APPENDIX 3

Answers to Selected Problems

Chapter 2

SECTION 2.1

1 **a** $-A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$

b $3A = \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \\ 21 & 24 & 27 \end{bmatrix}$

c $A + 2B$ is undefined.

d $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

e $B^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \end{bmatrix}$

f $AB = \begin{bmatrix} 4 & 6 \\ 10 & 15 \\ 16 & 24 \end{bmatrix}$

g BA is undefined.

2 $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.50 & 0 & 0.10 \\ 0.30 & 0.70 & 0.30 \\ 0.20 & 0.30 & 0.60 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

SECTION 2.2

1 $\begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$ or $\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 2 & 1 & 6 \\ 1 & 3 & 8 \end{array}\right]$

SECTION 2.3

1 No solution.

2 Infinite number of solutions of the form $x_1 = 2 - 2k$, $x_2 = 2 + k$, $x_3 = k$.

3 $x_1 = 2$, $x_2 = -1$.

SECTION 2.4

1 Linearly dependent.

2 Linearly independent.

SECTION 2.5

2 $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -1 & -2 & 3 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

3 A^{-1} does not exist.

8 **a** $\frac{1}{100}B^{-1}$.

SECTION 2.6

2 30.

REVIEW PROBLEMS

1 Infinite number of solutions of the form $x_1 = k - 1$, $x_2 = 3 - k$, $x_3 = k$.

3 $\begin{bmatrix} U_{t+1} \\ T_{t+1} \end{bmatrix} = \begin{bmatrix} 0.75 & 0 \\ 0.20 & 0.90 \end{bmatrix} \begin{bmatrix} U_t \\ T_t \end{bmatrix}$

4 $x_1 = 0$, $x_2 = 1$.

13 Linearly independent.

14 Linearly dependent.

15 **a** Only if a, b, c, and d are all nonzero will rank $A = 4$. Thus, A^{-1} exists if and only if all of a, b, c, and d are nonzero.

b Applying the Gauss-Jordan method, we find if a, b, c, and d are all nonzero,

$$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 & 0 \\ 0 & \frac{1}{b} & 0 & 0 \\ 0 & 0 & \frac{1}{c} & 0 \\ 0 & 0 & 0 & \frac{1}{d} \end{bmatrix}$$

18 -4.

Chapter 3

SECTION 3.1

1 $\max z = 30x_1 + 100x_2$

$$\begin{aligned} \text{s.t.}\quad x_1 + x_2 &\le 7 && \textbf{(Land constraint)}\\ 4x_1 + 10x_2 &\le 40 && \textbf{(Labor constraint)}\\ 10x_1 &\ge 30 && \textbf{(Government constraint)}\\ x_1, x_2 &\ge 0 \end{aligned}$$

2 No, the government constraint is not satisfied.

b No, the labor constraint is not satisfied.

c No, $x_2 \ge 0$ is not satisfied.

SECTION 3.2

1 $z = \$370$, $x_1 = 3$, $x_2 = 2.8$.

3 $z = \$14$, $x_1 = 3$, $x_2 = 2$.

4 a We want to make x_1 larger and x_2 smaller, so we move down and to the right.

b We want to make x_1 smaller and x_2 larger, so we move up and to the left.

b We want to make both x_1 and x_2 smaller, so we move down and to the left.

SECTION 3.3

1 No feasible solution.

2 Alternative optimal solutions.

3 Unbounded LP.

SECTION 3.4

1 For $i = 1, 2, 3$, let x_i = tons of processed factory i waste. Then the appropriate LP is

$$\min z = 15x_1 + 10x_2 + 20x_3$$

$$\begin{aligned} \text{s.t.}\quad 0.10x_1 + 0.20x_2 + 0.40x_3 &\ge 30 && \textbf{(Pollutant 1)}\\ 0.45x_1 + 0.25x_2 + 0.30x_3 &\ge 40 && \textbf{(Pollutant 2)}\\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

It is doubtful that the processing cost is proportional to the amount of waste processed. For example, processing 10 tons of waste is probably not 10 times as costly as processing 1 ton of waste. The Divisibility and Certainty Assumptions seem reasonable.

SECTION 3.5

1 Let x_1 = number of full-time employees (FTE) who start work on Sunday, x_2 = number of FTE who start work on Monday, . . . , x_7 = number of FTE who start work on Saturday; x_8 = number of part-time employees (PTE) who start work on Sunday, . . . , x_{14} = number of PTE who start work on Saturday. Then the appropriate LP is

$$\min z = 15(8)(5)(x_1 + x_2 + \cdots + x_7) + 10(4)(5)(x_8 + x_9 + \cdots + x_{14})$$

$$\begin{aligned} \text{s.t.}\quad & 8(x_1 + x_4 + x_5 + x_6 + x_7) + 4(x_8 + x_{11} + x_{12} + x_{13} + x_{14}) \ge 88 && \textbf{(Sunday)}\\ & 8(x_1 + x_2 + x_5 + x_6 + x_7) + 4(x_8 + x_9 + x_{12} + x_{13} + x_{14}) \ge 136 && \textbf{(Monday)}\\ & 8(x_1 + x_2 + x_3 + x_6 + x_7) + 4(x_8 + x_9 + x_{10} + x_{13} + x_{14}) \ge 104 && \textbf{(Tuesday)}\\ & 8(x_1 + x_2 + x_3 + x_4 + x_7) + 4(x_8 + x_9 + x_{10} + x_{11} + x_{14}) \ge 120 && \textbf{(Wednesday)}\\ & 8(x_1 + x_2 + x_3 + x_4 + x_5) + 4(x_8 + x_9 + x_{10} + x_{11} + x_{12}) \ge 152 && \textbf{(Thursday)}\\ & 8(x_2 + x_3 + x_4 + x_5 + x_6) + 4(x_9 + x_{10} + x_{11} + x_{12} + x_{13}) \ge 112 && \textbf{(Friday)}\\ & 8(x_3 + x_4 + x_5 + x_6 + x_7) + 4(x_{10} + x_{11} + x_{12} + x_{13} + x_{14}) \ge 128 && \textbf{(Saturday)} \end{aligned}$$

$$20(x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}) \le 0.25(136 + 104 + 120 + 152 + 112 + 128 + 88)$$

(The last constraint ensures that part-time labor will fulfill at most 25% of all labor requirements)

All variables ≥ 0

3 Let x_1 = number of employees who start work on Sunday and work five days, x_2 = number of employees who start work on Monday and work five days, . . . , x_7 = number of employees who start work on Saturday and work five days. Also let o_1 = number of employees who start work on Sunday and work six days, . . . , o_7 = number of employees who start work on Saturday and work six days. Then the appropriate LP is

$$\min z = 250(x_1 + x_2 + \cdots + x_7) + 312(o_1 + o_2 + \cdots + o_7)$$

$$\begin{aligned} \text{s.t.}\quad & x_1 + x_4 + x_5 + x_6 + x_7 + o_1 + o_3 + o_4 + o_5 + o_6 + o_7 \ge 11 && \textbf{(Sunday)}\\ & x_1 + x_2 + x_5 + x_6 + x_7 + o_1 + o_2 + o_4 + o_5 + o_6 + o_7 \ge 17 && \textbf{(Monday)}\\ & x_1 + x_2 + x_3 + x_6 + x_7 + o_1 + o_2 + o_3 + o_5 + o_6 + o_7 \ge 13 && \textbf{(Tuesday)}\\ & x_1 + x_2 + x_3 + x_4 + x_7 + o_1 + o_2 + o_3 + o_4 + o_6 + o_7 \ge 15 && \textbf{(Wednesday)}\\ & x_1 + x_2 + x_3 + x_4 + x_5 + o_1 + o_2 + o_3 + o_4 + o_5 + o_7 \ge 19 && \textbf{(Thursday)}\\ & x_2 + x_3 + x_4 + x_5 + x_6 + o_1 + o_2 + o_3 + o_4 + o_5 + o_6 \ge 14 && \textbf{(Friday)}\\ & x_3 + x_4 + x_5 + x_6 + x_7 + o_2 + o_3 + o_4 + o_5 + o_6 + o_7 \ge 16 && \textbf{(Saturday)} \end{aligned}$$

All variables ≥ 0

SECTION 3.6

2 NPV of investment 1

$$= -6 - \frac{5}{1.1} + \frac{7}{(1.1)^2} + \frac{9}{(1.1)^3} = \$2.00.$$

NPV of investment 2

$$= -8 - \frac{3}{1.1} + \frac{9}{(1.1)^2} + \frac{7}{(1.1)^3} = \$1.97.$$

Let x_1 = fraction of investment 1 that is undertaken and x_2 = fraction of investment 2 that is undertaken. If we measure NPV in thousands of dollars, we want to solve the following LP:

$$\begin{aligned}
\max z &= 2x_1 + 1.97x_2 \\
\text{s.t.}\quad 6x_1 + 8x_2 &\le 10 \\
5x_1 + 3x_2 &\le 7 \\
x_1 &\le 1 \\
x_2 &\le 1 \\
\text{All variables} &\ge 0
\end{aligned}$$

The optimal solution to this LP is $x_1 = 1$, $x_2 = 0.5$, $z = \$2{,}985$.

SECTION 3.7

1 $z = \$2{,}500$, $x_1 = 50$, $x_2 = 100$.

SECTION 3.8

1 Let ingredient 1 = sugar, ingredient 2 = nuts, ingredient 3 = chocolate, candy 1 = Slugger, and candy 2 = Easy Out. Let x_{ij} = ounces of ingredient i used to make candy j. (All variables are in ounces.) The appropriate LP is

$$\begin{aligned}
\max z &= 25(x_{12} + x_{22} + x_{32}) + 20(x_{11} + x_{21} + x_{31}) \\
\text{s.t.}\quad x_{11} + x_{12} &\le 100 && \textbf{(Sugar constraint)} \\
x_{21} + x_{22} &\le 20 && \textbf{(Nuts constraint)} \\
x_{31} + x_{32} &\le 30 && \textbf{(Chocolate constraint)} \\
x_{22} &\ge 0.2(x_{12} + x_{22} + x_{32}) \\
x_{21} &\le 0.1(x_{11} + x_{21} + x_{31}) \\
x_{31} &\ge 0.1(x_{11} + x_{21} + x_{31}) \\
\text{All variables} &\ge 0
\end{aligned}$$

SECTION 3.9

1 Let x_1 = hours of process 1 run per week

x_2 = hours of process 2 run per week

x_3 = hours of process 3 run per week

g_2 = barrels of gas 2 sold per week

o_1 = barrels of oil 1 purchased per week

o_2 = barrels of oil 2 purchased per week

$$\begin{aligned}
\max z &= 9(2x_1) + 10g_2 + 24(2x_3) - 5x_1 - 4x_2 \\
&\quad - x_3 - 2o_1 - 3o_2 \\
\text{s.t.}\quad o_1 &= 2x_1 + x_2 \\
o_2 &= 3x_1 + 3x_2 + 2x_3 \\
o_1 &\le 200 \\
o_2 &\le 300 \\
g_2 + 3x_3 &= x_1 + 3x_2 && \textbf{(Gas 2 production)} \\
x_1 + x_2 + x_3 &\le 100 && \textbf{(100 hours per week of cracker time)} \\
\text{All variables} &\ge 0
\end{aligned}$$

5 Let A = total number of units of A produced

B = total number of units of B produced

CS = total number of units of C produced (and sold)

AS = units of A sold

BS = units of B sold

$$\begin{aligned}
\max z &= 10AS + 56BS + 100CS \\
\text{s.t.}\quad A + 2B + 3C &\le 40 \\
A &= AS + 2B \\
B &= BS + CS \\
\text{All variables} &\ge 0
\end{aligned}$$

SECTION 3.10

1 Let x_t = production during month t and i_t = inventory at end of month t.

$$\begin{aligned}
\min z &= 5x_1 + 8x_2 + 4x_3 + 7x_4 \\
&\quad + 2i_1 + 2i_2 + 2i_3 + 2i_4 - 6i_4 \\
\text{s.t.}\quad i_1 &= x_1 - 50 \\
i_2 &= i_1 + x_2 - 65 \\
i_3 &= i_2 + x_3 - 100 \\
i_4 &= i_3 + x_4 - 70 \\
\text{All variables} &\ge 0
\end{aligned}$$

SECTION 3.11

3 Let A = dollars invested in A, B = dollars invested in B, c_0 = leftover cash at time 0, c_1 = leftover cash at time 1, and c_2 = leftover cash at time 2. Then a correct formulation is

$$\begin{aligned}
\max z &= c_2 + 1.9B \\
\text{s.t.}\quad A + c_0 &= 10{,}000 && \textbf{(Time 0 available = time 0 invested)} \\
0.2A + c_0 &= B + c_1 && \textbf{(Time 1 available = time 1 invested)} \\
1.5A + c_1 &= c_2 && \textbf{(Time 2 available = time 2 invested)} \\
\text{All variables} &\ge 0
\end{aligned}$$

The optimal solution to this LP is $B = c_0 = \$10{,}000$, $A = c_1 = c_2 = 0$, and $z = \$19{,}000$. Notice that it is optimal to wait for the "good" investment (B) even though leftover cash earns no interest.

SECTION 3.12

2 Let JAN1 = number of computers rented at beginning of January for one month, and so on. Also define IJAN = number of computers available to meet January demand, and so on. The appropriate LP is

$$\begin{aligned}
\min z = {} & 200(\text{JAN1} + \text{FEB1} + \text{MAR1} + \text{APR1} \\
& + \text{MAY1} + \text{JUN1}) + 350(\text{JAN2} + \text{FEB2} \\
& + \text{MAR2} + \text{APR2} + \text{MAY2} + \text{JUN2}) \\
& + 450(\text{JAN3} + \text{FEB3} + \text{MAR3} + \text{APR3}) \\
& + \text{MAY3} + \text{JUN3}) - 150\text{MAY3} \\
& - 300\ \text{JUN3} - 175\text{JUN2}
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\quad \text{IJAN} &= \text{JAN1} + \text{JAN2} + \text{JAN3} \\
\text{IFEB} &= \text{IJAN} - \text{JAN1} + \text{FEB1} + \text{FEB2} + \text{FEB3} \\
\text{IMAR} &= \text{IFEB} - \text{JAN2} - \text{FEB1} + \text{MAR1} \\
&\quad + \text{MAR2} + \text{MAR3} \\
\text{IAPR} &= \text{IMAR} - \text{FEB2} - \text{MAR1} - \text{JAN3} \\
&\quad + \text{APR1} + \text{APR2} + \text{APR3} \\
\text{IMAY} &= \text{IAPR} - \text{FEB3} - \text{MAR2} - \text{APR1} \\
&\quad + \text{MAY1} + \text{MAY2} + \text{MAY3} \\
\text{IJUN} &= \text{IMAY} - \text{MAR3} - \text{APR2} - \text{MAY1} \\
&\quad + \text{JUN1} + \text{JUN2} + \text{JUN3} \\
\text{IJAN} &\ge 9 \\
\text{IFEB} &\ge 5 \\
\text{IMAR} &\ge 7 \\
\text{IAPR} &\ge 9 \\
\text{IMAY} &\ge 10 \\
\text{IJUN} &\ge 5 \\
&\text{All variables} \ge 0
\end{aligned}$$

REVIEW PROBLEMS

2 Let x_1 = number of chocolate cakes baked and x_2 = number of vanilla cakes baked. Then we must solve

$$\begin{aligned}
\max z &= x_1 + \tfrac{1}{2}x_2 \\
\text{s.t.}\quad \tfrac{1}{3}x_1 + \tfrac{2}{3}x_2 &\le 8 \\
4x_1 + x_2 &\le 30 \\
x_1, x_2 &\ge 0
\end{aligned}$$

The optimal solution is $z = \frac{\$69}{7}$, $x_1 = \frac{36}{7}$, $x_2 = \frac{66}{7}$.

