CS 162 **Fall 2015**

Homework 2 Problems

October 8, 2015 *Timothy Johnson*

1. Exercise 2.5.2 on page 79 of Hopcroft et al.

   Consider the following ϵ-NFA.

	ϵ	a	b	c
$\to p$	$\{q, r\}$	$\emptyset$	$\{q\}$	$\{r\}$
q	$\emptyset$	$\{p\}$	$\{r\}$	$\{p, q\}$
$*r$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

   (a) Compute the ϵ-closure of each state.

   $p \to \{p, q, r\}$
   $q \to \{q\}$
   $r \to \{r\}$

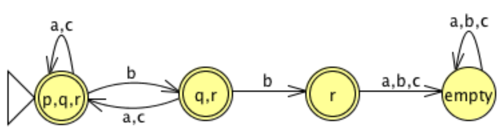
   (b) Give all the strings of length three or less accepted by the automaton.

   ϵ, a, b, c, ba, bb, bc, ca, cb, cc, aaa, aab, aac, aba, abb, baa, bab, bac, bca, bcb, bcc, caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc

   (c) Convert the automaton to a DFA. (Please construct the table, and then draw the diagram.)

   We first construct the table below:

	a	b	c
$\to *\{p, q, r\}$	$\{p, q, r\}$	$\{q, r\}$	$\{p, q, r\}$
$*\{q, r\}$	$\{p, q, r\}$	$\{r\}$	$\{p, q, r\}$
$*\{r\}$	$\emptyset$	$\emptyset$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$



2. Exercise 3.1.1 on page 91 of Hopcroft et al.

   Write regular expressions for the following languages.

   (a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

   $(\mathbf{A}+\mathbf{B}+\mathbf{C})^*(\mathbf{A}(\mathbf{A}+\mathbf{B}+\mathbf{C})^*\mathbf{B}+\mathbf{B}(\mathbf{A}+\mathbf{B}+\mathbf{C})^*\mathbf{A})(\mathbf{A}+\mathbf{B}+\mathbf{C})^*$

   (b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

   $(\mathbf{0}+\mathbf{1})^*\mathbf{1}(\mathbf{0}+\mathbf{1})^{\mathbf{9}}$

   (c) The set of strings of 0's and 1's with at most one pair of consecutive 1's.

   $(\mathbf{0}+\mathbf{10})^*(\mathbf{11}+\epsilon)(\mathbf{0}+\mathbf{10})^*$

3. Exercise 3.1.4 on page 92 of Hopcroft et al.

   Give English descriptions of the languages of the following regular expressions.

   (a) $(\mathbf{1}+\epsilon)(\mathbf{00}^*\mathbf{1})^*\mathbf{0}^*$

   This is the language of strings with no two consecutive 1's.

   (b) $(\mathbf{0}^*\mathbf{1}^*)^*\mathbf{000}(\mathbf{0}+\mathbf{1})^*$

   This is the language of strings with three consecutive 0's.

   (c) $(\mathbf{0}+\mathbf{10})^*\mathbf{1}^*$

   This is the language of strings in which there are no two consecutive 1's, except for possibly a string of 1's at the end.

4. Exercise 3.2.1 on page 107 of Hopcroft et al.

   Here is a transition table for a DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

   (a) Give all the regular expressions $R_{ij}^{(0)}$. Note: Think of state q_i as if it were the state with integer number i.

$$
\begin{aligned}
R_{11}^{(0)} &= \mathbf{1} + \epsilon \\
R_{12}^{(0)} &= \mathbf{0} \\
R_{13}^{(0)} &= \emptyset \\
R_{21}^{(0)} &= \mathbf{1} \\
R_{22}^{(0)} &= \epsilon \\
R_{23}^{(0)} &= \mathbf{0} \\
R_{31}^{(0)} &= \emptyset \\
R_{32}^{(0)} &= \mathbf{1} \\
R_{33}^{(0)} &= \mathbf{0} + \epsilon
\end{aligned}
$$

(b) Give all the regular expressions $R_{ij}^{(1)}$. Try to simplify the expressions as much as possible.

$$
\begin{aligned}
R_{11}^{(1)} &= R_{11}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\
&= (\mathbf{1} + \epsilon) + (\mathbf{1} + \epsilon)(\mathbf{1} + \epsilon)^*(\mathbf{1} + \epsilon) \\
&= \mathbf{1}^* \\
R_{12}^{(1)} &= R_{12}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\
&= \mathbf{0} + (\mathbf{1} + \epsilon)(\mathbf{1} + \epsilon)^* \mathbf{0} \\
&= \mathbf{1}^* \mathbf{0} \\
R_{13}^{(1)} &= R_{13}^{(0)} + R_{11}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\
&= \emptyset \\
R_{21}^{(1)} &= R_{21}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\
&= \mathbf{1} + \mathbf{1}(\mathbf{1} + \epsilon)^*(\mathbf{1} + \epsilon) \\
&= \mathbf{1}^+ \\
R_{22}^{(1)} &= R_{22}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\
&= \epsilon + \mathbf{1}(\mathbf{1}^*)\mathbf{0} \\
&= \epsilon + \mathbf{1}^+ \mathbf{0} \\
R_{23}^{(1)} &= R_{23}^{(0)} + R_{21}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\
&= \mathbf{0} \\
R_{31}^{(1)} &= R_{31}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{11}^{(0)} \\
&= \emptyset \\
R_{32}^{(1)} &= R_{32}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{12}^{(0)} \\
&= \mathbf{1} \\
R_{33}^{(1)} &= R_{33}^{(0)} + R_{31}^{(0)}(R_{11}^{(0)})^* R_{13}^{(0)} \\
&= \mathbf{0} + \epsilon
\end{aligned}
$$

