Lab 8 - Uncertaintity in MDA

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The 4 tool functions are shown below.

Tool 1: Describes mass balance of the aircraft according to the following equation:

$$m_t = 100 + m_f + 6 * S_w + 800$$

Where m_t is total aircraft mass, m_f is fuel mass and S_w is wing area in square meters

```
function [mt L V mf] = tool1(y_vect,x_vect);
% Split up y_vect into all CA variables
mt = y_vect(1);
L = y_vect(2);
V = y_vect(3);
mf = y_vect(4);
% Split up x_vect into all design variables
Sw = x_vect(1);
t = x_vect(2);

%  Tool X is made up of the following equation
wing_mass_slope = 6 ; % the wing mass scales as 6kg/m2
pilot_mass = 100; %kg
structure_mass = 800 ; %kg
mt = pilot_mass + mf + Sw*wing_mass_slope + structure_mass  ; % total mass is the sum of fuel mass, wing mass,
```

Tool 2 describes the lift force - assumes no acceleration in the y direction:

$$L = 9.81 * m_t$$

```
function [mt L V mf] = tool2(y_vect,x_vect);
% Split up y_vect into all CA variables
mt = y_vect(1);
L = y_vect(2);
V = y_vect(3);
mf = y_vect(4);
% Split up x_vect into all design variables
Sw = x_vect(1);
t = x_vect(2);

%  Tool X is made up of the following equation
```

```
L = 9.81 * mt;
```

Tool 3 describes the velcoity of the aircraft - again asumes no acceleration and a C_l of 0.5 and assumes a single altitude where air density = 1.2 $\frac{\mathrm{kg}}{m^3}$

```
function [mt L V mf] = tool3(y_vect,x_vect);
% Split up y_vect into all CA variables
mt = y_vect(1);
L = y_vect(2);
V = y_vect(3);
mf = y_vect(4);
% Split up x_vect into all design variables
Sw = x_vect(1);
t = x_vect(2);

%  Tool X is made up of the following equation
C_l = 0.5;
rho = 1.2;
V = sqrt(L/(0.5*rho*C_l*Sw));
```

Tool 4 describes the energy consumption of the aircraft. Assumes a C_d of 0.025, a BSFC of 400$\frac{g}{\mathrm{kW} * \mathrm{hr}}$ and propeller efficiency of 0.85

```
function [mt L V mf] = tool4(y_vect,x_vect);
% Split up y_vect into all CA variables
mt = y_vect(1);
L = y_vect(2);
V = y_vect(3);
mf = y_vect(4);
% Split up x_vect into all design variables
Sw = x_vect(1);
t = x_vect(2);

%  Tool X is made up of the following equation
C_d = 0.025;
rho = 1.2;
BSFC = 400;
efficiency = 0.85;
D = 0.5 * rho * C_d * Sw * V^2;
mf = BSFC/1000 * D*V/1000 * t * efficiency;
```

Now lets use these tools to solve the system of equations using MatLabs fsolve function:

```
CA_vars_converged = fsolve('List_of_CAs',CA_vars_guess,optimset('Display','iter','MaxFunEvals',
```

```
                                         Norm of      First-order   Trust-region
 Iteration  Func-count     f(x)          step         optimality    radius
     0          5      1.07006e+06                     1.04e+03               1
     1         10       1.0673e+06              1      1.03e+03               1
     2         15      1.06078e+06            2.5      1.03e+03             2.5
     3         20       1.0464e+06           6.25      1.01e+03            6.25
     4         25      1.01587e+06         15.625           979            15.6
     5         30           944473        39.0625           909            39.1
     6         35           880498        97.6562      1.66e+03            97.7
     7         40           798088        244.141           774             244
     8         45           672845        610.352      1.34e+03             610
     9         50           454228        1525.88           535        1.53e+03
```

```
    10         55         120075          3814.7             543        3.81e+03
    11         60        6.94914         4197.52            6.51        9.54e+03
    12         65     0.00892315         6.80382           0.201        1.05e+04
    13         70    3.41955e-14        0.974618        4.23e-07        1.05e+04

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
CA_vars_converged = 1×4
10^4 ×
    0.1062    1.0421    0.0039    0.0028
```

