TOI-OHOMAI Institute of Technology	**COMP.5202 Fundamentals of Programming and Problem Solving**	**Mathematics Algebra**

Algebra Part Two - Quadratic Equations

Overview

The document will cover:

- Factorising Quadratic Equations
- Solving Quadratic Equations

NOTE: Algebraic expressions are made up of letters, symbols and arithmetic symbols, eg. + - / * etc.

What is a Quadratic Equation?

A quadratic equation:

- Is one in which the highest power of x is x^2.
- Looks like "**$ax^2 + bx + c$**" where a, b and c are just numbers.

Examples of Quadratic Equations

- $2x^2 + 5x - 12 \quad (a = 2,\ b = 5,\ c = -12)$
- $4x^2 + 16 \quad (a = 4,\ b = 0,\ c = 16)$
- $3x^2 + 8x \quad (a = 3,\ b = 8,\ c = 0)$

Factorising SIMPLE Quadratic Equations

To factor a simple quadratic equation, "**$ax^2 + bx + c$**" where the value of **a** is not greater than 1, we need to find 2 factors of **c** that add together to get **b**.

Example 1: $x^2 + 5x + 6 = (x + ?)(x + ?)$

In this example, we need to find factors of 6, that add together to give us 5.

Factors of 6 are: 1 and 6, **2** and **3** (also -1 and -6, -2 and -3)

The factors that would add up to give us 5 are **2** and **3**.

We can then put these factors in two sets of brackets with an x on the left of each.

So $x^2 + 5x + 6 = (x + \mathbf{2})(x + \mathbf{3})$

CHECK:

We can expand (x + 2) (x + 3) to see if we end up with the original quadratic expression. Simplifying (by removing the brackets) can be done by using one of the techniques previous covered, eg. by using the first, lasts, inners and outers technique.

Expanding **(x + 2) (x + 3)** should end up with the original expression **$x^2 + 5x + 6$**.

Example 2: $x^2 – 6x + 8 = (x + ?)(x + ?)$

In this example, we need to find factors of 8, that add together to give us -6.

Factors of 8 are: 1 and 8, 2 and 4, -1 and -8, -**2** and **-4**

The factors that would add up to give us -6 are **-2** and **-4**.

We can then put these factors in two sets of brackets with an x on the left of each.

So $x^2 - 6x + 8 = (x - \mathbf{2})(x - \mathbf{4})$

CHECK:

We can expand (x - 2) (x - 4) to see if we end up with the original quadratic expression. Simplifying (by removing the brackets) can be done by using one of the techniques previous covered, eg. by using the first, lasts, inners and outers technique.

Expanding **(x - 2) (x - 3)** should end up with the original expression **$x^2 - 6x + 8$**.

Factorising Quadratics - Exercises

Try factorising the following simple quadratic equations:

1. $x^2 + 7x + 10$

2. $x^2 + 2x – 8$

3. $x^2 – 7x + 12$

4. $x^2 + 6x + 9$

5. $x^2 + 6x – 7$

6. $x^2 + 10x + 25$

7. $x^2 + 4x – 45$

8. $x^2 – 9$

9. $x^2 - 9x + 20$

10. $x^2 - x – 12$

Factorising Quadratics – Exercises: Sample Answers:

You could use an online "Factoring Calculator" to find the answers to the exercise questions.
https://www.mathpapa.com/factoring-calculator/

1. $x^2 + 7x + 10 = (x + 2)(x + 5)$

2. $x^2 + 2x - 8 = (x - 2)(x + 4)$

3. $x^2 - 7x + 12 = (x - 3)(x - 4)$

4. $x^2 + 6x + 9 = (x + 3)(x + 3)$

5. $x^2 + 6x - 7 = (x - 1)(x + 7)$

6. $x^2 + 10x + 25 = (x + 5)(x + 5)$

7. $x^2 + 4x - 45 = (x - 5)(x + 9)$

8. $x^2 - 9 = (x + 3)(x - 3)$

9. $x^2 - 9x + 20 = (x - 4)(x - 5)$

10. $x^2 - x - 12 = (x + 3)(x - 4)$

Note: Check by expanding the brackets.

