation of new medical services, adding to this the reactivation of the borders, which had been closed until August, in search of an economic reactivation, La República (2020).

3 Solution method / Methodology

3.1 Data collection and analysis

For the process of data collection and filtering, a total of eleven departments in Colombia which present particular sociodemographic characteristics were selected. For the analysis of departments with high population density, Antioquia, Atlántico, Bogotá, Bolívar and Valle del Cauca were used. In departments with intermediate population density, Meta, Nariño and Santander were considered. Finally, Amazonas, Chocó and San Andrés were chosen to study the behavior of places

with remote locations. Subsequently, data collection was supported by the project "Plataforma web para la recolección de datos, visualización, análisis, predicción y evaluación de estrategias de control de la enfermedad producida por SARS-CoV-2 mediante herramientas de modelación matemática, simulación e inteligencia artificial", led by Maria Eugenia Puerta Yepes. For this, different databases with daily records were accessed and a ten-week selection was made, from July 6, 2020 to September 13, 2020, to be used as inputs and outputs.

In the process of data selection, the congruence of the data with the expected behavior was analyzed. These procedures refer to the verification of population data, the fulfillment of the relation between the number of recoveries and deaths with the number of cases reported as positive, where the first two cannot exceed the last one per week, as well as having a greater or equal number of tests performed than confirmed cases. For this reason, the data to be considered refer to the average of the records reported as new each week, instead of considering the accumulated ones, since the latter presented significant inconsistencies. However, it should be clarified that for the data used, after analyzing the most consistent time periods, an unexpected relationship is presented for six of the records considered. In the case of the department of Atlántico in the first period, the number of tests carried out that were reported in the databases consulted was 497.00, while the confirmed cases for that week were 1367.43. Behavior of this type, although with much smaller differences in values, is presented in five of the records for the department of Bolívar for these same two categories.

3.2 Data Envelopment Analysis Implementation

In the search to analyze the behavior of the departments considered in this study for each period, the factors of the control of the contagion and the quality of the medical treatment were of great interest.

Contagion control

The first model developed, illustrated in the Figure 1, considering as inputs the population density and the weekly number of COVID-19 tests performed, and as output the number of confirmed positive cases, seeks to determine the efficiency of each department for each period (110 DMUs) in controlling the spread of the virus in its territory.

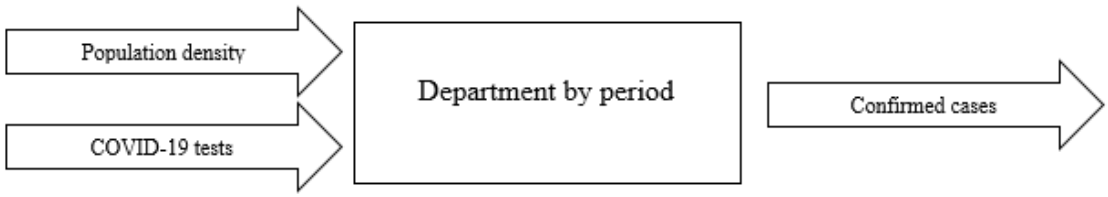

Figure 1: Model 1 - Virus spread control

Medical treatment

The second model developed, shown in the Figure 2, takes into account as inputs the number of ICU beds and confirmed positive cases, and as outputs the number of recovered cases and deaths.

It can be seen that, for the first model, the number of tests performed is an undesirable input, while the number of confirmed cases is an undesirable output; thus, the second model presents as

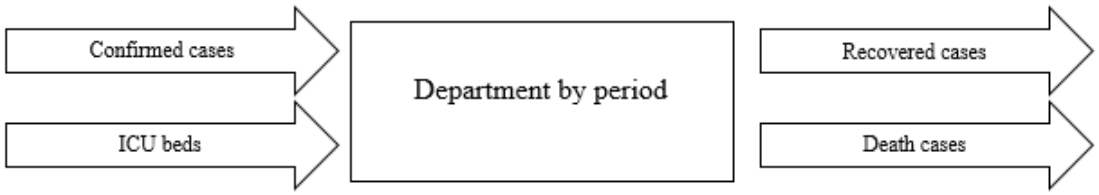

Figure 2: Model 2 - Quality in medical treatment

an undesirable input the number of ICU beds and as an undesirable output the number of deaths. For both models, since the return to scale is variable, the BCC-input optimization model present in (1) and proposed by Banker *et al.* (1984), was implemented to be tested in Gurobi with AMPL, and then automated in the MATLAB tool for each DMU considered .

$$
\begin{aligned}
\min \quad & \theta_0 \\
\text{s.t.} \quad & \\
& \theta_0 x_{i0} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0 \qquad \forall i, \quad i = 1, ..., m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - y_{r0} \geq 0 \qquad \forall r, \quad r = 1, ..., s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \qquad \forall j, \quad j = 1, ..., n \\
& \theta_0 \quad \text{free}
\end{aligned}
\tag{1}
$$

In the above model m, s and n are the number of inputs, outputs, and DMUs. x_{ij} and y_{rj} are the value of the i^{th} input and the r^{th} output for the j^{th} DMU. λ_j is the weight given to the j^{th} DMU, while θ_0 is the efficiency value for the DMU_0 to be optimized. A DMU with an efficiency value of 1 is considered efficient, otherwise it will be inefficient.