8 Let x_1 = acres of farm 1 devoted to corn, x_2 = acres of farm 1 devoted to wheat, x_3 = acres of farm 2 devoted to corn, x_4 acres of farm 2 devoted to wheat. Then a correct formulation is

$$\begin{aligned}
\min z = 100x_1 + 90x_2 + 120x_3 + 80x_4 & \\
\text{s.t.}\quad x_1 + x_2 \le 100 & \quad \textbf{(Farm 1 land)} \\
x_3 + x_4 \le 100 & \quad \textbf{(Farm 2 land)} \\
500x_1 + 650x_3 \ge 7{,}000 & \quad \textbf{(Corn requirement)} \\
400x_2 + 350x_4 \ge 11{,}000 & \quad \textbf{(Wheat requirement)} \\
x_1, x_2, x_3, x_4 \ge 0 &
\end{aligned}$$

9 Let x_1 = units of process 1, x_2 = units of process 2, and x_3 = modeling hours hired. Then a correct formulation is

$$\begin{aligned}
\max z = 5(3x_1 + 5x_2) - 3(x_1 + 2x_2) & \\
- 2(2x_1 + 3x_2) - 100x_3 & \\
\text{s.t.}\quad x_1 + 2x_2 \le 20{,}000 & \quad \textbf{(Limited labor)} \\
2x_1 + 3x_2 \le 35{,}000 & \quad \textbf{(Limited chemicals)} \\
3x_1 + 5x_2 = 1{,}000 + 200x_3 & \\
& \quad \textbf{(Perfume production = perfume demands)} \\
x_1, x_2, x_3 \ge 0 &
\end{aligned}$$

17 Let OT = number of tables made of oak, OC = number of chairs made of oak, PT = number of tables made of pine, and PC = number of chairs made of pine. Then the correct formulation is

$$\begin{aligned}
\max z = 40(OT + PT) + 15(OC + PC) & \\
\text{s.t.}\quad 17(OT) + 5(OC) \le 150 & \\
& \quad \textbf{(Use at most 150 board ft of oak)} \\
30PT + 13PC \le 210 & \\
& \quad \textbf{(Use at most 210 board ft of pine)} \\
OT, OC, PT, PC \ge 0 &
\end{aligned}$$

18 Let school 1 = Cooley High, and school 2 = Walt Whitman High. Let M_{ij} = number of minority students who live in district i who will attend school j, and let NM_{ij} = number of nonminority students who live in district i who will attend school j. Then the correct LP is

$$\begin{aligned}
\min z = {} & (M_{11} + NM_{11}) + 2(M_{12} + NM_{12}) \\
& + 2(M_{21} + NM_{21}) + (M_{22} + NM_{22}) \\
& + (M_{31} + NM_{31}) + (M_{32} + NM_{32}) \\
\text{s.t.}\quad & M_{11} + M_{12} = 50 \\
& M_{21} + M_{22} = 50 \\
& M_{31} + M_{32} = 100 \\
& NM_{11} + NM_{12} = 200 \\
& NM_{21} + NM_{22} = 250 \\
& NM_{31} + NM_{32} = 150
\end{aligned}$$

For school 1, we obtain the following blending constraints:

$$0.20 \le \frac{M_{11} + M_{21} + M_{31}}{M_{11} + M_{21} + M_{31} + NM_{11} + NM_{21} + NM_{31}} \le 0.30$$

This yields the following two LP constraints:

$$0.8M_{11} + 0.8M_{21} + 0.8M_{31} - 0.2NM_{11} - 0.2NM_{21} - 0.2NM_{31>} \ge 0$$

$$0.7M_{11} + 0.7M_{21} + 0.7M_{32} - 0.3NM_{11} - 0.3NM_{21} - 0.3NM_{31} \le 0$$

For school 2, we obtain the following blending constraints:

$$0.20 \le \frac{M_{12} + M_{22} + M_{32}}{M_{12} + M_{22} + M_{32} + NM_{12} + NM_{22} + NM_{32}} \le 0.30$$

This yields the following two LP constraints:

$$0.8M_{12} + 0.8M_{22} + 0.8M_{32} - 0.20NM_{12} - 0.20NM_{22} - 0.20NM_{32} \ge 0$$

$$0.7M_{12} + 0.7M_{22} + 0.7M_{32} - 0.30NM_{12} - 0.30NM_{22} - 0.30NM_{32} \le 0$$

We must also ensure that each school has between 300 and 500 students. Thus, we also need the following constraints:

$$300 \le M_{11} + NM_{11} + M_{21} + NM_{21} + M_{31} + NM_{31} \le 500$$

$$300 \le M_{12} + NM_{12} + M_{22} + NM_{22} + M_{32} + NM_{32} \le 500$$

To complete the formulation, add the sign restrictions that all variables are ≥ 0.

47 For $i < j$, let X_{ij} = number of workers who get off days i and j of week (day 1 = Sunday, day 2 = Monday, . . . , day 7 = Saturday).

$$\max z = X_{12} + X_{17} + X_{23} + X_{34} + X_{45} + X_{56} + X_{67}$$

s.t. $X_{17} + X_{27} + X_{37} + X_{47} + X_{57} + X_{67} = 2$ **(Saturday constraint)**

$X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} = 12$ **(Sunday constraint)**

$X_{12} + X_{23} + X_{24} + X_{25} + X_{26} + X_{27} = 12$ **(Monday constraint)**

$X_{13} + X_{23} + X_{34} + X_{35} + X_{36} + X_{37} = 6$ **(Tuesday constraint)**

$X_{14} + X_{24} + X_{34} + X_{45} + X_{46} + X_{47} = 5$ **(Wednesday constraint)**

$X_{15} + X_{25} + X_{35} + X_{45} + X_{56} + X_{57} = 14$ **(Thursday constraint)**

$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} + X_{67} = 9$ **(Friday constraint)**

All variables ≥ 0

49 Let X_{ij} = money invested at beginning of month i for a period of j months. After noting that for each month (money invested) + (bills paid) = (money available), we obtain the following formulation:

$$\max z = 1.08X_{14} + 1.03X_{23} + 1.01X_{32} + 1.001X_{41}$$

s.t. $X_{11} + X_{12} + X_{13} + X_{14} + 600 = 400 + 400$ **(Month 1)**

$X_{21} + X_{22} + X_{23} + 500 = 1.001X_{11} + 800$ **(Month 2)**

$X_{31} + X_{32} + 500 = 1.01X_{12} + 1.001X_{21} + 300$ **(Month 3)**

$X_{41} + 250 = 1.03X_{13} + 1.01X_{22} + 1.001X_{31} + 300$ **(Month 4)**

All variables ≥ 0

53 Let T_1 = number of type 1 turkeys purchased

T_2 = number of type 2 turkeys purchased

D_1 = pounds of dark meat used in cutlet 1

W_1 = pounds of white meat used in cutlet 1

D_2 = pounds of dark meat used in cutlet 2

W_2 = pounds of white meat used in cutlet 2

Then the appropriate formulation is

$$\max z = 4(W_1 + D_1) + 3(W_2 + D_2) - 10T_1 - 8T_2$$

s.t. $W_1 + D_1 \le 50$ **(Cutlet 1 demand)**

$W_2 + D_2 \le 30$ **(Cutlet 2 demand)**

$W_1 + W_2 \le 5T_1 + 3T_2$ **(Don't use more white meat than you have)**

$D_1 + D_2 \le 2T_1 + 3T_2$ **(Don't use more dark meat than you have)**

$W_1/(W_1 + D_1) \ge 0.7$ or $0.3W_1 \ge 0.7D_1$

$W_2/(W_2 + D_2) \ge 0.6$ or $0.4W_2 \ge 0.6D_2$

$T_1, T_2, D_1, W_1, D_2, W_2 \ge 0$

Chapter 4

SECTION 4.1

1 $\max z = 3x_1 + 2x_2$

s.t.
$$\begin{aligned} 2x_1 + x_2 + s_1 &= 100 \\ x_1 + x_2 + s_2 &= 80 \\ x_1 + s_3 &= 40 \end{aligned}$$

3 $\min z = 3x_1 + x_2$

$$\begin{aligned} \text{s.t.} \quad x_1 - e_1 &= 3 \\ x_1 + x_2 + s_2 &= 4 \\ 2x_1 - x_2 &= 3 \end{aligned}$$

SECTION 4.4

1 From Figure 2 of Chapter 3, we find the extreme points of the feasible region.

Point	Basic Variables
$H = (0, 0)$	$s_1 = 100, s_2 = 80, s_3 = 40$
$E = (40, 0)$	$x_1 = 40, s_1 = 20, s_2 = 40$
$F = (40, 20)$	$x_1 = 40, x_2 = 20, s_2 = 20$
$G = (20, 60)$	$x_1 = 20, x_2 = 60, s_3 = 20$
$D = (0, 80)$	$x_2 = 80, s_1 = 20, s_3 = 40$

SECTION 4.5

1 $z = 180, x_1 = 20, x_2 = 60.$

2 $z = \frac{32}{3}, x_1 = \frac{10}{3}, x_2 = \frac{4}{3}.$

SECTION 4.6

1 $z = -5, x_1 = 0, x_2 = 5.$

SECTION 4.7

2 Solution 1: $z = 6, x_1 = 0, x_2 = 1$; solution 2: $z = 6, x_1 = \frac{56}{17}, x_2 = \frac{45}{17}$. By averaging these two solutions, we obtain solution 3: $z = 6, x_1 = \frac{28}{17}, x_2 = \frac{31}{17}$.

SECTION 4.8

1 $x_1 = 4{,}999, x_2 = 5{,}000$ has $z = 10{,}000$.

SECTION 4.10

1 **a** Both very small numbers (for example, 0.000003) and large numbers (for example, 3,000,000) appear in the problem.

b Let x_1 = units of product i produced (in millions). If we measure our profit in millions of dollars, the LP becomes

$$\begin{aligned} \max = 6x_1 + 4x_2 + 3x_3 & \\ \text{s.t.} \quad 4x_1 + 3x_2 + 2x_3 \le 3 & \quad \textbf{(Million labor hours)} \\ 3x_1 + 2x_2 + x_3 \le 2 & \quad \textbf{(lb of pollution)} \\ x_1, x_2, x_3 \ge 0 & \end{aligned}$$

SECTION 4.11

1 $z = 16, x_1 = x_2 = 2$. The point where all three constraints are binding ($x_1 = x_2 = 2$) corresponds to the following three sets of basic variables:

$$\begin{aligned} \text{Set 1} &= \{x_1, x_2, s_1\} \\ \text{Set 2} &= \{x_1, x_2, s_2\} \\ \text{Set 3} &= \{x_1, x_2, s_3\} \end{aligned}$$

SECTIONS 4.12 AND 4.13

1 $z = 1, x_1 = x_2 = 0, x_3 = 1.$

4 Infeasible LP.

SECTION 4.14

1 Let $i_t = i'_t - i''_t$ be the inventory position at the end of month t. For each constraint in the original problem, replace i_t by $i'_t - i''_t$. Also add the sign restrictions $i'_t \ge 0$ and $i''_t \ge 0$. To ensure that demand is met by the end of quarter 4, add constraint $i''_4 = 0$. Replace the terms involving i_t in the objective function by

$$(100i'_1 + 100i''_1 + 100i'_2 + 100i''_2 + 100i'_3 + 110i''_3 + 100i'_4 + 110i''_4)$$

2 $z = 5, x_1 = 1, x_2 = 3.$

SECTION 4.16

2 Let x_i = number of lots purchased from supplier i. The appropriate LP is

$$\begin{aligned} \min z = 10s_1^- + 6s_2^- + 4s_3^- + s_4^+ & \\ \text{s.t.} \quad 60x_1 + 50x_2 + 40x_3 + s_1^- - s_1^+ = 5{,}000 & \quad \textbf{(Excellent chips)} \\ 20x_1 + 35x_2 + 20x_3 + s_2^- - s_2^+ = 3{,}000 & \quad \textbf{(Good chips)} \\ 20x_1 + 15x_2 + 40x_3 + s_3^- - s_3^+ = 1{,}000 & \quad \textbf{(Mediocre chips)} \\ 400x_1 + 300x_2 + 250x_3 + s_4^- - s_4^+ = 28{,}000 & \quad \textbf{(Budget constraint)} \\ \text{All variables} \ge 0 & \end{aligned}$$

REVIEW PROBLEMS

4 Unbounded LP.

5 $z = -6, x_1 = 0, x_2 = 3.$

6 Infeasible LP.

8 $z = 12, x_1 = x_2 = 2.$

10 Four types of furniture.

15 $z = \frac{17}{2}, x_2 = \frac{3}{2}, x_4 = \frac{1}{2}.$

17 **a** $-c \ge 0$ and $b \ge 0$.

b $b \ge 0$ and $c = 0$. Also need $a_2 > 0$ and/or $a_3 > 0$ to ensure that when x_1 is pivoted in, a feasible solution results. If only $a_3 > 0$, then we need b to be strictly positive.

c $-c < 0$, $a_2 \le 0$, $a_3 \le 0$ ensures that x_1 can be made arbitrarily large and z will become arbitrarily large.

20 Let c_t = net number of drivers hired at the beginning of the year t. Then $c_t = h_t - f_t$, where h_t = number of drivers hired at beginning of year t, and f_t = number of drivers fired at beginning of year t. Also let d_t = number of drivers after drivers have been hired or fired at beginning of year t. Then a correct formulation is (cost in thousands of dollars)

$$\begin{aligned}
\min z = {} & 10(d_1 + d_2 + d_3 + d_4 + d_5) \\
& + 2(f_1 + f_2 + f_3 + f_4 + f_5) \\
& + 4(h_1 + h_2 + h_3 + h_4 + h_5) \\
\text{s.t.} \quad & d_1 = 50 + h_1 - f_1 \\
& d_2 = d_1 + h_2 - f_2 \\
& d_3 = d_2 + h_3 - f_3 \\
& d_4 = d_3 + h_4 - f_4 \\
& d_5 = d_4 + h_5 - f_5 \\
& d_1 \ge 60,\ d_2 \ge 70,\ d_3 \ge 50,\ d_4 \ge 65, \\
& d_5 \ge 75 \\
& \text{All variables} \ge 0
\end{aligned}$$

26 Let

R_t = robots available during quarter t (after robots are bought or sold for the quarter)

B_t = robots bought during quarter t

S_t = robots sold during quarter t

I'_t = cars in inventory at end of quarter t

C_t = cars produced during quarter t

I''_t = backlogged demand for cars at end of quarter t

Then a correct formulation is

$$\begin{aligned}
\min z = {} & 500(R_1 + R_2 + R_3 + R_4) \\
& + 200(I'_1 + I'_2 + I'_3 + I'_4) \\
& + 5{,}000(B_1 + B_2 + B_3 + B_4) \\
& - 3{,}000(S_1 + S_2 + S_3 + S_4) \\
& + 300(I''_1 + I''_2 + I''_3 + I''_4) \\
\text{s.t.} \quad & R_1 = 2 + B_1 - S_1 \\
& R_2 = R_1 + B_2 - S_2 \\
& R_3 = R_2 + B_3 - S_3 \\
& R_4 = R_3 + B_4 - S_4 \\
& I'_1 - I''_1 = C_1 - 600 \\
& I'_2 - I''_2 = I'_1 - I''_1 + C_2 - 800 \\
& I'_3 - I''_3 = I'_2 - I''_2 + C_3 - 500 \\
& I'_4 - I''_4 = I'_3 - I''_3 + C_4 - 400 \\
& R_4 \ge 2 \\
& C_1 \le 200R_1 \\
& C_2 \le 200R_2 \\
& C_3 \le 200R_3 \\
& C_4 \le 200R_4,\ I''_4 = 0 \\
& B_1, B_2, B_3, B_4 \le 2 \\
& \text{All variables} \ge 0
\end{aligned}$$

Chapter 5

SECTION 5.1

1 Decision variables remain the same. New z-value is \$210.

4 **a** $\frac{50}{3} \le c_1 \le 350$.

c $4{,}000{,}000 \le \text{HIW} \le 84{,}000{,}000$; $x_1 = 3.6 + 0.15\Delta$, $x_2 = 1.4 - 0.025\Delta$.

f \$310,000.

