12th Sept '22
Math 104 - 01

Mathematics

* Derivatives
* Functions
* Range
* Real numbers

Quiz - 4/5 (10%)
MT - 1 (30%)
Att. - (10%)
+ Perf
Final - 50%

- 100

$$\mathbb{R} = \boxed{(-\infty, \infty)}$$

open Interval

$\mathbb{R} \to \mathbb{Q}$, $\mathbb{R} \to \mathbb{Q}^c$ | $\mathbb{Q} = p/q$ $[q \neq 0]$

∞ is defined

But $x/0$ = undefined

$$\mathbb{Q} \cup \mathbb{Q}^c = \mathbb{R}$$

$$\mathbb{Z} = \mathbb{Z}^+ \cup \mathbb{Z}^- \cup \{0\}$$

Booklist - 1. Calculus by Howard Anton*

2. Advanced Engineering Mathematics

14th Sept '22

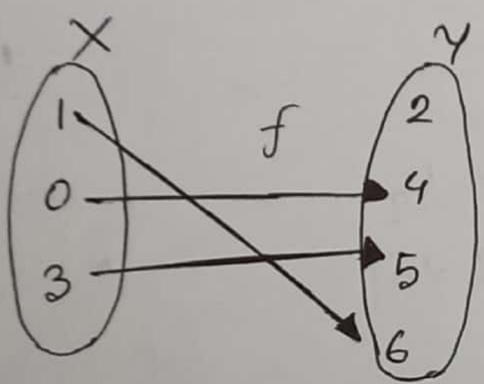
Math 104 - 02

= Functions =

* Relations and functions

Function -

For every input of an independent variable there is ~~an~~ a unique output (dependent variable) then the relation is a function.



$X \longrightarrow Y$

Dom f = $\{1, 0, 3\}$

Range f = $\{4, 5, 6\}$

For all $x \in X$ there is a unique $y \in Y$

$y = m$ [m is real]