3.3 Results classification

Seeking to have a better understanding of the results obtained for the two models described above, a classification of efficiencies is proposed for each DMU as shown in the Figure 3. On the x-axis of the graphic is the efficiency in the control of the propagation of the virus, and on the y-axis is the efficiency in the quality of the medical services. Thus, the region of the graph is divided into four zones, determined by a horizontal and vertical line that represent the average value of the efficiencies for each axis, where the behavior of a DMU will be considered high or low depending on the quadrant it is in.

The classification presented above allows to obtain a characterization of the behavior of each department in relation to the other, and thus evaluate how effective the strategies taken were. Given that the country has taken containment measures both dictated by the Presidency of the Republic and given by each territory, the evaluation of these implemented strategies is an individual process that must be carried out for each department in relation to its socio-demographic conditions.

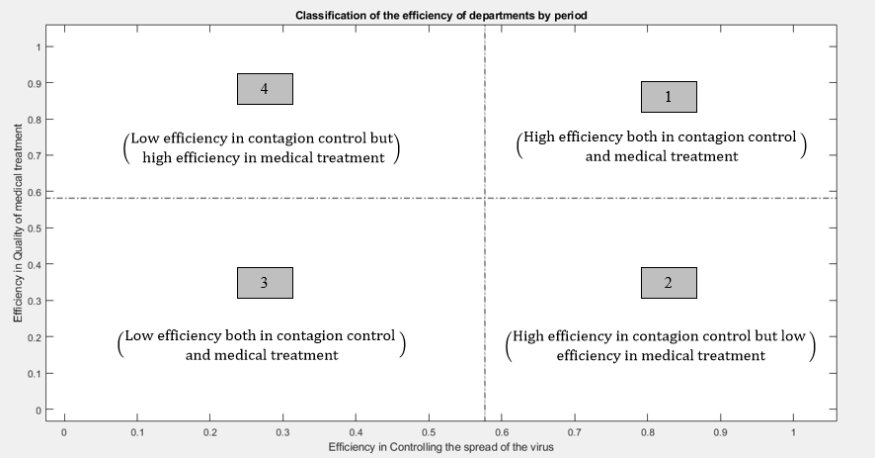

Figure 3: Classification zones for each DMU

4 Results

4.1 Control in the spread of the virus

For the implementation of the first model described above, a transformation of the undesirable inputs and outputs was carried out. Thus, the undesirable input of the tests performed was modeled as a desirable output, while the undesirable output of the confirmed cases was treated as a desirable input. The efficiency results for each department in each period are shown in the Figure 4.

Model 1 Efficuency- Contagion control											
Period	Department										
	Amazonas (1)	Antioquia (2)	Atlántico (3)	Bogotá (4)	Bolivar (5)	Chocó (6)	Meta (7)	Nariño (8)	San Andrés (9)	Santander (10)	Valle del Cauca (11)
Jul 6 / Jul 12 (1)	1.000000	0.912292	0.034235	1.000000	0.055820	0.399446	1.000000	0.348939	1.000000	1.000000	0.542904
Jul 13 / Jul 19 (2)	1.000000	1.000000	0.228954	0.734124	0.053083	0.516478	0.877484	0.199693	0.916700	0.508946	0.715684
Jul 20 / Jul 26 (3)	1.000000	0.700976	0.160518	0.921557	0.056209	0.441323	0.741242	0.349685	0.647158	0.334204	0.640284
Jul 27 / Aug 2 (4)	1.000000	1.000000	0.276164	0.742276	0.079303	1.000000	0.791447	0.316036	0.611214	0.506437	0.804142
Aug 3 / Aug 9 (5)	1.000000	0.972258	0.192767	1.000000	0.109054	0.405359	0.931155	0.448887	0.523918	0.422586	0.763103
Aug 10 / Aug 16 (6)	1.000000	0.928088	0.167049	1.000000	0.127520	0.870346	0.843422	0.285494	0.207618	0.422638	0.644474
Aug 17 / Aug 23 (7)	1.000000	0.784056	0.226749	0.721111	0.148956	0.672580	0.922703	0.185198	0.139292	0.444330	0.729066
Aug 24 / Aug 30 (8)	1.000000	0.651556	0.181482	0.891737	0.151630	1.000000	0.850331	0.200776	0.108953	0.503906	0.571376
Aug 31 / Sep 6 (9)	1.000000	0.705809	0.180583	0.868455	0.105528	0.299136	0.785765	0.157585	0.069927	0.613354	0.542840
Sep 7 / Sep 13 (10)	1.000000	0.628395	0.236870	1.000000	0.042194	0.570625	0.820580	0.206698	0.054244	0.612304	0.424214

