MATH 2116: Linear Algebra

Class Note: 01

Insitute of Information Technology (IIT), DU

BSSE (3rd Sem)

2024/2025

Linear Algebra

Wikipedia says, "**Linear algebra** is the branch of mathematics concerning **linear equations**. - - - Until the 19th century, linear algebra was introduced through **systems of linear equations** and **matrices**. In modern mathematics, the presentation through **vector spaces** is generally preferred, since it is more synthetic, more general, and conceptually simpler, although more abstract."

What is Linear Algebra?

Basic Definition

Linear Algebra is the branch of mathematics aimed at solving systems of linear equations with a finite number of unknowns. In particular, one would like to obtain answers to the following questions:

- Are there solutions to a given system of linear equations? How many solutions are there?
- How does the solution set look? What are the solutions?

Linear Algebra is a systematic theory regarding the solutions of systems of linear equations.

What is Linear Algebra?

Modern Definition

Linear algebra is about **linear combinations**. That is, using arithmetic on columns of numbers called vectors and arrays of numbers called matrices, to create new columns and arrays of numbers.

Linear algebra is a branch of mathematics. It has a wide range of applications in Physics and Mathematics. But the truth of it is that linear algebra is the mathematics of data. Matrices and vectors are the language of data. It is the basic concept for machine learning and data science.

Linear algebra is a field of mathematics that is universally agreed to be a prerequisite to a deeper understanding of machine learning.

Recall the school Math I

1. The sum of two numbers is 50 and the difference of them is 20. Find the numbers.
2. Mike and Sally are math teachers at the same high school. If Mike has 6 more years of teaching experience than Sally has, and the two of them have 16 years of teaching experience combined, how many years of teaching experience does Sally have?
3. The cost of a ticket to the circus is Tk. 25.00 for children and Tk. 50.00 for adults. On a certain day, attendance at the circus is 2,000 and the total gate revenue is Tk. 70,000. How many children and how many adults bought tickets?
4. Meal tickets at the circus cost Tk. 4.00 for children and Tk. 12.00 for adults. If 1,650 meal tickets were bought for a total of Tk. 14,200, how many children and how many adults bought meal tickets?

Recall the school Math II

5. A river cruise ship sailed 60 miles downstream for 4 hours and then took 5 hours sailing upstream to return to the dock. Find the speed of the ship in still water and the speed of the river current.
6. A farmer has two types of milk, one that is 24% butterfat and another which is 18% butterfat. How much of each should he use to end up with 42 gallons of 20% butterfat?

How did we solve these in school?

Cross Multiplication Method

Cross-multiplication is a technique to determine the solution of linear equations in two variables. It proves to be the simplest, easiest and fastest method to solve a pair of linear equations. For a given pair of linear equations in two variables:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Using following **cross multiplication formula** we can solve this linear system:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

How did we solve ...

Substitution Method

The substitution method involves substituting the value of any one of the variables from one equation into the other equations. The steps to apply are given below:

- Step-1: Solve any one variable from one of the equations.
- Step-2: Substitute that value in the other equation.
- Step-3: Now, simplify the new equation to obtain the value of one variable.
- Step-4: Now, substitute the value obtained in Step 3 in any of the given equations to solve for the other variable.

How did we solve ...

Elimination Method

The steps to use the elimination method are:

- Step-1: Multiply each equation with a non-zero number to get a common coefficient of any one of the variables in both equations.
- Step-2: Add or subtract both equations to eliminate the same terms.
- Step-3: Simplify the result to get a value for the left out variable.
- Step-4: Substitute this value in any of the given equations to find the value of the other variable.

Thanks

Thank you all.