With design variables of $S_w = 22.4\, m^2$ and $t = 4\,\text{hours}$ the result is a plane with the following parameters:

$m_t = 1062\,\text{kg}$

$V = 39\dfrac{m}{s^2}$

$m_f = 28\text{kg}$

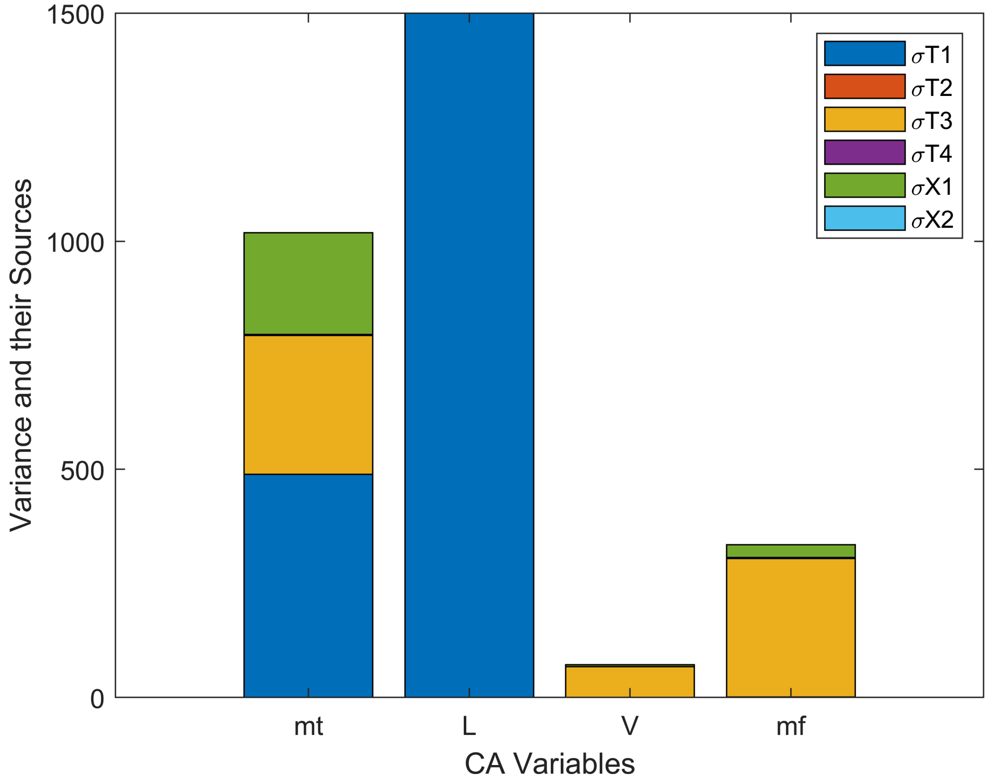
Ok that result is great and all, but how much confidence do we have in each of those numbers? Well we need to perform a System Sensitivity Analysis to find out.