Figure 4: Model 1 - Efficiency of each DMU

If we look at the efficiency of each department for all its periods in Figura 4, we can see that in general the change in efficiency over time is very marked. The San Andrés region shows its highest efficiency at the beginning of the periods and with time it is decreasing. In contrast, departments such as Atlántico and Bolívar had the lowest efficiency in period one and this was increasing. Also, the only department that maintains its effectiveness constant is Amazonas, while in general, for the other departments, we can see that there is oscillatory behavior, that is to say that they increase and decrease for each period, where the magnitude of these changes varies considerably for each department.

For intermediate departments, such as Santander, Nariño and Meta, we can see that the efficiencies for the first two are not very high, with certain exceptions, while the third presents better and more stable results. This is because, given the density population in the territories,

the number of reported cases is very high and the tests carried out have not been able to control those infected to prevent new infections. However, given that the density of the department of Meta is low compared to the others, its control of the spread is better. In the remote departments, i.e. Amazonas, Chocó and San Andrés, we can see that the patterns are very different. On the Amazonas side, since it has such a low population density in relation to the other regions, this makes its efficiency much higher and constant. On the other hand, Chocó represents a relatively high population density, so this, added to the few measures and resources for prevention, generates a very variable efficiency, the product of ineffective actions that last over time. In the case of San Andrés, we have a very high population density, with very few average tests carried out per week, which means that, as the number of confirmed cases increases, efficiency continues to fall, and these are the worst performers. In the case of the largest departments, these range from very low efficiencies due to the densities and the few tests performed, while in others there is greater management of the spread of the virus, thus generating greater efficiencies.

It can be observed that, around all periods shown in Figure 5, the departments of Bolívar and Atlántico are located in the lowest places in relation to their efficiency, this being the result of the few tests carried out in relation to the number of reported infections. Likewise, departments such as Nariño and Chocó show low effectiveness in several of their periods, due to ineffective strategies that last over time to control the spread. The last notable department, San Andrés, has the particularity for the last period that, after being isolated from the rest of the country, for the period nine there was a reopening of flights to the island, which caused a considerable increase in confirmed cases and, given its low capacity for containment, caused very low values of effectiveness.

