
SECTION – AThere are **FIVE** questions in this Section. Answer any **FOUR**.

1. (a) A beam is loaded as shown in Fig-1. Calculate tensile and compressive stresses at section 1-1 due to flexure only. Given that beam cross-section is $2'' \times 6''$. (16 ¼)
(b) Determine kern for a member having a hollow circular cross-section with outer diameter of D and inner diameter of d . (10)
2. (a) Using the moment area method, determine the deflection and the slope of the elastic curve at point A, due to the applied load for the beam shown in Fig-2. Specify the direction of deflection and rotation. Given that $E = 200 \text{ GPa}$, $I = 80 \times 10^6 \text{ mm}^4$. (16 ¼)
(b) Find the end moments (i.e., moments at the supports) for a fixed-end beam loaded with a uniformly distributed load of w_0 and span L . (10)
3. (a) Using direct integration method, determine the equation of elastic curves for segments AB and BC for the beam loaded as shown in Fig-3. The moment of inertia of the cross section of the beam is $3I$ for segment AB and I for segment BC. (26 ¼)
4. (a) For the state of stress shown in Fig-4, using the general equations for the transformation of stress find: (i) the principal stresses and show their sense on a properly oriented element (ii) the maximum shear stress with associated normal stress and show results on a properly oriented element. (16 ¼)
(b) Using Mohr's circle, transform the stresses shown in Fig-4 into stresses acting on the plane at an angle of $+ 30^\circ$ with the vertical axis. (10)
5. (a) Using equation of strain density, derive expression of total strain energy for shearing stress for a shaft subjected to torsional load. (10)
(b) For the beam loaded as shown in Fig-5 using strain energy method, calculate deflection at mid span of the beam. (16 ¼)

CE 213

SECTION – B

There are **FIVE** questions in this Section. Answer any **FOUR**.

6. Using AISC column specifications, determine the safe axial loads on a W 360 × 122 section used as a column under the following conditions: (26 ¼)
- (a) hinged ends and a length of 8.5 metre
- (b) fixed ends and a length of 12.0 metre
- given: $\sigma_{yp} = 300$ MPa and $E = 200$ GPa. From the table, $A = 15,500$ sq.mm and least radius of gyration, $r = 63.0$ mm for W 360 × 122 section.
7. For the suspension bridge shown in Fig. 6, determine maximum tension T_{max} , length of the cable and forces in the side cable. Consider the pier top support as saddle on roller. (26 ¼)
8. (a) Derive equation for maximum energy of distortion theory (Von Mises theory). Given that (10)
- (i) total strain energy for three principal stress is given by
- $$W = \frac{1}{2E} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \frac{\nu}{E} (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$
- (ii) Bulk modulus $k = \frac{E}{3(1-2\nu)}$ and $\nu = \frac{\sigma_m}{k}$
- (b) Establish the failure envelope from appropriate equation as per "Tresca' Theory. (16 ¼)
9. Bracket is loaded as shown in Fig. 7. If allowable shearing stress is 145 MN/m^2 , what size of fillet weld, nearest millimeter should be used? (26 ¼)
10. Find the allowable tensile force that the multiple-riveted structural joints shown in Fig. 8 can transmit. Also find the efficiency of the joint. All rivets are nominally 22 mm in 25 mm diameter holes. The allowable stresses are 150 MPa in tension, 100 MPa in shear and 335 MPa in bearing on the main as well as cover plates. (26 ¼)