Let's first run the code from Dr. Bradley to perform the SSA with all of the tool and input uncertaintities set to 0. We will expect the same results above with output of uncertainties of 0. To allow for easy running of the code with different input parameters, I created a function out of the m file.

```
CA_vars_guess = [1000,10000,50,50];
design_vars = [22.4, 4.0];
SSA(design_vars, CA_vars_guess)
```

```
Number of variables in each tool, each column is a tool

Tool x Variable

num_des_vars = 1×4
     1     0     1     2
num_CA_inputs = 1×4
     1     1     1     1
num_CA_outputs = 1×4
     1     1     1     1
num_CAs = 4
num_CA_vars = 1×4
     1     1     1     1
design_vars_in = 4×2
     1     0
     0     0
     1     0
     1     1
CA_in = 4×4
     0     0     0     1
```

```
     1     0     0     0
     0     1     0     0
     0     0     1     0
CA_out = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
CA_list_sorted_ordered = 4×1 cell array
'mt'
'L'
'V'
'mf'
design_vars_list_sorted_ordered = 2×1 cell array
'Sw'
't'
design_vars_errors_sorted_ordered = 1×2
    0.1000         0
CA_outputs_errors_sorted_ordered = 1×4
    0.0200    0.0100    0.2000    0.0500
ca_names_list = 1×4 cell array
'tool1'      'tool2'      'tool3'      'tool4'
                                         Norm of      First-order   Trust-region
 Iteration  Func-count     f(x)          step         optimality    radius
     0          5         43404.6                       1.95e+03               1
     1         10         39583.9              1        1.85e+03               1
     2         15         30890.7            2.5        1.61e+03             2.5
     3         20         14521.1           6.25             996            6.25
     4         25            3953         15.625            55.2            15.6
     5         30         1429.37        39.0625             195            39.1
     6         35         554.719        97.6563            24.4            97.7
     7         40         10.3818        244.141            10.3             244
     8         45       0.0001154        49.1087          0.0229             610
     9         50     4.72117e-17       0.108457        7.28e-09             610

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
These are the converged outputs of the MDA:
CA_vars = 1×4
10^4 ×
    0.1062    1.0421    0.0039    0.0028
These uncertainties are input to the SSA:
1) Tool uncertainties:
uncertT_fraction = 4×1
    0.0200
    0.0100
    0.2000
    0.0500
2) Design variable uncertainties:
uncertX_fraction = 2×1
    0.1000
         0
These uncertainties are output from the SSA:
1) Uncertainties in the CA variables, [ mt L V mf  ]:
uncertY_fraction = 4×1
    0.0300
    0.0318
    0.2150
    0.6555
```




```
CA_vars_guess = [1000,10000,50,50];
design_vars = [22.4, 4.0];
SSA(design_vars, CA_vars_guess)
```

```
Number of variables in each tool, each column is a tool

Tool x Variable

num_des_vars = 1×4
     1     0     1     2
num_CA_inputs = 1×4
     1     1     1     1
num_CA_outputs = 1×4
     1     1     1     1
num_CAs = 4
num_CA_vars = 1×4
     1     1     1     1
design_vars_in = 4×2
     1     0
     0     0
     1     0
     1     1
CA_in = 4×4
     0     0     0     1
     1     0     0     0
     0     1     0     0
     0     0     1     0
```