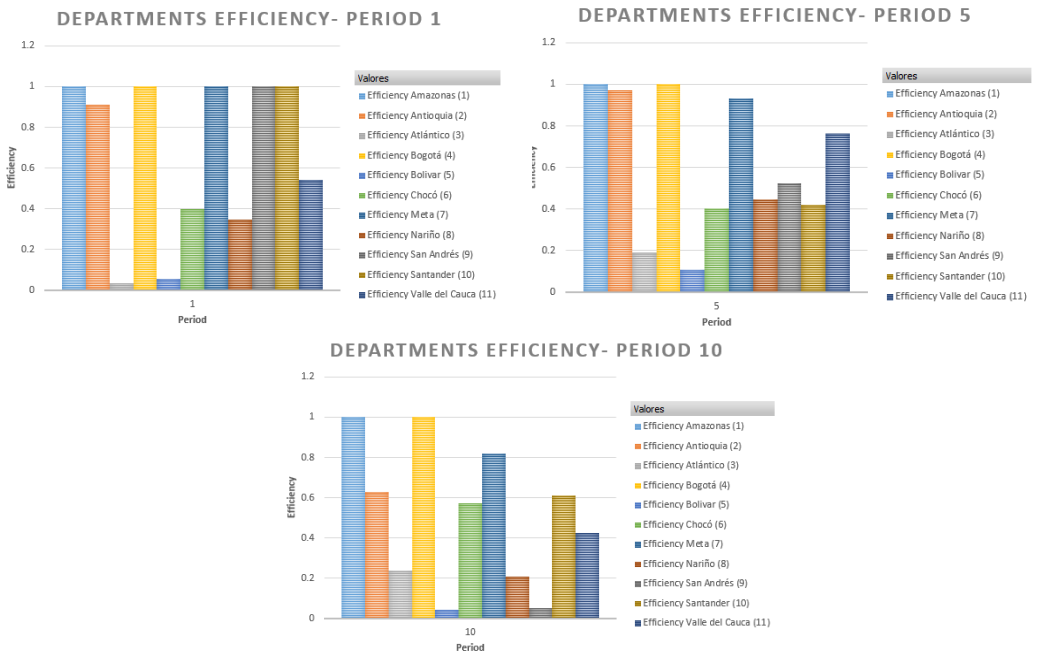

Figure 5: Model 1 - Departments Efficiency in Period 1, 5 y 10

4.2 Medical treatment

For the application of the second model described above, a transformation of the undesirable inputs and outputs was carried out. Thus, the undesirable input of the number of ICU beds was modeled as a desirable output, while the undesirable output of the weekly deaths was treated as a desirable input. The results of the efficiency of each department in each period are shown in the Figure 6.

Modelo 2 Efficiency - Quality in the medical treatment											
Period	Department										
	Amazonas (1)	Antioquia (2)	Atlántico (3)	Bogotá (4)	Bolivar (5)	Chocó (6)	Meta (7)	Nariño (8)	San Andrés (9)	Santander (10)	Valle del Cauca (11)
Jul 6 / Jul 12 (1)	0.5103607	1.0000000	0.2163517	0.9407407	0.3581272	0.3013965	1.0000000	0.2675193	1.0000000	1.0000000	1.0000000
Jul 13 / Jul 19 (2)	1.0000000	0.6317948	0.5499525	0.7326031	0.6471766	0.4659491	0.3881260	0.1932266	0.5000000	0.6934387	1.0000000
Jul 20 / Jul 26 (3)	0.1289641	0.4238773	0.5956703	0.6738969	0.4197131	0.2788060	0.3492758	0.1513656	1.0000000	0.4265289	0.6714707
Jul 27 / Aug 2 (4)	0.8347107	0.3608445	0.7771454	0.4986911	0.5993174	0.8324869	0.2242719	0.2746791	0.1250000	0.2578255	0.7042529
Aug 3 / Aug 9 (5)	0.0264510	0.4405149	0.7349049	0.6075544	0.7404482	0.4504527	0.1778956	0.2329720	0.2727273	0.1708785	0.6477341
Aug 10 / Aug 16 (6)	0.2695925	0.6428627	0.9485509	0.6392617	0.8223640	0.8076314	0.1918301	0.3027047	1.0000000	0.1906492	0.5462394
Aug 17 / Aug 23 (7)	1.0000000	0.7335016	1.0000000	1.0000000	0.8594398	0.4755167	0.3330239	0.6231400	0.0483858	0.2630968	0.7109851
Aug 24 / Aug 30 (8)	0.5259740	0.5752609	1.0000000	0.7426901	1.0000000	0.7240747	0.3240627	0.3819041	0.1058663	0.3090381	0.9257553
Aug 31 / Sep 6 (9)	0.2770563	0.7349306	0.8367433	0.9501247	1.0000000	0.7104573	0.4003529	0.4434376	0.0704728	0.3968583	1.0000000
Sep 7 / Sep 13 (10)	0.2561983	0.8712059	1.0000000	1.0000000	1.0000000	0.5892518	0.7226640	0.5081090	0.1339958	0.5468408	1.0000000

Figure 6: Model 2 - Efficiency of each DMU

If we look at the efficiency of each department for all its periods in Figure 6, we can see that in general the change in efficiency over time is very marked, as in Model 1. In some regions, such as Antioquia, Meta and Valle del Cauca, the periods of greatest efficiency occur in the first and last weeks, while in the central part, where it was closer to the peak, the performance is lower. On the other hand, there are regions such as San Andrés and Santander, which present their greatest efficiency at the beginning of the periods, but with time these begin to diminish, and where only Santander begins to recover in the end. On the other hand, departments such as Atlántico, Bolívar and Nariño present the lowest effectiveness in period one and this was on the rise. For the other cases we can see that their behavior is oscillatory, that is, it increases and decreases for each period considered.

For the intermediate departments, such as Santander, Nariño and Meta, we can see that efficiency is better in the first periods, while as time goes on they are not as good, especially in the intermediate periods. This is because, as the virus grew in each territory, the health systems were meeting the demand required in some cases, but by the time the epidemic reached its peak, they were at the top of their capacity, diminishing their effectiveness. In the remote departments, i.e., Amazonas, Chocó and San Andrés, different behaviors can be seen. In the first two departments, because they are so remote, their health systems are unable to cope with the spread of the virus in each territory, while in the third department was able to gradually adapt its resources to provide better medical service. In the case of the larger departments, we can see that they show certain oscillations in their efficiency, a product of the multiple strategies that the governments implement in each region with respect to the behaviors of previous weeks.

