Numerically Modelling Ocean Dissipation in Icy Satellites: A Comparison of Linear and Quadratic Friction

Hamish C. F. C. Hay[a], Isamu Matsuyama[a]

[a]*Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85719, United States*

Abstract

Abstract

1. Introduction

The primary sources of thermal energy in the interiors of the outer Solar System icy satellites are radiogenic decay and tidal dissipation. Radiogenic decay may play a significant role in heating large bodies (ref), but this role diminishes with decreasing mass of silicate material. Small satellites, those with a high surface area to volume ratio, lose their heat quickly. Thermal energy generated from radioactive decay is rapidly lost, cooling down the satellite's interior. Yet, several icy satellites have been confirmed to harbour global and potentially non-global subsurface oceans, suggesting significant interior heating.

Europa exhibits an induced magnetic field consistent with a global layer of salty, liquid water beneath its surface, as measured by the Galileo spacecraft (Zimmer et al., 2000; Kivelson et al., 2000; Hand and Chyba, 2007). This is also the case with Ganymede and Callisto, although these liquid oceans are likely to exist at least a few hundred kilometers beneath their surface (Zimmer et al., 2000; Kivelson et al., 2000).

The detection of a Schumann-like resonance at Saturn's largest moon, Titan, has been interpreted to suggest a conductive liquid layer beneath its surface (Béghin et al., 2010). This is supported by measurements of Titan's mean moment of inertia (Bills and Nimmo, 2011). Enceladus also shows signs of a liquid ocean confined to its southern hemisphere. Evidence for this includes the emission and composition of cryovolcanic plumes from a series of tidally induced shear faults over its south pole (Hansen et al., 2011), as well as the detection of a negative mass anomaly consistent with a subsurface ocean or sea (Iess et al., 2014).

Tidal dissipation is currently considered the most promising mechanism that supports the long-term existence of these oceans over geological time, and shall be the main focus of this paper.

1.1. Tidal Dissipation

Any satellite that passes through a varying gravitational potential will experience some form of tidal dissipation. The varying gravitational potential may be a result of the satellite's orbital eccentricity and/or obliquity, as well as any non-synchronous rotation. For a satellite in (near) synchronous rotation, the gravitational tidal potential will vary periodically with the satellite's orbit. The changing potential does mechanical work on the satellite, and a portion of this is converted to thermal energy, or heat. This is process is known as tidal dissipation. As long as sufficient orbital or rotational energy remains in the system (in the form of eccentricity, obliquity or non-synchronous rotation), tidal dissipation will occur.

Both the solid and fluid parts of an orbiting satellite will experience tidal dissipation. However, despite the overwhelming evidence for and abundance of subsurface oceans in the icy satellites, the majority of dissipation studies have focused on solid-body tides, (e.g., Moore and Schubert, 2000; Tobie et al., 2005; Roberts and Nimmo, 2008; Beuthe, 2013). The majority of terrestrial tidal dissipation occurs within the oceans, and while Earth has a complex dynamic between tidally-induced ocean flow and the continents, it illustrates the importance of considering ocean dissipation.

The importance of ocean dissipation in some outer planet satellites other was first recognised after Voyager measurements of the saturation and surface temperature of Titan ref. Sagan and Dermott (1982) tackled this problem analytically, deriving expressions to estimate dissipated energy in a hydrocarbon surface ocean on Titan. In doing so they attempted to explain Titan's high eccentricity.

The same problem was then approached numerically by Sears (1995), solving the shallow water equations to derive orbitally time averaged estimates of dissipated energy within a hydrocarbon of varying depths. Sears' numerical model is the basis for the model described in this paper.