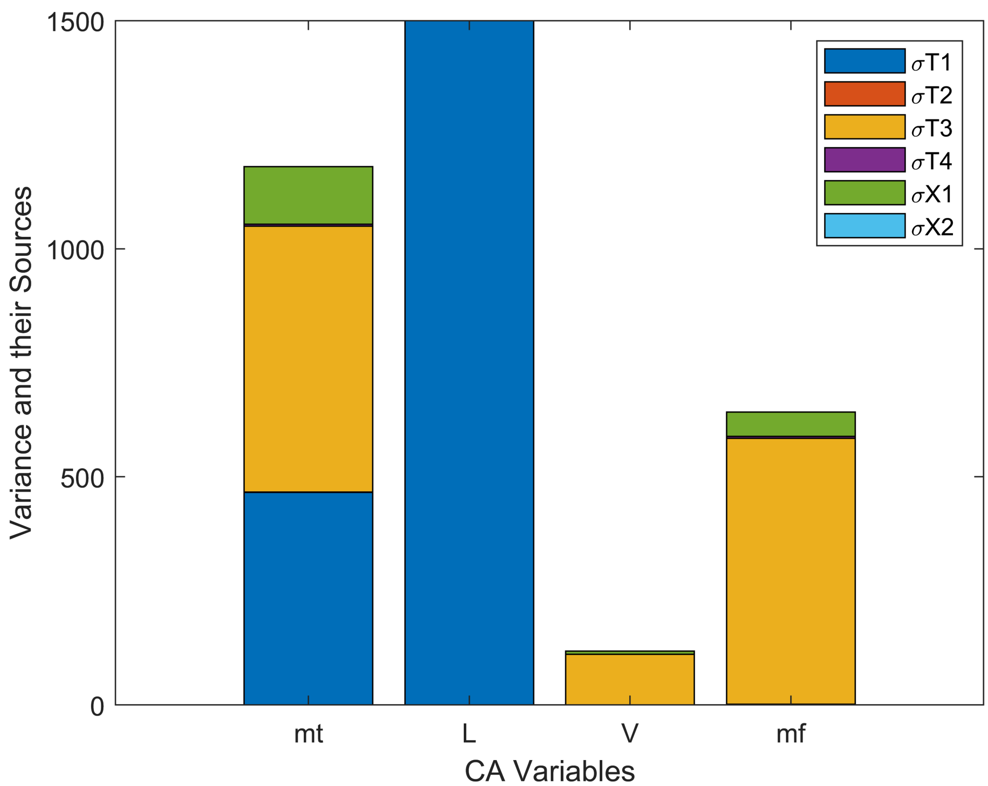
```
CA_out = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
CA_list_sorted_ordered = 4×1 cell array
'mt'
'L'
'V'
'mf'
design_vars_list_sorted_ordered = 2×1 cell array
'Sw'
't'
design_vars_errors_sorted_ordered = 1×2
    0.1000         0
CA_outputs_errors_sorted_ordered = 1×4
    0.0200    0.0100    0.2000    0.0500
ca_names_list = 1×4 cell array
'tool1'      'tool2'      'tool3'      'tool4'
                                          Norm of      First-order   Trust-region
 Iteration  Func-count     f(x)          step         optimality    radius
     0          5         43404.6                       1.95e+03             1
     1         10         39583.9              1        1.85e+03             1
     2         15         30890.7            2.5        1.61e+03           2.5
     3         20         14521.1           6.25             996          6.25
     4         25            3953         15.625            55.2          15.6
     5         30         1429.37        39.0625             195          39.1
     6         35         554.719        97.6563            24.4          97.7
     7         40         10.3818        244.141            10.3           244
     8         45       0.0001154        49.1087          0.0229           610
     9         50     4.72117e-17       0.108457        7.28e-09           610

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
These are the converged outputs of the MDA:
CA_vars = 1×4
10^4 ×
    0.1062    1.0421    0.0039    0.0028
These uncertainties are input to the SSA:
1) Tool uncertainties:
uncertT_fraction = 4×1
    0.0200
    0.0100
    0.2000
    0.0500
2) Design variable uncertainties:
uncertX_fraction = 2×1
    0.1000
         0
These uncertainties are output from the SSA:
1) Uncertainties in the CA variables, [ mt L V mf  ]:
uncertY_fraction = 4×1
    0.0300
    0.0318
    0.2150
    0.6555
```

```
CA_vars_guess = [1000,10000,50,50];
design_vars = [13.5, 4.5];
```

```
SSA(design_vars, CA_vars_guess)
```

```
Number of variables in each tool, each column is a tool

Tool x Variable

num_des_vars = 1×4
     1     0     1     2
num_CA_inputs = 1×4
     1     1     1     1
num_CA_outputs = 1×4
     1     1     1     1
num_CAs = 4
num_CA_vars = 1×4
     1     1     1     1
design_vars_in = 4×2
     1     0
     0     0
     1     0
     1     1
CA_in = 4×4
     0     0     0     1
     1     0     0     0
     0     1     0     0
     0     0     1     0
CA_out = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
CA_list_sorted_ordered = 4×1 cell array
'mt'
'L'
'V'
'mf'
design_vars_list_sorted_ordered = 2×1 cell array
'Sw'
't'
design_vars_errors_sorted_ordered = 1×2
    0.1000         0
CA_outputs_errors_sorted_ordered = 1×4
    0.0200    0.0100    0.2000    0.0500
ca_names_list = 1×4 cell array
'tool1'      'tool2'      'tool3'      'tool4'
                                        Norm of      First-order   Trust-region
 Iteration  Func-count     f(x)          step         optimality    radius
     0          5         37188.2                      1.89e+03               1
     1         10         33476.3              1        1.8e+03               1
     2         15         25055.8            2.5       1.55e+03             2.5
     3         20         9374.82           6.25            942            6.25
     4         25     0.000105424        15.2732         0.0236            15.6
     5         30     1.20133e-17       0.107781       8.18e-09            38.2

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
These are the converged outputs of the MDA:
CA_vars = 1×4
```

```
10^3  ×
    1.0190    9.9963    0.0497    0.0380
These uncertainties are input to the SSA:
1) Tool uncertainties:
uncertT_fraction = 4×1
    0.0200
    0.0100
    0.2000
    0.0500
2) Design variable uncertainties:
uncertX_fraction = 2×1
    0.1000
         0
These uncertainties are output from the SSA:
1) Uncertainties in the CA variables, [ mt L V mf  ]:
uncertY_fraction = 4×1
    0.0337
    0.0353
    0.2187
    0.6668
```