$y = x$

$y = x^2$

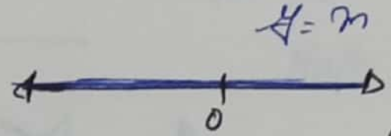
$y = x^3$

$y = |x|$

$y = \frac{1}{x}$

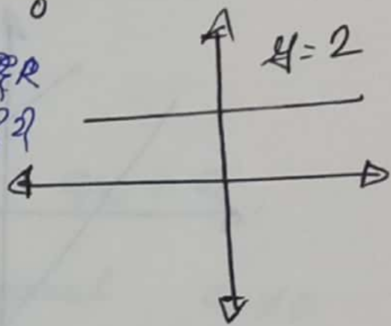
$y = \sqrt{x}$

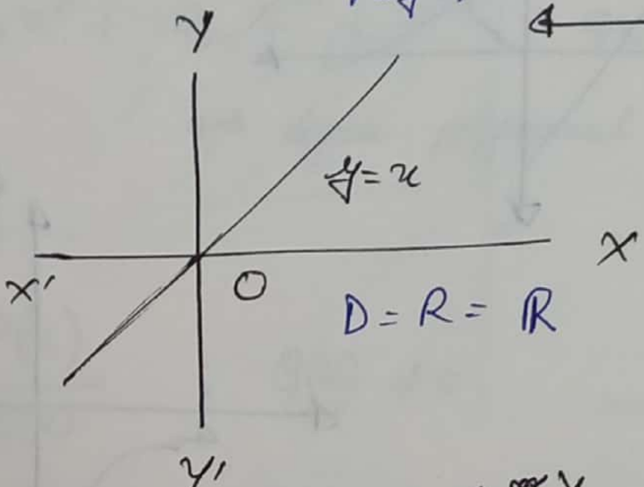
$y = 1/x^2$



Dom $f = \{R\}$

Range $f = \{2\}$

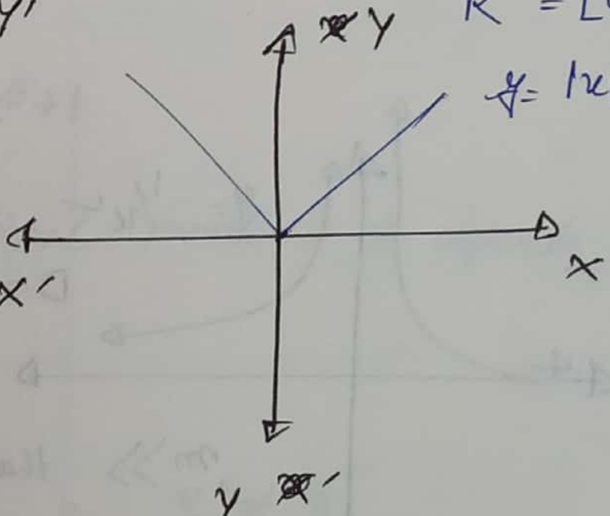




$D = R = \mathbb{R}$

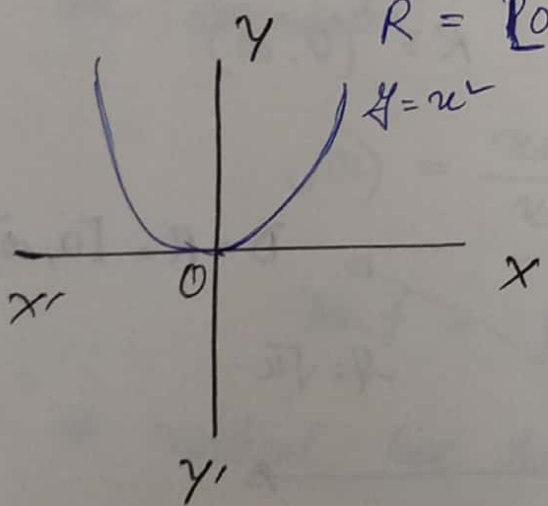
$D = \mathbb{R}$

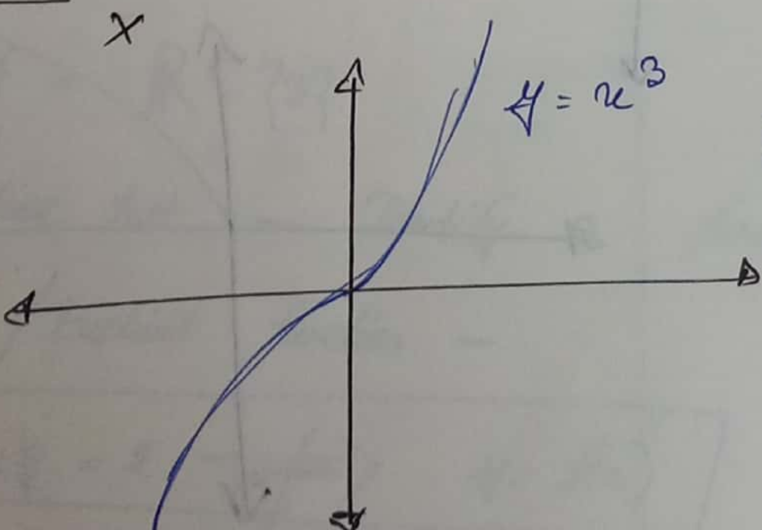
$R = [0, \infty)$



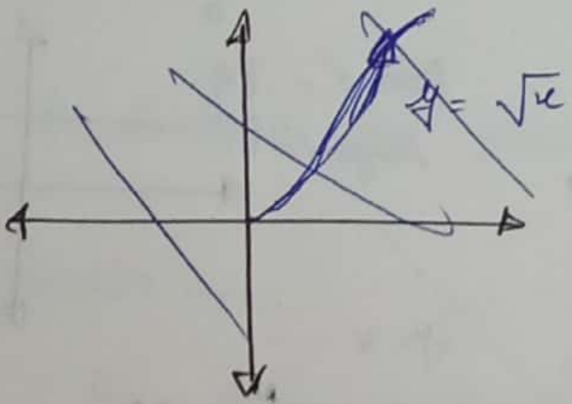
$D = R$

$R = [0, \infty)$

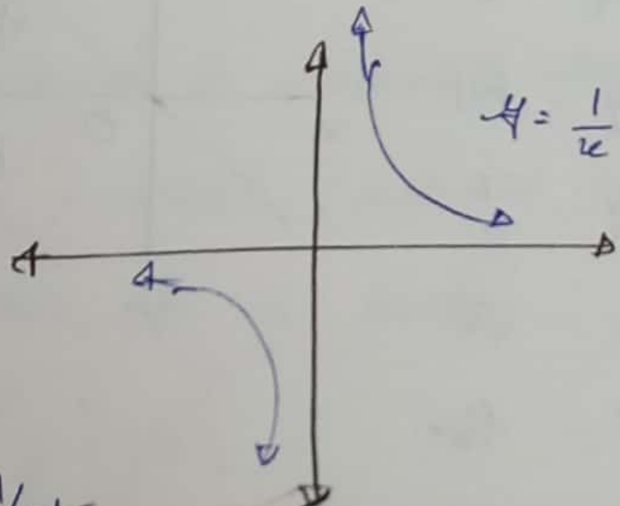


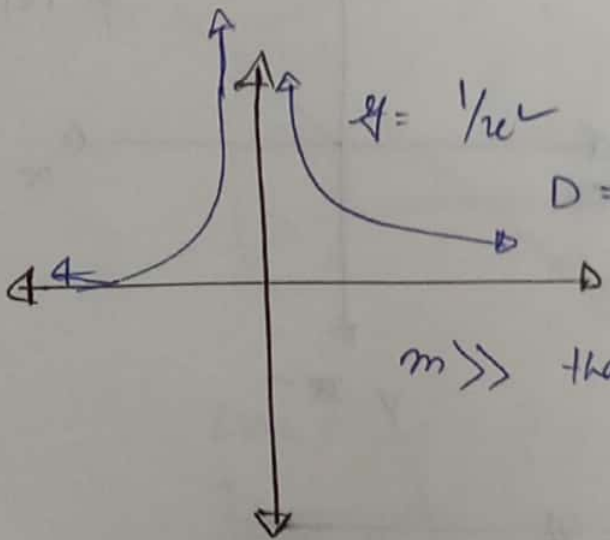


$D = R = \mathbb{R}$



$D = R = \mathbb{R} \setminus \{0\}$



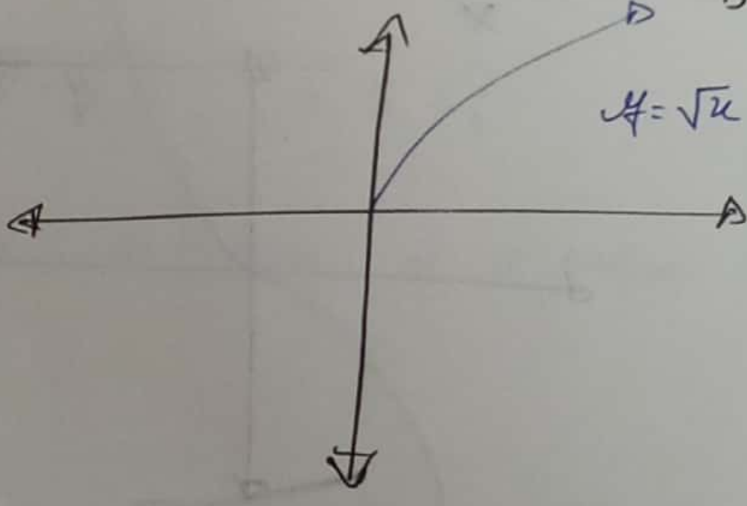


$D = \mathbb{R} \setminus \{0\}$

$R = (0, \infty)$

m >> than $\frac{1}{x}$

$D = R = [0, \infty)$



19th Sept' 22

Math 104 - 03

Function (single variable) —

* Polynomial function –

$$P(x) = y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

nth degree polynomial, $a_n \neq 0$

* Rational function –

$$R(x) = \frac{P(x)}{Q(x)} \quad ; \quad Q(x) \neq 0$$

example, $y = x^5 + 1$

* For the domain of rational function, it depends on the denominator.