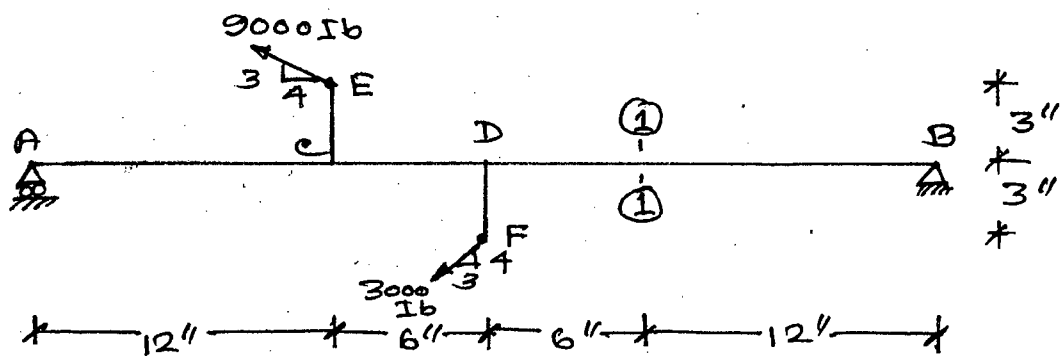


Fig-1

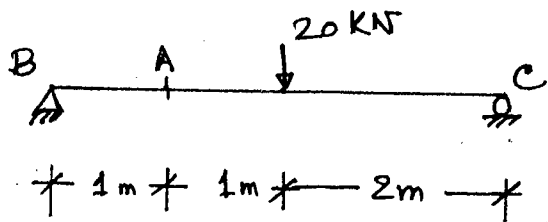


Fig-2

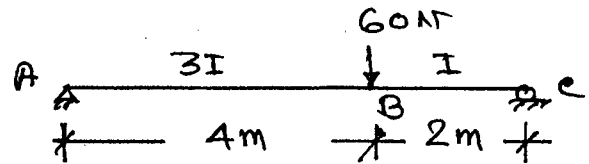


Fig-3

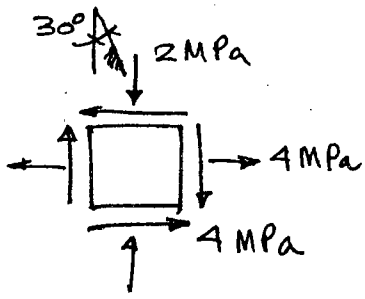


Fig-4

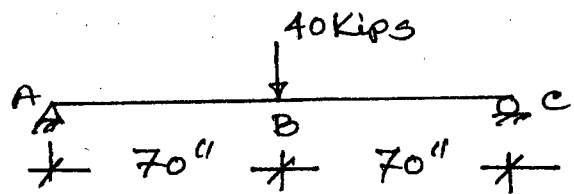
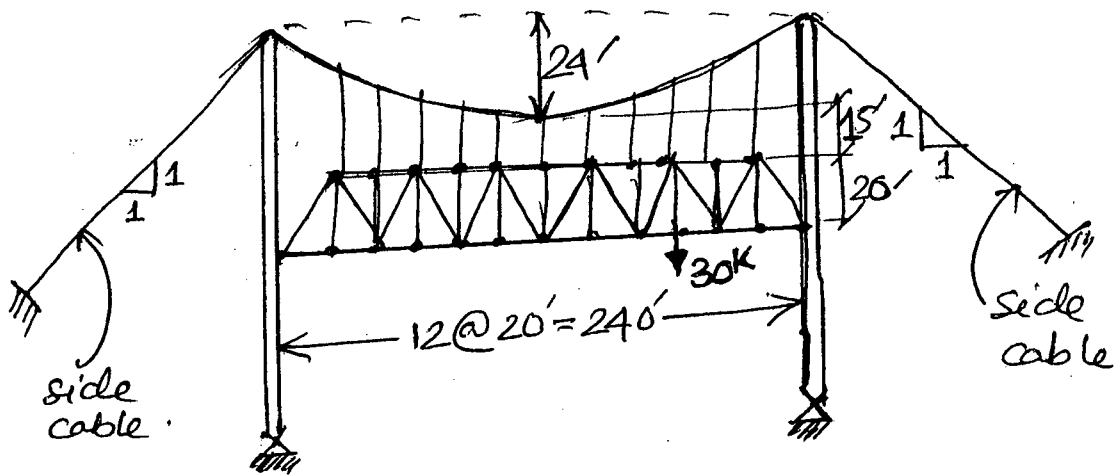


Fig-5



6
Fig. 4

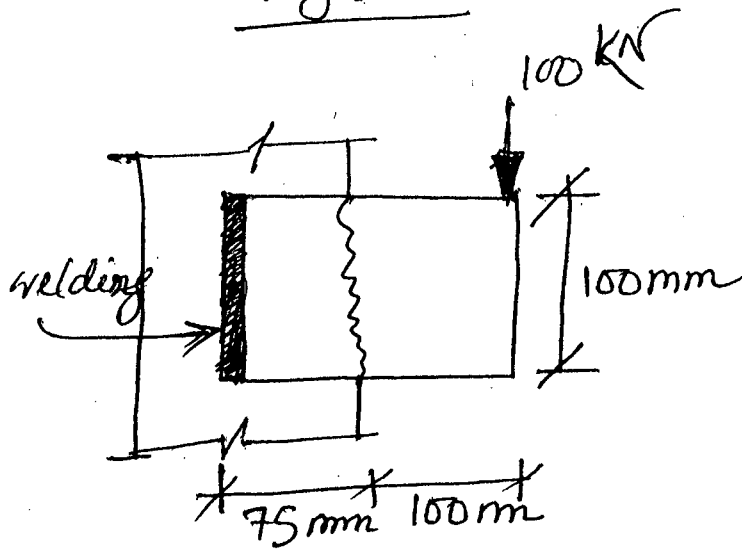


Fig. 5

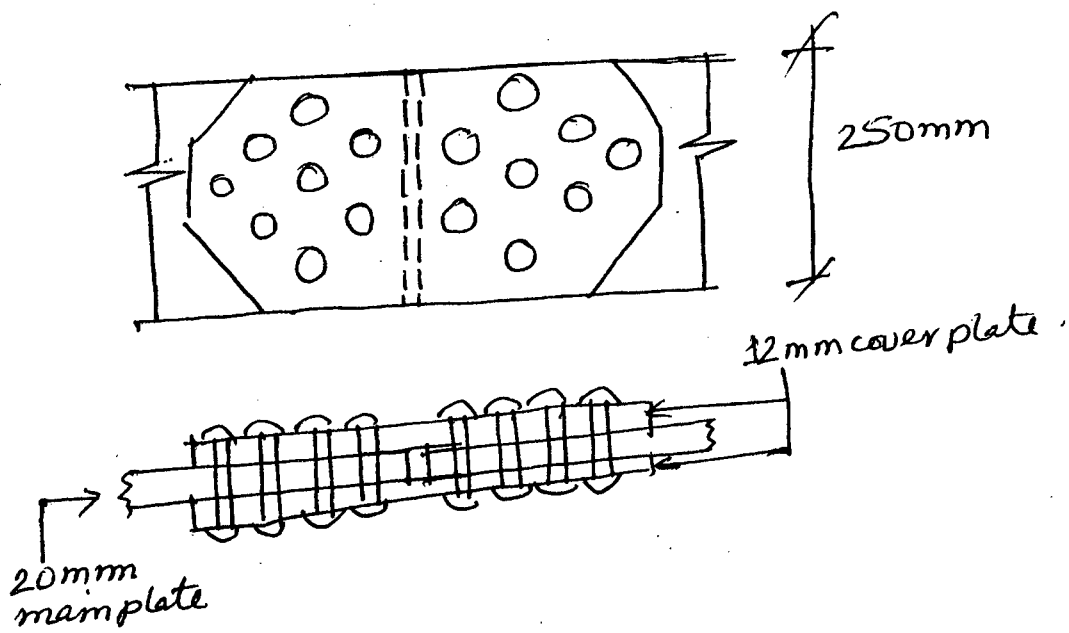


Fig. 6

SECTION - AThere are **FOUR** questions in this Section. Answer any **THREE**.

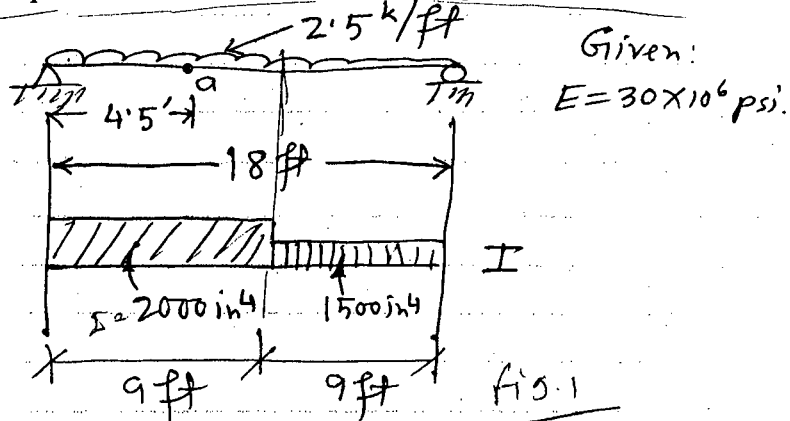
1. (a) Derive the final expression of Gregory-Newton Interpolation formula. (10 1/3)