Task 4 - Homework

Problem 1

Present the converged solution of the MDA. What are the values of each of the CA variables at a relevant set of values of the design variables? You can choose DVs that are of interest to you

For this problem lets assume we are designing the Cirrus SR22 airplane pictured below and specs taken from here:

Wing Area	13.5 m^2
Endurance	4.5 hours
m_t	1450 kg
m_f	300 kg
V	35 m/s
L	14230 N

```
CA_vars_guess = [1000,10000,50,50];
design_vars = [13.5, 4.5];
CA_vars_converged = fsolve('List_of_CAs',CA_vars_guess,optimset('Display','iter','MaxFunEvals',
```

```
                                         Norm of      First-order   Trust-region
 Iteration  Func-count     f(x)          step         optimality    radius
     0          5         37188.2                      1.89e+03          1
     1         10         33476.3              1        1.8e+03          1
     2         15         25055.8            2.5       1.55e+03        2.5
     3         20         9374.82           6.25            942       6.25
     4         25     0.000105424        15.2732         0.0236       15.6
     5         30     1.20133e-17       0.107781       8.18e-09       38.2

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
CA_vars_converged = 1×4
10^3 ×
    1.0190    9.9963    0.0497    0.0380
```

Running the simulation with the design vars specified above gave the following results, with the outputs of the

	Actual	Estimated	Error
Wing Area	13.5 m^2	13.5 m^2	-
Endurance	4.5 hours	4.5 hours	-
m_t	**1450 kg**	**1019 kg**	**-30%**
m_f	**300 kg**	**497 kg**	**66%**
V	**35 m/s**	**38 m/s**	**9%**
L	**14230 N**	**9996 N**	**-30%**

simulation bolded.

For the crudeness of the model that was used to predict the total mass, fuel mass, velocity and lift I would say that the model did a pretty good job of estimating. The largest error if 66% for fuel mass. At first glance without running any uncertaintity analysis I would say that this model does its job as intended. Which is to give a good starting design point for moving into a more detailed design process.