It can be observed in Figure 7 that, in the first periods, departments such as Bolívar and Chocó present a low efficiency, the first of these due to the rapid growth of confirmed cases in relation to those recovered, while, for the second, this behavior is due to the low hospital capacity to attend patients. Thus, the departments that have been most affected in recent periods are Amazonas, San Andrés and Meta. The first two are classified as remote, which implies that they are affected because their resources are not sufficient to counteract the spread of the virus and the medical demand that this generates. The third department, Meta, has not been very effective over time because, despite the problems it had at the beginning of the epidemic, its resources did not allow

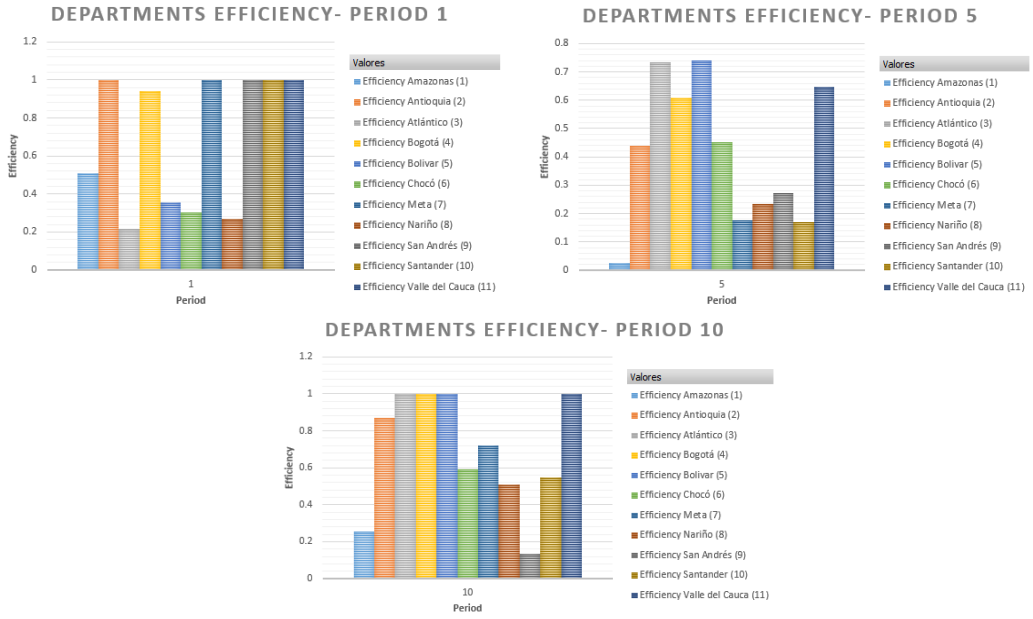

Figure 7: Model 2 - Departments Efficiency in Period 1, 5 y 10

for improvement in the performance of the territory's health systems. It can be seen that in the case of San Andrés, the economic reopening of the territory for the ninth week not only damaged the spread of the virus but also the quality of medical treatment provided to those infected.

4.3 Classification of departments

The Figure 8 shows the classification of the departments into four zones, determined by the averages of efficiency in each model, where you can observe the behavior of each department in relation to the others. The departments that were efficient in both models are: San Andrés, period 1, Amazonas, period 2 and 7, Santander, period 1, Meta, period 1, and Bogotá, period 10.