$$f(x) = \frac{x+1}{x-3}, \quad x \neq 3$$

$$\text{Dom} f = \mathbb{R} \setminus \{3\}$$

* Vertical line test — Identify a function
* Implicit / Explicit function –

$$\boxed{xy^2 + x^2y = 2 \not\Longrightarrow y = f(x)}$$

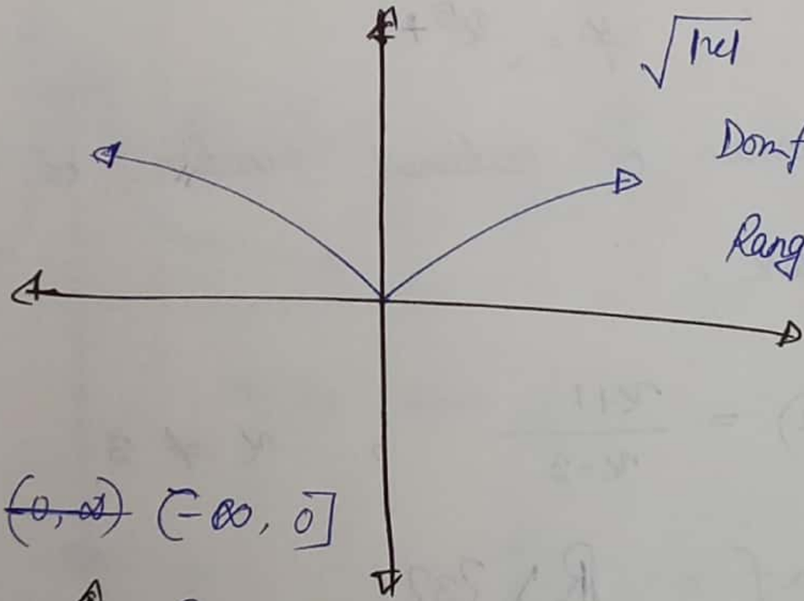
implicit equation (not a function)

$$\boxed{y = f(x) = x^2 + 2}$$ explicit equation / function

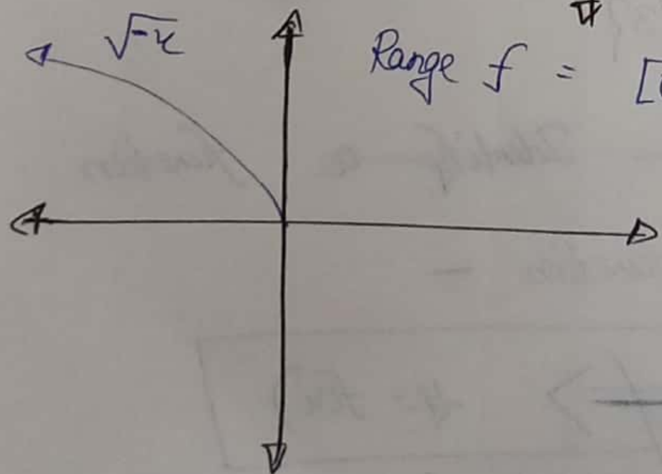
$\sqrt{|x|}$

Dom $f = (-\infty, \infty)$

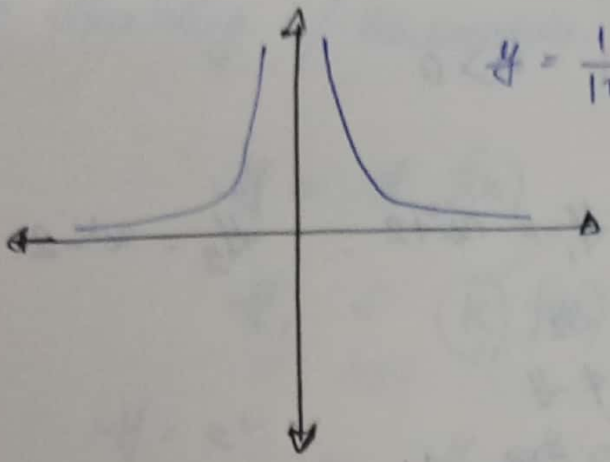
Range $f = [0, \infty)$



Dom $f =$ ~~$(0, \infty)$~~ $(-\infty, 0]$



Range $f = [0, \infty)$



$y = \frac{1}{|x|}$

~~Dom f = (0, ∞)~~

Range $f = (0, \infty)$

Dom $f = (-\infty, \infty) \setminus \{0\}$

Transformation –

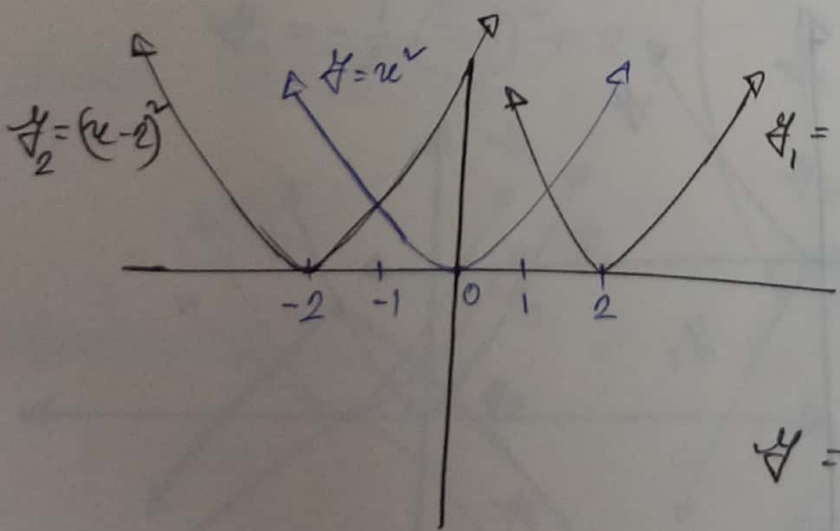
* vertical, horizontal, reflecting, stretching / compression

$y = f(x)$

$y = f(x \pm c)$

$x \pm c \rightarrow$ horizontal shifting

$y = x^2$, $y_1 = (x+2)^2$, $y_2 = (x-2)^2$

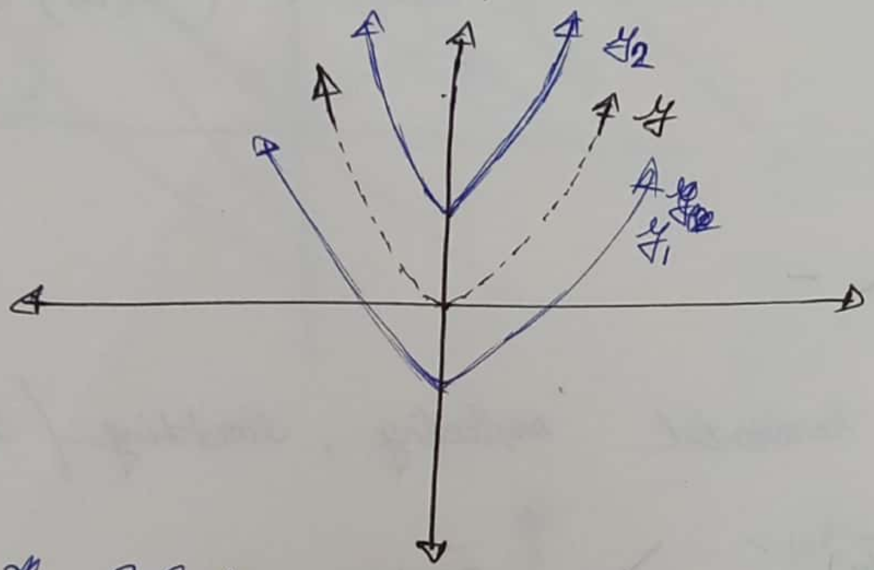


$y = f(x \pm c)$; $c > 0$

* Vertical Shifting

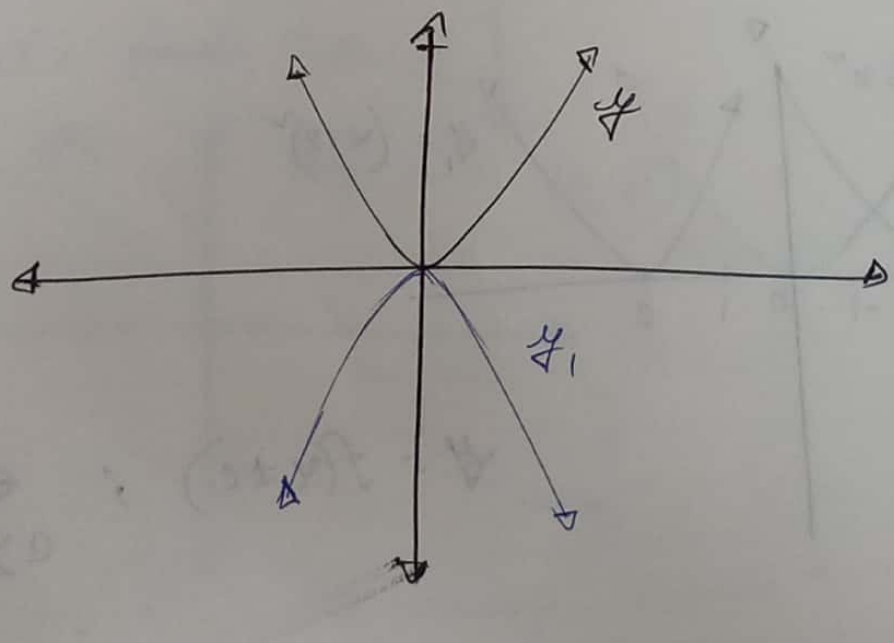
$$\boxed{y = f(x) \pm k} \; ; \quad k > 0$$