(b) Set a polynomial equation passing through the points provided in the following table and use it to find the interpolated value of $x = 185$ without using the difference table. (13)

x	0	50	100	150	200	250
$f(x)$	0	0.093	0.169	0.243	0.321	0.429

2. (a) Derive the general expression of $I = \int_a^b f(x)dx$ using Simpson's Rule. (10)

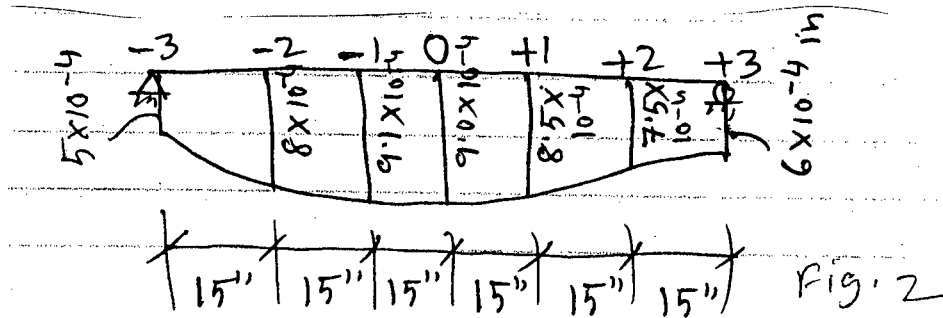
(b) Find the deflection at point 'a' for the beam shown in Fig. 1 (13 1/3)



3. (a) Explain Gauss-Quadrature method and derive the required parameters for $n = 4$. (11 1/3)

(b) The deflection at various points on a normally loaded beam is shown in Fig. 2.

Estimate bending moments at all the points and shear forces at -2 , 0 and $+2$. (12)



4. (a) Write short notes on (10)

(i) Gauss-Jordan elimination method

(ii) Cramer's Rule.

(b) Estimate $\int_1^3 \frac{1}{x} dx$ using Romberg's quadrature; use $\epsilon = 0.0001$. (13 1/3)

CE 205

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) In environmental engineering, the following equation can be used to compute the oxygen level c (in mg/L) in a river downstream from a sewage discharge

(16)

$$c = 10 - 20(e^{-0.15x} - e^{-0.5x})$$

Where x is the distance downstream in kilometers

- Determine the distance downstream where the oxygen level falls to a reading of 5mg/L.
 - Determine the distance downstream at which the oxygen level is at a minimum (i.e., $dc/dx = 0$) [In both cases use bisection method and determine your answer to a 1% error. As an initial guess, assume that your solution lies within $x = 0$ and $x = 5$ km.]
 - how many bisection iterations would it require to determine the distance downstream (x) to an absolute error of 0.01 km with your initial guess range?
- (b) What are the main motivations of learning numerical methods? What is the difference between round-off error and truncation error? Explain the relationship between total numerical error and step-size.

(7 1/3)

6. (a) The volume V of a liquid in a spherical tank of radius r is related to the depth h of the liquid by $V = \frac{\pi h^2(3r - h)}{3}$

(14 1/3)

- if $r = 1$ m and $V = 0.75$ m³, determine the depth of the tank, h . [Use Newton-Raphson method with an initial guess of $h = 1$ m]
- The following two formulas can be developed in order to solve the problem using fixed point iteration method:

$$h = \sqrt{\frac{h^3 + (3V/\pi)}{3r}} \quad \text{and} \quad h = \sqrt[3]{3(rh^2 - V/\pi)}$$

if $r = 1$ m and $V = 0.75$ m³, determine whether either of these is stable using the convergence criteria of fixed point iteration.

- (b) Use nonlinear regression to fit the equation $y = a 10^{bx}$ to the data shown in the following table:

(9)

x	4	8	12	16	20	24
y	1600	1320	1000	890	650	560

7. (a) Use Heun's method to solve $\frac{d^2x}{dt^2} + 5x \frac{dx}{dt} + (x+7)\sin(t) = 0$

over the range $t = 0$ to 12 using a step size of 3 with $x(0) = 6$ and $\frac{dx}{dt}(0) = 1.5$

(11)

CE 205

Contd ... Q. No. 7

(b) Solve the following equation for $y(1.0)$

$$\frac{dy}{dt} = 4e^{0.8t} - 0.5y$$

using a step size of 0.5 with $y(0) = 2$. Use the 4th order Runge-Kutta method.

(10 1/3)

(c) How can you improve the estimate of Euler's method?

(2)

8. (a) Suppose the position of a falling object is governed by the following differential equation.

$$\frac{d^2x}{dt^2} + \frac{c}{m} \frac{dx}{dt} - g = 0$$

where, c = drag coefficient = 12.5 kg/s, m = mass = 70 kg, $g = 9.81 \text{ m/s}^2$.

(10 1/3)

For the boundary conditions: $x(0) = 0$ and $x(12) = 500$ and step-size, $\Delta x = 2$, write down the finite difference equations (in matrix form) to solve the equation for position of the falling object. No need to solve the problem.

(b) Write down the steps for solving the problem in part (a) using Shooting method.

(5)

(c) Use the Taylor series to estimate $f(x) = e^{-x}$ at $x_{i+1} = 1$ using $x_i = 0.2$. Employ the zero-first, second- and third-order versions of the Taylor series and compute the true % error for each case.

(8)

L-2/T-2/CE

Date : 06/01/2015

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B.Sc. Engineering Examinations 2012-2013

Sub : **WRE 211** (Fluid Mechanics)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

Assume reasonable values if needed. Symbols have their usual meanings.

1. (a) Define (i) steady flow, (ii) unsteady flow, (iii) uniform flow, (iv) non-uniform flow. (4)
(b) Derive the general equation of continuity for flow through regions with fixed boundaries. (6)
(c) Water flows in a circular pipe of radius 10 cm. The maximum velocity at the centre of the pipe is 1 m/s and varies linearly to 0.1 m/s at the wall of the pipe. Compute momentum correction factor β . (10)
(d) Referring to Figure 1, compute the accelerations a , a_x , and a_y at points A and B of the bent stream tube. The radius of the bend is 1 m measured at center of the tube. (15)
2. (a) Define (i) streamline, (ii) turbulent flow. (4)
(b) Derive the general equation for steady flow of any fluid using the first law of thermodynamics. (6)
(c) A pump lifts water at a rate of $6 \text{ m}^3/\text{s}$ to a height of 120 m and the friction loss in the pipe is 10 m. What is the horsepower required if the pump efficiency is 90 percent. Also compute the change in water temperature due to the head loss. (c for water = $4.187 \text{ N.m}/(\text{kg}).(\text{K}).$) (10)
(d) A liquid with a specific gravity of 1.26 flows in a pipe at a rate of 700 L/s. At a point where the pipe diameter is 60 cm, the pressure is 100 kN/m^2 . Find the pressure at a second point where the pipe diameter is 30 cm and the second point is 1.0 m higher than the first point. If two pitot tubes are inserted at these two points and the tubes are connected by a differential manometer ($s = 0.82$), find the manometer reading. Show your result in a sketch. Neglect head loss. (15)
3. (a) Briefly describe the working principle of a siphon. (4)
(b) Derive the one dimensional Euler equation of steady motion along a streamline for ideal fluid and hence derive the Bernoulli's equation. What modifications do you need to make if the fluid is real? (6)
(c) Water is flowing under a sluice gate. The depth of flow far upstream of the gate is 2 m and downstream of the gate is 0.8 m. Compute discharge. Neglect head loss. (10)