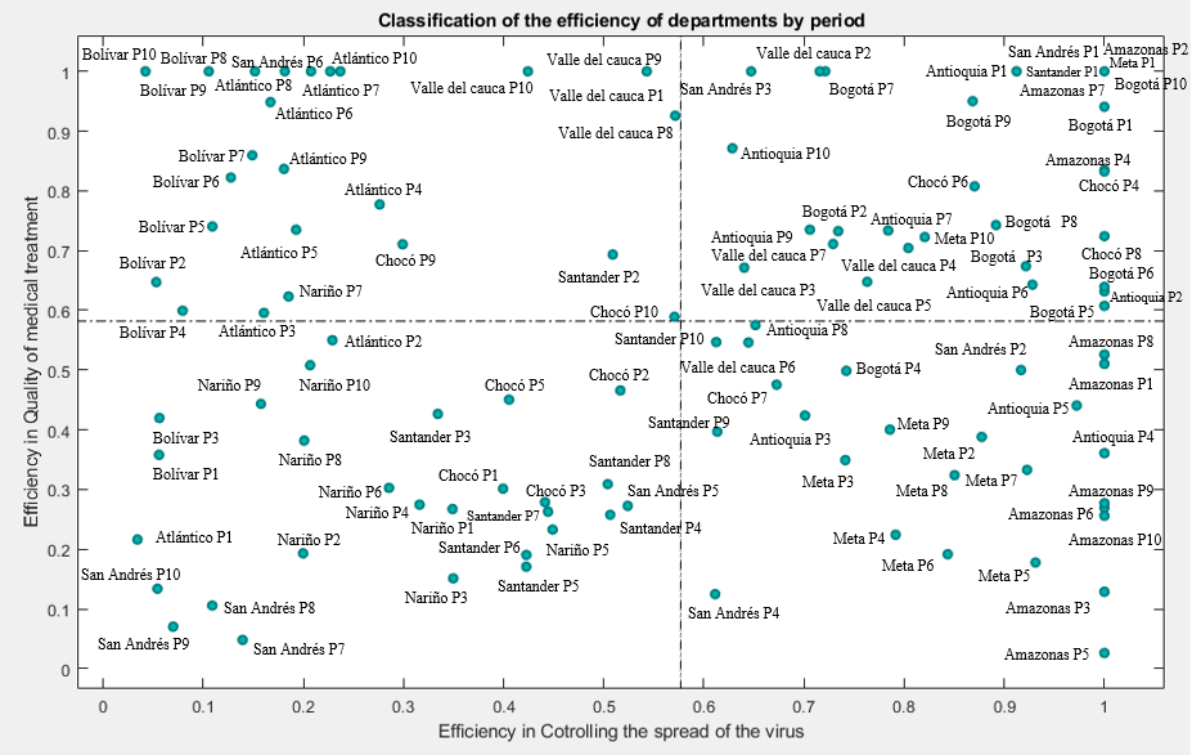

Figure 8: Classification the efficiency of departments by period

In the Figure 9, a classification of the departments for both models in relation to their efficiency values and average values is presented. There, it can be seen that the departments of Antioquia, Bogotá and Valle del Cauca present most of their periods in an above-average efficiency for the control of the propagation of infections and the medical treatment provided. On the other hand, the departments of Amazonas and Meta behave well in controlling infections, while Atlántico and Bolívar are characterized by providing above-average medical treatment. Finally, the departments of Nariño and Santander are the ones with the most alarming behavior, since they are below the average in both models for most of the periods, which represents a greater affectation of the virus in these territories.

4.4 Strategies analysis

The results obtained show that, for the efficient departments in both models in the first evaluation period, which present intermediate densities or are remote location territories, the virus affectation was not as high as in the biggest departments at that point. In addition, it can be seen that Amazonas presents an effective behavior in the period 7, product of the multiple aids received by the different cities of the country to harness its medical service and its geospatial location, that maintains the department isolated in great part of the cities with high number of infections. In turn, Bogotá presents an effective performance in the period 10, product of the great investment made in terms of strengthening the medical services, the reduction of sources of infection in public transport and quarantines in the most affected localities. Unfortunately, San Andrés presents the lowest levels of efficiency, product of the behavior in the last periods, due to the problems that it

classification of departments							
High efficiency both in contagion control and medical treatment		High efficiency in contagion control but low efficiency in medical treatment		Low efficiency in contagion control but high efficiency in medical treatment		Low efficiency both in contagion control and medical treatment	
Department	Period	Department	Period	Department	Period	Department	Period
Amazonas	2	Amazonas	1	Atlántico	3	Atlantico	1
Amazonas	4	Amazonas	3	Atlántico	4	Atlantico	2
Amazonas	7	Amazonas	5	Atlántico	5	Bolivar	1
Antioquia	1	Amazonas	6	Atlántico	6	Bolivar	3
Antioquia	2	Amazonas	8	Atlántico	7	Chocó	1
Antioquia	6	Amazonas	9	Atlántico	8	Chocó	2
Antioquia	7	Amazonas	10	Atlántico	9	Chocó	3
Antioquia	9	Antioquia	3	Atlántico	10	Chocó	5
Antioquia	10	Antioquia	4	Bolivar	2	Nariño	1
Bogotá	1	Antioquia	5	Bolivar	4	Nariño	2
Bogotá	2	Antioquia	8	Bolivar	5	Nariño	3
Bogotá	3	Bogotá	4	Bolivar	6	Nariño	4
Bogotá	5	Chocó	7	Bolivar	7	Nariño	5
Bogotá	6	Meta	2	Bolivar	8	Nariño	6
Bogotá	7	Meta	3	Bolivar	9	Nariño	8
Bogotá	8	Meta	4	Bolivar	10	Nariño	9
Bogotá	9	Meta	5	Chocó	9	Nariño	10
Bogotá	10	Meta	6	Chocó	10	San andrés	5
Chocó	4	Meta	7	Nariño	7	San andrés	7
Chocó	6	Meta	8	San Andrés	6	San andrés	8
Chocó	8	Meta	9	Santander	2	San andrés	9
Meta	1	San Andrés	2	Valle del Cauca	1	San andrés	10
Meta	10	San Andrés	4	Valle del Cauca	8	Santander	3
San Andrés	1	Santander	9	Valle del Cauca	9	Santander	4
San Andrés	3	Santander	10	Valle del Cauca	10	Santander	5
Santander	1	Valle del Cauca	6			Santander	6
Valle del Cauca	2					Santander	7
Valle del Cauca	3					Santander	8
Valle del Cauca	4						
Valle del Cauca	5						
Valle del Cauca	7						