Factorising and Solving Quadratics

Solving Quadratics - Overview

We have looked at simplifying and factorising. Now we will look at solving quadratic equations. That basically means finding the value of "x" (ie. the roots of the quadratic equation).

So we are solving for "x" in the expression ax2 + bx + c = 0
In a quadratic we will have two answers for "x", ie. two roots.

Another way of looking at it:
In the equation y = ax2 + bx + c, we want to find the values of "x" when "y = 0". These values of "x" are the "roots" of the equation.

Solving Quadratics - Steps

The steps for solving a quadratic equation by factorising are as follows:

1. Collect all terms on the left so the quadratic equation is in the following format: $ax^2 + bx + c = 0$
2. Factor the quadratic equation
3. Set each factor to zero
4. Solve the resulting linear equations (these numbers are called the roots of the quadratic equation)
5. Check the solutions in the original equation

Solving Quadratics - Example

$x^2 - x = 20$ < find the roots

$x^2 - x - 20 = 0$ Step 1

$(x + 4)(x - 5) = 0$ Step 2

$x + 4 = 0$ $x - 5 = 0$ Step 3

$x = -4$ $x = 5$ Step 4

The roots of this quadratic equation are x = -4 and x = 5.

Step 5:

We can check them in the original equation by substitution (substituting the each root value in place of x)

Substitute x = -4 in the equation $x^2 - x - 20 = 0$

$(-4)^2 - (-4) - 20 = 0$

$16 + 4 - 20 = 0$

$0 = 0$

Substitute x = 5 in the equation $x^2 - x - 20 = 0$

$(5)^2 - (5) - 20 = 0$

$25 - 5 - 20 = 0$

$0 = 0$

Both roots satisfy the original equation.

Solving Quadratics - Exercises

Solve the following quadratic equations:

1. $x^2 + x - 6 = 0$

2. $x^2 - 4 = 0$

3. $x^2 + 30 = 11x$

4. $x^2 = 36$

5. $x^2 - 7x + 12 = 0$

6. $x^2 + 6x + 9 = 0$

7. $x^2 + 6x = 7$

8. $x^2 + 25 = -10x$

Solving Quadratics – Exercises: Sample Answers

You could use an online "Quadratic Equations Solver" to find the answers to the exercise questions.
https://www.mathgoodies.com/calculators/quadratic_equations