Contd P/2

WRE 211

Contd ... Q. No. 3

- (d) Determine the magnitude and direction of the force exerted by the water on the double nozzle of Figure 2. Both nozzle jets have a velocity of 10 m/s. The axes of the pipe and both nozzles all lie the same axis. Neglect head loss. (15)
4. (a) Write short notes on piezometer, pitot tube and Prandtl tube. (4)
- (b) Briefly describe the procedure to compute force on a single and series of moving vanes. (6)
- (c) Referring to Figure 3, compute the horizontal component of force of water jet on the blade if (i) the blade is fixed in position (ii) moving to the right at 12 m/s (iii) moving to the left at 12 m/s. $v_1 = 10$ m/s, $v_2 = 9$ ms. Initial jet diameter = 5 cm. Flow occurs in on a horizontal plane. (10)
- (d) The diameter of a horizontal pipe at point A is 90 cm and reduces to 30 cm at point B. The pressure at point A is 70 kPa. Compute the force of a flowing fluid ($s = 0.86$ on the pipe section AB if cavitation just occurs at point B neglecting head loss. P_{vap} of the fluid = 26 kPa, abs. (15)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Difference between (10)
- (i) Adhesion and cohesion.
 - (ii) Ideal and real fluid.
 - (iii) Simple and Differential manometer.
 - (iv) Compressible and incompressible fluid
- (b) Two reservoirs with a difference in water surface elevation of 7 m are connected by two pipes in series as shown in Figure 4. The equivalent roughness heights are 2.0 mm and 0.4 mm respectively. Find discharge by equivalent velocity method. Given $\nu = 3 \times 10^{-6}$ m²/s. Use Moody diagram for friction factor. (15)
- (c) Derive the expression for friction loss with laminar flow. (10)
6. (a) The pipes in the system shown in Figure 5 are all new Cast iron ($e = 0.25$ mm). With a flow of 0.8 m³/s, find the head loss from B to C. Given $\nu = 1.14 \times 10^{-6}$ m²/s. (18)
- (b) Write briefly (i) Why is mercury used in Barometer? (9)
- (ii) Show that center of pressure is below the center of gravity.
 - (iii) Relationship between gas (and liquid) and temperature.

WRE 211

Contd ... Q. No. 6

- (c) A cubical block weighing 200 gm and having a 20 cm edge is allowed to slide down on an inclined plane surface making an angle of 30° with the horizontal on which there is a thin film having a viscosity of $2.2 \times 10^{-3} \text{ N-s m}^2$. What terminal velocity will be attained if the film thickness is estimated to be 0.025 mm. (8)
7. (a) Define absolute pressure, vacuum pressure and gage pressure. How are they interrelated? (6)
- (b) If the flows into and out of a two-loop pipe system are shown in Figure 6, determine the flow in each pipe. The K-values for each pipe are to be calculated from the data given for each pipe. Use Hardy-Cross method. Make one trial. (20)
- (c) Find the resultant force acting on the curved section of the open tank. The shape of the curve is a quadrant of a circle of radius 2 m. The upper portion of the curve is at a depth of 2 m from the free surface. (9)
8. (a) Write short notes on (6)
- (i) Viscous sublayer
 - (ii) Buoyancy
 - (iii) Center of pressure
- (b) The head loss in 60 m of 15 cm diameter pipe is known to be 8 m when oil ($s = 0.85$) of viscosity 0.04 N-s/m^2 flows at $0.06 \text{ m}^3/\text{s}$. Determine the centerline velocity, the shear stress at the wall of the pipe and the velocity at 5 cm from the centerline. (14)
- (c) The gate shown in Figure 7 is 60 cm wide perpendicular to the sketch. It is pivoted at O. The gate weighs 2224 N. Its center of gravity is 36 cm to the right and 27 cm above O. For what values of water depth X above O will the gate remain closed. Neglect friction at the pivot and neglect thickness of the gate. (15)
-

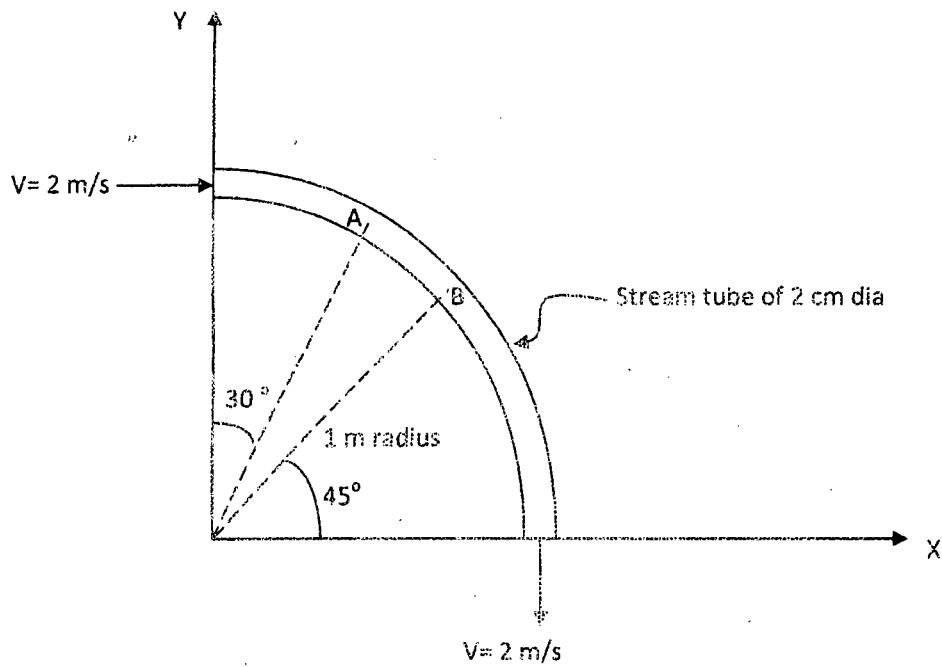


Figure 1 for Question No. 1(d)

