This Linear Programming Problem (LPP) can be solved using the **Big M Method** variant of the Simplex Algorithm, as it is a minimization problem with "$\ge$" constraints.

1. Convert to Standard Form (Big M Method)

The LPP is:
Minimize $z = 2x_1 + x_2$
Subject to:

1. $3x_1 + x_2 \ge 9$
2. $x_1 + x_2 \ge 6$
   $x_1, x_2 \ge 0$

A. Objective Function

Convert minimization to maximization:

$$\text{Maximize } z' = -z = -2x_1 - x_2$$

B. Constraints

For $\ge$ constraints, we introduce **surplus variables** (s_1, s_2) to make them equalities, and **artificial variables** (A_1, A_2) to establish an initial basis.

1. $3x_1 + x_2 - s_1 + A_1 = 9$
2. $x_1 + x_2 - s_2 + A_2 = 6$
   $x_1, x_2, s_1, s_2, A_1, A_2 \ge 0$

C. Modified Objective Function

Add the penalty terms (M is a very large positive number) for the artificial variables to the objective function:

$$\text{Maximize } z' = -2x_1 - x_2 - M A_1 - M A_2$$
Rearranging for the initial tableau (eliminating A_1 and A_2 from the z' row):

$$z' + (2 - 4M)x_1 + (1 - 2M)x_2 + M s_1 + M s_2 = -15M$$

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2. Simplex Iterations

Initial Tableau

Basis	z′	x1	x2	s1	s2	A1	A2	RHS	Ratio (θ)
z'	1	$\mathbf{2-4M}$	$1-2M$	M	M	0	0	$-15M$	
A_1	0	$\mathbf{3}$	1	-1	0	1	0	9	$9/3 = \mathbf{3}$
A_2	0	1	1	0	-1	0	1	6	$6/1 = 6$

- **Entering Variable:** x_1 (most negative coefficient in z' row: $2-4M$).
- **Leaving Variable:** A_1 (smallest positive ratio: 3).
- **Pivot Element:** 3.

Iteration 1

A_1 leaves, x_1 enters. $R_2' = R_2/3$.

Basis	z′	x1	x2	s1	s2	A1	A2	RHS	Ratio (θ)
z'	1	0	$\mathbf{(1-2M)/3}$	$(2-M)/3$	M	$(4M-2)/3$	0	$-3M-6$	
x_1	0	1	$1/3$	$-1/3$	0	$1/3$	0	3	$3/(1/3) = 9$
A_2	0	0	$\mathbf{2/3}$	$1/3$	-1	$-1/3$	1	3	$3/(2/3) = \mathbf{4.5}$

- **Entering Variable:** x_2 (most negative coefficient in z' row: $(1-2M)/3$).
- **Leaving Variable:** A_2 (smallest positive ratio: 4.5).
- **Pivot Element:** $2/3$.

Iteration 2 (Optimal Tableau)

A_2 leaves, x_2 enters. $R_3'' = R_3' \times 3/2$.

Basis	z′	x1	x2	s1	s2	A1	A2	RHS

z'	1	0	0	$1/2$	$1/2$	$M+1/2$	$M+1/2$	$\mathbf{-7.5}$
x_1	0	1	0	$-1/2$	$1/2$	$1/2$	$-1/2$	$\mathbf{1.5}$
x_2	0	0	1	$1/2$	$-3/2$	$-1/2$	$3/2$	$\mathbf{4.5}$

Since all coefficients in the z' row are non-negative, the tableau is **optimal**.

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3. Final Solution

The maximum value for z' is -7.5.
Since $\text{Minimize } z = -(\text{Maximize } z')$, we have:

$$\text{Minimize } z = -(-7.5) = \mathbf{7.5}$$

The optimal solution is:

- $x_1 = 1.5$
- $x_2 = 4.5$
- Minimum $z = 7.5$