SECTION 5.2

1 **a** \$3,875.

b Decision variables remain the same. New z-value is \$3,750.

c Solution remains the same.

3 **a** Still 90¢.

b 95¢.

c 95¢.

d Still 90¢.

e 82.5¢.

f 30¢ or less.

g 22.5¢ or less.

SECTION 5.3

3 2.5¢.

4 \$2.

5 Buy raw material, because it will reduce cost by \$6.67.

SECTION 5.4

3 See Figures 1–4.

FIGURE 1

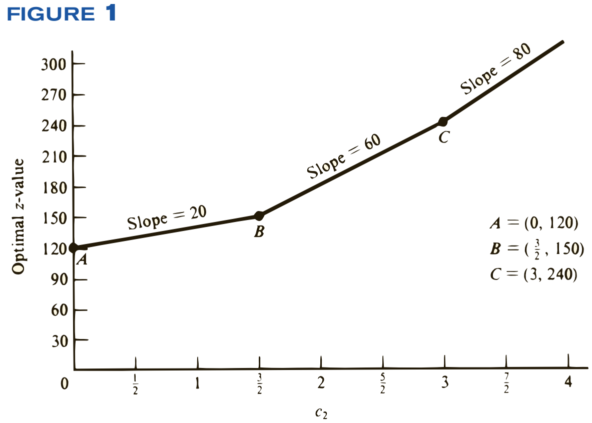


FIGURE 2

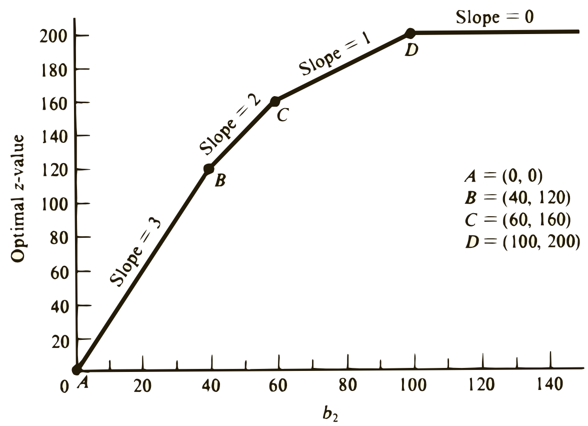


FIGURE 3

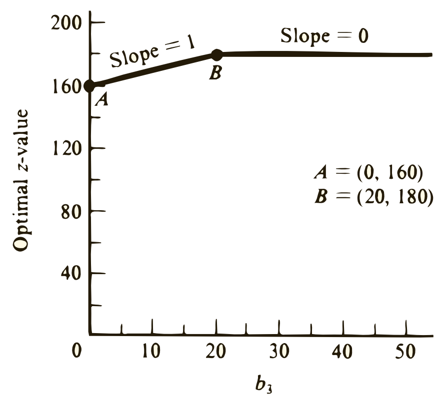
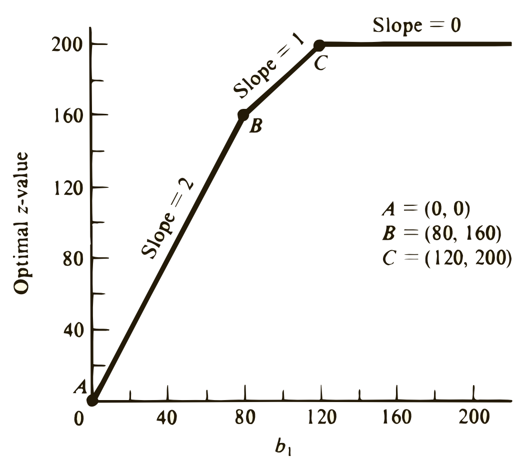


FIGURE 4



REVIEW PROBLEMS

1 **a** $1,046,667.
b Yes.
c $33.33.
b $333.33.

7 **a** Decision variables remain unchanged. New z-value is $1,815,000.
b Pay $0 for an additional 100 board ft of lumber. Pay $1,350 for an additional 100 hours of labor.
c $1,310,000.
d $1,665,000.

Chapter 6

SECTION 6.1

1 Decision variables remain the same. New z-value is $210.

SECTION 6.2

1
$$\begin{aligned} z \quad &+ 4s_1 + 5s_2 = 28 \\ x_1 \quad &+ s_1 + s_2 = 6 \\ x_2 &+ s_1 + 2s_2 = 10 \end{aligned}$$

SECTION 6.3

3 $x_1 = 2$, $x_2 = 0$, $x_3 = 8$, $z = 280$ (same as original solution).

5 Home computer tables should not be produced.

6 **a** Profit for candy bar 1 $\leq$ 6¢. If type 1 candy bar earns 7¢ profit, new optimal solution is $z = \$3.50$, $x_1 = 50$, $x_2 = x_3 = 0$.
b 5¢ $\leq$ candy bar 2 profit $\leq$ 15¢. If candy bar 2 profit is 13¢, decision variables remain the same, but profit is now $4.50.
c $\frac{100}{3} \leq$ sugar $\leq$ 100.
d $z = \$3.40$, $x_1 = 0$, $x_2 = 20$, $x_3 = 40$. If 30 oz of sugar is available, current basis is no longer optimal, and problem must be solved again.
e Make type 1 candy bars.
f Make type 4 candy bars.

8 **a** $16,667.67 $\leq$ comedy cost $\leq$ $350,000.
b 4 million $\leq$ HIW $\leq$ 84 million. For 40 million HIW exposures, new optimal solution is $x_1 = 5.4$, $x_2 = 1.1$, $z = \$350{,}000$.
c Advertise on news program.

SECTION 6.4

1 Yes.

2 No.

4 Yes.

SECTION 6.5

1 $\min w = y_1 + 3y_2 + 4y_3$

$$\begin{aligned} \text{s.t.}\quad -y_1 + y_2 + y_3 &\geq 2 \\ y_1 + y_2 - 2y_3 &\geq 1 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}$$

2 $\max z = 4x_1 + x_2 + 3x_3$

$$\begin{aligned} \text{s.t.}\quad 2x_1 + x_2 + x_3 &\leq 1 \\ x_1 + x_2 + 2x_3 &\leq -1 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

3 $\min w = 5y_1 + 7y_2 + 6y_3 + 4y_4$

$$\begin{aligned} \text{s.t.}\quad y_1 + 2y_2 + y_4 &\geq 4 \\ y_1 + y_2 + 2y_3 &= -1 \\ y_3 + y_4 &= 2 \\ y_1, y_2 \geq 0;\ y_3 \leq 0;\ y_4 &\text{ urs} \end{aligned}$$

4 $\max z = 6x_1 + 8x_2$

$$\begin{aligned} \text{s.t.}\quad x_1 + x_2 &\leq 4 \\ 2x_1 - x_2 &\leq 2 \\ 2x_2 &= -1 \\ x_1 \leq 0;\ x_2 &\text{ urs} \end{aligned}$$

SECTION 6.7

1 **a** $\min w = 100y_1 + 80y_2 + 40y_3$

$$\begin{aligned} \text{s.t.}\quad 2y_1 + y_2 + y_3 &\geq 3 \\ y_1 + y_2 &\geq 2 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}$$

b and c $y_1 = 1$, $y_2 = 1$, $y_3 = 0$, $w = 180$. Observe that this solution has a w-value that equals the optimal primal z-value. Since this solution is dual feasible, it must be optimal (by Lemma 2) for the dual.

2 **a** $\min w = 3y_1 + 2y_2 + y_3$

$$\begin{aligned} \text{s.t.}\quad y_1 + y_3 &\geq -2 \\ y_1 + y_2 &\geq -1 \\ y_1 + y_2 + y_3 &\geq 1 \end{aligned}$$

b $y_1 =$ coefficient of s_1 in optimal row $0 = 0$

$y_2 = -$(coefficient of e_2 in optimal row 0)

$= -1$

$y_3 =$ coefficient of a_3 in optimal row $0 - M$

$= 2$

Optimal w-value $= 0$.

9 Dual is $\max w = 28y_1 + 24y_2$

$$\begin{aligned} \text{s.t.}\quad 7y_1 + 2y_2 &\leq 50 \\ 2y_1 + 12y_2 &\leq 100 \\ y_1, y_2 &\geq 0 \end{aligned}$$

Optimal dual solution is $w = \$320{,}000$, $y_1 = 5$, $y_2 = 7.5$.

SECTION 6.8

2 **b** New z-value $= \$3.40$.

c New z-value $= \$2.60$.

d Since current basis is no longer optimal, the current shadow prices cannot be used to determine the new z-value.

5 **b** Skilled labor shadow price $= 0$, unskilled labor shadow price $= 0$, raw material shadow price $= 15$, and product 2 constraint shadow price $= -5$.

We would be willing to pay \$0 for an additional hour of either type of labor. We would pay up to \$15 for an extra unit of raw material. Reducing the product 2 marketing requirement by 1 unit will save the company \$5.

c $\Delta b_3 = 5$, so new z-value $= 435 + 5(15) = \$510$.

d Since shadow price of each labor constraint is zero, the optimal z-value remains unchanged.

e For a 5-unit requirement, $\Delta b_4 = 2$. Thus, new z-value $= 435 + 2(-5) = \$425$. For a 2-unit requirement, $\Delta b_4 = -1$. Thus, new z-value $= 435 + (-1)(-5) = \$440$.

6 **a** If purchased at the given price of \$1, an extra unit of raw material increases profits by \$2.50. Thus, the firm would be willing to pay up to $1 + 2.5 = \$3.50$ for an extra unit of raw material.

b Both labor constraints are nonbinding. All we can say is that if an additional hour of skilled labor were available at \$3/hour, we would not buy it, and if an additional hour of unskilled labor were available at \$2/hour, we would not buy it.

7 **a** New z-value $= \$380{,}000$.

b New z-value $= \$290{,}000$.

SECTION 6.9

1 The current basis is no longer optimal. We should make computer tables, because they sell for \$35 each and use only \$30 worth of resources.

2 **a** Current basis remains optimal if type 1 profit $\leq 6¢$.

SECTION 6.10

1 **a** $\min w = 600y_1 + 400y_2 + 500y_3$

$$\begin{aligned} \text{s.t.}\quad 4y_1 + y_2 + 3y_3 &\geq 6 \\ 9y_1 + y_2 + 4y_3 &\geq 10 \\ 7y_1 + 3y_2 + 2y_3 &\geq 9 \\ 10y_1 + 40y_2 + y_3 &\geq 20 \\ y_1, y_2, y_3, y_4 &\geq 0 \end{aligned}$$

b $w = \frac{2{,}800}{3}$, $y_1 = \frac{22}{5}$, $y_2 = \frac{2}{15}$, $y_3 = 0$.

SECTION 6.11

1 $z = -9$, $x_1 = 0$, $x_2 = 14$, $x_3 = 9$.

2 **a** The current solution is still optimal.

b The LP is now infeasible.

c The new optimal solution is $z = 10$, $x_1 = 1$, $x_2 = 4$.

SECTION 6.12

4 Only HPER is inefficient.

REVIEW PROBLEMS

1 **a**
$$\begin{aligned} \min w &= 6y_1 + 3y_2 + 10y_3 \\ \text{s.t.}\quad y_1 + y_2 + 2y_3 &\ge 4 \\ 2y_1 - y_2 + y_3 &\ge 1 \\ y_1 \text{ urs};\ y_2 \le 0;\ y_3 &\ge 0 \end{aligned}$$

Optimal dual solution is $w = \frac{58}{3}$, $y_1 = -\frac{2}{3}$, $y_2 = 0$, $y_3 = \frac{7}{3}$.

b $9 \le b_3 \le 12$. If $b_3 = 11$, the new optimal solution is $z = \frac{65}{3}$, $x_1 = \frac{16}{3}$, $x_2 = \frac{1}{3}$.

2 $c_1 \ge \frac{1}{2}$.

3 **a**
$$\begin{aligned} \min w &= 6y_1 + 8y_2 + 2y_3 \\ \text{s.t.}\quad y_1 + 6y_2 &\ge 5 \\ y_1 + y_3 &\ge 1 \\ y_1 + y_2 + y_3 &\ge 2 \\ y_1, y_2, y_3 &\ge 0 \end{aligned}$$

Optimal dual solution is $w = 9$, $y_1 = 0$, $y_2 = \frac{5}{6}$, $y_3 = \frac{7}{6}$.

b $0 \le c_1 \le 6$.

c $c_2 \le \frac{7}{6}$.

4 **a** New z-value $= 32{,}540 + 10(88) = \$33{,}420$. Decision variables remain the same.

b Can't tell, since allowable increase is < 1.

c \$0.

d $32{,}540 + (-2)(-20) = \$32{,}580$.

e Produce jeeps.

8 **a** New z-value $= \$266.20$.

b New z-value $= \$270.70$. Decision variables remain the same.

c \$12.60.

d 20¢.

e Produce product 3.

17 $z = -16$, $x_1 = 8$, $x_2 = 0$.

20 Optimal primal solution: $z = 13$, $x_1 = 1$, $x_2 = x_3 = 0$, $x_4 = 2$. Optimal dual solution: $w = 13$, $y_1 = 1$, $y_2 = 1$.

21 **a** $c_1 \ge 3$.

b $c_2 \le \frac{4}{3}$.

c $0 \le b_1 \le 9$.

d $b_2 \ge 10$.

28 LP 2 optimal solution: $z = 550$, $x_1 = 0.5$, $x_2 = 5$. Optimal solution to dual of LP 2: $w = 550$, $y_1 = y_2 = \frac{100}{3}$.

36 $b_2 \ge 3$.

Chapter 7

SECTION 7.1

1

	CUSTOMER 1	CUSTOMER 2	CUSTOMER 3	SUPPLY
Warehouse 1	15	35	25	40
Warehouse 2	10	50	40	30
Shortage	90	80	110	20
DEMAND	30	30	30	

3

1-RT	7	8	9	10	11	12	0	200
1-OT	11	12	13	14	15	16	0	100
2-RT	*M*	7	8	9	10	11	0	200
2-OT	*M*	11	12	13	14	15	0	100
3-RT	*M*	*M*	7	8	9	10	0	200
3-OT	*M*	*M*	11	12	13	14	0	100
4-RT	*M*	*M*	*M*	7	8	9	0	200
4-OT	*M*	*M*	*M*	11	12	13	0	100
5-RT	*M*	*M*	*M*	*M*	7	8	0	200
5-OT	*M*	*M*	*M*	*M*	11	12	0	100
6-RT	*M*	*M*	*M*	*M*	*M*	7	0	200
6-OT	*M*	*M*	*M*	*M*	*M*	11	0	100
	200	260	240	340	190	150	420	

5

	MONTH 1	MONTH 2	DUMMY	SUPPLY
Daisy	800	720	0	5
Laroach	710	750	0	5
DEMAND	3	4	3	

7 This is a maximization problem, so number in each cell is a revenue, not a cost.

	CLIFF	BLAKE	ALEXIS	SUPPLY
Site 1	1000	900	1100	100,000
Site 2	2000	2200	1900	100,000
Dummy	0	0	0	40,000
DEMAND	80,000	80,000	80,000	

12 **a** Replace the M's by incorporating a backlogging cost. For example, month 3 regular production can be used to meet month 1 demand at a cost of 400 + 2(30) = \$460.

b Add a supply point called "lost sales," with cost of shipping a unit to any month's demand being \$450. Supply of "lost sales" supply point should equal total demand. Then adjust dummy demand point's demand to rebalance the problem.

c A shipment from month 1 production to month 4 demand should have a cost of M.

d For each month, add a month i subcontracting supply point, with a supply of 10 and a cost that is \$40 more than the cost for the corresponding month i regular supply point. Then adjust the demand at the dummy demand point so that the problem is balanced.

