TOI-OHOMAI Institute of Technology	**COMP.5202** **Fundamentals of Programming and Problem Solving**	**Mathematics Algebra**

Algebra Part One (SAMPLE ANSWERS)

Overview

The document will cover:

- Simplifying
- Factorising
- Solving Linear Equations

NOTE: Algebraic expressions are made up of letters, symbols and arithmetic symbols, eg. + - / * etc.

Simplification (aka Simplifying) of Algebraic Expressions

Is making the equation simpler (the most compact) by bringing together like values.

NOTE: Adding and subtracting in algebra has different rules to multiplying and dividing.

Example 1:

$x + x = 2x$	$x + x + x = 3x$	$3x – x = 2x$
$x * x = x^2$	$x * x * x = x^3$	$x^3 / x = x^2$

Example 2:

$x + y = x + y$	$x + y + y = x + 2y$	$3x + 2y – x – y = 2x + y$
$x * y = xy$	$x * y * y = xy^2$	$x^3y^2 / xy = x^2y$

Addition and Subtraction

When solving algebraic expressions that include addition and subtraction, we have to find the like terms. These are multiples of the same symbol. For example, 3x, 5x and 2.1x are all multiples of the same symbol x. It is the same with $2x^2$ and $7x^2$ as these are all multiples of x^2 and even 8xy, -4xy are all multiples of xy.

Example 1: Simplify $3x + 7x – 2x$

These are all like terms so the answer = 8x

Example 2: Simplify $4x^2 - 3x^2 + 5x^2$

These are all like terms so the answer = $6x^2$

Example 3: Simplify $3yx^2 - yx^2 + 6yx^2$

These are all like terms so the answer = $8yx^2$

Try simplifying these:

1. $x + 7x + x^2$

   $x^2 + 8x$

2. $3x + 4y$

   $3x = 4y$

3. $ab + a^2 – 7b^2 + 9ab + 8b^2$

   $a^2 + b^2 + 10ab$

4. $6b^2 + 3 – 2b^2 – 1$

   $4b^2 + 2$

5. $2by - 3c + 6c + by + 9$

   $3by + 3c + 9$

6. $5a^2 + 2ab - a^2 + 8ab$

   $4a^2 + 10ab$

7. $2av + 3a + 2v - av - a$

   $av + 2a + 2v$

8. $v^2 + 5 - 2 - v^2$

   3

9. $3a - 2b + c + 7a - 5b + c$

   $10a - 7b + 2c$

10. $5x - 2y + 4y - 6x + 7z - 2x$

    $-3x + 2t + 7z$

11. $5x^2 - 7x - 4 - x^2 - 2 + 5x$

    $4x^2 -2x - 6$

12. $7x^3 + 8x^2 - 2xy - 6 - 3x^3 + 6x^2 - yx + 9$

    $4x^3 + 14x^2 - 3xy + 3$

13. $7xy^2z + 3y^2zx + 6zxy^2$

    $16xy^2z$

14. $5x^2y - 3xyz + 7x - 3yx^2 + yzx - 9x$

    $2x^2y - 2xyz - 2x$

15. $2 - 4x + y + 5 + a + 7z + 3a + 4y + 10 + y$

    $-4x + 6y + 4a + 17$

Multiplying and Removing Brackets

You know when we multiply any two numbers together that it doesn't matter which order we do it in. For example 2 x 10 and 10 x 2 both equal 20.
The order still does not matter when we are multiplying 3 or more numbers together. We could do (2 * 4) x 5, or 2 * (4 * 5) and either way we would get the answer 40.

We could also have written this as (2)(4)(5)

What are the rules for the following:

- positive * positive = ? *positive*
- negative * negative = ? *positive*
- positive * negative = ? *negative*
- negative * positive = ? *negative*

So let's trying removing the brackets by multiplying.

NOTE - Multiplication: Although you can only add and subtract like terms, in multiplication you can multiply different terms together i.e.
x * y = xy,
whereas with x + y nothing can be done, the answer is still simply; x + y or y + x.