More recently, Tyler (2008, 2009, 2011, 2014) has done extensive tidal dissipation work, showing that thermal energy released in ocean dissipation can theoretically prevent subsurface liquid from freezing. By running his model over a large parameter space, Tyler (2011) discovered ocean dissipation resonances. These resonances tend to occur at particular values of ocean depth, where the geometry of the ocean massively enhances tidal flow and consequently tidal dissipation. Matsuyama (2014) developed a similar model to that used by Tyler (2011), adding the effects of ocean loading, self-attraction, and deformation of the solid regions. These effects tend to shift dissipative resonances to different ocean depths.

Both Tyler (2011) and Matsuyama (2014) used Rayleigh friction in their models. While useful, this is somewhat unrealistic as ocean dissipation is unlikely to scale linearly with velocity. The numerical model presented in this work will instead solve the shallow water equations using bottom (quadratic) friction, as is typically the case with terrestrial ocean dissipation studies (ref). Sears (1995) used both Rayleigh and bottom friction simultaneously in his model. We, however, shall adopt a more bimodal approach to illustrate the differences between Rayleigh and bottom friction.

1.2. Bottom Friction

References from Chen and Nimmo

This paper can be divided into three sections. Firstly, we compare our model against the semi-analytical solutions of Matsuyama (2014), minus the effects of ocean loading, self-attraction, and deformation of the solid regions. Our test cases span from the 'canonical' ocean scenario outlined by Sagan and Dermott (1982) and modelled in Sears (1995), to extensive modelling throughout a large parameter space in ocean depth and Rayleigh friction coefficient. Secondly, we illustrate the differences between Rayleigh friction and bottom friction by comparing numerical model results for each friction type. Finally, we compare our results against scaling laws developed by (Chen et al., 2013).

2. Methodology

This section describes some of the theory and methods involved in this work. The governing equations of the numerical model presented here are discussed in section 2.1. Following this, a description of the equations discretisation and the numerical scheme required to solve them are outlined in section 2.2.

2.1. Laplace Tidal Equations

The equations of motion and mass conservation that describe ocean flow in the shallow water limit are called the Laplace Tidal Equations (LTE) (also known as the shallow water equations) (Lamb, 1932). The main assumption in their derivation is that radial (vertical) ocean flow is negligible when compared to lateral flow. This is indeed a good approximation for global oceans, where lateral flow spans a much greater distance than the depth of the ocean. The conservation of mass (eq. 1) and momentum (eq. 2) that make up the LTE, including both Rayleigh and bottom friction, are given as (Sears, 1995; Tyler, 2008; Matsuyama, 2014):

$$\partial_t \eta + \nabla \cdot (h\boldsymbol{u}) = 0 \,, \tag{1}$$

$$\begin{aligned} \partial_t \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} + \alpha \boldsymbol{u} + \frac{c_D}{h} |\boldsymbol{u}| \, \boldsymbol{u} + g\nabla\eta \\ = (1 + k_2 - h_2)\nabla U_2 \,. \end{aligned} \tag{2}$$

Equation 1 consists of two terms. The first is the time rate of change of vertical sea surface displacement about some equilibrium level. η denotes this displacement. The second term represents the divergence of the surface velocity vector, $\boldsymbol{u} \equiv (u, v)$, where u and v are the eastward and northward velocity components respectively. In our calculations, we assume the ocean's undisturbed depth, h, to be constant.

The term on the right hand side of Equation 2 is an applied force per unit mass. ∇U_2 is the gradient of the degree-2 tidal potential (discussed in section ...). It is multiplied by Love's reduction factor involving the degree-2 tidal Love numbers, k_2 and h_2. Love's first number, k_2, is a proportionality constant accounting for the additional tidal potential due to the elastic redistribution of mass on the satellite. h_2, the second Love number, accounts for the tidal potential arising from solid body surface displacement of the satellite (Love, 1911).