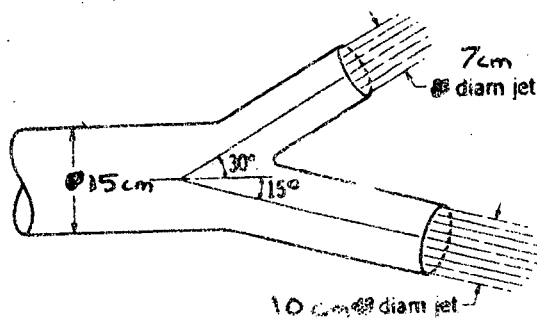


Figure 2 for Question No 3(d)



Figure 3 for Question No 4 (c) for fixed blade condition.

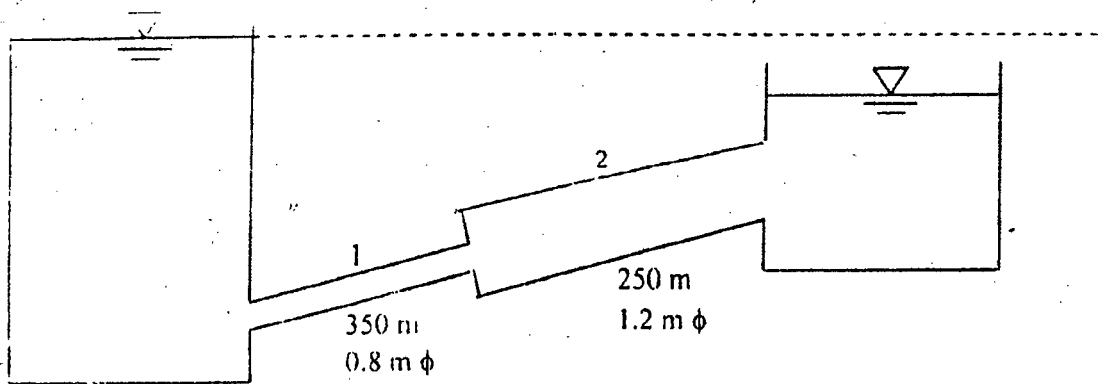


Figure 4 for Q. No. 5(b)

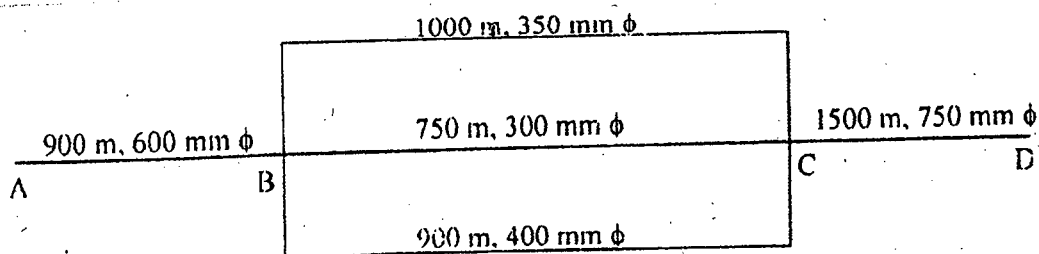


Figure 5 for Q. No. 6(a)

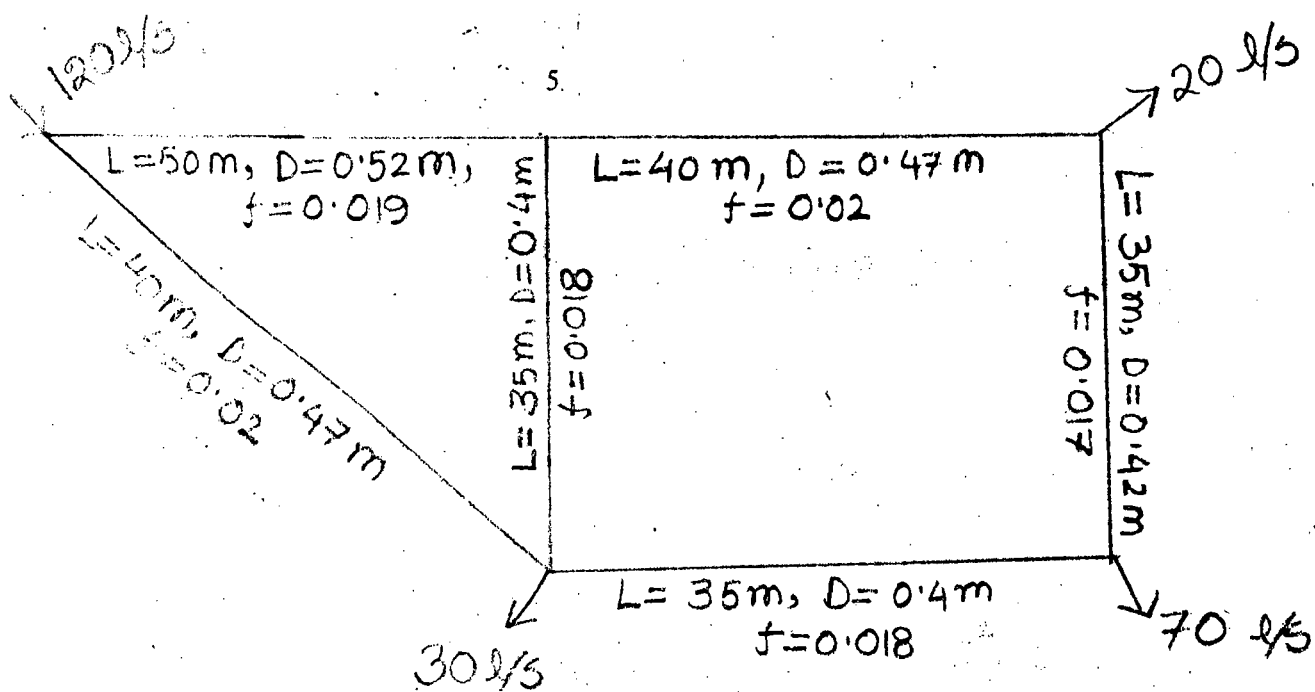


Figure 6 for Q. No. 7(b)

