MATH5605 ASSIGNMENT 2

DUE APR 17, 2014

Problem 1: Prove that every positive operator $T \in B(\mathcal{H})$, i.e., the operator satisfying

$$\langle Tx, x \rangle \geq 0, \quad \forall x \in \mathcal{H}$$

is self-adjoint.

Problem 2: Let $T = T_A$ be a Toeplitz operator on ℓ^2, i.e., a matrix-type operator T_A with respect to the matrix

$$A = \left\{ a_{j-k} \right\}_{j,k=1}^{\infty}$$

where $\{a_m\}_{m \in \mathbb{Z}}$ is a sequence of complex numbers.

1) Prove that T is bounded if

$$\sum_{m \in \mathbb{Z}} |a_m| < +\infty.$$

2) Is this condition sharp?[1]
3) When is a Toeplitz operator Hilbert-Schmidt?

Problem 3: For $T \in B(\mathcal{H})$ show that

1) $(\operatorname{Im} T)^{\perp} = \operatorname{Ker} T^*$;
2) $(\operatorname{Ker} T)^{\perp} = \overline{\operatorname{Im} T^*}$.

Here

$$\operatorname{Im} T = \{Tx, \quad x \in \mathcal{H}\} \quad \text{and} \quad \operatorname{Ker} T = \{x \in \mathcal{H} : \quad Tx = 0\}.$$

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Date: April 9, 2014.

[1]This is very hard!