Example: Simplify 3(4x)
This is just the same as 3 * 4 * x, which equals 12x

NOTE – Multiplying Exponents: When multiplying exponents we can simple add the exponents together but only if they have the ***same base***.

Example: Simplify $2x^2 * 4x^3$
This can be rewritten as 2 * x * x * 4 * x * x * x
$= 8 * x^5$
$= 8x^5$

Removing Brackets From a(b + c), a(b – c) and (a + b)(c + d)

Although with multiplication the order in which we multiply our numbers is not important this is not the same with addition and subtraction. BEDMAS has some input here and it states we must solve anything in the brackets first. So if we solved (5 – 2) + 7, we could get a different result from this 5 – (2 + 7). It might contain the same numbers and symbols but the brackets are in a different position, causing us to solve a different part of the expression first, thus giving us a different answer.

1 set of brackets:

But when multiplication is involved like in this expression **a(b + c)** we are expected to multiply everything inside the brackets by what is outside the brackets (i.e. the a).

This gives us the answer **ab + ac**.

Example:

2(3x – 3)

= 2 * 3x + 2 * (-3)

= 6x – 6

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2 sets of brackets

In the expression **(a + b) (c + d)** the first set of brackets must be multiplied by everything in the second set of brackets.

Example 1: (a + b) (c + d)

(a + b)c + (a + b)d	or	a(c +d) + b(c + d)
= ac + bc + ad + bd		= ac + ad + bc + bd

Or

We could multiply the brackets out by doing firsts, lasts, inners and outers

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Example 2: (a + b)(c + d)

Firsts: ac

Lasts: bd

Inners: bc

Outers: ad

= ac + bd + bc + ad

Try simplifying these by removing the brackets:

1. $5(3y)$

2. $x(7 + 2)$

3. $4x^2(7x^5)$

4. $7(2b^2)$

5. $a(-b)$

6. $2(6v – 1)$

7. $2b(9b – 6)$

8. $-2c(7c + 9)$

9. $(3n + 3)6$

10. a. $8z(6z)$

    b. $8z + 6z$

11. $4(x + 1)$

12. $-5(x + 2)$

13. $7(x – 5)$

14. $-3(x – 8)$

15. $5(x – y)$

16. $19(x + 3y)$

17. $8(a + b)$

18. $(5 + x)y$

19. $12(x + 4)$

20. $(x + 1)(x + 6)$

21. $(x + 4)(x + 5)$

22. $(x + m)(m + n)$

23. $(5 - x)(5 + x)$

24. $(4 + y)(3 + x)$

25. $(17x + 2)(3x - 5)$

26. $(-3x + 2)(2x - 4)$

27. $(2xy + 3)(2x - 4)$

28. $(ac + 3b)(2a - b)$

Dividing of Algebraic Expressions

At this stage we are only going to look at the rules for dividing Monomials. A Monomial is an algebraic expression consisting of a single term. It doesn't contain any addition or subtraction.

Here are some examples of monomials: x, 5xy, $7x^3$

These are **NOT** monomials: 5x + 9, 6x + 2y -5

The correct terms in division are as follows:

6 (Dividend)

$$\frac{6}{2} = 3$$

3 (Quotient)

2 (Divisor)

When we want to find the quotient of one monomial divided by another, we use the laws of exponents and the laws for dividing numbers.

If the base of the exponents are the same, then we can combine the exponents.