(c) Give all the regular expressions $R_{ij}^{(2)}$. Try to simplify as much as possible.

$$
\begin{aligned}
R_{11}^{(2)} &= R_{11}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{21}^{(1)} \\
&= \mathbf{1}^* + \mathbf{1} * \mathbf{0}(\epsilon + \mathbf{1}^+\mathbf{0})^*\mathbf{1}^+ \\
&= (\mathbf{1} + \mathbf{01})^* \\
R_{12}^{(2)} &= R_{12}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\
&= R_{12}^{(1)}(R_{22}^{(1)})^* \\
&= \mathbf{1}^*\mathbf{0}(\epsilon + \mathbf{1}^+\mathbf{0})^* \\
&= (\mathbf{1} + \mathbf{01})^*\mathbf{0} \\
R_{13}^{(2)} &= R_{13}^{(1)} + R_{12}^{(1)}(R_{22}^{(1)})^* R_{23}^{(1)} \\
&= \emptyset + \mathbf{1}^*\mathbf{0}(\epsilon + \mathbf{1}^+\mathbf{0})^*\mathbf{0} \\
&= (\mathbf{1} + \mathbf{01})^*\mathbf{00} \\
R_{21}^{(2)} &= R_{21}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{21}^{(1)} \\
&= (R_{22}^{(1)})^* R_{21}^{(1)} \\
&= (\epsilon + \mathbf{1}^+\mathbf{0})\mathbf{1}^+ \\
&= \mathbf{1}^+(\epsilon + \mathbf{01}^+) \\
R_{22}^{(2)} &= R_{22}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\
&= (R_{22}^{(1)})^+ \\
&= (\epsilon + \mathbf{1}^+\mathbf{0})^+ \\
&= (\mathbf{1}^+\mathbf{0})^* \\
R_{23}^{(2)} &= R_{23}^{(1)} + R_{22}^{(1)}(R_{22}^{(1)})^* R_{23}^{(1)} \\
&= (R_{22}^{(2)})^* R_{23}^{(1)} \\
&= (\epsilon + \mathbf{1}^+\mathbf{0})^*\mathbf{0} \\
&= (\mathbf{1}^+\mathbf{0})^*\mathbf{0} \\
R_{31}^{(2)} &= R_{31}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{21}^{(1)} \\
&= \emptyset + \mathbf{1}(\epsilon + \mathbf{1}^+\mathbf{0})^*\mathbf{1}^+ \\
&= \mathbf{1}(\mathbf{1}^+\mathbf{0})^*\mathbf{1}^+ \\
R_{32}^{(2)} &= R_{32}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{22}^{(1)} \\
&= \mathbf{1} + \mathbf{1}(\epsilon + \mathbf{1}^+\mathbf{0})^+ \\
&= \mathbf{1}(\mathbf{1}^+\mathbf{0})^* \\
R_{33}^{(2)} &= R_{33}^{(1)} + R_{32}^{(1)}(R_{22}^{(1)})^* R_{23}^{(1)} \\
&= (\mathbf{0} + \epsilon) + \mathbf{1}(\epsilon + \mathbf{1}^+\mathbf{0})^*\mathbf{0} \\
&= \mathbf{0} + \mathbf{1}(\mathbf{1}^+\mathbf{0})^*\mathbf{0} + \epsilon
\end{aligned}
$$

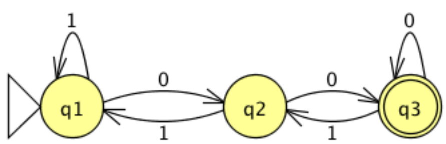
(d) Give a regular expression for the language of the automaton.

The language of our DFA is $R_{13}^{(3)}$.

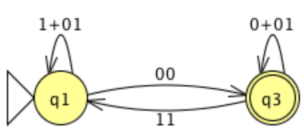
$$\begin{aligned}
R_{13}^{(3)} &= R_{13}^{(2)} + R_{13}^{(2)}(R_{33}^{(2)})^* R_{33}^{(2)} \\
&= R_{13}^{(2)}(R_{33}^{(2)})^* \\
&= (\mathbf{1} + \mathbf{01})^*\mathbf{00}(\mathbf{0} + \mathbf{1}(\mathbf{1}^+\mathbf{0})^*\mathbf{0} + \epsilon))^* \\
&= (\mathbf{1} + \mathbf{01})^*\mathbf{00}(\mathbf{0} + \mathbf{1}(\mathbf{1}^+\mathbf{0})^*\mathbf{0})^*
\end{aligned}$$

(e) Construct the transition diagram for the DFA and give a regular expression for its language by eliminating state q_2.

The transition diagram is:



When we eliminate q_2, we get the following diagram:



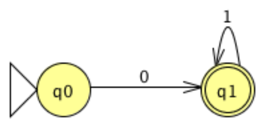
This gives us the following regular expression for the language of our DFA:

$$[\mathbf{1} + \mathbf{01} + \mathbf{00}(\mathbf{0} + \mathbf{10})^*\mathbf{11}]^*\mathbf{00}(\mathbf{0} + \mathbf{10})^*$$

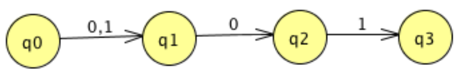
5. Exercise 3.2.4 on page 108 of Hopcroft et al.

Convert the following regular expressions to NFA's with ϵ-transitions. (I've simplified my solutions somewhat, but some students may turn in equivalent solutions that are more complicated because they followed the book exactly, which is fine.)

(a) $\mathbf{01}^*$.



(b) $(\mathbf{0}+\mathbf{1})\mathbf{01}$.



(c) $\mathbf{00}(\mathbf{0}+\mathbf{1})^*$.