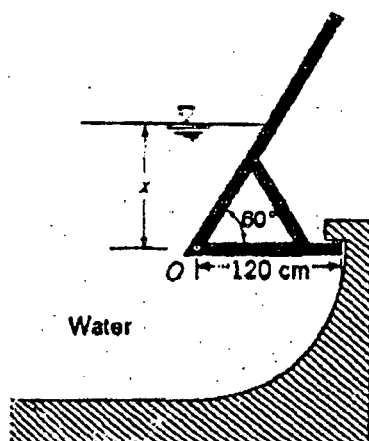
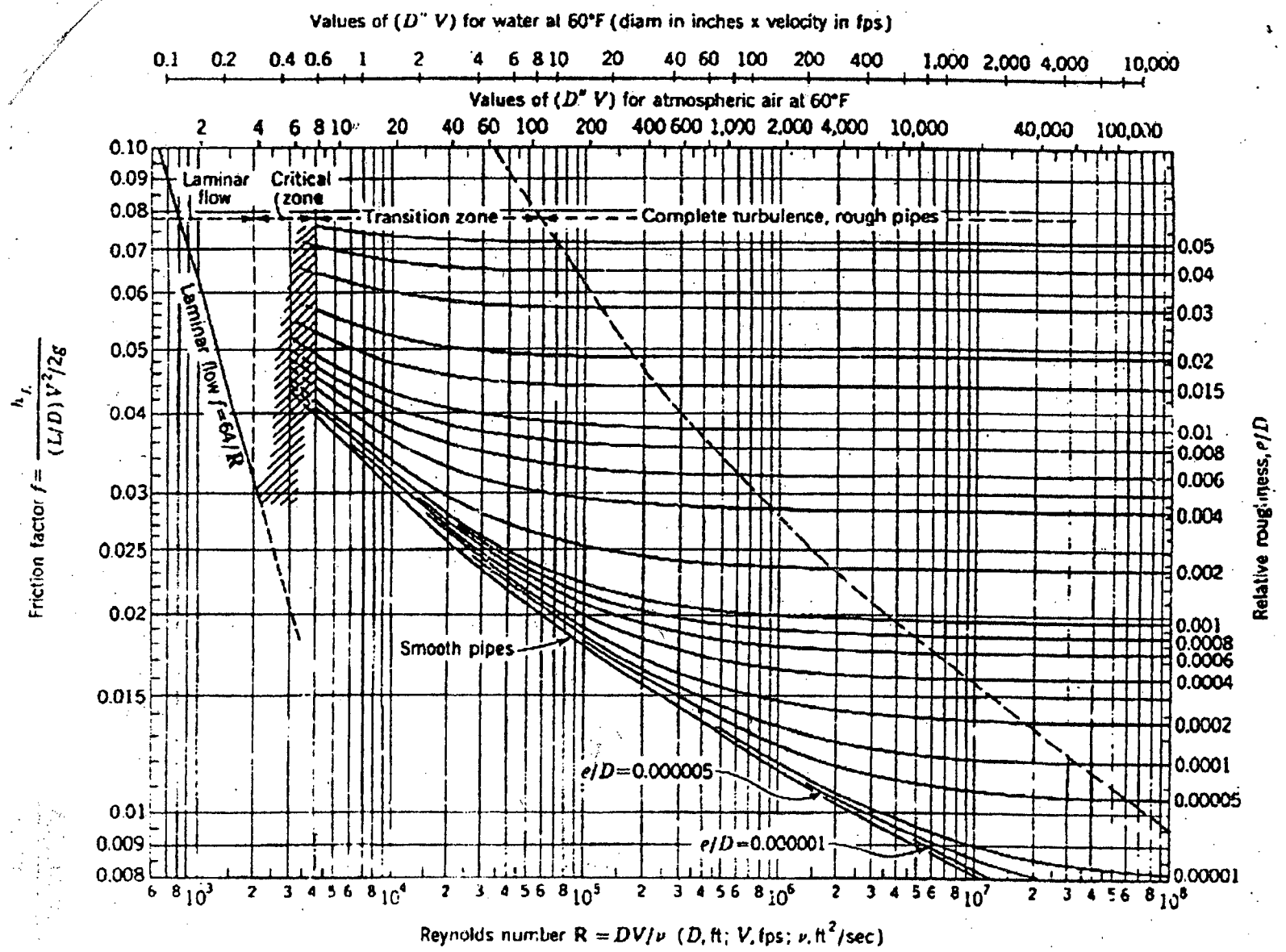


Figure 7 for Q. No. 8(c)



SECTION – AThere are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) What are the factors that influence the shifting of a demand curve? Explain. (10)
- (b) What are the exceptions to the law of demand? Explain them. (10)
- (c) From the following demand function, make a hypothetical demand schedule and plot the graph. $Q = 60 - 15p + p^2$. (3 ⅓)

2. (a) Show that price elasticity of demand varies from zero to infinity along any straight line demand curve. Explain graphically. (10)
- (b) From the following table calculate elasticity of demand if you move from point B to C and explain what you understand from the result. (10)

Point	P_x	Q_y
A	1200	250
B	1500	200
C	1800	180

- (c) What are the determinants of price elasticity of demand? (3 ⅓)
3. (a) How is price determined in an economy under competition? What will happen to the equilibrium price and quantity due to change in supply? (10)
- (b) From the following demand and supply functions calculate equilibrium price and quantity and show the result in a graph. What will happen to the equilibrium price and quantity if government imposes a unit tax of Tk. 2 per unit? (13 ⅓)

$$P = 0.1Q + 8$$

$$P = -0.5Q + 50$$

4. (a) Explain the properties of an indifference curve. (10)
- (b) Explain consumer's equilibrium with the help of budget line and indifference curve. (13 ⅓)

= 2 =

CE 303**SECTION – B**

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) What is called production function? A labor intensive firm used 10 units of labor in its daily production process, hypothetically show some total product, marginal product and average product of that firm in a table. (10)
- (b) What do you know about Law of Variable Proportions? (3 ⅓)
- (c) How can you show the exhaustion of factors income according to the Euler's Theorem? (10)
6. (a) If firm 'A' produces 7 units output and its Total Fixed Cost (TFC) is 50 taka then find out the Total Variable Cost (TVC) and Total Cost (TC) of each unit of firm 'A'. Draw a graph of Total Cost, Total Variable Cost and Total Fixed Cost curve of firm 'A'. (10)
- (b) What are the assumptions of a perfect competition market? (3 ⅓)
- (c) Under which conditions can a firm attain an equilibrium position in a perfectly competitive market? How can a short run firm reach in an equilibrium position with profits under perfect competition market? (10)
7. (a) What do you mean by NI, GDP, GNP and NNP? (6)
- (b) How can you determine equilibrium level of national income with the help of basic Keynesian Model? (8)
- (c) What are difficulties of measurement of national income in least developed countries, like Bangladesh? (6)
- (d) What do you mean by Value Added Method? (3 ⅓)
8. (a) What is called Inflation? What are the causes behind Inflation in Bangladesh? (5)
- (b) Distinguish between Demand-Pull Inflation and Cost-Push Inflation. (4 ⅓)
- (c) What are the ways to control Inflation in Bangladesh? Explain. (6)
- (d) What is Inflationary and Deflationary gaps? Illustrate with graph. (8)

The figures in the margin indicate full marks.