Example: $$\frac{16x^7}{4x^2} = 4x^{7-2} = 4x^5$$

Another thing to remember:

$$\frac{4a^2 + 8a}{2a} \quad \textit{is the same as} \quad \frac{4a^2}{2a} + \frac{8a}{2a}$$

$$\textit{which reduces to} \quad \mathbf{2a + 4}$$

Let's try a more complicated one:

$$\frac{2p + V^2d + 2ydg}{2dg}$$

$$= \frac{2p}{2dg} + \frac{V^2d}{2dg} + \frac{2ydg}{2dg}$$

$$= \frac{p}{dg} + \frac{V^2}{2g} + y$$

Let's have some practise:

1. $\frac{6x^5}{2x^3}$ *$3x^2$*

2. $\frac{10a^7 b^4}{5a^3 b^2}$ *$5a^4b^2$*

3. $\frac{20z^6 y^9}{4z^3 y^5}$ *$5z^3y^4$*

4. $\frac{6x^2 – 4x}{2x}$ *$3x – 2$*

5. $\frac{8x^2 y^4 + 16x^4 y^5 – 12x^3y^3}{4xy}$ *$2xy^3 + 4x^3y^4 + 3x^2y^2$*

Factorising Algebraic Expressions

What is factorising? Think about normal multiplication using the correct terminology. We say that ***factor * factor = product***.

So, we can say 12 can be factorised into 2 x 6. The numbers 2 and 6 are factors of 12. There are also other factors, what are they?

An algebraic expression can also be factorised.

Factorising is finding what can be multiplied together to give us an expression.
It is doing the opposite of simplifying, instead of multiplying out the brackets, we are trying to put things back into brackets.

There are a few techniques in factorising, some of them we will learn are:

- Factorising by a common factor
- Factorising by grouping
- Factorising Quadratic equations.

Factorising by a Common Factor

This technique involves finding the largest number and/or letter which form part of <u>ALL</u> of the terms.

Example: $5x + 20y$
$= 5\,(x + 4y)$

NB: Your answer can then be checked by expanding the brackets again.

Looking at the above example 5x + 20y, although they don't have a letter (ie. an x or y) in common, the numbers are both multiples of 5, this means that the number 5 is a common factor here.

We then factor the common factors, whether it be a number or a letter or both and put them outside the brackets. If we put the 5 outside the brackets in this example, then we end up with x + 4y inside the brackets e.g. 5(x + 4y). If we expand out the brackets to check our answer we would end up with the original expression.

In summary, $5x + 20y = 5\,(x + 4y)$

Try factorising the following by finding the common factors:

1. $8x^2 - 12x$

   $4x(2x – 3)$

2. $6x + 3x^2 + 9xy$

   $3x(x + 3y + 2)$

3. $2xy + 10x – 6y$

   $2(xy + 5x – 3y)$

4. $3x + 18$

   $3(x + 6)$

5. $3y – 9$

   $3(y – 3)$

   $-3y -9$

   $-3(y + 3)$

6. $-5t – 20$

   $-5(t + 4)$

7. $3x + 12$

   $3(x + 4)$

8. $17t + 34$

   $17(t + 2)$

9. $7x + 21x^2$

   $7x(1 + 3x)$

Factorising by Grouping

In the previous example we looked at taking the common factor and putting it outside the brackets. But when you **can't** find any common factors for ALL of the terms, providing there is an even number of terms; we can group the terms into pairs and find the common factor for each set of pairs.

Example: ax + ay + bx + by
= a(x + y) + b(x + y)
= (a + b) (x + y)

The first pair of terms have a common factor of a, so we can factor ax + ay to look like a(x + y).

And the second pair of terms have a common factor of b, so we can factor bx + by to look like b(x + y).

Which gives us:
= a(x + y) + b(x + y)

We now see that there is a common factor with the sets of brackets (x + y), so we simply take the remaining terms and put them in a second set of bracket of their own and we get the answer:

= (a + b) (x + y)

Note: If there is a common bracket, try grouping your terms differently.