$y = x^2$, $y_1 = x^2+2$, $y_2 = x^2-2$



* Reflection —
about the x-axis

$y = f(x)$ / $y_1 = -f(x)$



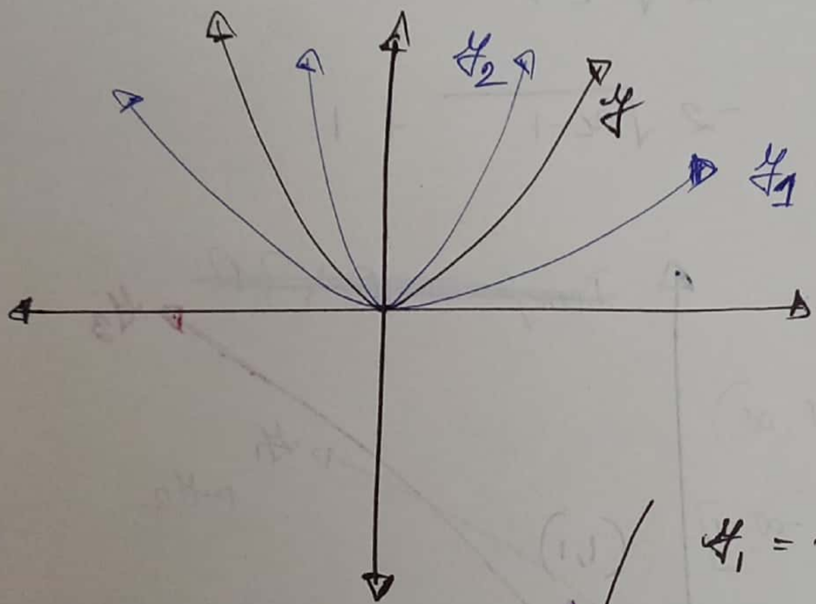
* Stretching / Compression —

$$y = f(x)$$

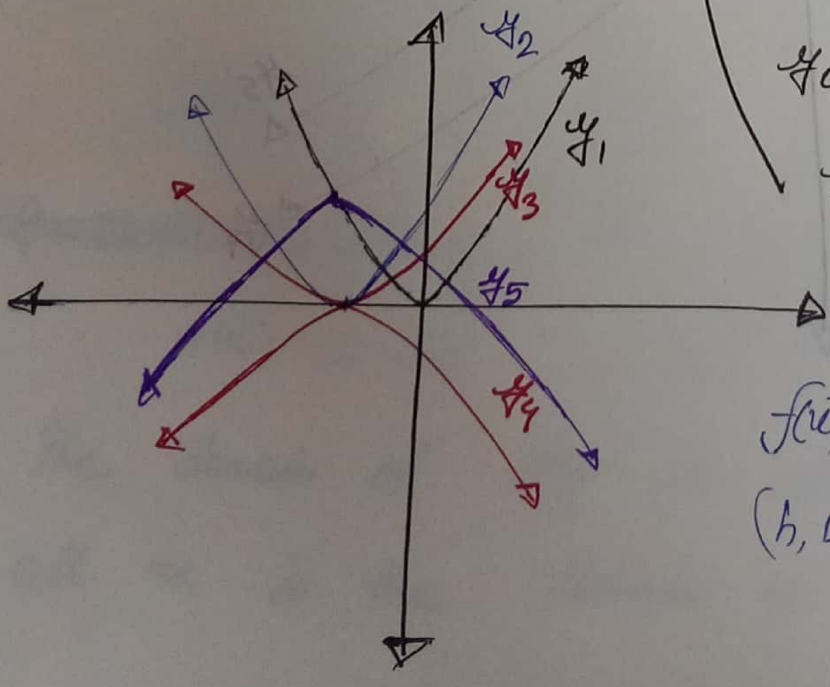
multiplier

$$y_1 = k\, f(x) \;;\; \cancel{k \neq 0} \quad k > 0$$

$y = x^2$, $y_1 = 2x^2$, $y_2 = \frac{1}{2}x^2$



$$y = -\frac{1}{2}(x+1)^2 + 2$$



$y_1 = x^2$

$y_2 = (x+1)^2$

$y_3 = \frac{1}{2}(x+1)^2$

$y_4 = -\frac{1}{2}(x+1)^2$

$y_5 = -\frac{1}{2}(x+1)^2 + 2$

$$f(x) = a(x \pm h)^2 \pm k$$

$(h, k) \longrightarrow$ vertex

$y = -2\sqrt{x-1} + 1$

$y_1 = \sqrt{x}$

$y_2 = \sqrt{x-1}$

$y_3 = 2\sqrt{x-1}$

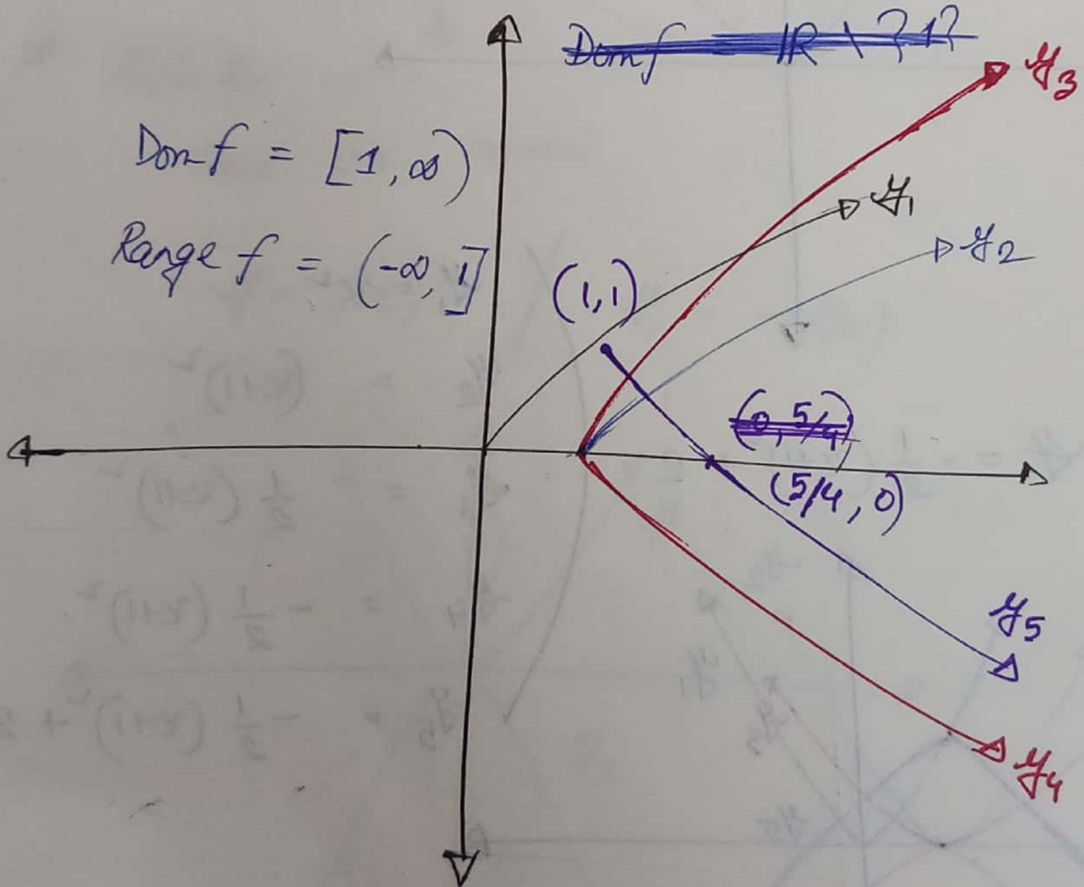
$y_4 = -2\sqrt{x-1}$

$y_5 = -2\sqrt{x-1} + 1$

~~Dom f = ℝ \ {1}~~

Dom f = [1, ∞)

Range f = (−∞, 1]



21st Sept' 22
Math 104 - 04

Composite function –

$y = \underline{\sin e^x}$ |

* not a root function
* ~~[illegible]~~ combination of functions
* function of functions

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$$

$$g(f(x)) = (g \circ f)(x)$$

Given function f and g

The composite of f with, denoted by $f \circ g$ is the function defined by,

$$(f \circ g)(x) = f(g(x))$$

~~$g = \sin\sqrt{x}$, $f = \sqrt{x}$~~ $y = \sin\sqrt{x} = g(f(x))$

$f(x) = \sqrt{x}$, $g(x) = \sin x$

The domain of $f \circ g$ is defined to consist of all x in the domain of g for which

$g(x)$ is in the dom (f).

1. $f(x) = x^2+1 \longrightarrow D = \mathbb{R}$

$g(x) = \sqrt{x+2} \longrightarrow D = [-2, \infty]$

$R = [0, \infty)$

$D\left(\left(f \circ g\right)(x)\right) = ?$

$= \text{dom}\ (g) = [-2, \infty)$

$D\left(\left(g \circ f\right)(x)\right) = ?$

$f(x) \longrightarrow D = \mathbb{R}$

~~$R = \mathbb{R} \setminus \{0\}$~~

$R = [1, \infty)$

$D\left(\left(g \circ f\right)(x)\right) = \text{Dom}\ (f) = \mathbb{R}$

2. $f(x) = \sqrt{x-2}$ $D = [2, \infty)$

$R = [0, \infty)$

$g(x) = \sqrt{x+2}$

$D = [-2, \infty)$

$R = [0, \infty)$

~~$D(fog) = D(f) = [2, \infty)$~~

$\sqrt{\sqrt{x-2}+2}$

$\sqrt{\sqrt{x+2}-2}$

$2, \infty$

~~$D(gof) = D(g) = [2, \infty)$~~

~~$D(fog) = \sqrt{\sqrt{x+2}-2}$~~

$D(fog) = D(f) = [2, \infty)$

$D(gof) = [2, \infty)$

$D(fog) = R(g) \subset D(f)$

$= [2, \infty)$

$D(gof) = R(f) \subset D(g)$

$D(fog)$

$= [2, \infty) \subset [-2, \infty)$

Odd / Even function

$$f(-x) = f(x)$$

Even $\longrightarrow$ $\cos x$, x^2+1

$$f(-x) = -f(x)$$

Odd $\longrightarrow$ $\sin x$, x

$$y = x^2+x+1$$

$$f(-x) = x^2-x+1 \neq f(x) \neq -f(x)$$

$\longrightarrow$ neithere odd nore even.