1. $x^2 + x - 6 = 0$

   $(x - 2)(x + 3) = 0$ $x = 2$ and $x = -3$

2. $x^2 - 4 = 0$

   $(x + 2)(x - 2) = 0$ $x = -2$ and $x = 2$

3. $x^2 + 30 = 11x$

   $x^2 - 11x + 30 = 0$ $(x - 5)(x - 6) = 0$ $x = 5$ and $x = 6$

4. $x^2 = 36$

   $x^2 - 36 = 0$ $(x + 6)(x - 6) = 0$ $x = -6$ and $x = 6$

5. $x^2 - 7x + 12 = 0$

   $(x - 3)(x - 4) = 0$ $x = 3$ and $x = 4$

6. $x^2 + 6x + 9 = 0$

   $(x + 3)(x + 3) = 0$ $x = -3$ and $x = -3$

7. $x^2 + 6x = 7$

   $x^2 + 6x - 7 = 0$ $(x - 1)(x + 7) = 0$ $x = 1$ and $x = -7$

8. $x^2 + 25 = -10x$

   $x^2 + 10x + 25 = 0$ $(x + 5)(x + 5) = 0$ $x = -5$ and $x = -5$

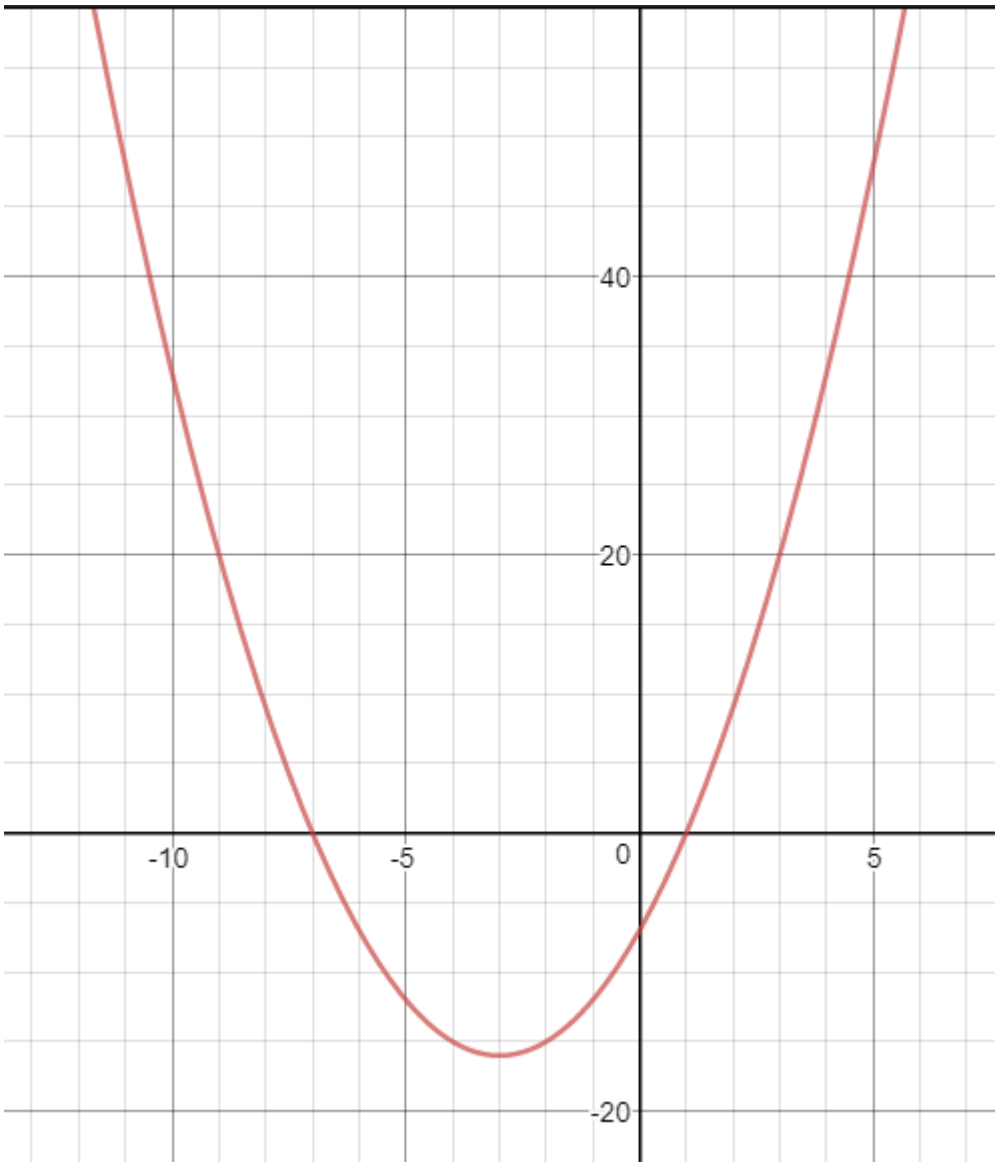
Additional Info: Graphing Quadratic Equations

For students who are interested ….