SECTIONS 7.2 AND 7.3

The optimal solution to Problem 1 of Section 7.1 is to ship 10 units from warehouse 1 to customer 2, 30 units from warehouse 1 to customer 3, and 30 units from warehouse 2 to customer 1.

The optimal solution to Problem 5 of Section 7.1 is to buy 4 gallons from Daisy in month 2 and 3 gallons from Laroach in month 1.

In Problem 7 of Section 7.1, Cliff gets 20,000 acres at site 1 and 20,000 acres at site 2. Blake gets 80,000 acres at site 2. Alexis gets 80,000 acres at site 1.

SECTION 7.4

2 Current basis remains optimal if $c_{34} \leq 7$.

4 New optimal solution is $x_{12} = 12$, $x_{13} = 23$, $x_{21} = 45$, $x_{23} = 5$, $x_{32} = 8$, $x_{34} = 30$, and $z = 1{,}020 - 2(3) - 2(10) = 994$.

SECTION 7.5

1 Person 1 does job 2, person 2 does job 1, person 3 does no job, person 4 does job 4, and person 5 does job 3.

8 **a** Company 1 does route 1, company 2 does route 2, company 3 does route 3, and company 4 does route 4.

b Company 3 does routes 3 and 1, company 2 does routes 2 and 4.

SECTION 7.6

1 **a**

	L.A.	DETROIT	ATLANTA	HOUSTON	TAMPA	DUMMY	
L.A.	0	140	100	90	225	0	5,100
Detroit	145	0	111	110	119	0	6,900
Atlanta	105	115	0	113	78	0	4,000
Houston	89	109	121	0	M	0	4,000
Tampa	210	117	82	M	0	0	4,000
	4,000	4,000	4,000	6,400	5,500	100	

b

	L.A.	DETROIT	ATLANTA	HOUSTON	TAMPA	DUMMY	
L.A.	0	M	100	90	225	0	5,100
Detroit	M	0	111	110	119	0	6,900
Atlanta	105	115	0	113	78	0	4,000
Houston	89	109	121	0	M	0	4,000
Tampa	210	117	82	M	0	0	4,000
	4,000	4,000	4,000	6,400	5,500	100	

c

	L.A.	DETROIT	ATLANTA	HOUSTON	TAMPA	DUMMY	
L.A.	0	140	100	90	225	0	5,100
Detroit	145	0	111	110	119	0	6,900
Atlanta	105	115	0	113	78	0	4,000
Houston	89	109	121	0	5	0	4,000
Tampa	210	117	82	5	0	0	4,000
	4,000	4,000	4,000	6,400	5,500	100	

REVIEW PROBLEMS

3 Meet January demand with 30 units of January production. Meet February demand with 5 units of January production, 10 units of February production, and 15 units of March production. Meet March demand with 20 units of March production.

4 Maid 1 does the bathroom, maid 2 straightens up, maid 3 does the kitchen, maid 4 gets the day off, and maid 5 vacuums.

7 Shipping 1 unit from W_i to W_j means one white student from district i goes to school in district j. Shipping 1 unit from B_i to B_j means one black student from district i goes to school in district j. The costs of M ensure that shipments from W_i to B_j or B_i to W_j cannot occur (table on next page).

8 Optimal solution is $z = 1{,}580$, $x_{11} = 40$, $x_{12} = 10$, $x_{13} = 10$, $x_{22} = 50$, $x_{32} = 10$, $x_{34} = 30$.

13 Optimal solution is $z = 98$, $x_{13} = 5$, $x_{21} = 3$, $x_{24} = 7$, $x_{32} = 3$, $x_{33} = 7$, $x_{34} = 5$.

25 Sell painting 1 to customer 1, painting 2 to customer 2, painting 3 to customer 3, and painting 4 to customer 4.

	W_1	B_1	W_2	B_2	W_3	B_3	SUPPLY
W_1	0	M	3	M	5	M	210
B_1	M	0	M	3	M	5	120
W_2	3	M	0	M	4	M	210
B_2	M	3	M	0	M	4	30
W_3	5	M	4	M	0	M	180
B_3	M	5	M	4	M	0	150
DEMAND	200	100	200	100	200	100	

Chapter 8

SECTION 8.2

2 1–2–5 (length 14).

3

	NODE 2	NODE 3	NODE 4	NODE 5	SUPPLY
Node 1	2	8	M	M	1
Node 2	0	5	4	12	1
Node 3	M	0	6	M	1
Node 4	M	M	0	10	1
DEMAND	1	1	1	1	

M = large number to prevent shipping a unit through a nonexistent arc.

5 Replace the car at times 2, 4, and 6. Total cost = \$14,400.

SECTION 8.3

1 max $z = x_0$

s.t. $x_{so,1} \le 6$, $x_{so,2} \le 2$, $x_{12} \le 1$, $x_{32} \le 3$,

$x_{13} \le 3$, $x_{3,si} \le 2$, $x_{24} \le 7$, $x_{4,si} \le 7$

$x_0 = x_{so,1} + x_{so,2}$ **(Node so)**

$x_{so,1} = x_{13} + x_{12}$ **(Node 1)**

$x_{12} + x_{32} + x_{so,2} = x_{24}$ **(Node 2)**

$x_{13} = x_{32} + x_{3,si}$ **(Node 3)**

$x_{24} = x_{4,si}$ **(Node 4)**

$x_{3,si} + x_{4,si} = x_0$ **(Node si)**

All variables ≥ 0

Maximum flow $= 6$. Cut associated with $V' = \{2, 3, 4, si\}$ has capacity 6.

2 max $z = x_0$

s.t. $x_{so,1} \le 2,\ x_{12} \le 4,\ x_{1,si} \le 3,\ x_{2,si} \le 2,$

$x_{23} \le 1,\ x_{3,si} \le 2,\ x_{so,3} \le 1$

$x_0 = x_{so,1} + x_{so,3}$ **(Node *so*)**

$x_{so,1} = x_{1,si} + x_{12}$ **(Node 1)**

$x_{12} = x_{23} + x_{2,si}$ **(Node 2)**

$x_{23} + x_{so,3} = x_{3,si}$ **(Node 3)**

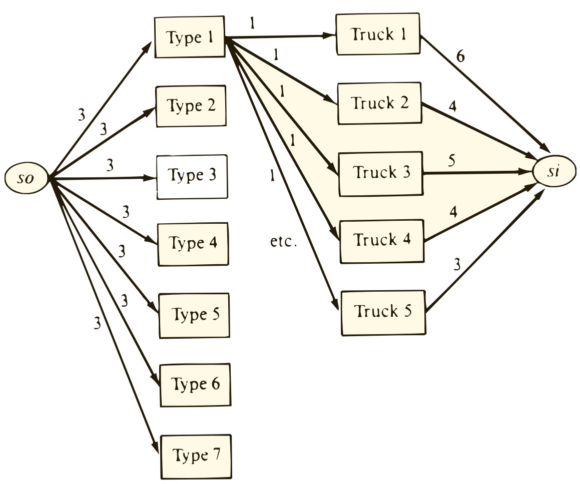
$x_{1,si} + x_{2,si} + x_{3,si} = x_0$ **(Node *si*)**

All variables ≥ 0

Maximum flow $= 3$. Cut associated with $V' = \{1, 2, 3, si\}$ has capacity 3.

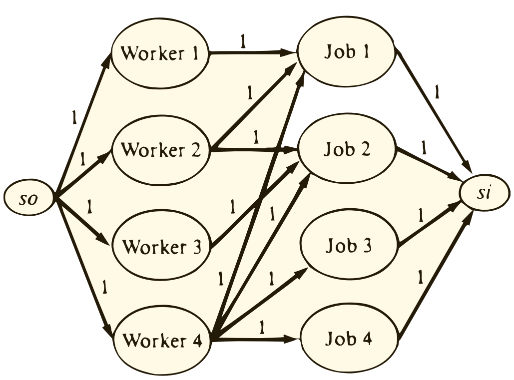
6 See Figure 5. An arc of capacity 1 goes from each package type node to each truck node. If maximum flow $= 21$, all packages can be delivered.

FIGURE 5



7 See Figure 6. If maximum flow $= 4$, then all jobs can be completed.

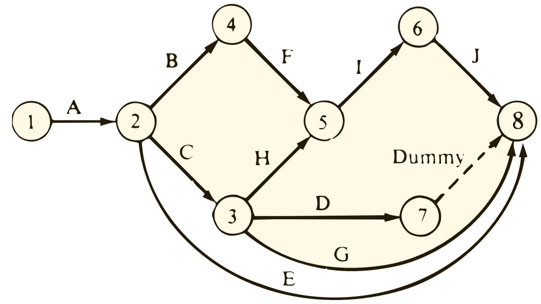
FIGURE 6



SECTION 8.4

4 a See Figure 7.

FIGURE 7



b Critical path is A–C–G (project duration is 14 days).

c Start project by June 13.

d min $z = x_8 - x_1$

s.t. $x_2 \ge x_1 + 3$

$x_3 \ge x_2 + 6$

$x_4 \ge x_2 + 2$

$x_5 \ge x_4 + 3$

$x_5 \ge x_3 + 1$

$x_6 \ge x_5 + 1.5$

$x_8 \ge x_6 + 2$

$x_8 \ge x_7$ $(x_7 \ge x_3 + 2)$

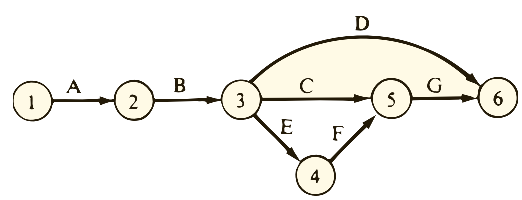
$x_8 \ge x_2 + 3$

$x_8 \ge x_3 + 5$

All variables urs

5 a See Figure 8. A–B–E–F–G and A–B–C–G are critical paths. Duration of project is 26 days.

FIGURE 8



Activity	Total Float	Free Float
A	0	0
B	0	0
C	0	0
D	8	8
E	0	0
F	0	0
G	0	0

b $\min z = 30A + 15B + 20C + 40D$
$+ 20E + 30F + 40G$

s.t. $x_2 \geq x_1 + 5 - A$

$x_3 \geq x_2 + 8 - B$

$x_4 \geq x_3 + 4 - E$

$x_5 \geq x_3 + 10 - C$

$x_5 \geq x_4 + 6 - F$

$x_6 \geq x_3 + 5 - D$

$x_6 \geq x_5 + 3 - G$

$x_6 - x_1 \leq 20$

$A \leq 2, B \leq 3, C \leq 1, D \leq 2, E \leq 2,$

$F \leq 3, G \leq 1$

$A, B, C, D, E, F, G \geq 0$

All other variables urs

8 b From the LP output, we find the critical path 1–2–3–4–5–6. This implies that activities A, B, E, F, and G are critical. (Since 1–2–3–5–6 is also a critical path, activity C is also a critical activity, but the LP does not give us this information.)

SECTION 8.5

1 $\min z = 4x_{12} + 3x_{24} + 2x_{46} + 3x_{13}$
$+ 3x_{35} + 2x_{25} + 2x_{56}$

s.t. $x_{12} + x_{13} = 1$ **(Node 1)**

$x_{12} = x_{24} + x_{25}$ **(Node 2)**

$x_{13} = x_{35}$ **(Node 3)**

$x_{24} = x_{46}$ **(Node 4)**

$x_{25} = x_{56}$ **(Node 5)**

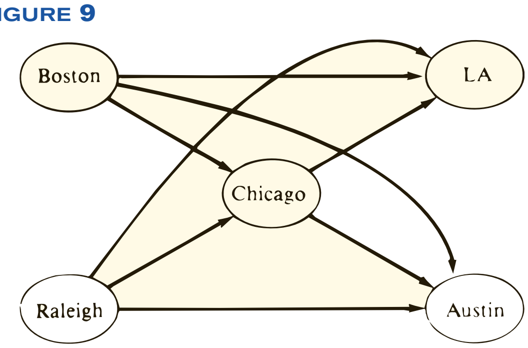
$x_{46} + x_{56} = 1$ **(Node 6)**

$x_{ij} \geq 0$

If $x_{ij} = 1$, the shortest path from node 1 to node 6 contains arc (i, j); if $x_{ij} = 0$, the shortest path from node 1 to node 6 does not contain arc (i, j).

4 a See Figure 9. All arcs have infinite capacity.

FIGURE 9



Arc	Shipping Cost
Bos.–Chic.	800 + 80 = $ 880
Bos.–Aus.	800 + 220 = $1,020
Bos.–L.A.	800 + 280 = $1,080
Ral.–Chic.	900 + 100 = $1,000
Ral.–Aus.	900 + 140 = $1,040
Ral.–L.A.	900 + 170 = $1,070
Chic.–Aus.	$40
Chic.–L.A.	$50

Problem is balanced, so no dummy point is needed.

City	Net Outflow
Boston	400
Raleigh	300
Chicago	0
L.A.	−400
Austin	−300

5

Arc	Unit Cost
S.D.–Dal.	$420
S.D.–Hous.	$100
L.A.–Dal.	$300
L.A.–Hous.	$110
S.D.–Dummy	$0
L.A.–Dummy	$0
Dal.–Chic.	700 + 550 = $1,250
Dal.–N.Y.	700 + 450 = $1,150
Hous.–Chic.	900 + 530 = $1,430
Hous.–N.Y.	900 + 470 = $1,370

City	Net Outflow (100,000 barrels/day)
San Diego	5
L.A.	4
Dallas	0
Houston	0
Chicago	−4
N.Y.	−3
Dummy	−2

SECTION 8.6

2 The MST consists of the arcs (1, 3), (3, 5), (3, 4), and (3, 2). Total length of MST is 15.

SECTION 8.7

1 c $z = 8$, $x_{12} = x_{25} = x_{56} = 1$, $x_{13} = x_{24} = x_{35} = x_{46} = 0$.

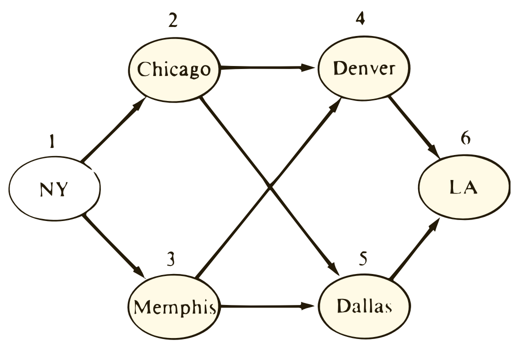
3 $z = 590$, $x_{12} = 20$, $x_{24} = 20$, $x_{34} = 2$, $x_{35} = 2$, $x_{13} = 12$, $x_{23} = 0$, $x_{25} = 0$, $x_{45} = 0$.

REVIEW PROBLEMS

1 **a** N.Y.–St. Louis–Phoenix–L.A. uses 2,450 gallons of fuel.

2 **a** See Figure 10.