Even $\longrightarrow$ Symmetry wrt y-axis

odd $\longrightarrow$ Symmetry wrt origin

Calculus — Howard Anton and Davis

10th Oct '22
Math 104 - 05

Inverse Function –

* Invertible function –

(i) $f(g(x)) = x$ for all x in dom (g)

(ii) $g(f(x)) = x$ for all x in dom (f)

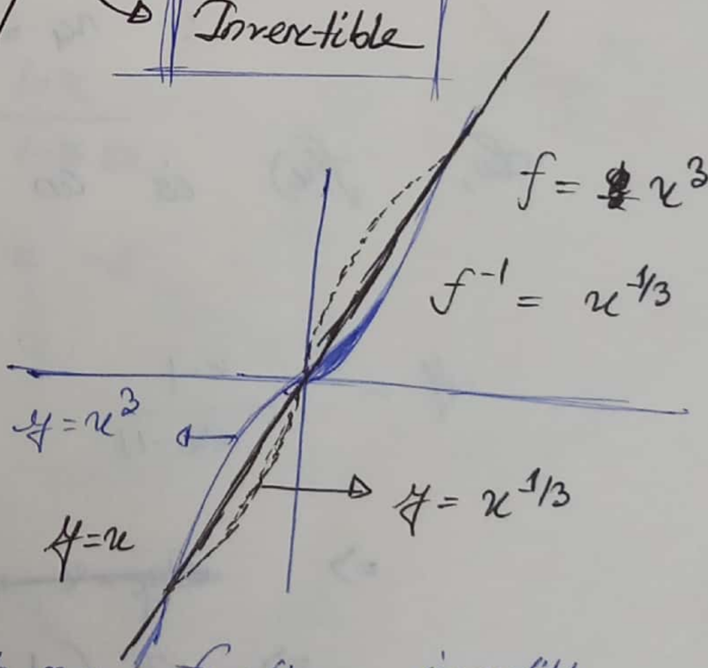
then f and g are said to be invertible functions and they are inverse to each other.

$f(x_1) = f(x_2)$
$\Updownarrow$
$x_1 = x_2$

One - to one functions
→ Invertible

* graphical inverse

Reflection of the Curve about y-axis



* To make a not one to one function invertible, restrict the domain.

$$f(x) = \frac{x-1}{2x+1} = y$$

let

$$f(x_1) = f(x_2)$$

$$\frac{x_1-1}{2x_1+1} = \frac{x_2-1}{2x_2+1}$$

$$\Rightarrow (x_1-1)(2x_2+1) = (x_2-1)(2x_1+1)$$

$$\Rightarrow 2x_1x_2 - 2x_2 + x_1 - 1 = 2x_1x_2 - 2x_1 + x_2 - 1$$

$$\Rightarrow 3x_1 = 3x_2$$

$$\therefore x_1 = x_2$$

So, $f(x)$ is an invertible function.

$$y = \frac{x-1}{2x+1} \Rightarrow 2xy + y = x - 1$$

$$\Rightarrow \cancel{2xy - x =} \; x - 2xy = 1 + y$$

$$\Rightarrow x(1-2y) = 1 + y$$

$$\therefore x = \frac{1+y}{1-2y}$$

$$y = \frac{1+x}{1-2x} = f^{-1}(x)$$

$$\text{Dom}(f) = \text{Range}\,(f^{-1})$$

$$\text{Range}\,(f) = \text{Dom}\,(f^{-1})$$

$$\text{Dom}\,(f) = \mathbb{R} \setminus \{-1/2\}$$

$$\text{Dom}\,(f^{-1}) = \mathbb{R} \setminus \{1/2\}$$

$$\therefore \text{Range}\,(f) = \mathbb{R} \setminus \{1/2\}$$

<u>Verification</u>

Let $y = \frac{1}{2} = \frac{1+x}{1-2x}$

$$\Rightarrow \quad 1 = -2$$

$$\therefore \quad y \neq 1/2$$

[4] Exponential / logarithmic functions —

$$y = \log_a x, \quad a > 0, \quad a \neq 1, \quad x > 0$$

If $a = 10 \longrightarrow$ natural logarithm

If $a = e \longrightarrow \ln$

$$y = a^x, \quad a>0, \quad a \neq 1, \quad [y>0]$$

$$x = \log_a y$$

* For log functions —

$$\text{Dom} = (0, \infty)$$

$$\text{Range} = \mathbb{R}$$

* For exp functions —

$$\text{Dom} = \mathbb{R}$$

$$\text{Range} = (0, \infty)$$

$$\boxed{y = 2^x}$$

$$\Rightarrow x = \log_2 y$$

$$\boxed{y = \log_2 x}$$

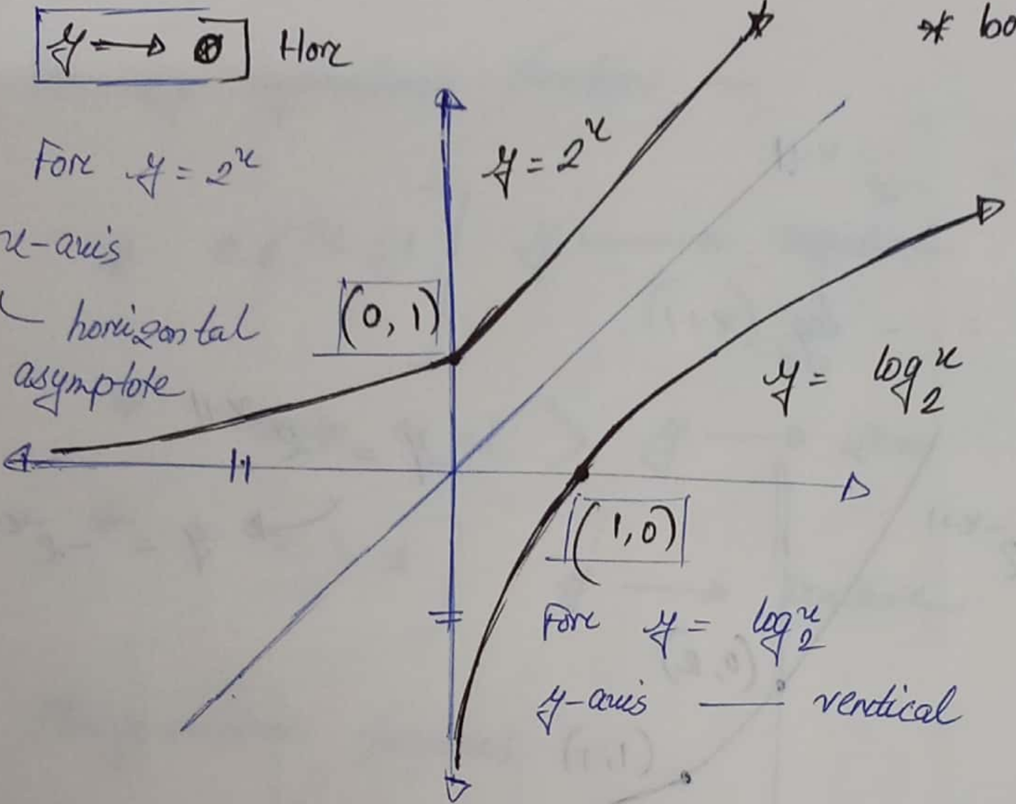
* When $x \to -\infty$

$\boxed{y \to 0}$ Hor

* both are increasing curves

For $y = 2^x$

x-axis — horizontal asymptote



When $\boxed{x \to 0}$ vert

$y \to -\infty$

For $y = \log_2 x$

y-axis — vertical asymptote

base change —

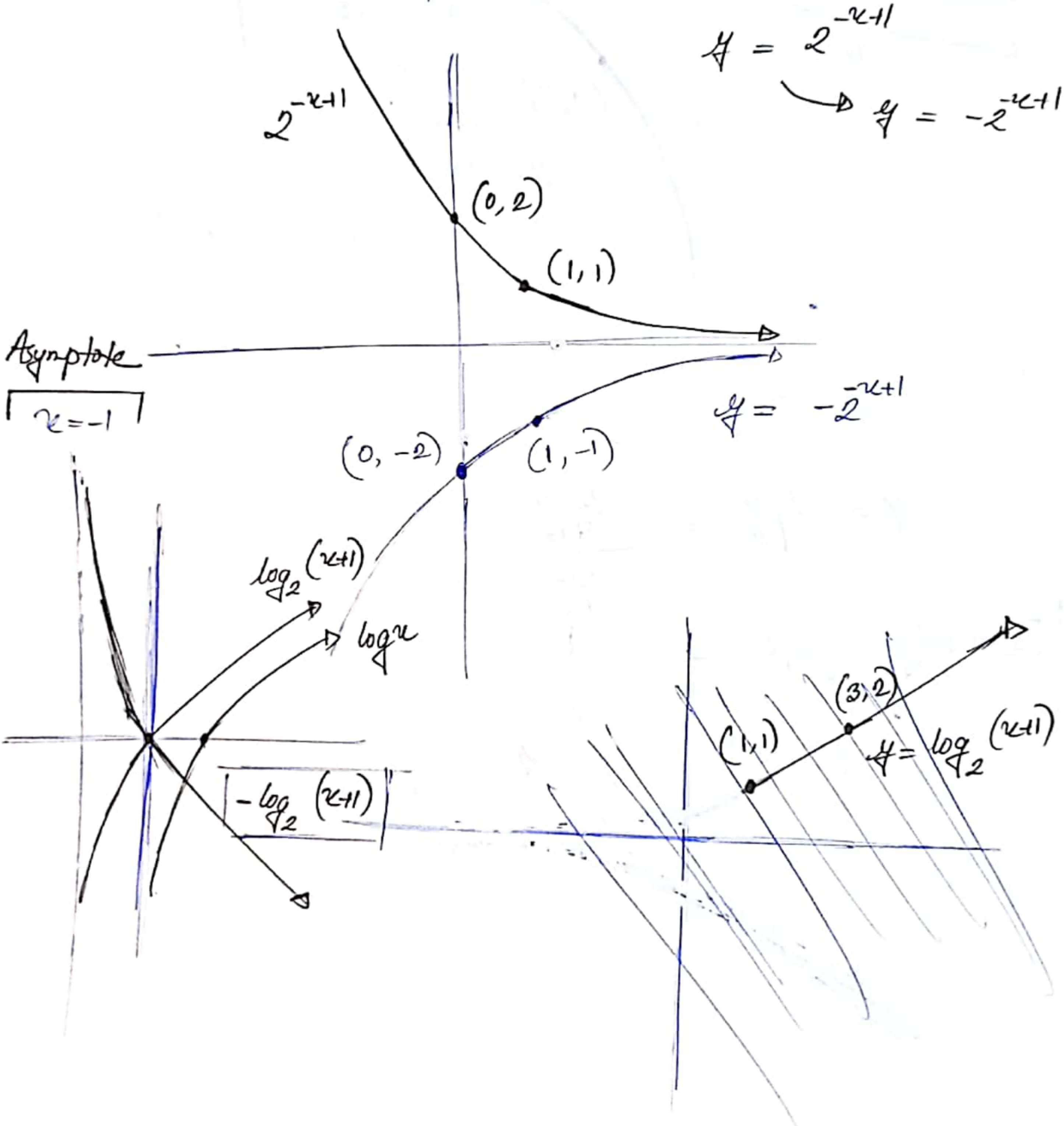
$$y = \log_2 x = \frac{\log_{10} x}{\log_{10} 2}$$

