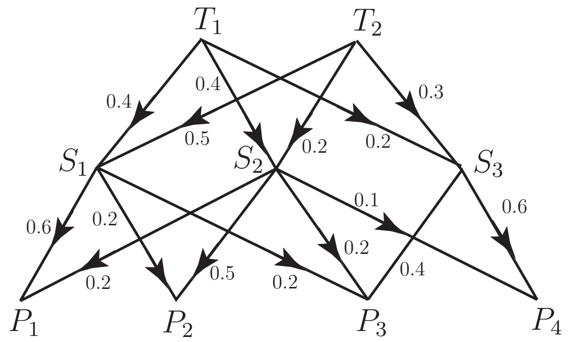
6.

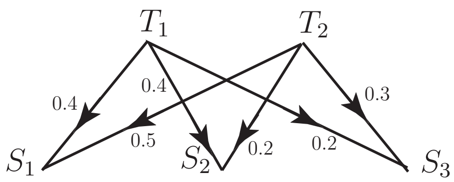
Petrol is delivered to terminals T_1 and T_2. They distribute the fuel to 3 storage depots ($S_1,\ S_2,\ S_3$). The network diagram below shows what fraction of the fuel goes from each terminal to the three storage depots. In turn the 3 depots supply fuel to 4 petrol stations (P_1, P_2, P_3, P_4) as shown below:



Question 1. Show how this situation may be described using matrices

Question 2.

Denote the amount of fuel, in litres, flowing from T_1 by t_1 and from T_2 by t_2 and the quantity being received at S_i by s_i for $i = 1, 2, 3$. This situation is described in the following diagram:

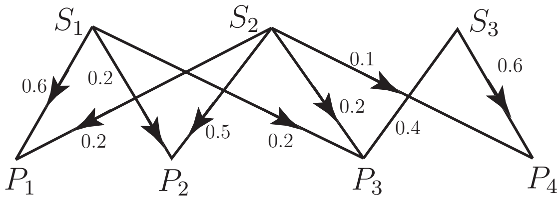


From this diagram we see that

$$\begin{aligned} s_1 &= 0.4t_1 + 0.5t_2 \\ s_2 &= 0.4t_1 + 0.2t_2 \\ s_3 &= 0.2t_1 + 0.3t_2 \end{aligned} \qquad \text{or, in matrix form:} \quad \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} =$$

Question 3.

In turn the 3 depots supply fuel to 4 petrol stations as shown in the next diagram:



$$\begin{aligned} p_1 &= 0.6s_1 + 0.2s_2 \\ p_2 &= 0.2s_1 + 0.5s_2 \\ p_3 &= 0.2s_1 + 0.2s_2 + 0.4s_3 \\ p_4 &= 0.1s_2 + 0.6s_3 \end{aligned}$$ or, in matrix form: $\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} =$

Exercises

1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ $\quad C = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$ find

   (a) AB, (b) AC, (c) $(A+B)C$, (d) $AC+BC$ (e) $2A-3C$

2. If a rotation through an angle θ is represented by the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and a second rotation through an angle ϕ is represented by the matrix $B = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$ show that both AB and BA represent a rotation through an angle $\theta + \phi$.

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ -1 & 2 \\ 5 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find AB and BC.

4. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, $\quad B = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 0 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, $\quad C = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$,

   verify $A(BC) = (AB)C$.

5. If $A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 2 \\ 4 & 5 & 6 \end{bmatrix}$ then show that AA^T is symmetric.