I can solve the Linear Programming Problem (LPP) using the **Simplex Method**.

📝 Problem Formulation

The LPP is:
Maximize $Z = x_1 + 2x_2 + 4x_3$
Subject to:

1. $3x_1 + x_2 + 5x_3 \le 10$
2. $x_1 + 4x_2 + x_3 \le 8$
3. $2x_1 + 2x_3 \le 7$
4. $x_1, x_2, x_3 \ge 0$

⚙️ Standard Form and Initial Tableau

To use the Simplex Method, we convert the inequalities into equations by introducing **slack variables** (s_1, s_2, s_3).
Objective Function (Standard Form):
Maximize $Z = x_1 + 2x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$
$Z - x_1 - 2x_2 - 4x_3 = 0$ (This is Row 0)
Constraints (Equations):

1. $3x_1 + x_2 + 5x_3 + s_1 = 10$
2. $x_1 + 4x_2 + x_3 + s_2 = 8$
3. $2x_1 + 2x_3 + s_3 = 7$

Initial Simplex Tableau:

Basis	Z	x1	x2	x3	s1	s2	s3	RHS (bi)
Z	1	-1	-2	**-4**	0	0	0	0
s_1	0	3	1	**5**	1	0	0	10
s_2	0	1	4	1	0	1	0	8
s_3	0	2	0	2	0	0	1	7

🔄 Iteration 1

1. Identify Entering Variable (EV)

The most negative value in the Z-row (Row 0) is -4 (under x_3).
$\implies$ x_3 is the Entering Variable (Pivot Column).

2. Identify Leaving Variable (LV)

Calculate the **minimum non-negative ratio** ($\frac{RHS}{x_3 \text{ coefficient}}$):

- Row 1 (s_1): $10 / 5 = \mathbf{2}$ (Minimum ratio)
- Row 2 (s_2): $8 / 1 = 8$
- Row 3 (s_3): $7 / 2 = 3.5$
  $\implies$ s_1 is the Leaving Variable (Pivot Row).

The **Pivot Element** is **5** (at the intersection of the x_3 column and s_1 row).

3. Pivot Operation

- **New Pivot Row (R_1'):** $R_1 / 5$ (Divide the old s_1 row by the pivot element 5)
- **New R_0':** $R_0 + 4 R_1'$
- **New R_2':** $R_2 - 1 R_1'$
- **New R_3':** $R_3 - 2 R_1'$

Tableau after Iteration 1:

Basis	**Z**	**x1**	**x2**	**x3**	**s1**	**s2**	**s3**	**RHS**
Z	1	$7/5$	$-6/5$	0	$4/5$	0	0	**8**
x_3	0	$3/5$	$1/5$	1	$1/5$	0	0	2
s_2	0	$2/5$	**$19/5$**	0	$-1/5$	1	0	6
s_3	0	$4/5$	$-2/5$	0	$-2/5$	0	1	3

(Current $Z = 8$. $x_3=2, s_2=6, s_3=3$. $x_1=x_2=s_1=0$).

🔄 Iteration 2

1. Identify Entering Variable (EV)

The only negative value in the Z-row (Row 0) is -6/5 (under x_2).
$\implies$ x_2 is the Entering Variable (Pivot Column).

2. Identify Leaving Variable (LV)

Calculate the minimum non-negative ratio:

- Row 1 (x_3): $2 / (1/5) = 10$
- Row 2 (s_2): $6 / (19/5) = 30/19 \approx \mathbf{1.57}$ (Minimum ratio)
- Row 3 (s_3): $3 / (-2/5)$ (Ignore, as the denominator is negative)
  $\implies$ s_2 is the Leaving Variable (Pivot Row).

The **Pivot Element** is **$19/5$**.

3. Pivot Operation

- **New Pivot Row (R_2''):** $R_2' / (19/5)$
- **New R_0'':** $R_0' + (6/5) R_2''$
- **New R_1'':** $R_1' - (1/5) R_2''$
- **New R_3'':** $R_3' + (2/5) R_2''$

Tableau after Iteration 2:

Basis	Z	x1	x2	x3	s1	s2	s3	RHS
Z	1	$29/19$	0	0	$14/19$	$6/19$	0	**$160/19$**
x_3	0	$11/19$	0	1	$4/19$	$-1/19$	0	$32/19$
x_2	0	$2/19$	1	0	$-1/19$	$5/19$	0	$30/19$
s_3	0	$16/19$	0	0	$-8/19$	$2/19$	1	$75/19$

✅ Final Solution

Since there are no negative values in the Z-row (Row 0), the current solution is **optimal**.

- **Optimal Value of Z:** $Z = \mathbf{160/19} \approx 8.421$
- **Optimal Values of Variables:**
  - x_1 (Nonbasic) $= 0$
  - x_2 (Basic) $= 30/19 \approx 1.579$
  - x_3 (Basic) $= 32/19 \approx 1.684$
- **Slack Values:**
  - s_1 (Nonbasic) $= 0$
  - s_2 (Nonbasic) $= 0$
  - s_3 (Basic) $= 75/19 \approx 3.947$ (This means the third constraint is not binding/tight).

The **Maximum** value of Z is $\mathbf{160/19}$.