$\boxed{\text{Decreasing graph}}$ —

$y = 2^{-x}$ / $y = \log_{1/2} x$

(iii) $y = -2^{-x+1}$

$y = -\log_2(x+1)$



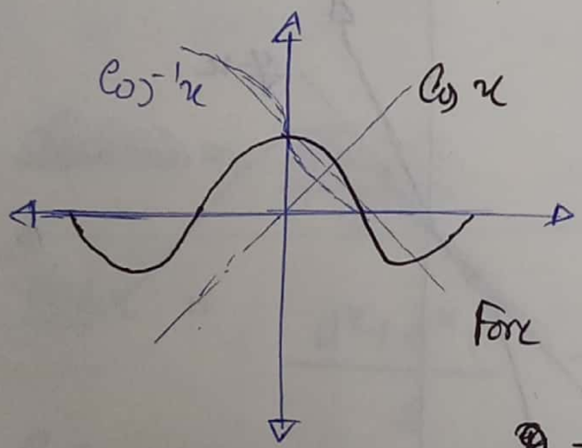
For any logarithmic functions —

* $0 \leq x < 1$] $y \longrightarrow$ negative

* ~~⟹~~ $x = 1$ > $y \longrightarrow$ zero

* $x > 1$] $y \longrightarrow$ positive

Trigonometric functions —



$D = \mathbb{R}$

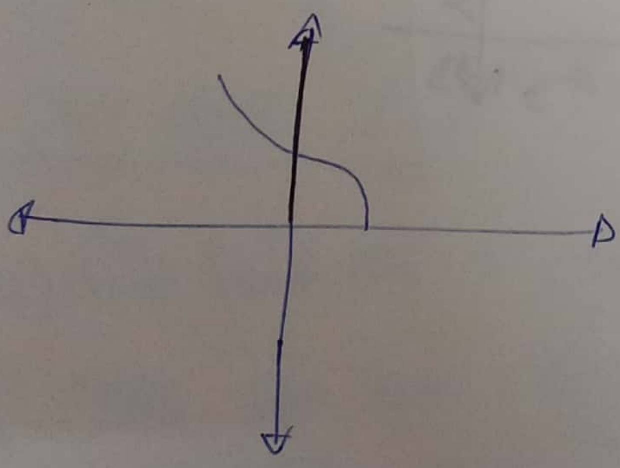
$R = [-1, 1]$

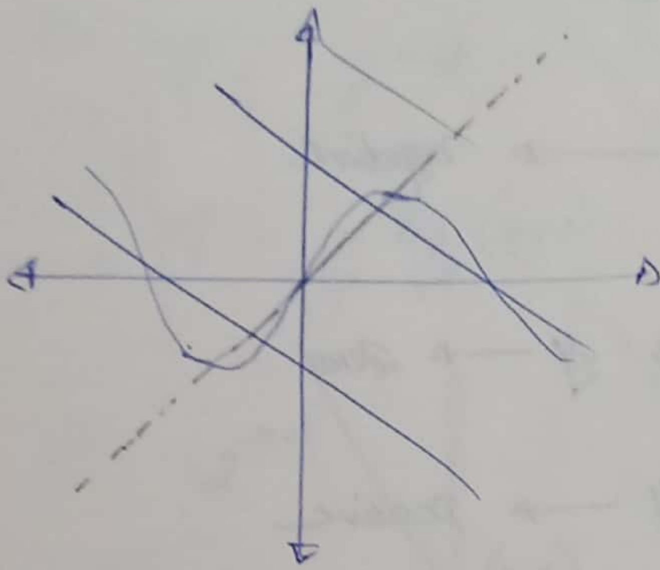
For $\cos^{-1}x$,

$\longrightarrow$ piecewise function

Domain — ~~[0, 1]~~ $[-1, 1]$

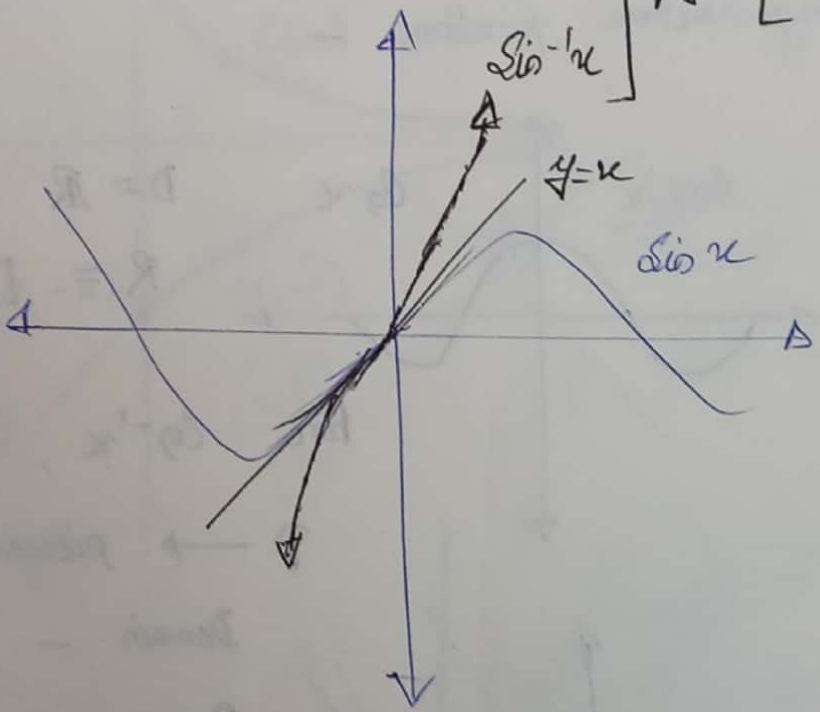
Range —



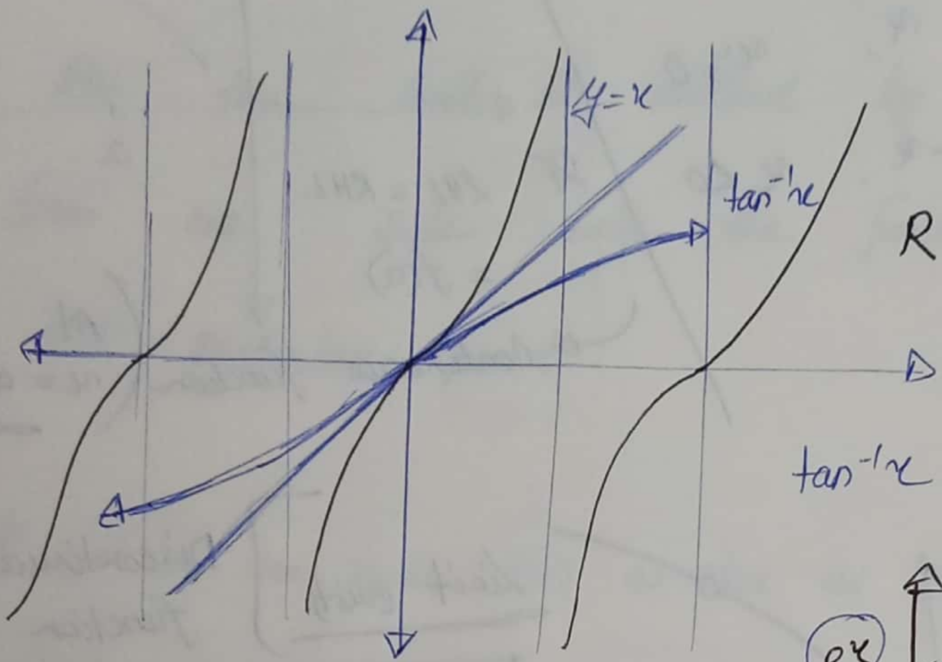


$D = [-1, 1]$

$R = [-\pi/2, \pi/2]$



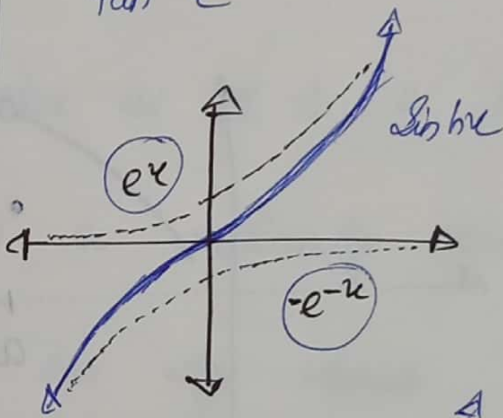
tan x



$D = \mathbb{R} \setminus \{ n\pi/2 : n = 1, 3, \cdots \}$

$R = \mathbb{R}$

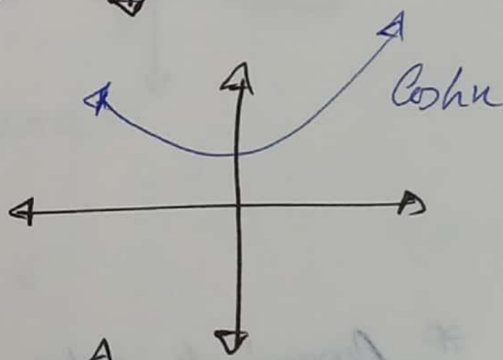
tan⁻¹x

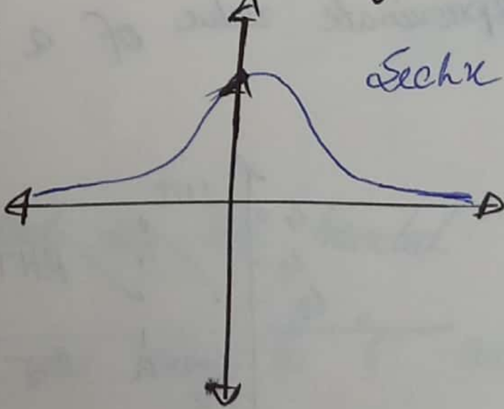


$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

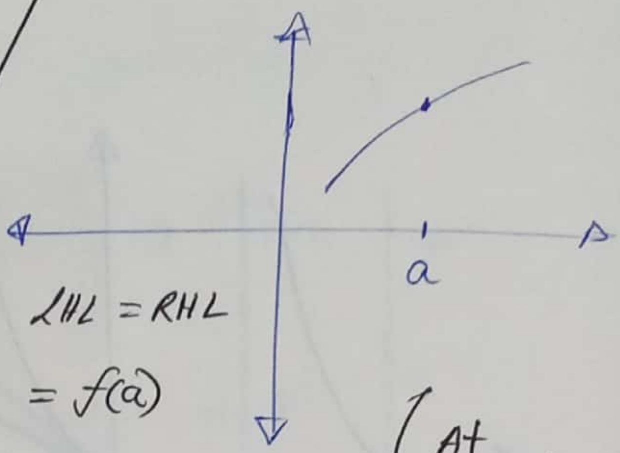




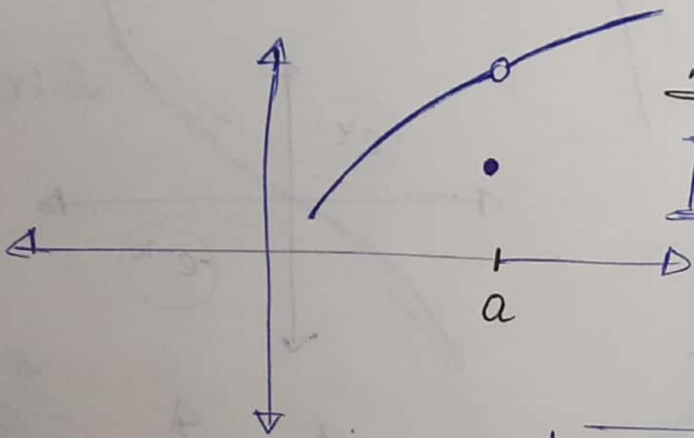
$y = |x|$

$= \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

If $LHL = RHL$
$= f(a)$
→ Continuous function (At $x = a$)



$\boxed{\text{Limit}}$

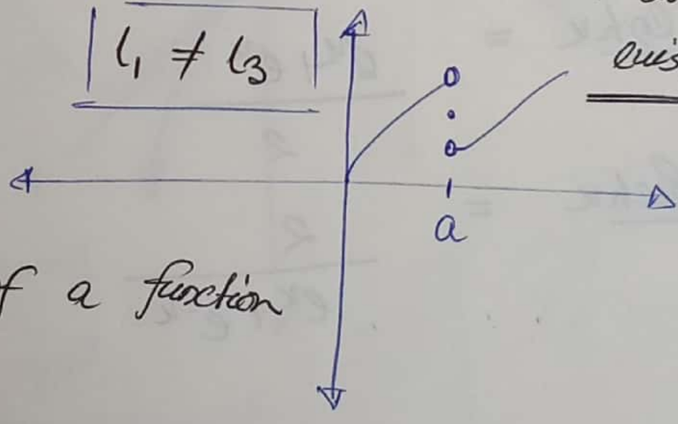


Limit exists] Discontinuous function

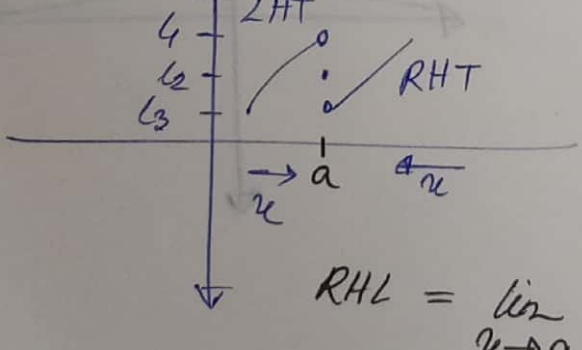
$\boxed{LHL = RHL} \neq f(a)$

$\boxed{l_1 \neq l_3}$

Limit doesn't exist



* Approximate value of a function



Left Handed Limit (LHL)

$= \lim_{x \to a^-} f(x) = l_1$

$RHL = \lim_{x \to a} f(x) = l_3$

If a function doesn't have either LHL or RHL then limit is checked by continuity from one side with the function value on that particular point.

* If we can make $f(x)$ as close as to a number 'l' as desired by choosing x sufficiently close to 'a' and $x \neq a$ then we can say ~~limit~~ limit of $f(x)$ exists at $x=a$

& we write $\lim_{x \to a} f(x) = l$

One Sided Limit

$\lim_{x \to a^-} f(x) = l$ is called the left handed limit if we ~~write~~ can make $f(x)$ as close to 'l' as desired by choosing 'x' sufficiently close to 'a' & $x < a$. Similarly RHL $= \lim_{x \to a^+} f(x) = l$

Limit at infinity

$$\boxed{\lim_{x \to \pm\infty} f(x) = l}$$

$\longrightarrow \lim_{x \to \pm\infty} \frac{1}{x} = 0$

Infinite limit

$$\boxed{\lim_{x \to a, a^+, a^-} f(x) = \pm\infty}$$

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

☐ Infinite limit at infinity —

$$\boxed{\lim_{x \to \pm\infty} f(x) = \pm\infty}$$

$\lim_{x \to \infty} x = \infty$ / $\lim_{x \to \pm\infty} x = \pm\infty$

* Infinite limit $\longrightarrow$ Limit does not exist.

4) $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

$\lim_{x \to 0} f(x) = ?$

$\lim_{x \to 0^-} \frac{|x|}{x} = \frac{-x}{x} = -1 = LHL$

$\lim_{x \to 0^+} \frac{|x|}{x} = \frac{x}{x} = 1 = RHL$

$f(0) = 1$.

Here, $LHL \neq RHL$.

So, limit doesn't exist.

$\begin{cases} x/x, & x > 0 \\ x/-x, & x < 0 \\ 1, & x = 0 \end{cases}$

$= \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 1, & x = 0 \end{cases}$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+2}}{x+2}$$

* Rationalization

$$= \lim_{x \to -\infty} \frac{|x|\sqrt{1+\frac{2}{x^2}}}{x\left(1+\frac{2}{x}\right)}$$

$$= \cancel{\lim_{x \to -\infty} \frac{\sqrt{1+2/x^2}}{1+2/x}} = \cancel{\sqrt{1}}$$

$$= \lim_{x \to -\infty} \frac{-x\sqrt{1+2/x^2}}{x\left(1+2/x\right)}$$

$$\sqrt{x^2} = |x|$$

$$= -1$$

$\epsilon - \delta$ Definition of limit of a function —

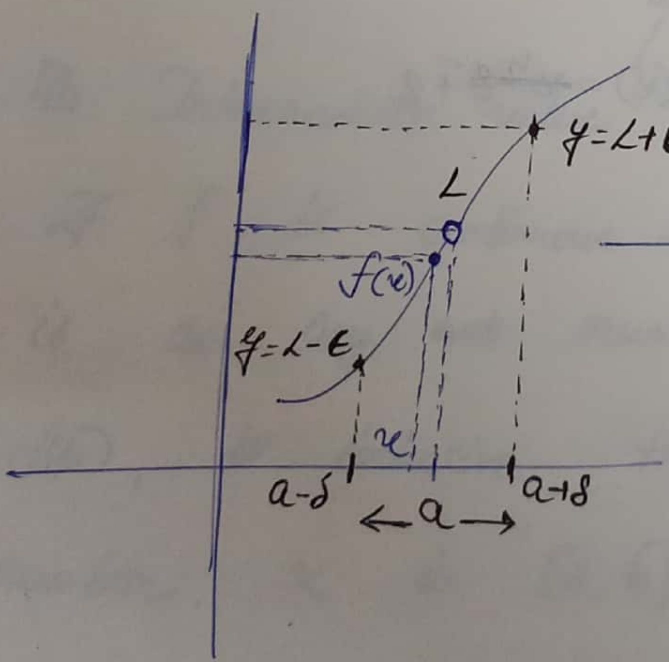
Let a function f be defined on an open interval containing 'a', except possibly at 'a' itself, and let 'L' be a real number,

then $\lim_{x \to a} f(x) = L$ means for every $\epsilon > 0$

there exists a $\delta > 0$ such that

if $\underbrace{0 < |x-a| < \delta}_{a-\delta < x < a+\delta}$, then $\underbrace{|f(x) - L| < \epsilon}_{L-\epsilon < f(x) < L+\epsilon}$

* ϵ and δ has to very very close to zero.