FIGURE 10



$$\max z = x_0$$

$$\begin{aligned}
\text{s.t.}\quad & x_{12} + x_{13} = x_0 && \textbf{(Node 1)}\\
& x_{12} = x_{24} + x_{25} && \textbf{(Node 2)}\\
& x_{13} = x_{34} = x_{35} && \textbf{(Node 3)}\\
& x_{24} + x_{34} = x_{46} && \textbf{(Node 4)}\\
& x_{25} + x_{35} = x_{56} && \textbf{(Node 5)}\\
& x_{46} + x_{56} = x_0 && \textbf{(Node 6)}\\
& x_{12} \le 500,\ x_{13} \le 400,\ x_{24} \le 300, &&\\
& x_{25} \le 250,\ x_{34} \le 200,\ x_{35} \le 150, &&\\
& x_{46} \le 400,\ x_{56} \le 350 &&
\end{aligned}$$

$$\text{All variables} \ge 0$$

5 The MST consists of the following arcs: N.Y.–Clev., N.Y.–Nash., Nash.–Dal., Dal.–St.L., Dal.–Pho., Pho.–L.A., and S.L.C.–L.A. Total length of the MST is 4,300.

Chapter 9

SECTION 9.2

1 Let $x_1 = \begin{cases} 1 & \text{if player } i \text{ starts} \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP is

$$\max z = 3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + x_7$$

$$\begin{aligned}
\text{s.t.}\quad & x_1 + x_3 + x_5 + x_7 \ge 4 && \textbf{(Guards)}\\
& x_3 + x_4 + x_5 + x_6 + x_7 \ge 2 && \textbf{(Forwards)}\\
& x_2 + x_4 + x_6 \ge 1 && \textbf{(Center)}\\
& x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5 &&\\
& 3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + 3x_7 \ge 10 && \textbf{(Ballhandling)}\\
& 3x_1 + x_2 + 3x_3 + 3x_4 + 3x_5 + x_6 + 2x_7 \ge 10 && \textbf{(Shooting)}\\
& x_1 + 3x_2 + 2x_3 + 3x_4 + 3x_5 + 2x_6 + 2x_7 \ge 10 && \textbf{(Rebounding)}\\
& x_6 + x_3 \le 1 &&\\
& -x_4 - x_5 + 2 \le 2y && \textbf{(If } x_1 > 0\textbf{, then } x_4 + x_5 \ge 2\textbf{)}\\
& x_1 \le 2(1 - y) &&\\
& x_2 + x_3 \ge 1 &&
\end{aligned}$$

$$x_1, x_2, \ldots, x_7, y \text{ all 0–1 variables}$$

3 Let x_1 = units of product 1 produced

x_2 = units of product 2 produced

$y_i = \begin{cases} 1 & \text{if any product } i \text{ is ordered} \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP is

$$\max z = 2x_1 + 5x_2 - 10y_1 - 20y_2$$

$$\begin{aligned}
\text{s.t.}\quad & 3x_1 + 6x_2 \le 120\\
& x_1 \le 40y_1\\
& x_2 \le 20y_2\\
& x_1, x_2 \ge 0;\ y_1, y_2 = 0 \text{ or } 1
\end{aligned}$$

6 $y_1 = \begin{cases} 1 & \text{if calculus is taken} \\ 0 & \text{otherwise} \end{cases}$

$y_2 = \begin{cases} 1 & \text{if operations research is taken} \\ 0 & \text{otherwise} \end{cases}$

$y_3 = \begin{cases} 1 & \text{if data structure is taken} \\ 0 & \text{otherwise} \end{cases}$

$y_4 = \begin{cases} 1 & \text{if business statistics is taken} \\ 0 & \text{otherwise} \end{cases}$

$y_5 = \begin{cases} 1 & \text{if computer simulation is taken} \\ 0 & \text{otherwise} \end{cases}$

$y_6 = \begin{cases} 1 & \text{if introduction to computer programming is taken} \\ 0 & \text{otherwise} \end{cases}$

$y_7 = \begin{cases} 1 & \text{if forecasting is taken} \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP is

$$\min z = y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7$$

$$\begin{aligned}
\text{s.t.}\quad & y_1 + y_2 + y_3 + y_4 + y_7 \ge 2 && \textbf{(Math)}\\
& y_2 + y_4 + y_5 + y_7 \ge 2 && \textbf{(OR)}\\
& y_3 + y_5 + y_6 \ge 2 && \textbf{(Computers)}\\
& y_4 \le y_1 &&\\
& y_5 \le y_6 &&
\end{aligned}$$

$$y_3 \le y_6$$
$$y_7 \le y_4$$
$$y_1, y_2, \ldots, y_7 = 0 \text{ or } 1$$

10 Add the constraints $x + y - 3 \le Mz$, $2x + 5y - 12 \le M(1 - z)$, $z = 0$ or 1, where M is a large positive number.

11 Add the constraints $y - 3 \le Mz$, $3 - x \le (1 - z)M$, $z = 0$ or 1, where M is a large positive number.

13 Let x_i = number of workers employed on line i

$$y_i = \begin{cases} 1 & \text{if line } i \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Then the appropriate IP is

$$\begin{aligned} \min z &= 1{,}000y_1 + 2{,}000y_2 + 500x_1 + 900x_2 \\ \text{s.t.} \quad & 20x_1 + 50x_2 \ge 120 \\ & 30x_1 + 35x_2 \ge 150 \\ & 40x_1 + 45x_2 \ge 200 \\ & x_1 \le 7y_1 \\ & x_2 \le 7y_2 \\ & x_1, x_2 \ge 0;\ y_1, y_2 = 0 \text{ or } 1 \end{aligned}$$

14 **a** Let $x_i = \begin{cases} 1 & \text{if disk } i \text{ is used} \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP is

$$\begin{aligned} \min z &= 3x_1 + 5x_2 + x_3 + 2x_4 + x_5 + 4x_6 \\ &\quad + 3x_7 + x_8 + 2x_9 + 2x_{10} \\ \text{s.t.} \quad & x_1 + x_2 + x_4 + x_5 + x_8 + x_9 \ge 1 && \textbf{(File 1)} \\ & x_1 + x_3 \ge 1 && \textbf{(File 2)} \\ & x_2 + x_5 + x_7 + x_{10} \ge 1 && \textbf{(File 3)} \\ & x_3 + x_6 + x_8 \ge 1 && \textbf{(File 4)} \\ & x_1 + x_2 + x_4 + x_6 + x_7 + x_9 + x_{10} \ge 1 && \textbf{(File 5)} \\ & x_i = 0 \text{ or } 1 \quad (i = 1, 2, \ldots, 10) \end{aligned}$$

b Add the constraints $1 - x_2 \le 2y$, $x_3 + x_5 \le 2(1 - y)$, $y = 0$ or 1 (need $M = 2$, because $x_3 + x_5 = 2$ is possible). We could also have added the constraints $x_2 \ge x_3$ and $x_2 \ge x_5$.

SECTION 9.3

1 $z = 20$, $x_1 = 4$, $x_2 = 0$.

2 $z = \$400{,}000$. ($x_1 = 6$, $x_2 = 1$) and ($x_1 = 4$, $x_2 = 2$) are both optimal solutions.

4 $z = 8$, $x_1 = 2$, $x_2 = 0$.

SECTION 9.4

1 $z = 5.6$, $x_1 = 1.2$, $x_2 = 2$.

SECTION 9.5

2 $\max z = 60x_1 + 48x_2 + 14x_3 + 31x_4 + 10x_5$

$$\text{s.t.} \quad 800x_1 + 600x_2 + 300x_3 + 400x_4 + 200x_5 \le 1{,}100$$
$$x_i = 0 \text{ or } 1$$

Optimal solution is $z = 79$, $x_2 = x_4 = 1$, $x_1 = x_3 = x_5 = 0$.

SECTION 9.6

1 Do jobs in the following order: job 2, job 1, job 3, and job 4. Total delay is 20 minutes.

2 LFR—LFP—LP—LR—LFR has a total cost of \$330.

8 Warehouse 1 to factory 1, warehouse 2 to factory 3, warehouse 3 to factory 4, warehouse 4 to factory 2, warehouse 5 to factory 5 has a total cost of \$35,000.

SECTION 9.7

1 $z = 4$, $x_1 = x_2 = x_4 = x_5 = 1$, $x_3 = 0$.

2 $z = 3$, $x_1 = x_3 = 1$, $x_2 = 0$.

SECTION 9.8

1 $z = 110$, $x_1 = 4$, $x_2 = 3$.

REVIEW PROBLEMS

3 Let $z_i = \begin{cases} 1 & \text{if gymnast } i \text{ enters both events} \\ 0 & \text{otherwise} \end{cases}$

$$x_i = \begin{cases} 1 & \text{if gymnast } i \text{ enters only balance beam} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if gymnast } i \text{ enters only floor exercises} \\ 0 & \text{otherwise} \end{cases}$$

Then the appropriate IP is

$$\begin{aligned} \max z &= 16.7z_1 + 17.7z_2 + \cdots + 17.7z_6 + 8.8x_1 \\ &\quad + 9.4x_2 + \cdots + 9.1x_6 + 7.9y_1 \\ &\quad + 8.3y_2 + \cdots + 8.6y_6 \\ \text{s.t.} \quad & z_1 + z_2 + \cdots + z_6 = 3 \\ & x_1 + x_2 + \cdots + x_6 = 1 \\ & y_1 + y_2 + \cdots + y_6 = 1 \\ & x_1 + y_1 + z_1 \le 1 \\ & x_2 + y_2 + z_2 \le 1 \\ & x_3 + y_3 + z_3 \le 1 \\ & x_4 + y_4 + z_4 \le 1 \\ & x_5 + y_5 + z_5 \le 1 \\ & x_6 + y_6 + z_6 \le 1 \\ & \text{All variables 0 or 1} \end{aligned}$$

4 Let $x_{ij} = \begin{cases} 1 & \text{if students from district } i \text{ are sent to school } j \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP is

$$\begin{aligned} \min z = {} & 110x_{11} + 220x_{12} + 37.5x_{21} + 127.5x_{22} \\ & + 80x_{31} + 80x_{32} + 117x_{41} + 36x_{42} \\ & + 135x_{51} + 54x_{52} \end{aligned}$$

s.t. $110x_{11} + 75x_{21} + 100x_{31} + 90x_{41} + 90x_{51} \ge 150$ **(School 1 $\ge$ 150 students)**

$110x_{12} + 75x_{22} + 100x_{32} + 90x_{42} + 90x_{52} \ge 150$ **(School 2 $\ge$ 150 students)**

$$0.20 \le \frac{30x_{11} + 5x_{21} + 10x_{31} + 40x_{41} + 30x_{51}}{110x_{11} + 75x_{21} + 100x_{31} + 90x_{41} + 90x_{51}},$$

or $0 \le 8x_{11} - 10x_{21} - 10x_{31} + 22x_{41} + 12x_{51}$

$$0.20 \le \frac{30x_{12} + 5x_{22} + 10x_{32} + 40x_{42} + 30x_{52}}{110x_{12} + 75x_{22} + 100x_{32} + 90x_{42} + 90x_{52}},$$

or $0 \le 8x_{12} - 10x_{22} - 10x_{32} + 22x_{42} + 12x_{52}$

$x_{11} + x_{12} = 1$

$x_{21} + x_{22} = 1$

$x_{31} + x_{32} = 1$

$x_{41} + x_{42} = 1$

$x_{51} + x_{52} = 1$

All variables 0 or 1

5 Let $x_1 = \begin{cases} 1 & \text{if RS is signed} \\ 0 & \text{otherwise} \end{cases}$

$x_2 = \begin{cases} 1 & \text{if BS is signed} \\ 0 & \text{otherwise} \end{cases}$

$x_3 = \begin{cases} 1 & \text{if DE is signed} \\ 0 & \text{otherwise} \end{cases}$

$x_4 = \begin{cases} 1 & \text{if ST is signed} \\ 0 & \text{otherwise} \end{cases}$

$x_5 = \begin{cases} 1 & \text{if TS is signed} \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP is

$$\begin{aligned} \max z = {} & 6x_1 + 5x_2 + 3x_3 + 3x_4 + 2x_5 \\ \text{s.t.} \quad & 6x_1 + 4x_2 + 3x_3 + 2x_4 + 2x_5 \le 12 \\ & x_2 + x_3 + x_4 \le 2 \\ & x_1 + x_2 + x_3 + x_5 \le 2 \\ & x_1 + x_2 \le 1 \end{aligned}$$

All variables 0 or 1

10 Use two 20¢ coins, one 50¢ coin, and one 1¢ coin.

13 Infeasible.

26 Let

$x_{it} = \begin{cases} 1 & \text{if building } i \text{ is started during year } t \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP (in thousands) is

$\max z = 100x_{11} + 50x_{12} + 60x_{21} + 30x_{22} + 40x_{31}$

s.t. $30x_{11} + 20x_{21} + 20x_{31} \le 60$ **(Year 1 workers)**

$30(x_{11} + x_{12}) + 20(x_{21} + x_{22}) + 20(x_{31} + x_{32}) \le 60$ **(Year 2 workers)**

$30(x_{12} + x_{13}) + 20(x_{22} + x_{23}) + 20(x_{31} + x_{32} + x_{33}) \le 60$ **(Year 3 workers)**

$30(x_{13} + x_{14}) + 20(x_{23} + x_{24}) + 20(x_{32} + x_{33} + x_{34}) \le 60$ **(Year 4 workers)**

$$\left.\begin{aligned} x_{11} + x_{21} + x_{31} \le 1 \\ x_{12} + x_{22} + x_{32} \le 1 \\ x_{13} + x_{23} + x_{33} \le 1 \end{aligned}\right\}$$ **(No more than one building begins during each year)**

$$\left.\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} \le 1 \\ x_{21} + x_{22} + x_{23} + x_{24} \le 1 \\ x_{31} + x_{32} + x_{33} + x_{34} \le 1 \end{aligned}\right\}$$ **(Each building is started at most once)**

$x_{21} + x_{22} + x_{23} = 1$ **(Building 2 is finished by end of year 4)**

All variables 0 or 1

27 Let $y_i = \begin{cases} 1 & \text{if truck } i \text{ is used} \\ 0 & \text{otherwise} \end{cases}$

$x_{ij} = \begin{cases} 1 & \text{if truck } i \text{ is used to deliver to grocer } j \\ 0 & \text{otherwise} \end{cases}$

Then the appropriate IP is

$\min z = 45y_1 + 50y_2 + 55y_3 + 60y_4$

s.t. $100x_{11} + 200x_{12} + 300x_{13} + 500x_{14} + 800x_{15} \le 400y_1$

$100x_{21} + 200x_{22} + 300x_{23} + 500x_{24} + 800x_{25} \le 500y_2$

$100x_{31} + 200x_{32} + 300x_{33} + 500x_{34} + 800x_{35} \le 600y_3$

$100x_{41} + 200x_{42} + 300x_{43} + 500x_{44} + 800x_{45} \le 1{,}100y_4$

$x_{11} + x_{21} + x_{31} + x_{41} = 1$

$x_{12} + x_{22} + x_{32} + x_{42} = 1$

$x_{13} + x_{23} + x_{33} + x_{43} = 1$

$x_{14} + x_{24} + x_{34} + x_{44} = 1$

$x_{15} + x_{25} + x_{35} + x_{45} = 1$

All variables 0 or 1

Chapter 10

SECTIONS 10.1 AND 10.2

1 $z = 11, x_1 = 3, x_2 = 0, x_3 = 2.$

2 $z = 10, x_1 = 2, x_2 = 2.$

SECTION 10.3

2 Let x_i = number of 15-ft boards cut according to combination i, where

Combination	3-Ft Boards	5-Ft Boards	8-Ft Boards
1	0	1	1
2	2	0	1
3	0	3	0
4	1	2	0
5	3	1	0
6	5	0	0

Then we want to solve

$$\begin{aligned} \min z &= x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \\ \text{s.t.}\quad & 2x_2 + x_4 + 3x_5 + 5x_6 \geq 10 \\ & x_1 + 3x_3 + 2x_4 + x_5 \geq 20 \\ & x_1 + x_2 \geq 15 \\ & x_i \geq 0 \end{aligned}$$

The optimal solution is $z = \frac{55}{3}, x_1 = 10, x_2 = 5, x_3 = \frac{10}{3}.$

SECTION 10.4

1 $z = 40, x_1 = 3, x_2 = 2, x_3 = 3.$

3 $z = 15, x_1 = 0, x_2 = 0, x_3 = 3.$

SECTION 10.5

1 $z = 29.5, x_1 = 2, x_2 = 0.5, x_3 = 4, x_4 = x_5 = 0.$

3 $z = \frac{81}{13}, x_1 = \frac{12}{13}, x_2 = \frac{11}{13}.$

SECTION 10.6

1 $\mathbf{y}^1 = \mathbf{x}^1 = [\frac{3}{8}\ \frac{1}{4}\ \frac{3}{8}].$

REVIEW PROBLEMS

1 $z = 9, x_1 = x_3 = 0, x_2 = 3.$

3 $z = -12, x_2 = 2, x_4 = 10, x_1 = x_3 = 0.$

5 Maximum profit is \$540. Optimal production levels are

Product 1 at plant 1 = $\frac{5}{3}$ units

Product 1 at plant 2 = $\frac{100}{3}$ units

Product 2 at plant 1 = $\frac{290}{9}$ units

Product 2 at plant 2 = 0 units

Chapter 11

SECTION 11.1

1 3.