The time derivative of velocity is given by the first term on the left hand side of the momentum equation. It is balanced by four other acceleration per unit mass terms on the left hand side. The first of these terms is the coriolis acceleration per unit mass, where Ω is the satellite's rotational angular velocity. Rayleigh and bottom friction are described in the next two terms, where α and c_D are the Rayliegh (linear)and bottom (quadratic) friction coefficients respectively (Sears, 1995; Chen et al., 2013). The last term on the right hand side of Equation 2 is the gravitational restoring acceleration per unit mass. It acts to balance changes in sea surface displacement, given by the gradient of equilibrium displacement, $\nabla\eta$.

2.2. Numerical Model

The numerical model outlined below is based on the models extensively discussed in Zahel (1973, 1978) and Sears (1994, 1995).In this section we first provide a description of the numerical grid setup, before giving an overview of the finite difference scheme itself.

2.2.1. Discretized Grid

We made grid.

2.2.2. Finite Difference Expansions

As in (Sears, 1995), we expand equations 1 and 2 in a semi-implicit finite difference scheme in spherical coordinates. By expanding $\boldsymbol{u}$ into its components, the momentum equation becomes,

$$u_{ij}^{t+1} \approx \Bigg[2\Omega \bar{v}_{ij} \sin\lambda_i - \alpha u_{ij}^t - \frac{c_D}{h}\sqrt{\left(u_{ij}^t\right)^2 + \left(\bar{v}_{ij}^t\right)^2} \cdot \left(u_{ij}^t\right)^2 - \frac{g}{R\cos\lambda_i}\frac{\partial \eta_{ij}^t}{\partial\phi_j} + (1+k_2-h_2)\frac{1}{R\cos\lambda_i}\frac{\partial U_{2,ij}^t}{\partial\phi_j}\Bigg]\Delta t + u_{ij}^t , \quad (3)$$

$$v_{ij}^{t+1} \approx \Bigg[2\Omega \bar{u}_{ij} \sin\lambda_i - \alpha v_{ij}^t - \frac{c_D}{h}\sqrt{\left(\bar{u}_{ij}^t\right)^2 + \left(v_{ij}^t\right)^2} \cdot \left(v_{ij}^t\right)^2 - \frac{g}{R}\frac{\partial \eta_{ij}^t}{\partial\lambda_i} + (1+k_2-h_2)\frac{1}{R}\frac{\partial U_{2,ij}^t}{\partial\lambda_i}\Bigg]\Delta t + v_{ij}^t , \quad (4)$$

and the mass conservation equation becomes,

$$\eta_{ij}^{t+1} \approx -\frac{h}{R\cos\lambda_i}\left(\frac{\partial\left(v_{ij}^{t+1}\cos\lambda_i\right)}{\partial\lambda_i} + \frac{\partial u_{ij}^{t+1}}{\partial\phi_j}\right)\Delta t + \eta_{ij}^t . \quad (5)$$

Latitude and longitude are denoted by λ and ϕ respectively. i and j represent the ith and jth latitude and longitude positions within the grid. The time index is given by t, and Δt represents the time-step. Overbars correspond to averages, a necessity given the staggered nature of the grid.

All derivatives of the degree-2 tidal potential have analytical solutions (section ...), and thus do not require further finite difference expansions. Derivatives of all other quantities, however, do require further expansion. The expansions take the general form of either,

$$\frac{\partial w_{ij}}{\partial\lambda} \approx \frac{w_{i+1/2,j} - w_{i-1/2,j}}{\Delta\lambda} , \quad (6)$$

or,

$$\frac{\partial w_{ij}}{\partial\phi} \approx \frac{w_{i,j+1/2} - w_{i,j-1/2}}{\Delta\phi} , \quad (7)$$

where w represents u, v or η. $\Delta\lambda$ and $\Delta\phi$ are the latitude and longitude grid spacing, respectively.

Examining equations 6 and 7 reveals that each derivative is evaluated halfway between the grid points where w is stored. Consequently, any derivative of w is calculated at the grid position held by a different function. For example, $\partial_\phi u_{ij}$ is always evaluated at the position held by η_{ij} as $u_{i,j-1/2}$ and $u_{i,j+1/2}$ lie to the left and right of η_{ij}, respectively. This is best visualized in figure ...