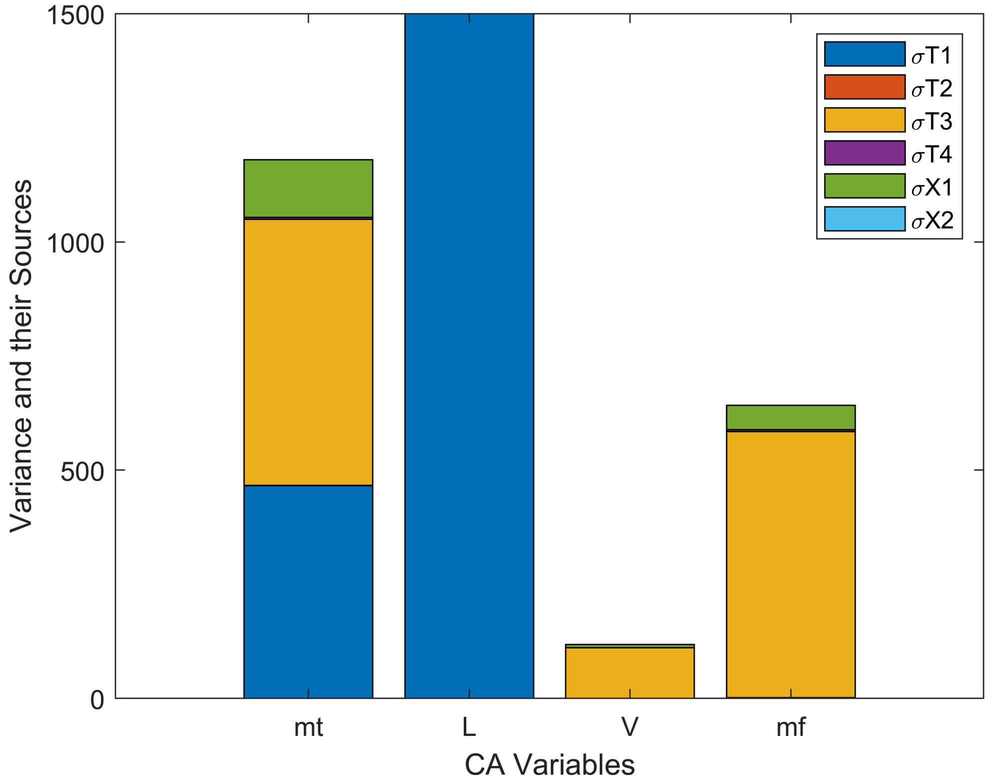
Problem 2

Present the results of the SSA in terms of the relevant total relative uncertainty in mt, and the contributors of uncertainty to mt. Make sure you make a nice plot to show me the contributors to uncertainty in mt. If you could spend money and time to reduce one of the tool errors or one of the design variables errors, which tool or design variable would you choose, and why?

```
CA_vars_guess = [1000,10000,50,50];
design_vars = [13.5, 4.5];
SSA(design_vars, CA_vars_guess)
```

```
Number of variables in each tool, each column is a tool

Tool x Variable

num_des_vars = 1×4
     1     0     1     2
num_CA_inputs =
     1     1     1     1
num_CA_outputs =
     1     1     1     1
num_CAs = 4
num_CA_vars =
     1     1     1     1
design_vars_in =
     1     0
     0     0
     1     0
     1     1
CA_in =
     0     0     0     1
     1     0     0     0
     0     1     0     0
     0     0     1     0
CA_out =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
CA_list_sorted_ordered = 4×1 cell array
    {'mt'}
    {'L' }
    {'V' }
    {'mf'}
design_vars_list_sorted_ordered = 2×1 cell array
    {'Sw'}
    {'t' }
design_vars_errors_sorted_ordered =
    0.1000         0
CA_outputs_errors_sorted_ordered =
    0.0200    0.0100    0.2000    0.0500
ca_names_list = 1×4 cell array

    {'tool1'}    {'tool2'}    {'tool3'}    {'tool4'}
                                         Norm of      First-order   Trust-region
 Iteration  Func-count     f(x)          step         optimality    radius
     0          5         37188.2                       1.89e+03               1
     1         10         33476.3              1         1.8e+03               1
```

```
     2         15         25055.8            2.5      1.55e+03             2.5
     3         20         9374.82           6.25           942            6.25
     4         25     0.000105424        15.2732        0.0236            15.6
     5         30     1.20133e-17       0.107781      8.18e-09            38.2

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
These are the converged outputs of the MDA:
CA_vars =
    1.0190    9.9963    0.0497    0.0380
These uncertainties are input to the SSA:
1) Tool uncertainties:
uncertT_fraction =
    0.0200
    0.0100
    0.2000
    0.0500
2) Design variable uncertainties:
uncertX_fraction =
    0.1000
         0
These uncertainties are output from the SSA:
1) Uncertainties in the CA variables, [ mt L V mf  ]:
uncertY_fraction =
    0.0337
    0.0353
    0.2187
    0.6668
```



Variance and their source for each of the Contributing Analysis Outputs

Since we are most interested in the total mass output of the CA, let's examine its error further. The total mass has a total variance of about 1200 kg^2. The variance due to Tool 1 and Tool 3 contribute the most to the variance of the mass. However The variance due to tool 1 is much larger than all the other variances because it is scaled up by 10 times due to the inner workings of tool one. However the error due to Tool 3 contributes just as much to the variance in the mass out put. Error due to Tool 3 also shows up in the CA variables V and fuel mass wheras error due to Tool 1 does this. Because of these 2 factors I would choose to perform experiments to **reduce uncertaintity in Tool 3.**