Symbols carry their common meanings. Assume reasonable values for missing data, if any.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Distinguish between dependent variable and independent variable. (7)
 - (b) An airplane flies at a velocity, v . Explain the phenomena, identify the systems affecting the flight and make a list. In this course (i) Derive the equations in differential form for the attained velocity overcoming the wind drag. (ii) In the equation mark out the dependent and independent variables (iii) Which solution strategy will you adopt to solve the equation you derived? (20)
 - (c) Define with examples: (i) Explicit solution (ii) Singular solution. Write one example for each from your engineering perspective. (8)
2. (a) A tank in Fig. 1 contains 1000 gal. of water in which 200 lb of salt are dissolved. 50 gallons of brine containing $(1 + \cos t)$ lb of dissolved salt run into the tank per minute. The mixture, kept uniform by stirring, runs out also at the same rate. Find out the amount of salt $y(t)$ in the tank at any time, t . In the process write down standard solution steps that you will logically consider. (25)

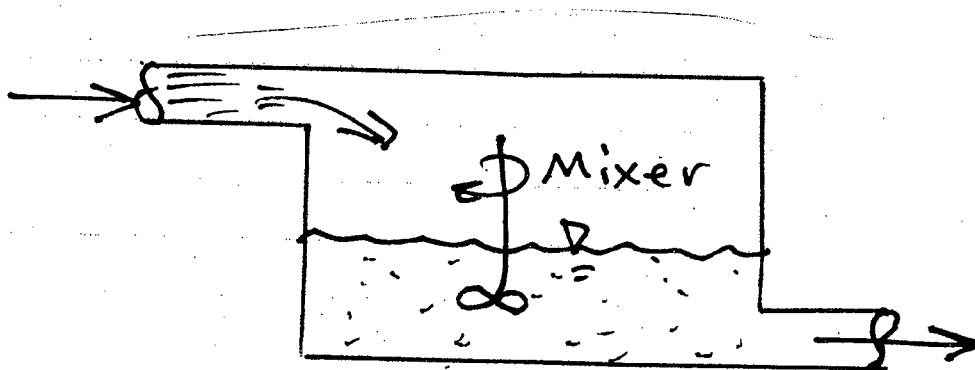


Figure 1.

- (b) Write down the conditions that are necessary to confirm the existence of a power series solution of a differential equation. (5)
- (c) Write a short note on "Orthogonality" of Legendre Polynomials. (5)

Contd P/2

CE 207

3. (a) What are the cases when you will consider "Power series Method" in solving a differential equation. (10)
- (b) Write down the Bessel's differential equation. Mention the conditions at which it can be solved using Frobenius Method. (15)
- (c) What is indicial equation? Derive it from Bessel's equation and describe the roots with physical significance. (10)
4. (a) Determine Fourier Transform of $\frac{2x}{x^2 + 4x + 8}$. (15)
- (b) An Oscillatory system is subjected to a dynamic force $\beta(t) = H(t) - 2H(t-1) + H(t-2)$; where, H is the Heaviside Step Function. Equation of motion of the oscillatory system is given by $4\ddot{u} + 16u = \beta(t)$. Determine the response $u(t)$. (20)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) If X_1, X_2, \dots, X_n are independent Normal random variables with each having the same parameter μ and σ , determine the maximum likelihood estimators of μ and σ (Derive expressions). (14)
- (b) Civil Engineers believe that W , the amount of weight (in units of 1000 pounds) that a certain span of a bridge can withstand without structural damage resulting, is normally distributed with mean 400 and standard deviation 40. Suppose that the weight (again, in units of 1000 pounds) of a car is a random variable with mean 3 and standard deviation 0.3. How many cars would have to be on the bridge span for the probability of structural damage to exceed 0.1? (14)
- (c) Suppose the probability that an item produced by a certain machine will be defective is 0.2. Find the probability that a sample of 9 items will contain at most one defective item. Assume that the quality of successive items is independent. (7)
6. (a) A communication system consists of n components, each component functions independently with a probability p . The total system can operate effectively if at least one half of its components function. For what values of p will a 5 component system be more likely to operate effectively than a 3 component system? (6)
- (b) A company supplies plastic sheets for industrial use. A new type of plastic has been produced in the company would like to claim that the average stress resistance of this new product is at least 30.0, where stress resistance is measured in pounds per square inch (psi) necessary to crack the sheet. The following random sample was drawn off the production line: 30.1, 32.7, 22.5, 27.5, 27.7, 29.8, 28.9, 31.4, 31.2, 24.3, 26.4, 22.8, 29.1, 33.4, 32.5, 21.7. Based on this sample, would the claim clearly be unjustified? Assume normal distribution and use the five percent level of significance. (15)

Contd P/3

CE 207

Contd ... Q. No. 6

(c) The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 40$, $\sigma = 4$. What is the probability that in 2 of the next 4 years the rainfall will exceed 50 inches? Assume that the rainfalls in different years are independent.

(14)

7. (a) What do you mean by standard residuals? Discuss with diagram how plot of residuals can be helpful in assessing validity of the linear regression model.

(6)

(b) An engineering construction firm is currently working on power plants at three different sites. Let A_i denote the event that the plant at site i is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of A_1, A_2, A_3 , draw a Venn diagram, and shade the region corresponding to each one: (i) At least one plant is completed by the contract date; (ii) All plants are completed by the contract date; (iii) Only the plant at site one is completed by the contract date; (iv) Exactly one plant is completed by the contract date; (v) Either the plant at site 1 or both of the other two plants are completed by the contract date.