3 **a** $-xe^{-x} + e^{-x}.$

e $\frac{3}{x}.$

5 $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3p^3}$ for some p between 1 and $1 + x$.

7 **a** k.

b The maximum size of the market (as measured in sales per year).

9 The machine time is the better buy.

SECTION 11.2

1 **a** Let S = soap opera ads and F = football ads. Then we want to solve the following LP:

$$\begin{aligned} \min z &= 50S + 100F \\ \text{s.t.}\quad & 5S^{1/2} + 17F^{1/2} \geq 40 \quad \textbf{(Men)} \\ & 20S^{1/2} + 7F^{1/2} \geq 60 \quad \textbf{(Women)} \\ & S, F \geq 0 \end{aligned}$$

b Since doubling S does not double the contribution of S to the constraints, we are violating the Proportionality Assumption. Additivity is not violated.

5 $a = b = c = 20.$

SECTION 11.3

1 Convex.

2 Neither convex nor concave.

5 Concave.

8 Concave.

SECTION 11.4

1 If fixed cost is \$5,000, spend \$10,000 on advertising. If fixed cost is \$20,000, don't spend any money on advertising.

2 Without tax, produce 12.25 units; with tax, produce 12 units.

5 $z = 1, x = 1.$

SECTION 11.5

2 After four iterations, the interval of uncertainty is $[-0.42, 0.17)$.

SECTION 11.6

1 $x = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$,

$y = \dfrac{y_1 + y_2 + \cdots + y_n}{n}$.

3 $q_1 = 98.5$, $q_2 = 1$.

6 (0, 0) is a saddle point. $(\frac{3}{2}, \frac{3}{2})$ and $(\frac{3}{2}, -\frac{3}{2})$ are each a local minimum.

SECTION 11.7

3 Successive points are $(\frac{1}{2}, \frac{3}{4})$, $(\frac{3}{4}, \frac{3}{4})$, $(\frac{3}{4}, \frac{7}{8})$.

SECTION 11.8

2 $L = K = \frac{10}{3}$; produce 10 machines.

4 $x_1 = \dfrac{900}{13}$, $x_2 = \dfrac{400}{13}$, $\lambda = \dfrac{13^{1/2}}{2} - 1$. An extra dollar spent on promotion would increase profit by approximately $\$\left(\dfrac{13^{1/2}}{2} - 1\right)$.

SECTION 11.9

1 Capacity = 27.5 kwh. Peak price = \$65. Off-peak price = \$20.

2 $z = 2^{1/2}$, $x_1 = \dfrac{2^{1/2}}{2}$, $x_2 = -\dfrac{2^{1/2}}{2}$.

6 $z = \frac{1}{2}$, $x_1 = \frac{1}{2}$, $x_2 = \frac{3}{2}$.

SECTION 11.10

1
$$\begin{aligned}
\min z &= 0.09x_1^2 + 0.04x_2^2 + 0.01x_3^2\\
&\quad + 0.012x_1x_2 - 0.008x_1x_3 + 0.010x_2x_3\\
\text{s.t.}\quad & x_2 - x_3 \geq 0\\
& x_1 + x_2 + x_3 = 100\\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

4 $p_1 = \$292.81$, $p_2 = \$158.33$. Pay no money for an additional hour of labor. Pay up to (approximately) \$53.81 for another chip.

SECTION 11.11

1 Using grid points 0, 0.5, 1, 1.5, and 2 for x_1 and grid points 0, 0.5, 1, 1.5, 2, and 2.5 for x_2, we obtain the following approximating problem:

$$\begin{aligned}
\min z &= 0.25\delta_{12} + \delta_{13} + 2.25\delta_{14} + 4\delta_{15} + 0.25\delta_{22}\\
&\quad + \delta_{23} + 2.25\delta_{24} + 4\delta_{25} + 6.25\delta_{26}\\
\text{s.t.}\quad & 0.25\delta_{12} + \delta_{13} + 2.25\delta_{14} + 4\delta_{15} + 2(0.25\delta_{22}\\
&\quad + \delta_{23} + 2.25\delta_{24} + 4\delta_{25} + 6.25\delta_{26}) \leq 4\\
& 0.25\delta_{12} + \delta_{13} + 2.25\delta_{14} + 4\delta_{15} + 0.25\delta_{22}\\
&\quad + \delta_{23} + 2.25\delta_{24} + 4\delta_{25} + 6.25\delta_{26} \leq 6\\
& \delta_{11} + \delta_{12} + \delta_{13} + \delta_{14} + \delta_{15} = 1\\
& \delta_{21} + \delta_{22} + \delta_{23} + \delta_{24} + \delta_{25} + \delta_{26} = 1
\end{aligned}$$

All variables nonnegative

Adjacency Assumption

SECTION 11.12

1 $\mathbf{x}^1 = [0 \quad 1]$ and $\mathbf{x}^2 = [\frac{1}{3} \quad \frac{5}{6}]$.

SECTION 11.13

2 First we attempt to maximize output by solving

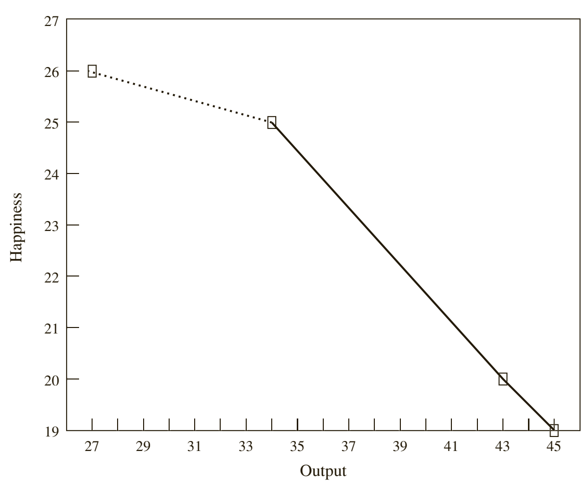
$$\begin{aligned}
\max z &= 20x_{11} + 12x_{12} + 10x_{13} + 12x_{21} + 15x_{22}\\
&\quad + 9x_{23} + 6x_{31} + 5x_{32} + 10x_{33}\\
\text{s.t.}\quad & x_{11} + x_{12} + x_{13} = 1\\
& x_{21} + x_{22} + x_{23} = 1\\
& x_{31} + x_{32} + x_{33} = 1\\
& x_{11} + x_{21} + x_{31} = 1\\
& x_{12} + x_{22} + x_{32} = 1\\
& x_{13} + x_{23} + x_{33} = 1\\
& \text{All } x_{ij} \geq 0
\end{aligned}$$

Here x_{ij} fraction of day worker i spends working on product j. This LP has optimal solution $x_{11} = x_{22} = x_{33} = 1$, with Output = 45 and Happiness = 6 + 5 + 8 = 19. Thus, point (45, 19) is on a tradeoff curve. Now add constraint

$$6x_{11} + 8x_{12} + 10x_{13} + 6x_{21} + 5x_{22} + 9x_{23} + 9x_{31} + 10x_{32} + 8x_{33} \geq \text{HAPP}$$

where HAPP = 20, 25, and 26. (HAPP cannot exceed 26.) This yields four points on the tradeoff curve in Figure 11. Value of Output is optimal z-value for each LP.

FIGURE 11



REVIEW PROBLEMS

2 Locate the store at point 7. In general, locate store at arithmetic mean of the location of all customers.

3 **a** Use $\frac{93}{8}$ units of raw material, sell $\frac{93}{4}$ units of product 1, and sell $\frac{15}{2}$ units of product 2.
c Pay slightly less than \$5 for an extra unit of raw material.

5 [1.18, 1.63).

7 Produce 20 units during each of the three months.

18 Locate the store at point 5; in general, locate store at the median of the customer's locations.

Chapter 12

SECTION 12.1

1 $\dfrac{e^{10}-1}{2}$.

3 $Ih - \dfrac{dh}{2}$.

SECTION 12.2

1 $2y(2y + y^2) - 3y + 2(y^2 - y)$.

SECTION 12.3

1 **a** $\frac{2}{9}$.
c No.
e $\frac{1}{4}$.

SECTION 12.4

1 $\frac{2}{3}$.

3 .001.

SECTION 12.5

1 **a** Let $\mathbf{S}$ = number sold. Then $E(\mathbf{S}) = \frac{290}{3}$, and var $\mathbf{S} = \frac{200}{9}$.

3 **a** $F(a) = 0$ for $a \le 0$, $F(a) = 1 - e^{-a}$ for $a \ge 0$.
b $E(\mathbf{X}) = 1$ var $\mathbf{X} = 1$.
c $e^{-1} - e^{-2}$.

SECTION 12.6

2 .8749.

REVIEW PROBLEMS

3 $\frac{25}{2}$.

5 **a** $E(\mathbf{X}) = 85$; var $\mathbf{X} = 9{,}000$.
b $P\left(\mathbf{Z} \ge \dfrac{91-85}{(9{,}000)^{1/2}}\right) = .476$.

7 **a** $\frac{1}{6}$.
b .004996.

Chapter 13

SECTION 13.1

1 Maximin decision; small campaign. Maximax decision: large campaign. Minimax regret decision: large campaign.

2 Maximin decision; don't build. Maximax decision: build. Maximax regret decision: build. Expected value decision: build.

5 Maximin decision: \$6,000, \$8,000, or \$11,000 bid. Maximax decision: \$11,000 bid. Minimax regret decision: \$11,000 bid. Expected value decision: \$11,000 bid.

SECTION 13.2

1 **a** Risk-averse.
b Prefer L_1; risk premium for L_2 = \$339.

2 **a** Risk-seeking.
b Prefer L_2; risk premium for $L_2 = -\$235$.

6 **b** \$1,900.

7 Take statistics course.

13 L_1 is preferred.

SECTION 13.4

1 Hire the geologist. If she gives a favorable report, drill; if she gives an unfavorable report, don't drill. Expected profit is \$180,000; EVSI = \$20,000; EVPI = \$55,000.

4 Market without testing. Expected profit = \$16,000; EVSI = \$3,800; EVPI = \$14,000.

9 Play daringly during the first game. If he wins the first game, play conservatively during the second game. If he loses the first game, play daringly during the second game. If tied after two games, play daringly during the third game.

12 **a** Buy the gold now.
b Wait for Congress and (if possible) buy the gold later.

SECTION 13.5

2 Hire the geologist. If he predicts an earthquake, build at Roy Rogers; if he predicts no earthquake, build at Diablo. Expected total cost = \$13,900,000. EVSI = \$1,100,000; EVPI = \$2,000,000.

4 Hire the firm. If it predicts a hit, air the show. If it predicts a flop, don't air the show. Expected profit = \$35,000; EVSI = \$50,000; EVPI = \$75,000.

SECTION 13.6

1 **c** National's utility function is of the form $.3u_1(x_1) + .5u_2(x_2) + .2u_1(x_1)u_2(x_2)$.

6 d $k_3 > 0$.

SECTION 13.7

1 a Professor 2 should receive the bigger raise.
c Pairwise comparison matrix is consistent.

REVIEW PROBLEMS

1 a Invest in money market fund.
b Invest in gold.
c Money market fund has a maximum regret of \$500.
d All investments have the same expected return.

2 Because of the risk-averse nature of the utility function, invest in the least risky investment (the money market fund).

5 a Invest all money in stocks.
b Indifferent between hiring and not hiring forecaster. Expected final asset position = \$1,160,000; EVSI = \$10,000; EVPI = \$20,000.

12 If all potential litterers are risk-averse, then raising the fine will result in the larger decrease in littering.

Chapter 14

SECTION 14.1

1 Value to row player = 2. Row player plays row 1, and column player plays column 1.

2 Value to row player = 6. Row player plays row 2, and column player plays column 1 or column 3.

SECTION 14.2

1 Value to row player = $\frac{4}{3}$. Row player's optimal strategy is $(\frac{2}{3}, \frac{1}{3})$ and column player's optimal strategy is $(\frac{2}{3}, \frac{1}{3}, 0)$.

8 State's optimal strategy is, with probability $\frac{1}{2}$, play A first and B second; with probability $\frac{1}{2}$, play B first and A second. Ivy's optimal strategy is, with probability $\frac{1}{2}$, play X first and Y second; with probability $\frac{1}{2}$, play X second and Y first. Value of game to State = $\frac{1}{2}$.

SECTION 14.3

1 a

	Soldier				
Gunner	1	2	3	4	5
Spot A	1	1	0	0	0
Spot B	0	1	1	0	0
Spot C	0	0	1	1	0
Spot D	0	0	0	1	1

b Columns 2 and 4 are dominated.
c Expected value to the gunner = $\frac{1}{3}$.
d Always firing at A.

e Gunner's LP is

$$\begin{aligned}
&\max z = v \\
&\text{s.t.} \quad v \le x_1 \\
&\qquad v \le x_1 + x_2 \\
&\qquad v \le x_2 + x_3 \\
&\qquad v \le x_3 + x_4 \\
&\qquad v \le x_4 \\
&\qquad x_1 + x_2 + x_3 + x_4 = 1 \\
&\quad x_1, x_2, x_3, x_4 \ge 0;\ v \text{ urs}
\end{aligned}$$

Soldier's LP is

$$\begin{aligned}
&\min w \\
&\text{s.t.} \quad w \ge y_1 + y_2 \\
&\qquad w \ge y_2 + y_3 \\
&\qquad w \ge y_3 + y_4 \\
&\qquad w \ge y_4 + y_5 \\
&\text{s.t.} \quad y_1 + y_2 + y_3 + y_4 + y_5 = 1 \\
&\quad y_1, y_2, y_3, y_4, y_5 \ge 0;\ w \text{ urs}
\end{aligned}$$

3 Value to row player = $\frac{5}{2}$. Row player's optimal strategy is $(\frac{1}{2}, \frac{1}{2})$, and column player's optimal strategy is $(\frac{3}{4}, \frac{1}{4}, 0)$.

SECTION 14.4

1 $(9, -1)$ is an equilibrium point.

3 This is a Prisoner's Dilemma game, with the equilibrium point occurring where each borough opposes the other borough's bond issues. Reward is \$0 to each borough.

SECTION 14.7

2 Core consists of the point (25, 25, 25, 25).

3 a $v(\{\ \}) = \$0$; $v(\{1\}) = v(\{2\}) = v(\{3\}) = -\$2$; $v(\{1, 2\}) = v(\{2, 3\}) = v(\{1, 3\}) = \$2$; $v(\{1, 2, 3\}) = \$3$.
b and c The Shapley value gives \$1 to each player. The core is (\$1, \$1, \$1).