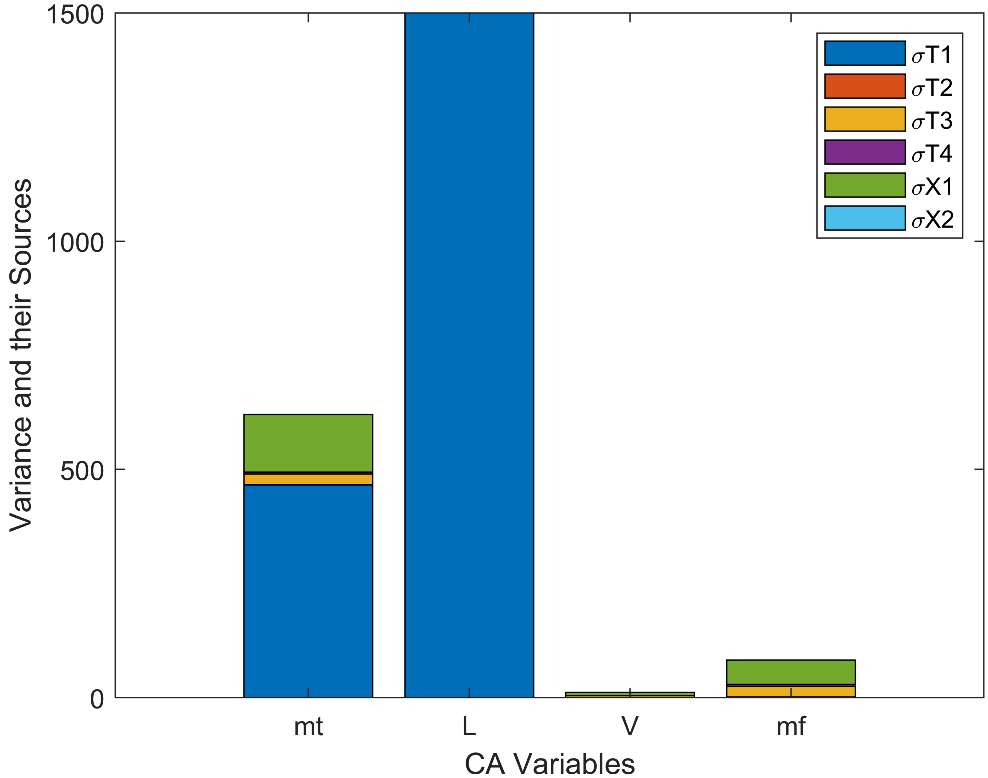
Problem 3

Assume that you can reduce the relative uncertainty associated with one of the tool errors OR one of the design variables error by a factor of 5, by performing some experiment. Perform the SSA using that reduced value of uncertainty to show that your experimental will be successful in reducing both the total uncertainty and allocation of uncertainty to mt, as you expected.

Let's assume that we went out and did some experiments relating to tool 3 and that we were able to gather data to support that our tool is actually accurate to within 4% instead of 20%. Now lets rerun the same analysis as above, while making the change to the Tool 3 uncertaintity.

```
CA_vars_guess = [1000,10000,50,50];
design_vars = [13.5, 4.5];
SSA(design_vars, CA_vars_guess)
```

```
Number of variables in each tool, each column is a tool

Tool x Variable


                                         Norm of      First-order   Trust-region
 Iteration  Func-count     f(x)          step         optimality    radius
     0          5         37188.2                       1.89e+03               1
     1         10         33476.3              1         1.8e+03               1
     2         15         25055.8            2.5        1.55e+03             2.5
     3         20         9374.82           6.25             942            6.25
     4         25     0.000105424        15.2732          0.0236            15.6
     5         30     1.20133e-17       0.107781        8.18e-09            38.2

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
```



Variance and their Sources for CA Variables after reducing Tool 3 Uncertaintity

As can be seen above, by reducing the uncertaintity in Tool 3 from 20% to 4%, the variance associated with Tool 3 in the total mass of the airplane was cut in half. The total variances in CA variables V and fuel mass were also greatly reduced. I would say that money was well spent.