Try factorising the following by grouping:

1. $6x + 9 + 2ax + 3a$

   $3(2x + 3) + a(2x + 3) = (3 + a)(2x + 3)$

2. $x^2 - 6x + 5x - 30$

   $x(x - 6) + 5(x - 6) = (x + 5)(x - 6)$

3. $5x + 10y - ax - 2ay$

   $5(x + 2y) - a(x + 2y) = (5 - a)(x + 2y)$

4. $a^2 - 2a - ax + 2x$

   $a(a - 2) - x(a - 2) = (a - x)(a - 2)$

5. $2x + 8y - 3px - 12py$

   $2(x + 4y) - 3p(x + 4y) = (2 - 3p)(x + 4y)$

6. $3x - 3y + 4ay - 4ax$

   $3(x - y) + 4a(y - x) = 3(x - y) - 4a(x - y) = (3 - 4a)(x - y)$

7. $7x + 14y + bx + 2by$

   $7(x + 2y) + b(x + 2y) = (7 + b)(x + 2y)$

http://www.onlinemathlearning.com/factoring-by-grouping.html

LINEAR EQUATIONS

A Linear Equation is: A mathematical expression that has an equal sign and linear expressions and usually contains a variable. Linear equations are the simplest equations that we will deal with.

A Linear expression is: "A mathematical statement that performs functions of addition, subtraction, multiplication, and division."

LINEAR EQUATION RULES

You cannot have any of the following in a linear equation:

- No exponents (or powers) on variables eg: x^2
- Multiplication or division on two different variables eg: x * y (or xy) or x / y
- A variable cannot be found under a square root sign √ eg: √ x

Examples

These are examples of the linear expressions:

- x + 4
- 2x + 4
- 2x + 4y

Solving Linear Equations

The most important rule when solving linear equations:

- Whatever you do to one side of an equation, you must also do to the other side of the equation.

In order to solve a Linear Equation for a particular variable, you need to get that variable by itself. In order to get the variable by itself, you need to "undo" whatever has been done to the variable.

Example 1:

Solve: $x + 6 = -3$

I want to isolate x, get it by itself on one side of the equals sign.
I don't want the plus 6 to be there, since the 6 is added to the x, we need to subtract it to get rid of it. In other words we need to subtract a 6 from the x in order to undo the adding of it to the x.

Remember; what we do to one side of the equation, we must also do to the other side of the equation.

So if we subtract 6 from the left side of the equals sign, we must also subtract it from the right hand side, which looks like this:

$$x + 6 - 6 = -3 - 6$$

Solved we end up with:

$$x = -9$$

Example 2:

Solve: $2x = 5$

To isolate x, which is multiplied by two, we will need to divide both sides by 2, which looks like this:

$$2x = 5$$

$$\frac{2x}{2} = \frac{5}{2}$$

$$x = \frac{5}{2} \quad \text{or} \quad x = 2\tfrac{1}{2}$$

Example 3:

Solve: $\frac{1}{2}x = 20$ or $\frac{x}{2} = 20$

To isolate x which is multiplied by the fraction ½ , we need to divide both sides of the equation by that fraction.

This is a special case of undoing multiplication, which we call flip-n-multiply.

So we multiply both sides of the equation by the flip reciprocal (opposite) of that fraction as shown below:

$$\frac{2}{1}\left(\frac{1}{2}x\right) = \frac{2}{1}\left(\frac{20}{1}\right)$$

$$x = 40$$

Note: You could view it a different way.

Solve: $\frac{1}{2}\,x = 20 \quad is\ the\ same\ as \quad \frac{x}{2} = 20$

To isolate x which is divided by 2, we need to multiply both sides of the equation by that number.

$$\frac{x}{2} = 20$$

$$\frac{x}{2} \times 2 = 20 \times 2$$

$$x = 40$$

Solve the following Linear Equations:

1. $x - 3 = -5$

   x = -5 + 3　　*x = -2*

2. $\frac{x}{5} = 6$

   *x = 6 * 5*　　*x = 30*

3. $\frac{3}{5}x = 10$

   *3x = 10 * 5*　　*3x = 50*　　*x = 50 / 3*　　*x = 16 2/3*

4. $2x + 4 = -8$

   2x = -8 -4　　*2x = -12*　　*x = -12 /2*　　*x = -6*

5. $3x + 2x = 15$

   5x = 15　　*x = 15 / 5*　　*x = 3*

6. $32 = 16x + 8x$

   32 = 24x　　*32 / 24 = x*　　*x = 32 / 24*　　*x = 4 /3*