MATH5605 ASSIGNMENT 1

DUE MAR 27, 2014

Problem 1: Let $f(x) = B_k(x)$ be the k-th Bernoulli polynomial, i.e.,

1) $B_k'(x) = kB_{k-1}(x)$;
2) $B_k(0) = B_k(1)$, $k > 1$;
3) $B_1(x) = x - \frac{1}{2}$.

Find the expansion

$$f(x) = \sum_{n\in\mathbb{Z}} \langle f, e_n \rangle \, e_n,$$

where $\{e_n\}_{n\in\mathbb{Z}}$ is the trigonometric basis of $L^2(0,1)$, i.e., $e_n(t) = e^{2\pi i n t}$.

Problem 2: Prove that the (normalized) Legendre polynomials are the orthonormal system resulting form the Gram-Schmidt orthogonalisation process applied to the system of monomials $\left\{t^k\right\}_{k=0}^{\infty}$ in the space $L^2(-1,1)$.

Problem 3: For every monomial $p_k(t) = t^k$, $k = 0, 1, \ldots$ in $L^2(-1,1)$, find the expansion

$$p = \sum_{n=0}^{\infty} \langle p, e_n \rangle \, e_n,$$

where $\{e_n\}_{n=0}^{\infty}$ is the system of normalized Legendre polynomials.

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