$$u_{[k]} = u_{[0]} + \Delta u_{[k]} \qquad \text{forward} \qquad v_{[k]} = M\left(u_{[k]}\right) \qquad \text{forward} \qquad \mathcal{J}_{[k]} = \mathcal{J}\left(M\left(u_{[k]}\right)\right)$$
 
$$\downarrow \qquad \qquad \qquad \downarrow$$
 
$$\nabla_u \mathcal{J}_{[k]}(\delta \mathcal{J}) = T^* \cdot \nabla_v \mathcal{J}|_{v_{[k]}}(\delta \mathcal{J}) \qquad \text{adjoint} \qquad ad \, v_{[k]}(\delta \mathcal{J}) = \nabla_v \mathcal{J}|_{v_{[k]}}(\delta \mathcal{J}) \qquad \text{adjoint} \qquad ad \, \mathcal{J} = \delta \mathcal{J}$$
 
$$\downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow$$
 
$$\mathcal{J}_{[k]}, \quad \nabla_u \mathcal{J}_{[k]} \qquad \longrightarrow \qquad \text{minimisation} \qquad \longrightarrow \qquad \Delta u_{[k+1]}$$

 $u_{[0]}$  ,  $\Delta u_{[k]}$