Use the online graphing tool to plot a graph of the following quadratic equations.

https://www.desmos.com/calculator

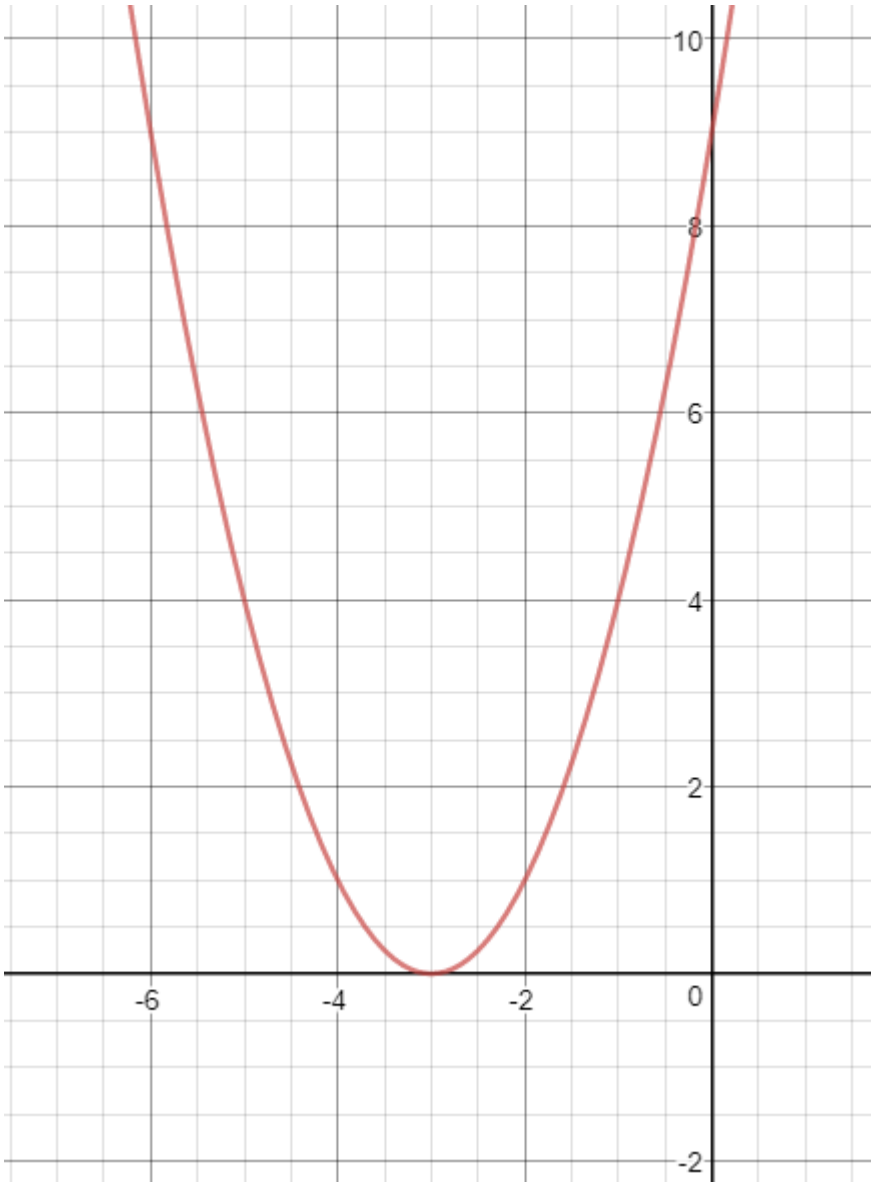
$y = x^2 + 6x - 7$



The graph **intersects** the x axis (y = 0) at x = -7 and x = 1.

This implies that the roots of the quadratic equation are x = -7 and x = 1.