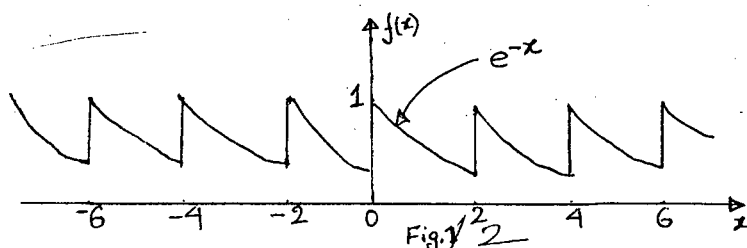
(14)

(c) The determination of the shear strength of spot welds is relatively difficult, whereas measuring the weld diameter of spot welds is relatively simple. As a result, it would be advantageous if shear strength could be predicated from a measurement of weld diameter. The data are as follows: Shear Strength (psi): 370, 780, 1210, 1560, 1980, 2450, 3070, 3550, 3940, 3950 and the corresponding Weld Diameter (in 0.0001 inch): 400, 800, 1250, 1600, 2000, 2500, 3100, 3600, 4000, 4000. Based on the given data: (i) Draw a scatter diagram; (ii) Find the least square estimates of the regression coefficients; and (iii) Predict the shear strength of spot welds whose diameter is 0.135 inch.

(15)

8. (a) Determine Fourier Series of the periodic function shown in Fig. 2.

(15)

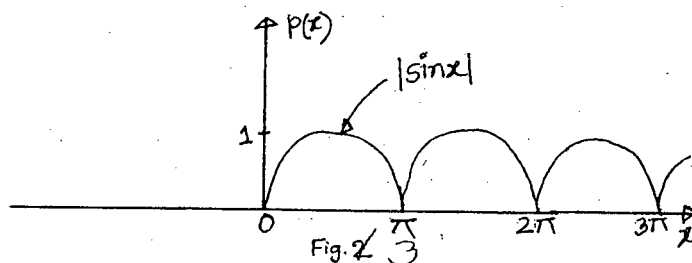


(b) Suppose a string on an elastic foundation is subjected to the force as shown in Fig. 3.

The governing equation of deformation of a string is given by: $T \frac{d^2 y}{dx^2} - ky = p(x)$. Where

T is the string tension, y is deformation of string, x is the longitudinal axis of the string, k is stiffness of the elastic base, $p(x)$ is the applied force. Solve the equation for y .

(20)



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

PERCENTAGE POINTS OF THE T DISTRIBUTION

Tail Probabilities		0.10	0.05	0.025	0.01	0.005	0.001	0.0005
One Tail		0.10	0.05	0.025	0.01	0.01	0.002	0.001
Two Tails		0.20	0.10	0.05	0.02	0.01	0.002	0.001
D	1	3.078	6.314	12.71	31.82	63.66	318.3	637
E	2	1.886	2.920	4.303	6.965	9.925	22.330	31.6
G	3	1.638	2.353	3.182	4.541	5.841	10.210	12.92
R	4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
E	5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
E	6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
S	7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
	8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
O	9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
F	10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
	11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
F	12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
R	13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
E	14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
E	15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
D	16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
O	17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
M	18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
	19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
	20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
	21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
	22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
	23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
	24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
	25	1.316	1.708	2.060	2.485	2.787	3.450	3.725

Appendix

Table of Fourier Transforms

$f(x)$	$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$
1. $\frac{1}{x^2 + a^2} \quad (a > 0)$	$\frac{\pi}{a} e^{-a \omega }$
2. $H(x) e^{-ax} \quad (\text{Re } a > 0)$	$\frac{1}{a + i\omega}$
3. $H(-x) e^{ax} \quad (\text{Re } a > 0)$	$\frac{1}{a - i\omega}$
4. $e^{-a x } \quad (a > 0)$	$\frac{2a}{\omega^2 + a^2}$
5. e^{-x^2}	$\sqrt{\pi} e^{-\omega^2/4}$
6. $\frac{1}{2a\sqrt{\pi}} e^{-x^2/(2a)^2} \quad (a > 0)$	$e^{-a^2\omega^2}$
7. $\frac{1}{\sqrt{ x }}$	$\sqrt{\frac{2\pi}{ \omega }}$
8. $e^{-a x /\sqrt{2}} \sin\left(\frac{a}{\sqrt{2}} x + \frac{\pi}{4}\right) \quad (a > 0)$	$\frac{2a^3}{\omega^4 + a^4}$
9. $H(x+a) - H(x-a)$	$\frac{2 \sin \omega a}{\omega}$
10. $\delta(x-a)$	$e^{-i\omega a}$
11. $f(ax+b) \quad (a > 0)$	$\frac{1}{a} e^{i\omega b/a} \hat{f}\left(\frac{\omega}{a}\right)$
12. $\frac{1}{a} e^{i\omega b/a} f\left(\frac{x}{a}\right) \quad (a > 0, b \text{ real})$	$\hat{f}(a\omega + b)$
13. $f(ax) \cos cx \quad (a > 0, c \text{ real})$	$\frac{1}{2a} \left[\hat{f}\left(\frac{\omega - c}{a}\right) + \hat{f}\left(\frac{\omega + c}{a}\right) \right]$
14. $f(ax) \sin cx \quad (a > 0, c \text{ real})$	$\frac{1}{2ai} \left[\hat{f}\left(\frac{\omega - c}{a}\right) - \hat{f}\left(\frac{\omega + c}{a}\right) \right]$
15. $\hat{f}(x+c) + \hat{f}(x-c) \quad (c \text{ real})$	$2\hat{f}(\omega) \cos \omega c$
16. $\hat{f}(x+c) - \hat{f}(x-c) \quad (c \text{ real})$	$2i\hat{f}(\omega) \sin \omega c$
17. $x^n f(x) \quad (n = 1, 2, \dots)$	$i^n \frac{d^n}{d\omega^n} \hat{f}(\omega)$
Linearity of transform and inverse:	
18. $\alpha f(x) + \beta g(x)$	$\alpha \hat{f}(\omega) + \beta \hat{g}(\omega)$
Transform of derivative:	
19. $f^{(n)}(x)$	$(i\omega)^n \hat{f}(\omega)$
Transform of integral:	
20. $f(x) = \int_{-\infty}^x g(\xi) d\xi,$ where $f(x) \rightarrow 0$ as $x \rightarrow \infty$	$\hat{f}(\omega) = \frac{1}{i\omega} \hat{g}(\omega)$
Fourier convolution theorem:	
21. $(f * g)(x) = \int_{-\infty}^{\infty} f(x-\xi) g(\xi) d\xi$	$\hat{f}(\omega) \hat{g}(\omega)$