7 Assuming that the runway costs \$1/ft, the Shapley value recommends the following fees per landing: type 1, \$20; type 2, $\$\frac{240}{7}$; type 3, $\$\frac{440}{7}$; type 4, $\$\frac{1,140}{7}$.

REVIEW PROBLEMS

1 Both stores will be located at point B, and the two firms will each have 26 customers.

3 b Value to row player = $-\frac{5}{11}$. For each player, the optimal strategy is $(\frac{4}{11}, \frac{4}{11}, \frac{3}{11})$.

6 a $v(\{\ \}) = v(\{49\}) = v(\{50\}) = v(\{1\}) = v(\{1, 49\}) = 0$; $v(\{1, 50\}) = v(\{49, 50\}) = v(\{1, 49, 50\}) = 1$.
b Core consists of point (0, 0, 1).
c Shapley value gives $\frac{1}{6}$ to player 1, $\frac{1}{6}$ to player 2, and $\frac{2}{3}$ to player 3.

Chapter 15

SECTION 15.2

1 **a** 4,000 gallons.

b 12 orders per year.

c One month.

e For a 2-week lead time, reorder point $= \frac{48,000}{26} =$ 1,846.15 gallons. For a 10-week lead time, reorder point = 1,230.77 gallons.

3 **a** Send out $\frac{30}{7.07} = 4.24$ trucks per hour.

b Send out $\frac{30}{5} = 6$ trucks per hour.

12 **b** Six trainees in each program.

c Run 4.5 programs per year.

d 2.25 trainees.

SECTION 15.3

1 Order 300 boxes per year. Place 3.2 orders per year.

5 Order 100 thermometers.

SECTION 15.4

2 Optimal run size = 692.82. Do 34.64 runs per year.

SECTION 15.5

2 Whenever dealer is 10 cars short, an order for 50 cars should be placed. Maximum shortage will be 10 cars.

SECTION 15.6

1 Demand is too lumpy to justify using EOQ.

REVIEW PROBLEMS

1 **a** 268.33 desks.

b 22.36 orders per year.

c 2(22.36)(300) = \$13,416.

d For 1-week lead time, reorder point = 115.38 desks; for 5-week lead time, reorder point = 40.93 desks.

e Order 342.05 desks 17.54 times per year.

3 The EOQ for the lowest price is optimal. Thus, 417.79 cameras should be ordered.

Chapter 16

SECTION 16.2

1 **a** $q = 6$.

b $q = 2$.

c $E(q)$ is not a convex function of q.

SECTION 16.3

1 Order 35 cars.

3 q^* will decrease.

5 **a** Order 60 cells.

SECTION 16.4

2 \$775,000, using $F(.25) = .60$.

3 107.5 trees, using $F(.25) = .60$.

SECTION 16.5

1 Locate at $\frac{2^{1/2}}{2}$.

SECTION 16.6

1 Order quantity = 45.61, reorder point = 32.6, safety stock = 12.6.

2 **a** Reorder print = 57.9, safety stock = 17.9.

b Reorder print = 60.60, safety stock = 20.60.

SECTION 16.7

1

SLM_1	Reorder Point
80%	11.06
90%	16.46
95%	20.24
99%	26.30

For SLM_2 = 0.5 stockout per year, the reorder point is 32.06.

3 **a** $SLM_1 = 98.75\%$.

b $r = 20$ is the smallest reorder point with SLM_1 exceeding 95%.

c Need a reorder point of at least 30 units.

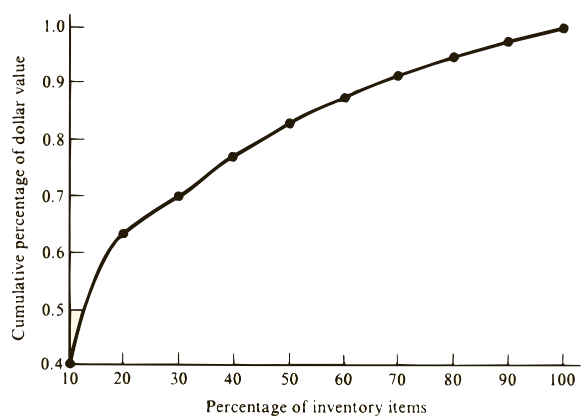
SECTION 16.8

1 $R = 0.23$ years and $S = 3,241$.

SECTION 16.9

1 Type A items: 1 and 2; type B items: 3 through 6; type C items: 7 through 10. See Figure 12.

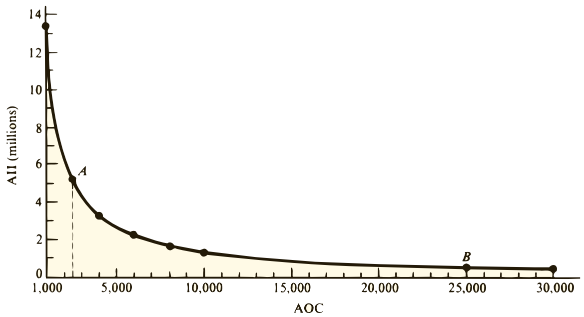
FIGURE 12



SECTION 16.10

1 **a** See Figure 13.

FIGURE 13



REVIEW PROBLEMS

1 **a** Bake 40 dozen cookies.
b Bake 58.6 dozen cookies.
c Bake 55 dozen cookies.

3 **a** Order quantity = 707.11, shortage cost = \$12.41.
b Order quantity = 707.11. Assuming a penalty for lost sales of 8 − 5 = \$3, shortage cost = \$9.12.
c Reorder point of zero will do the job.

Chapter 17

SECTION 17.2

1

$$\begin{array}{c} \\ \text{Sunny} \\ \text{Cloudy} \end{array}\begin{array}{c} \begin{array}{cc}\text{Sunny} & \text{Cloudy}\end{array} \\ \begin{bmatrix} .90 & .10 \\ .20 & .80 \end{bmatrix} \end{array}$$

2 State

$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 2 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 3 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 4 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}$$

4 Let *SC* denote that yesterday was sunny and today is cloudy, and so on.

$$\begin{array}{c|cccc} & SS & SC & CS & CC \\ \hline SS & .95 & .05 & 0 & 0 \\ SC & 0 & 0 & .40 & .60 \\ CS & .70 & .30 & 0 & 0 \\ CC & 0 & 0 & .20 & .80 \end{array}$$

SECTION 17.3

1 **a** Urban, .651; suburban, .258; rural, .091.
b 31.5%.

SECTION 17.4

1 2.

2 Yes.

3 **a** State 4.
b States 1, 2, 3, 5, and 6.
c {1, 3, 5} and {2, 6}.

4 P_1 is ergodic; P_2 is not ergodic.

SECTION 17.5

1 Urban, $\frac{38}{183}$; suburban, $\frac{90}{183}$; rural, $\frac{55}{183}$.

3 **a** State 1, $\frac{3}{5}$; state 2, $\frac{2}{5}$.

4 Replace a fair car.

7 Expected price of stock 1 = \$16.67; expected price of stock 2 = \$16.00.

SECTION 17.6

1 **a** 1.11 + 0.99 + 0.99 + 0.88 = 3.97.
b .748.

2 1 + 0.80 + 18 = 19.80 years.

11 **a** 14.3%.
b Reducing warranty period will save \$715,000 − \$392,500 = \$322,500.

SECTION 17.7

1 7,778 freshmen, 7,469 sophomores, 8,057 juniors, and 7,162 seniors.

2 Each working adult must contribute \$2,000 more.

REVIEW PROBLEMS

1 An average of $\frac{260}{3}$ tools per day will be produced.

2 **a** .815.
b Annual profit without warranty = (3,000)(total market size)($\frac{1}{4}$). Annual profit with warranty = (2,700)(total market size)($\frac{4}{9}$). Profit with warranty is larger.

3 Value of star = \$4,400,000; value of starter = \$3,199,000; value of substitute = \$1,600,000.

4 **a** Each year, 1,638,270 new books, 1,474,443 once-used books, 1,179,554 twice-used books, and 707,733 thrice-used books will be sold.

Chapter 18

SECTION 18.1

1 Begin by picking up 4 matches. On each successive turn, pick up 5 − (number of matches picked up by opponent on last turn).

2 The players began with \$16.25, \$8.75, and \$5.00.

SECTION 18.2

1 1–3–5–8–10, 1–4–6–9–10, and 1–4–5–8–10 are all shortest paths from node 1 to node 10 (each has length 11). The path 3–5–8–10 is the shortest path from node 3 to node 10 (this path has length 7).

3 Bloomington–Indianapolis–Dayton–Toledo–Cleveland takes 8 hours.

SECTION 18.3

1 Produce no units during month 1, 3 units during month 2, no units during month 3, and 4 units during month 4. Total cost is $15.00

2 Month 1, 200 radios; month 2, 600 radios; month 3, no radios. Total cost is $8,950.

3 **a** Produce 1 unit. Cost associated with arc is $4.50.

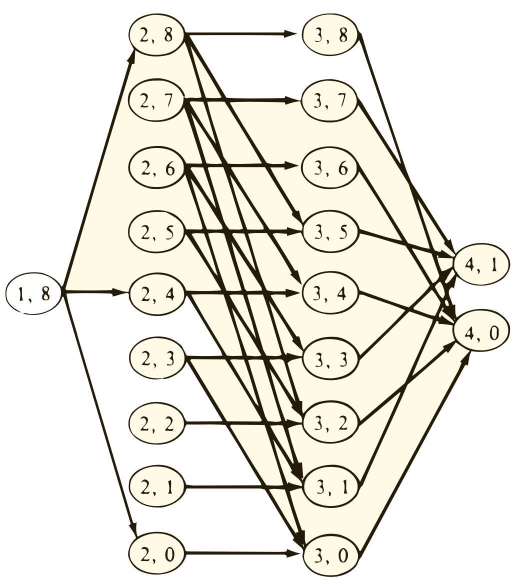
SECTION 18.4

1 Site 1, $1 million; site 2, $2 million; site 3, $1 million. Total revenue is $24 million.

2 Obtain a benefit of 10 with two type 1 items or two type 2 and one type 3 item.

3 **a** See Figure 14.

FIGURE 14



5 One type 2 item and one type 3 item yield a benefit of 75.

SECTION 18.5

2 Trade in car whenever it is two years old (at times 2, 4, and 6). Net cost is $14,400.

SECTION 18.6

1 Let $f_t(i)$ be the maximum expected net profit earned during years $t, t+1, \ldots, 10$, given that Sunco has i barrels of reserves at the beginning of year t. Then

$$f_{10}(i) = \max_x \{xp_{10} - c(x)\}$$

where x must satisfy $0 \le x \le i$. For $t \le 9$,

$$f_t(i) = \max_x \{xp_t - c(x) + f_{t+1}(i + b_t - x)\} \qquad (1)$$

where $0 \le x \le i$. We use Equation (1) to work backward until $f_1(i_0)$ is determined. If discounting is allowed, let $\beta =$ the discount factor. Then we redefine $f_t(i)$ to be measured in terms of year t dollars. Then we replace (1) with (1′):

$$f_t(i) = \max_x \{xp_t - c(x) + \beta f_{t+1}(i + b_t - x)\} \qquad (1')$$

where $0 \le x \le b$.

2 **b** Let $f_t(d)$ be the maximum utility that can be earned during years $t, t+1, \ldots, 10$, given that d dollars are available at the beginning of year t (including year t income). During year 10, it makes sense to consume all available money (after all, there is no future). Thus, $f_{10}(d) = \ln d$. For $t \le 9$,

$$f_t(d) = \max_c \{\ln c + f_{t+1}(1.1(d - c) + i)\}$$

where $0 \le c \le d$. We work backward from the $f_{10}(\cdot)$'s to $f_1(D)$.

5 French, 1 hour; English, no hours; statistics, 3 hours. There is a .711 chance of passing at least one course.

7 Define $f_t(w)$ to be the maximum net profit (revenues less costs) obtained from the steer during weeks $t, t+1, \ldots, 10$, given that the steer weighs w pounds at the beginning of week t. Now

$$f_{10}(w) = \max_p \{10g(w, p) - c(p)\}$$

where $0 \le p$. Then for $t \le 9$,

$$f_t(w) = \max_p \{-c(p) + f_{t+1}(g(w, p))\}$$

Farmer Jones should work backward until $f_1(w_0)$ has been computed.

8 Define $f_t(i, d)$ to be the maximum number of loyal customers at the end of month 12, given that there are i loyal customers at the beginning of month t and d dollars available to spend on advertising during months $t, t+1, \ldots, 12$. If there is only one month left, all available funds should be spent during that month. This yields

$$f_{12}(i, d) = (1 - p(d))i + (N - i)q(d)$$

For $t \le 11$,

$$f_t(i, d) = \max_x \{f_{t+1}[(1 - p(x))i + (N - i)q(x), d - x]\}$$

where $0 \le x \le d$. We work backward until $f_1(i_0, D)$ has been determined.

10 Let $f_t(i_t, x_{t-1})$ be the minimum cost incurred during months $t, t+1, \ldots, 12$, given that inventory at the beginning of month t is i_t, and production during month $t-1$ was x_{t-1}. Then

$$f_{12}(i_{12}, x_{11}) = \min_{x_{12}} \{c_{12}x_{12} + 5|x_{12} - x_{11}| + h_{12}(i_{12} + x_{12} - d_{12})\}$$

where x_{12} must satisfy $x_{12} \geq 0$ and $i + x_{12} \geq d_{12}$. For $t \leq 11$,

$$f_t(i_t, x_{t-1}) = \min_{x_t} \{c_t x_t + 5|x_t - x_{t-1}| + h_t(i + x_t - d_t) + f_{t+1}(i_t + x_t - d_t, x_t)\}$$

where x_t must satisfy $x_t \geq 0$ and $i_t + x_t \geq d_t$. We work backward until $f_1(20, 20)$ has been computed.

SECTION 18.7

1 If initial inventory is 200 units, only modification is to produce 200 fewer units during period 1.

2 The Wagner–Whitin and Silver–Meal methods both yield the following production schedule: period 1, 90 units; period 4, 230 units. Total cost is \$176.

REVIEW PROBLEMS

1 Shortest path from node 1 to node 10 is 1–4–8–10. Shortest path from node 2 to node 10 is 2–5–8–10.

2 Month 2, 1 unit; month 3, 4 units. Total cost is \$12.

4 For 6 flights, the airline earns \$540 with 3 Miami, 2 L.A. and 1 N.Y. flight; or 3 Miami and 3 L.A. flights. For four flights, the airline earns \$375 with 2 Miami and 2 L.A. flights.

5 Without the 20¢ piece, use one 50¢, one 25¢, one 10¢, one 5¢, and one 1¢ piece. With the 20¢ piece, use one 50¢, two 20¢, and one 1¢ piece.

7 **a** Let $f_t(w)$ be the minimum cost incurred in meeting demands for the years $t, t + 1, \ldots, 5$, given that (before hiring and firing for year t) w workers are available.

h_t = workers hired at beginning of year t

d_t = workers fired at beginning of year t

w_t = workers required during year t

Then

$$f_t(w) = \min_{h_t, d_t} \{10{,}000h_t + 20{,}000d_t + 30{,}000(w + h_t - d_t) + f_{t+1}(h_t + .9(w - d_t))$$

where h_t and d_t must satisfy $0 \leq h_t$, $0 \leq d_t \leq w$, and $w + h_t - d_t \geq w_t$.

Chapter 19

SECTION 19.1

1 Two gallons of milk to each store.

2 \$2 million to investment 1, \$0 to investment 2, and \$2 million to investment 3.

SECTION 19.2

1 Produce 3 units during period 1, no units during period 2, and 1 unit during period 3.

2 At any time, produce the number of units needed to bring the period's stock level (before the period's demand is met) to 2 units.