Figure 9: Classification the efficiency of departments by period

presents in the promotion of its medical services, added to this the low number of tests made weekly and the reopening of the flights towards this place, which diminished the control in the propagation of the virus. On the other hand, Nariño is another territory where most of its periods are below the average for each model, this due to the delay presented in the implementation of screening services for the vulnerable population and a low capacity in the provision of medical services.

5 Conclusions and future research

In this paper, the efficiency of some departments in Colombia regarding the COVID-19 is studied using DEA. The departments that present a higher efficiency level are those of great size and economic power, since these possess greater resources to implement containment plans that regulate the propagation of the virus in relation to the daily tests carried out, as well as they have the capacity to adapt quickly to the increasing demand of the medical services that the growth of the cases requires. On the other hand, as shown by Amazonas and Meta, the number of inhabitants due to the territorial extension of a department plays a very important role in the spread of the virus, directly benefiting those with a lower population density. Additionally, it is observed that the departments most affected are those with an intermediate population density, as evidenced by the results for Nariño and Santander. The latter is due to the fact that these intermediate departments are territories with high population, which are considerably exposed to contact with each other and, since they do not have as many resources as the major departments, their medical services are not sufficient to counteract the demand for medical services. In the case of San Andrés, where the few

action plans at the beginning of the pandemic have brought about quite large consequences towards the end of the periods analyzed.

In this process, it is possible to perceive the great impact that certain measures have caused in the different departments. As expected, isolation, quarantines and monitoring of active cases are the plans that present a better performance in the spread of the virus. Also, the action plans to strengthen the tests carried out in the territories have a great influence on the control of cases where the origin of infection is not known, and where it is clear that the early adaptation of the health systems to face the pandemic has a great positive impact on the results. Finally, it can be seen that in most of the departments the number of confirmed cases is alarming, where the governments do not prioritize enough the spread of the virus but rather those measures that increase the number of deaths. Additionally, it is clear that those departments that present more confirmed cases are not necessarily the ones that present less efficiency in time. On the contrary, those territories where there is not an adequate control of the spread of the virus are the places with the worst behavior in the last periods, because they have slower growth of infected people but that, given their little capacity in the medical services, the can not response with increasing of the cases in the hospitals.

Future implementations include the application of this type of methodology for the current COVID-19 situation at the city level for different time periods, either a longer period or an updated one, thus seeking to include new behaviors that at the time of this analysis were not yet recorded. In addition, this new study will be implemented with an DEA model that can handle undesirable inputs and outputs from its very nature, without having to consider these in an inverse relationship of inputs and outputs to meet the restrictions of the classical models.

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References

BAL, HASAN, & GÖLCÜKCÜ, AYHAN. 2016. ACADEMIC EFFICIENCY COMPARISON OF COUNTRIES VIA DEA. *Data Envelopment Analysis and its Applications*, 9.

Banker, Rajiv D, Charnes, Abraham, & Cooper, William Wager. 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management science*, **30**(9), 1078–1092.

Charnes, Abraham, Cooper, William W, & Rhodes, Edwardo. 1978. Measuring the efficiency of decision making units. *European journal of operational research*, **2**(6), 429–444.

Charnes, Abraham, Clark, Charles T, Cooper, William W, & Golany, Boaz. 1984. A developmental study of data envelopment analysis in measuring the efficiency of maintenance units in the US air forces. *Annals of operations Research*, **2**(1), 95–112.

Consultor Salud. 2020. *MEDELLÍN, EJEMPLO MUNDIAL EN EL MANEJO DEL COVID-19*. Retrieved November 28, from https://bit.ly/3o9SqlO.

Cooper, William W, Seiford, Lawrence M, & Tone, Kaoru. 2000. Data envelopment analysis. *Handbook on data envelopment analysis*, 1–40.

DA CUNHA, RENATO BARBOSA, BANDEIRA, DENISE LINDSTROM, & BORNIA-POULSEN, CAMILO JOSÉ. 2016. MEASURING EFFICIENCY OF SALES TERRITORIES FOR MEDICAL DEVICES IN BRAZIL THROUGH DATA ENVELOPMENT ANALYSIS (DEA). *Data Envelopment Analysis and its Applications*, 111.

Doyle, John, & Green, Rodney. 1994. Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. *Journal of the operational research society*, **45**(5), 567–578.