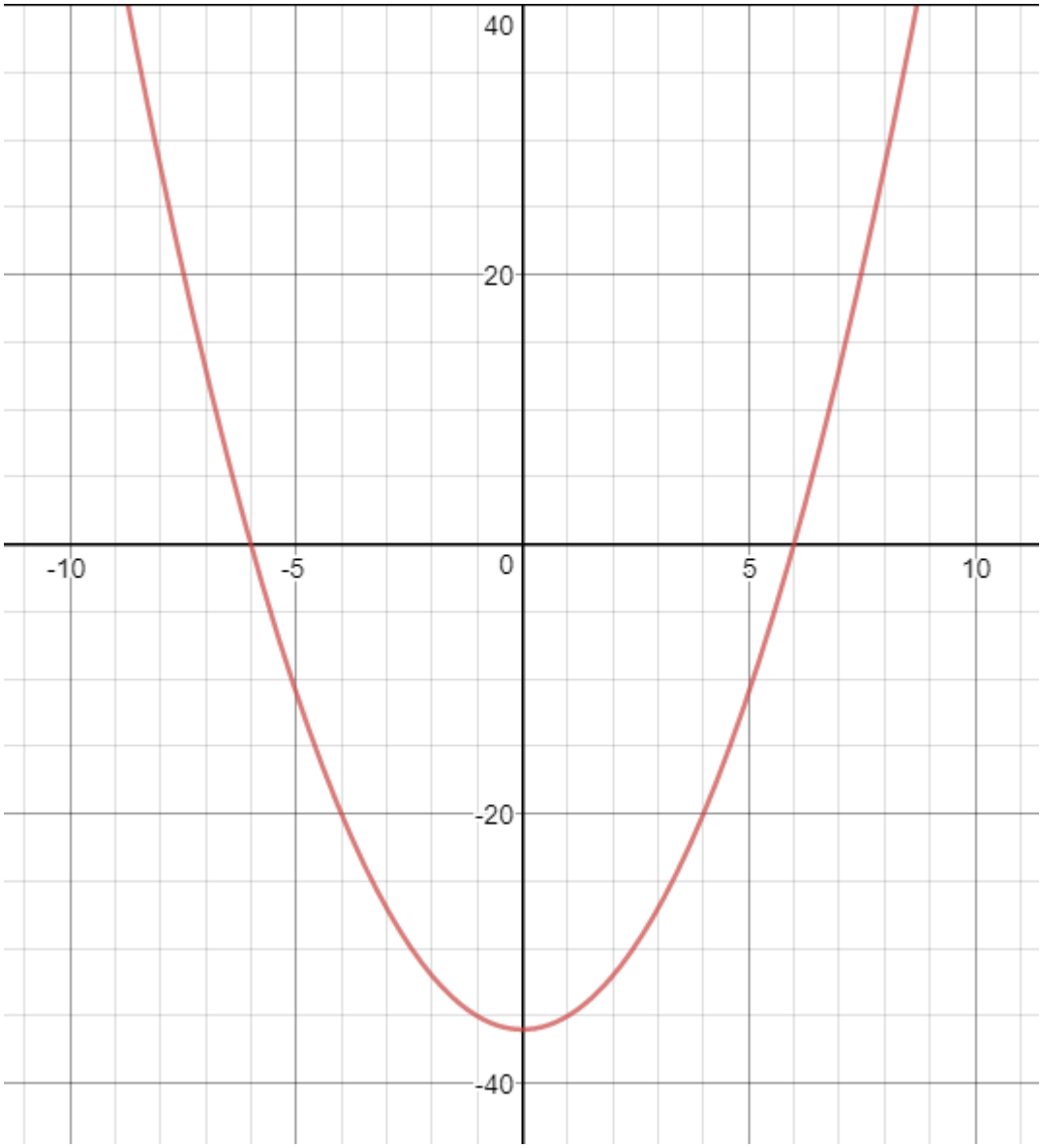
$y = x^2 + 6x + 9$



The graph **touches** the x axis (y = 0) at x = -3.

This implies that the roots of the quadratic equation are x = -3 and x = -3.

$y = x^2 - 36$



The graph **intersects** the x axis (y = 0) at x = -6 and x = 6.

This implies that the roots of the quadratic equation are x = -6 and x = 6.