SECTION 19.3

1 Ulanowsky should play boldly during the first game. If he wins the first game, then he should play conservatively in the second game. If he loses the first game, then he should play boldly in the second game. If there is a tie-breaking game, Ulanowsky should play boldly. His chance of winning the match is .537.

2 On the first toss, Dickie should bet \$2. If he loses, he is wiped out, but if he wins, he bets \$1 on the second toss. If he wins on the second toss, he stops. If he loses on the second toss, he bets \$2 on the third toss.

SECTION 19.4

2 Let $f_t(d)$ be the maximum expected asset position of the firm at the end of year 10, given that at the beginning of year t, the firm has d dollars in assets. Then

$$f_{10}(d) = \max_i \left\{p \sum_y q_y(d + i + y) + (1 - p)\sum_y q_y(d - i + y)\right\}$$

where $0 \leq i \leq d$. For $t \leq 9$,

$$f_t(d) = \max_i \left\{p \sum_y q_y f_{t+1}(d + i + y) + (1 - p)\sum_y q_y f_{t+1}(d - i + y)\right\}$$

We work backward until $f_1(10{,}000)$ has been computed.

5 We should always do maintenance on a running machine and always repair a broken machine.

6 Let $f_t(p)$ be the maximum expected revenue earned from selling a share of Wivco stock during days $t, t + 1, \ldots, 30$, given that the price of a share at the beginning of day t is p dollars. Then

$$f_{30}(p) = p \qquad \textbf{(Sell stock)}$$

and for $t \leq 29$,

$$f_t(p) = \max \begin{Bmatrix} p & \textbf{(Sell stock)} \\ \sum_x q(x) f_{t+1}((1 + x/100)p) & \textbf{(Keep stock)} \end{Bmatrix}$$

We work backward until $f_1(10)$ has been determined, and we continue until the optimal action is to sell the stock.

7 Sara should not accept the first cat, but any later cat that is the best Sara has seen so far should be accepted. The probability that Sara will get her preferred cat is $\frac{11}{24}$.

8 Define $f_t(i)$ to be the minimum net expected cost incurred during periods $t, t + 1, \ldots, 100$, given that the inventory level is i at the beginning of period t. Then

$$f_{100}(i) = \min_x \left\{ \sum_{d \leq i+x} q_d(i + x - d - rd) + \sum_{d > i+x} q_d(p(d - i - x) - r(i + x)) + c(x)\right\}$$

where $x \geq 0$. For $t \leq 99$,

$$f_t(i) = \min_x \left\{ \sum_{d \leq i+x} q_d(i + x - d - rd) + \sum_{d > i+x} (p(d - i - x) - r(i + x))q_d + c(x) + \sum_{d \leq i+x} q_d f_{t+1}(i + x - d) + \sum_{d > i+x} q_d f_{t+1}(0) \right\}$$

where $x \geq 0$. We work backward until $f_1(0)$ has been determined.

10 Define $f_t(d)$ to be the maximum expected number of units sold in markets $t, t + 1, \ldots, T$, given that d dollars are available to spend on these markets. Then

$$f_T(d) = c_T k_T p_T(d)$$

and for $t \leq T - 1$,

$$f_t(d) = \max_x \{c_t k_t p_t(x) + f_{t+1}(d - x)\}$$

where $0 \leq x \leq d$. We work backward until $f_1(D)$ has been computed.

SECTION 19.5

1 a

Period's Beginning Inventory	Period's Production Level
0	4
1	3
2	0
3	0

2 a Always charge 11% interest rate on loans.

REVIEW PROBLEMS

1 Assign 2 sales reps to district 1, 1 sales rep to district 2, and 2 sales reps to district 3.

2 a Produce 2 units during period 1. During period 2, produce 1 or 2 units if beginning inventory is 0; if beginning inventory for period 2 is 1 or 2 units, produce no units during period 2. During period 3, produce 1 unit if beginning inventory is 0; otherwise, produce no units during period 3.

4 Let $f_t(b)$ be the maximum discounted net benefit earned during years $t, t + 1, \ldots, 2039$, given that b barrels of oil are available at the beginning of year t. Then

$$f_t(b) = \max_{d,x} \{u(x) - d + \beta p(d) f_{t+1}(b - x + 500{,}000) + \beta(1 - p(d)) f_{t+1}(b - x)\}$$

where $0 \leq d$ and $0 \leq x \leq b$. We work backward until $f_{2004}(B)$ is determined and then compute the optimal consumption strategy.

5 a Try to answer the first two questions and then stop. The expected amount of money won is $9,000.

Chapter 20

SECTION 20.2

1 $\frac{6}{7}$.

2 $\frac{555}{11}$ minutes.

4 a $\frac{2e^{-2}}{3} = .09$.

b $1 - e^{-2} - 2e^{-2} = .594$.

SECTION 20.3

2 b $\frac{121}{144}$.

c $\frac{1}{144}$.

SECTION 20.4

1 a $\frac{5}{6}$.

b $\frac{25}{6}$ passengers.

c $\frac{1}{2}$ minute.

3 a Unchanged.

b Cut in half.

c Unchanged.

8 Two checkpoints.

10 b 1 taxi.

c $120 per hour.

SECTION 20.5

1 a 1.75 customers per hour.

b $\frac{114}{130}$.

6 Barber 1's average hourly revenue = $53.23. Barber 2's average hourly revenue = $40.00.

SECTION 20.6

1 Two registers.

3 a Finance, $\frac{1}{5}$ day; Marketing, $\frac{1}{10}$ day.

b 0.078 day.

d .07.

4 4 servers.

13 $\frac{3}{4}$ mechanic.

14 System 1 is more efficient.

SECTION 20.7

1 5,200 members.

3 200 firms. The probability that there are at least 3,200 firms is zero.

SECTION 20.8

1 $\frac{2}{3}$ car.

4 **a** $\frac{5}{6}$ customer.

b 9 minutes.

c $\frac{1}{3}$.

SECTION 20.9

1 Superworker is better.

2 **a** $\frac{81}{125}$.

b $\frac{6}{5}$ puppies.

SECTION 20.10

2 **a** 2.73 cars.

b 0.06 hour.

4 $\frac{11}{3}$ students.

SECTION 20.11

1 15 fire engines.

7 **b** 2 copies.

SECTION 20.12

1 Using four categories (each having $e_i = 6$), we accept the hypothesis that the length of a telephone call is exponential with mean $\frac{1{,}024}{24}$ seconds.

SECTION 20.15

1 Tests spend an average of $\frac{19}{8}$ hours in the system, research papers spend an average of $\frac{27}{4}$ hours in the system, and class handouts spend an average of $\frac{23}{2}$ hours in the system.

2 Highest priority to $k = 1$ customers, next highest priority to $k = 2$ customers, and so on.

REVIEW PROBLEMS

2 **a** 8 minutes.

b $\dfrac{(1.25)^2 e^{-1.25}}{2\,!} = .22.$

c $e^{-0.75} = .47.$

3 **a** 3 windows.

b 3 windows.

6 2 copiers

7 **a** $\frac{2}{3}$ car.

b 8.2 minutes.

8 Rent the vacant lot. Then expected daily lost profit is $21(20)(0.008) = \$3.36$.

9 $\frac{1}{2}$ hour.

11 Have one crew of 100 workers.

16 **a** Both lines are free $\frac{8}{13}$ of time, one line is free $\frac{4}{13}$ of the time, and both lines are busy $\frac{1}{13}$ of the time.

b $\frac{6}{13}$ line.

c $30(\frac{1}{13}) = \frac{30}{13}$ callers per hour.

Chapter 21

SECTION 21.4

1 Approximately 0.76 minute. The answer may vary according to the random numbers used in the computations.

3 See Figure 15. Variables used in the model:

VALUE = face value from the roll of the dice

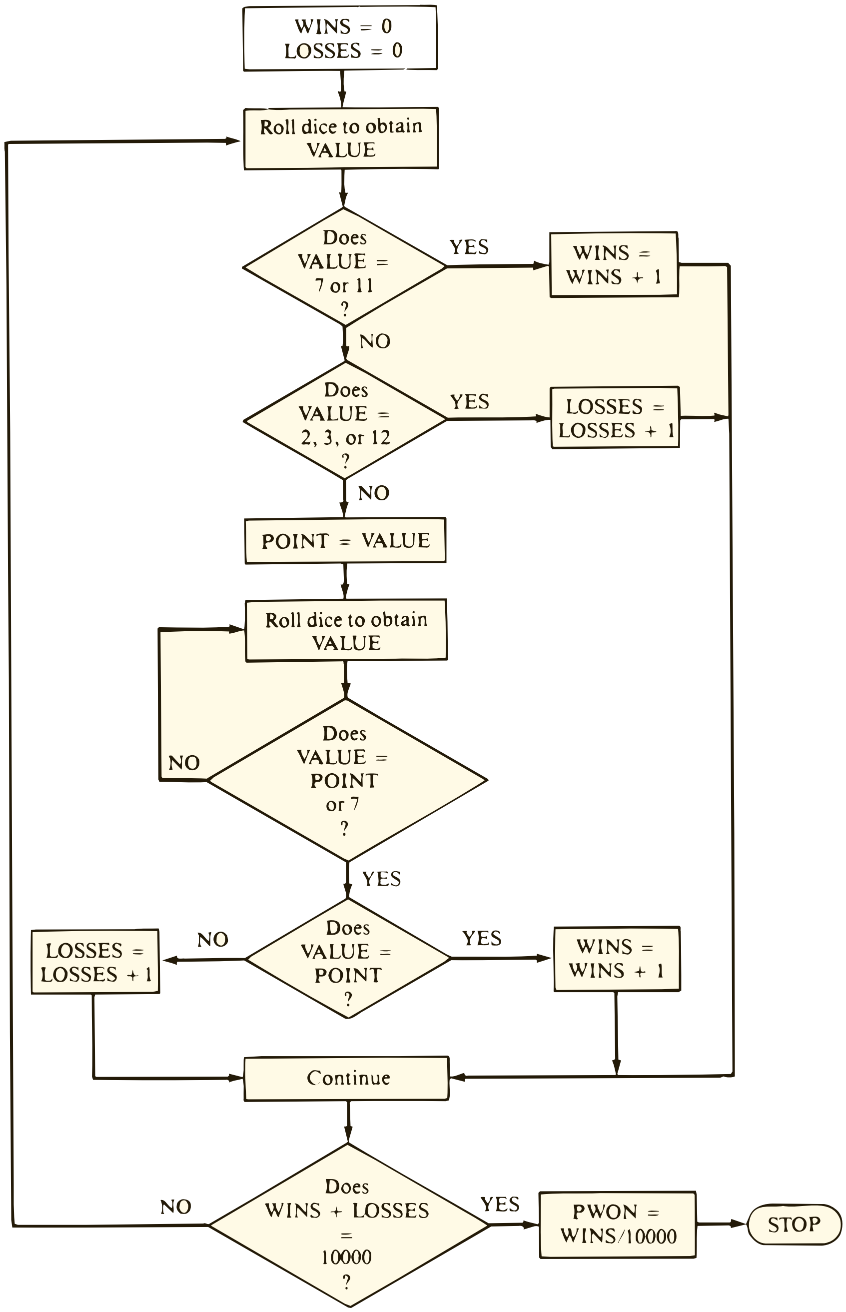
WINS = total number of wins up to current simulation

LOSSES = total number of losses up to current simulation

POINT = face value from the first roll

PWON = proportion of wins

FIGURE 15



SECTION 21.5

1 ITM: Generate a random number r.

If ($r \le 0.5$) then

$x = 2r$

else

$x = 3 - 2\sqrt{2 - 2r}$

endif

ARM: Generate two random numbers, r_1 and r_2.

Set $x^* = 3r_1$

If ($x^* \le 1$) then

Accept x^* as the random variate

else

If ($r_2 \le \frac{3}{2}(1 - r_1)$) then

Accept x^* as the random variate

else

Reject x^* and repeat the process

endif

endif

2 Generate a random number r.

If ($r \le 0.25$) then

$x = 2 + 4\sqrt{r}$

else

$x = 10 - 4\sqrt{3 - 3r}$

endif

4

	$E(x)$	var x
After 250 variates	5.274	2.823
After 500 variates	5.318	2.755
After 1,000 variates	5.364	2.856
After 5,000 variates	5.344	2.887
Theoretical values	5.333	2.889

REVIEW PROBLEMS

1 0.70, 0.33, 0.04, 0.11, 0.30, 0.53, 0.44, 0.91, 0.90, 0.73.

3 See Figure 16. Variables used in the model:

DAY = current day in the simulation

EXC = total number of days with net demand greater than 100

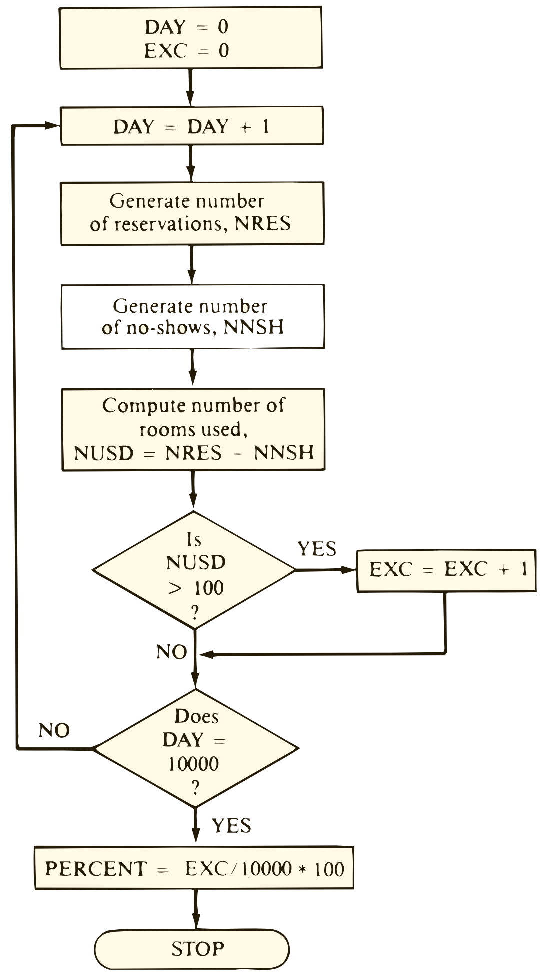
NRES = number of rooms reserved for the current day in simulation

NNSH = number of no-shows for the current day in simulation

NUSD = number of rooms used on the current day in simulation

PERCENT = percentage of days when net demand exeeded 100

FIGURE 16



Chapter 24

SECTION 24.4

1 **a** 4,000 cases.
b 3,980 cases.

2 **a** New base = 452; new trend = 43.6; new summer seasonality = 1.37.
b Forecast for winter quarter = 377.44.

4 95% sure that December sales will be between 172.5 and 297.5.

12 **a** 106.
b 115.26.

SECTION 24.5

1 For each professor's payday, compute

$$\frac{\text{Actual customers}}{\text{Forecast customers}}$$

Average these ratios. Suppose we obtain 1.3. Then to obtain forecast for a day on which professors are paid, compute a forecast by our basic method and multiply this forecast by 1.3.

SECTION 24.8

1 **a** Estimate beta = 0.88.

b Yes.

c 45%.

e 16.1%.

3 **a** $\widehat{\text{SALES}} = 52{,}900 + 912.5T - 9{,}859Q_1 - 8{,}467Q_2 + 20{,}129Q_4$, where T = quarter number and Q_i = dummy variable for quarter i.

d The part (b) model has smaller standard error than the part (a) model. Thus, the part (b) model will yield a better forecast.

4 $\widehat{\text{SALES}} = e^7\ \text{PRICE}^{-0.67}$.

5 **b** Indicates positive autocorrelation.