Discuss semi-implicitness, over-sampling, pole discontinuity.

3. Results

4. Discussion

5. Conclusions

References

Béghin, C., Sotin, C., Hamelin, M., 2010. Titan's native ocean revealed beneath some 45km of ice by a Schumann-like resonance. Comptes Rendus Geoscience 342, 425–433.

Beuthe, M., 2013. Spatial patterns of tidal heating. Icarus 223, 308–329.

Bills, B.G., Nimmo, F., 2011. Rotational dynamics and internal structure of Titan. Icarus 214, 351–355.

Chen, E., Nimmo, F., Glatzmaier, G., 2013. Tidal heating in icy satellite oceans. Icarus 229, 11–30.

Hand, K.P., Chyba, C.F., 2007. Empirical constraints on the salinity of the Europan ocean and implications for a thin ice shell. Icarus 189, 424–438.

Hansen, C., Shemansky, D., Esposito, L., Stewart, A., Lewis, B., Colwell, J., Hendrix, A., West, R., Waite, J., Teolis, B., et al., 2011. The composition and structure of the Enceladus plume. Geophysical Research Letters 38.

Iess, L., Stevenson, D., Parisi, M., Hemingway, D., Jacobson, R., Lunine, J., Nimmo, F., Armstrong, J., Asmar, S., Ducci, M., et al., 2014. The gravity field and interior structure of enceladus. Science 344, 78–80.

Kivelson, M.G., Khurana, K.K., Russell, C.T., Volwerk, M., Walker, R.J., Zimmer, C., 2000. Galileo magnetometer measurements: A stronger case for a subsurface ocean at Europa. Science 289, 1340–1343.

Lamb, H., 1932. Hydrodynamics. Cambridge university press.

Love, A.E.H., 1911. Some Problems of Geodynamics: Being an Essay to which the Adams Prize in the University of Cambridge was Adjudged in 1911. CUP Archive.

Matsuyama, I., 2014. Tidal dissipation in the oceans of icy satellites. Icarus 242, 11–18.

Moore, W.B., Schubert, G., 2000. The tidal response of Europa. Icarus 147, 317–319.

Roberts, J.H., Nimmo, F., 2008. Tidal heating and the long-term stability of a subsurface ocean on Enceladus. Icarus 194, 675–689.

Sagan, C., Dermott, S.F., 1982. The tide in the seas of Titan. Nature 300, 731–733.

Sears, W.D., 1994. Tidal effects on the evolution of Titan. Ph.D. thesis. University of Arizona.

Sears, W.D., 1995. Tidal dissipation in oceans on Titan. Icarus 113, 39–56.

Tobie, G., Mocquet, A., Sotin, C., 2005. Tidal dissipation within large icy satellites: Applications to Europa and Titan. Icarus 177, 534–549.

Tyler, R., 2008. Strong ocean tidal flow and heating on moons of the outer planets. Nature 456, 770–772.

Tyler, R., 2009. Ocean tides heat Enceladus. Geophysical Research Letters 36.

Tyler, R., 2011. Tidal dynamical considerations constrain the state of an ocean on Enceladus. Icarus 211, 770–779.

Tyler, R., 2014. Comparative estimates of the heat generated by ocean tides on icy satellites in the outer solar system. Icarus 243, 358–385.

Zahel, W., 1973. The diurnal K1 tide in the world oceana numerical investigation. Pure and applied geophysics 109, 1819–1825.

Zahel, W., 1978. The influence of solid earth deformations on semidiurnal and diurnal oceanic tides, in: Tidal friction and the Earths rotation. Springer, pp. 98–124.

Zimmer, C., Khurana, K.K., Kivelson, M.G., 2000. Subsurface oceans on Europa and Callisto: Constraints from Galileo magnetometer observations. Icarus 147, 329–347.