$\longrightarrow$ $L \approx f(x)$ Provided $x \approx a$

$[\delta \approx 0 \longrightarrow \epsilon \approx 0]$

17th Oct '22

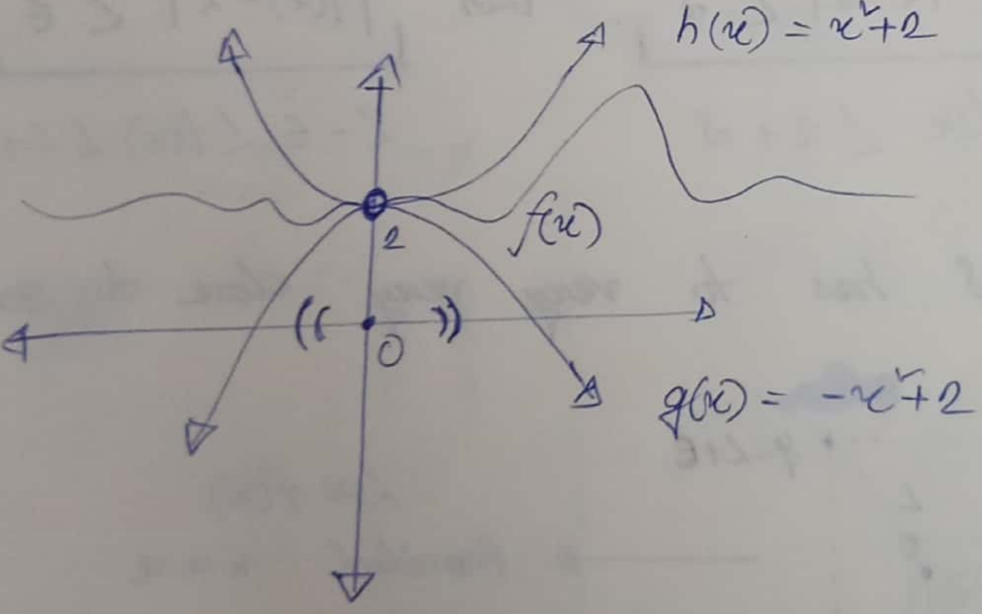
Math 104 - 06

The squeezing theorem –

Let f, g, h be function satisfying $g(x) \le f(x) \le h(x)$, $\forall x$ in some open interval containing the point 'a' possibly $x \neq a$,

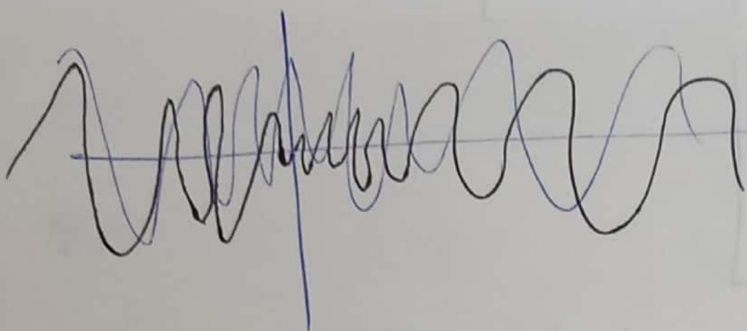
If $\lim_{x \to a} g(x) = L = \lim_{x \to a} h(x)$

then $\lim_{x \to a} f(x) = L$



$\therefore \lim_{x \to 0} f(x) = 2$

$$\lim_{x \to 0} x^2 \sin^2 \frac{1}{x}$$



$$-1 \le \sin x \le 1$$

$$0 \le \sin^2 x \le 1$$

$$0 \le \sin^2 \frac{1}{x} \le 1$$

$$\underbrace{0}_{g(x)} \le \underbrace{x^2 \sin^2 \frac{1}{x}}_{f(x)} \le \underbrace{x^2}_{h(x)}$$

(I) $\lim_{x \to 0} g(x) = 0$

(II) $\lim_{x \to 0} h(x) = 0$

$\therefore \lim_{x \to 0} f(x) = 0$

# Indicates continuity //

Intermediate value theorem –

If f is continuous ~~on~~ on $[a, b]$ and c is an any ~~no~~ number between $f(a)$ and $f(b)$, ~~to~~ exclusive, then there is at least one number x in $[a, b]$ such that $f(x) = c$.

// $f(x) = \frac{1}{x}$ > Dom = $\underline{R \setminus \{0\}}$

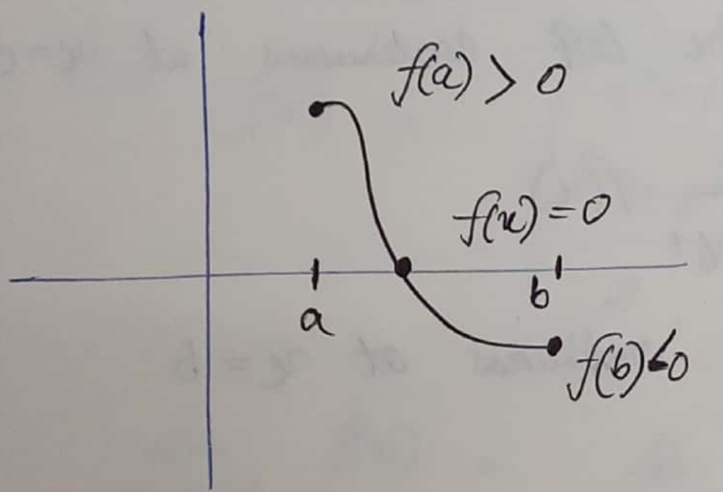
$g(x) = x + 3$ > Dom = R
Range = R

$f(g(x)) = \frac{1}{x+3}$

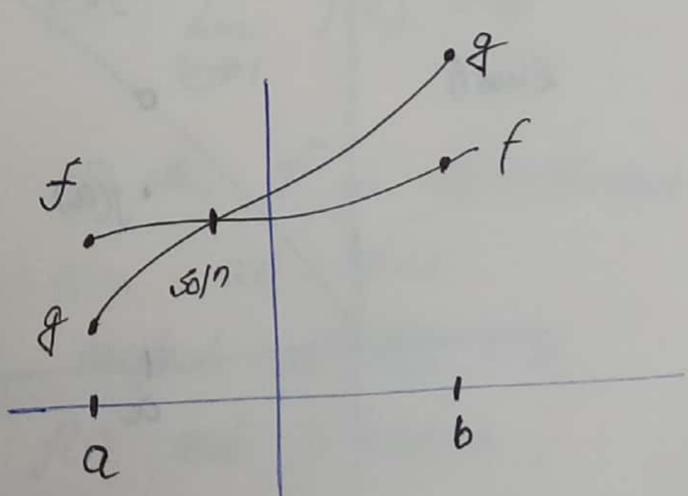
* Inside function → Dom, range
* Outer function → Dom
* Inf range ← outf dom

19th Oct '22
Math 104 - 07

\# If f is continuous on $[a, b]$ and if $f(a)$ and $f(b)$ have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in (a, b)



If f and g are continuous on $[a, b]$ and $f(a) > g(a)$, $f(b) < g(b)$, then there is at least one solution of the equation $f(x) = g(x)$ in (a, b)



Considering $F(x) = f(x) - g(x) = 0$,

$F(a) = f(a) - g(a) > 0$

$F(b) = f(b) - g(b) < 0$

$\therefore$ $F(x) = 0$ has at least one solution in the given interval

$\therefore$ $f(x) = g(x)$ has a solution in (a, b)

One-sided Continuity

i) $f(c)$ is defined

ii) $\lim_{x \to c^-} f(x)$ exists

iii) $f(c) = \lim_{x \to c} f(x)$

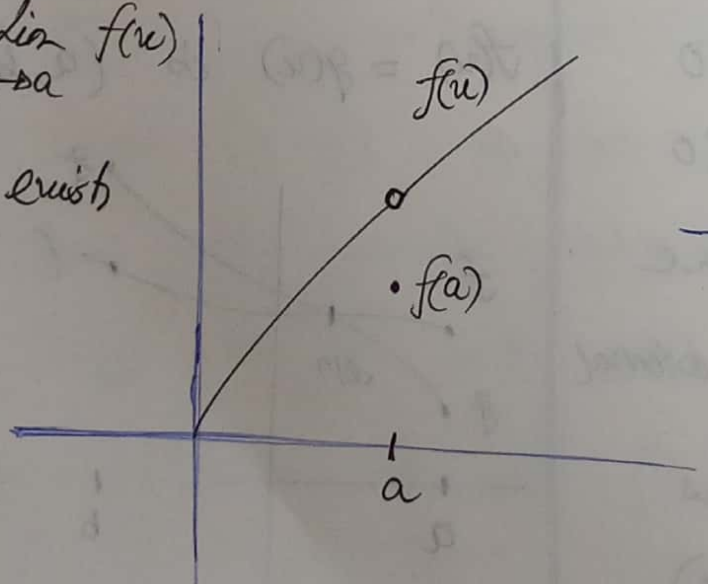
f is said to be left continuous at $x = c$

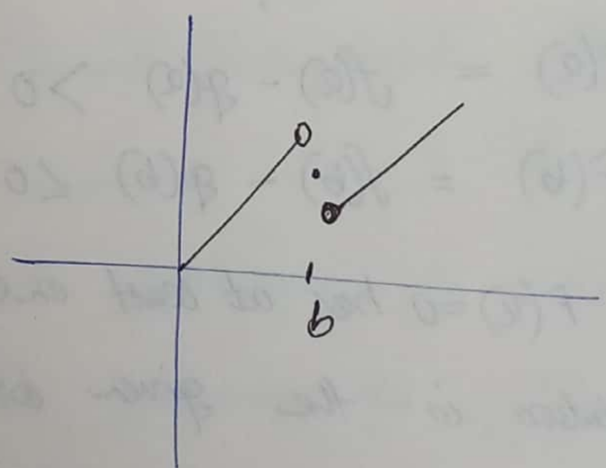
Similarly, $f(b) = \lim_{x \to b^+} f(x)$

then f is right continuous at $x = b$

Removable discontinuity –

$\lim_{x \to a} f(x)$ exist





Necessary condition —

* Existence of limit.

If we can make $f(a) = \lim_{x \to a} f(x)$ then the function becomes continuous.

Ex $$f(x) = \begin{cases} x^2-1, & x>1 \\ 2x-2, & x<1 \\ 2, & x=1 \end{cases}$$

This type of discontinuity is termed as removable discontinuity.

