MATH5605 ASSIGNMENT 3

DUE MAY 15, 2014

Problem 1: Let $\mathcal{X}_0 \subseteq \mathcal{X}$ be a subspace in a normed space $\mathcal{X}$. Prove that $\mathcal{X}/\mathcal{X}_0 \subseteq (\mathcal{X}_0^\perp)^*$ isometrically.

Problem 2: Let $\mathcal{X}$ be a normed space. A subset $U \subseteq \mathcal{X}$ is called *open* if and only if for any $x \in U$ there is $\epsilon > 0$ such that

$$\{x' \in \mathcal{X} : \quad \|x - x'\| < \epsilon\} \subseteq U.$$

That is, every point of U belongs to U together with a open ball centred at that point.

Let $\mathcal{X}$ and $\mathcal{Y}$ be normed spaces and let $T : \mathcal{X} \mapsto \mathcal{Y}$ is a linear mapping. Prove that T is bounded if and only if the pre-image $T^{-1}(U)$ is open for every open subset $U \subseteq \mathcal{Y}$. Here,

$$T^{-1}(U) = \{x \in \mathcal{X} : \quad T(x) \in U\}.$$

Problem 3: Let $\mathcal{X} = \ell^1$ and $\mathcal{X}_0 = \left\{(\xi_k) \in \ell^1 : \sum_k \xi_k = 0\right\}$. Find $\mathcal{X}_0^\perp$ and $\mathcal{X}/\mathcal{X}_0$.

E-mail address: d.potapov@unsw.edu.au

Date: May 7, 2014.