El Mundo. 2020. *Mapa del coronavirus: expansión en cifras del Covid-19 en el mundo*. Retrieved November 27, from https://bit.ly/2YOyKtH.

El Tiempo. 2020a. *Antioquia aumentará la búsqueda activa para detectar casos de covid*. Retrieved November 28, from https://bit.ly/2KMZnuF.

El Tiempo. 2020b. *Así está superando el Amazonas sus horas más oscuras por covid-19*. Retrieved November 28, from https://bit.ly/3fPEBGb.

El Tiempo. 2020c. *San Andrés, en crisis sanitaria por aumento de contagios de covid-19*. Retrieved November 28, from https://bit.ly/36m7mHK.

Farrell, Michael James. 1957. The measurement of productive efficiency. *Journal of the Royal Statistical Society: Series A (General)*, **120**(3), 253–281.

Gobierno de Bogotá. 2020. *Alcaldesa Claudia López anunció nuevas medidas a partir del 16 de agosto*. Retrieved November 28, from https://bit.ly/3fNt0aT.

Gobierno de Pasto. 2020. *Alcaldía de Pasto y Gobernación de Nariño avanzan en iniciativa para desacelerar la expansión del Covid-19*. Retrieved November 28, from https://bit.ly/3fQyLV7.

HUŽVÁR, MIROSLAV, RIGOVÁ, ZUZANA, & SÝKOROVÁ, KATARÍNA. 2016. EFFICIENCY OF MUNICIPAL GOVERNMENTS IN SLOVAKIA. *Data Envelopment Analysis and its Applications*, 61.

Ibargüen, Victor Manuel Quesada. 2003. Estimación de la eficiencia mediante análisis envolvente de datos - DEA-. *Panorama Económico No.18*. Retrieved from https://bit.ly/2XMDgZb.

Jahanshahloo, Gholam Reza, Lotfi, F Hosseinzadeh, Shoja, N, Tohidi, Ghasem, & Razavyan, Shabnam. 2005. Undesirable inputs and outputs in DEA models. *Applied mathematics and computation*, **169**(2), 917–925.

La República. 2020. *Pasajeros que viajen a San Andrés y Providencia ya no deberán practicarse la prueba covid-19*. Retrieved November 28, from https://bit.ly/3o6VkYD.

Ministerio de Salud. 2020. *Situación actual: nuevo Coronavirus (COVID-19)*. Retrieved November 20, from https://bit.ly/3kssSiH.

Presidencia de la República. 2020. *Medidas sectoriales adoptadas por el gobierno nacional frente al coronavirus*. Retrieved November 27, from https://bit.ly/3gG8ZBT.

Procuraduría. 2020. *Procuraduría urgió a gobernador y alcaldes de Nariño la adopción de estrategias inmediatas y efectivas para controlar el contagio con covid-19*. Retrieved November 28, from https://bit.ly/2VhD7ev.

Rodriguez, Gabriel Damiaá, & Loaiza, Melissa Velásquez. 2020. *¿Por qué han aumentado tanto los casos de covid-19 en Colombia, type VIDEO*. Retrieved November 27, from https://cnn.it/2YOYTsf.

SALUDATA. 2020. *Porcentaje de ocupación de unidades de cuidados intensivos para la atención del COVID-19 en Bogotá D.C.* Retrieved September 1, from https://bit.ly/31O9dCD.

Serrano, Vicente Coll, & Blasco, Olga M. Blasco. 2006. *Evaluación de la eficiencia mediante el análisis envolvente de datos: introducción a los modelos básicos*.

Shirouyehzad, Hadi, Jouzdani, Javid, & Khodadadi Karimvand, Mazdak. 2020. Fight against COVID-19: a global efficiency evaluation based on contagion control and medical treatment. *Journal of Applied Research on Industrial Engineering*, **7**(2), 109–120.

Statista. 2020. *Covid-19: número de muertes por país 2020*. Retrieved November 27, from https://bit.ly/34Y5Uen.

Talluri, Srinivas. 2000. Data Envelopment Analysis: Models and Extensions. *Decision Line Vol. 31*. Retrieved from https://bit.ly/3ismh62.

Thanassoulis, Emmanuel. 2001. Introduction to the theory and application of data envelopment analysis. *Kluwer Academic Publishers*.

Transmilenio. 2020. *Incremento en capacidad del Sistema pasa del 35% al 50%*. Retrieved November 28, from https://bit.ly/3mz5ZuA.

Wang, Ke, Wei, Yi-Ming, & Zhang, Xian. 2012. A comparative analysis of China's regional energy and emission performance: Which is the better way to deal with undesirable outputs? *Energy Policy*, **46**, 574–584.