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2-1) = 0$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x-2) = 0$$

$\therefore$ LHL = RHL $\Rightarrow \lim_{x \to 1} f(x) = 0$

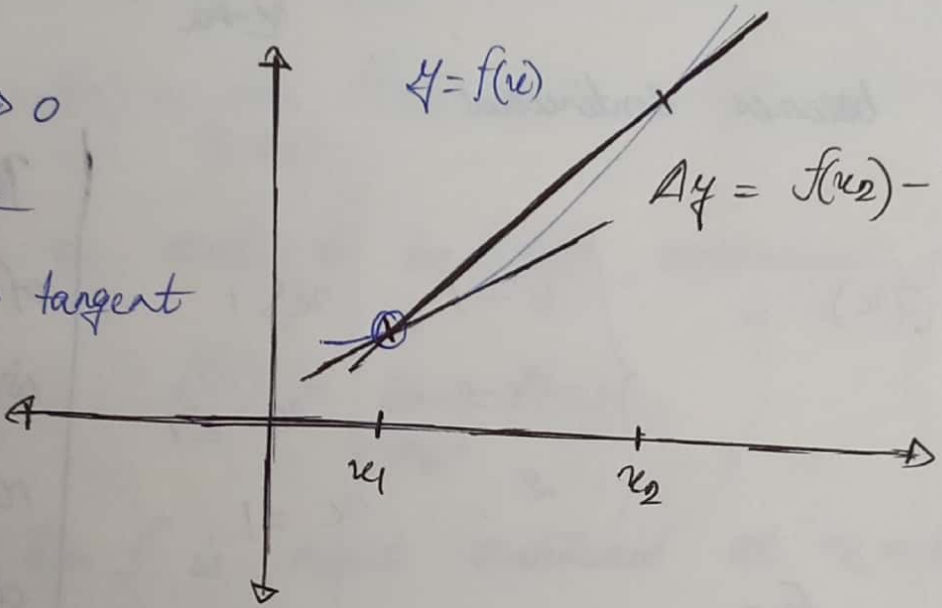
$f(1) = 2 \neq 0 = \lim_{x \to 1}$. So, f is discontinuous at $x=1$.

But if we redefine $f(1) = 0$ instead of $f(1) = 2$ then we have $\lim_{x \to 1} f(x) = 0 = f(1)$ and f becomes continuous at $x=1$.

24th Oct' 22
Math 104 - 08

Derivative —

$\Delta x \longrightarrow 0$
secant line
$\quad\longrightarrow$ tangent



$$\frac{\Delta y}{\Delta x} = \text{average rate of change}$$

Δx

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \text{instantaneous rate of change}$$

derivative $\longrightarrow$ slope to the tangent of the curve at $x = x_1$

= instantaneous rate of change of the dependent variable with respect to the independent variable.

average velocity $= \dfrac{\text{distance travelled}}{\text{time elapsed}}$

velocity = instantaneous rate of change

$$= \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$m_{sec} = \frac{f(x_2) - f(x_1) = \Delta y}{x_2 - x_1 = \Delta x} = \frac{\text{rise}}{\text{run}}$$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Let, $x_2 - x_1 = h = \Delta x$

$\Rightarrow x_2 = x_1 + h$

→ Provided limit exists

and $h \to 0 \Rightarrow x_2 \to x_1$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\Delta h}{\Delta x}$$

$$= \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = \frac{dy}{dx}$$

// $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, Provided limit exists.

Ex $y = \sqrt{x}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(\sqrt{x+h}) - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^{1/2} - x^{1/2}}{h}$$

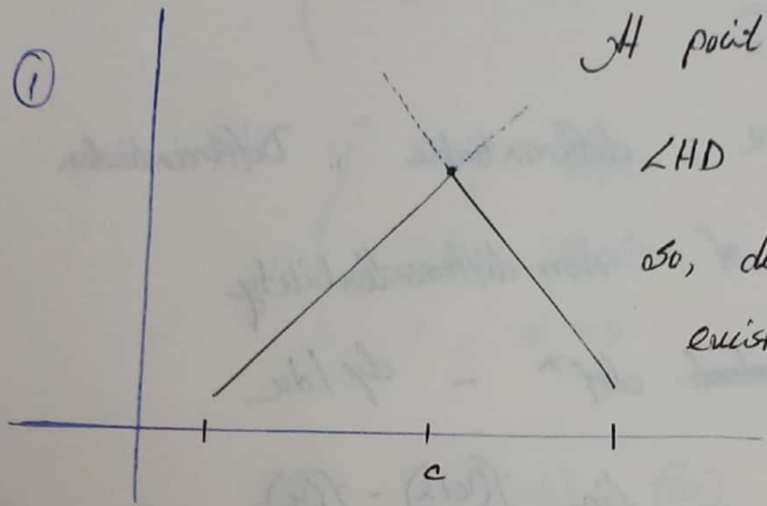
$$= \lim_{h \to 0} \frac{(\sqrt{(x+h)} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

Differentiability —

(i)



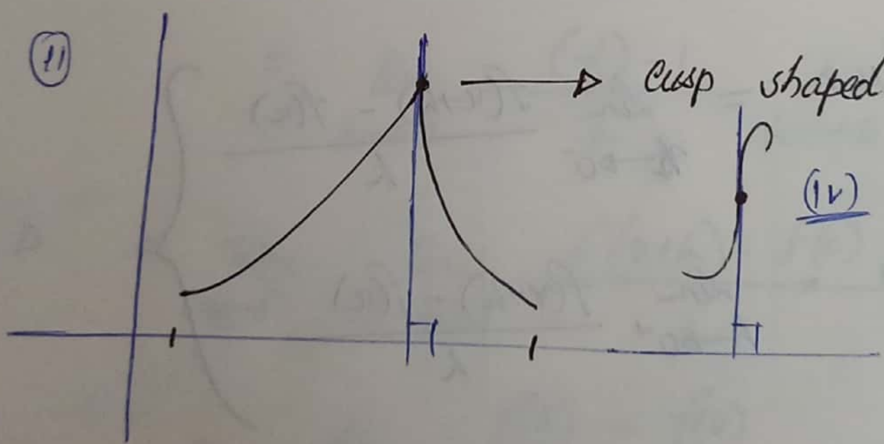
At point c,

$LHD \neq RHD$

so, derivative doesn't exist at c.

→ graphically if there is a corner point, then the function is not differentiable at the point.

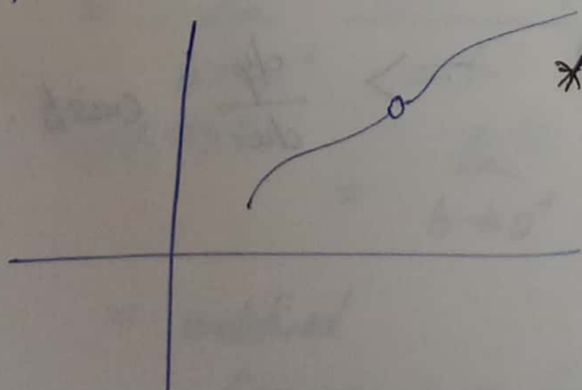
(ii)



→ cusp shaped curve

(iv) vertical tangent

(iii)



* Discontinuity

Math 104 - 09
26th Oct '22

// Derivative ; differentiable ; Differentiation

// Cases of non-differentiability

// Use ~~def~~ def^n - dy/dx

// $$\frac{dy}{dx} = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h},$$

provided limit exists.

// One-sided Derivative

$$\left.\begin{aligned} L.H.D &= \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} \\ RHD &= \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} \end{aligned}\right\}$$

$$L.H.D = R.H.D \implies \frac{dy}{dx} \text{ exists.}$$

$$f(x) = y = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

$$= \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 1, & x = 0 \end{cases}$$

L.H.D $\boxed{\text{at } x = 0}$ $= \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$

$$= \lim_{\substack{h \to 0^- \\ h < 0}} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(-1) - 1}{h} = \lim_{h \to 0^+} {}^{-2}/h = \infty$$

R.H.D $= \cancel{\lim_{x \to}} \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$

$$= \lim_{h \to 0^+} \frac{f(h) - f(0)}{h}$$

$h > 0$

$$= \lim_{h \to 0^+} \frac{1-1}{h} = \lim_{h \to 0^+} 0/h$$

$= \infty$

= undefined

(Indeterminate form)

L' Hospital Rule :

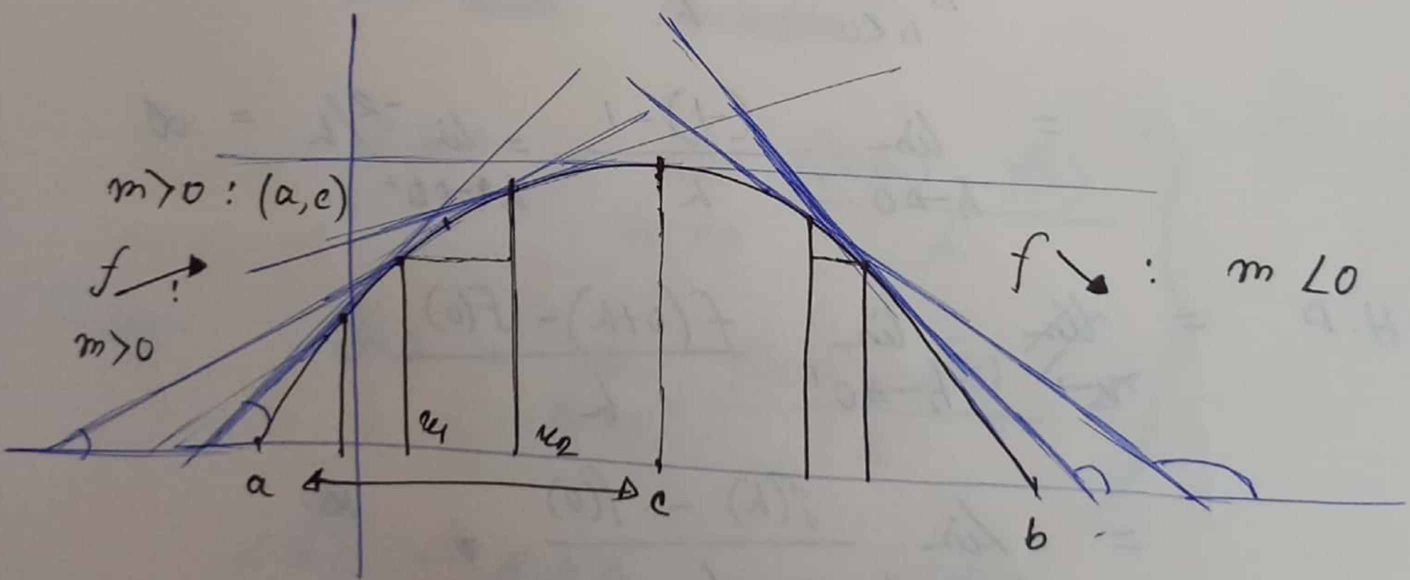
Differentiate to determine the limit.

Indeterminable form —

$0/0$ or $\frac{\infty}{\infty}$

Application of derivative —

* Increasing / Decreasing / Constant function —



increasing : $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$