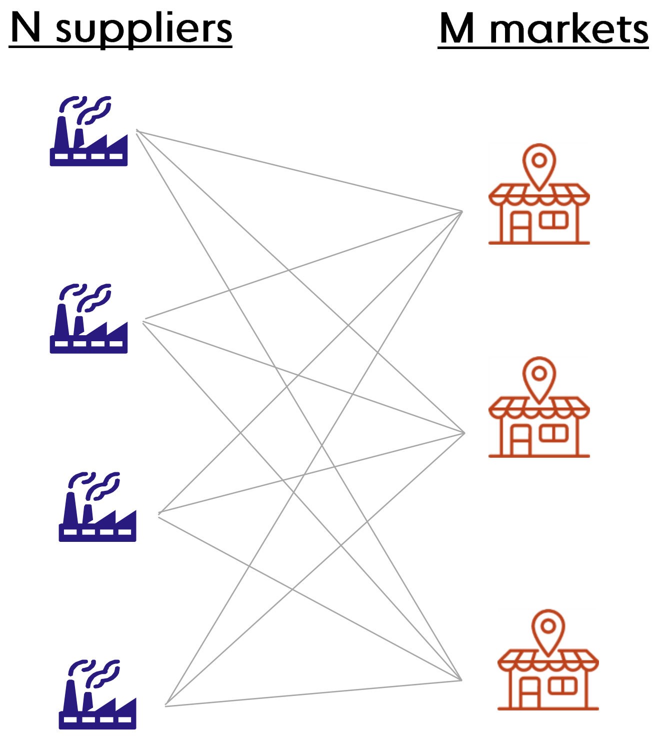
AFRICA BUSINESS SCHOOL – UM6P

Program FEOR – Module 2 – Optimization I

TP : the transportation problem

Projet : the transportation problem (I)

Problem description



Consider the market of an unique product

- This market has N suppliers (or plants), each one of capacity K_i with a production cost C_i
- They have to serve the demand D_j of M markets
- The unit cost for sending a ton of product from plant i to market j is TC_{ij}

Key question : which plant shall serve which market in order to minimize the total (production + transportation) cost ?

Projet : the transportation problem (II)

Questions

1. **Model the market as if a « big owner » owned all the suppliers and aims at satisfying the demands at least cost.** Solve this optimization problem with PuLP
2. **Use PuLP to get the dual variables.** Why does it make sense to interpret the dual variables associated with the Demand constraints as prices ?
3. Observe that for the obtained price, the obtained flows are such that **each supplier is maximising its margin**
4. Impose a constraint stating that a supplier shall have **not more than 50% of market share** on any market. What is the impact on the prices ?
5. **Write the dual of the problem, and solve it with PuLP.** Check that the value of the variables are the same that what was obtained with the primal
6. **Solve the problem by using the strong duality theorem :** find the solution by defining both dual variables and constraints, then setting that primal objective = dual objective. Use a dummy objective
7. Use this to find a solution that **minimizes at the same time** the cost and the flows arising from plant P4
8. Can you use the same trick to **both miminise the cost and maximise the margin of a given plan** ?

Bonus

1. **How to interpret the primal model in the case where the suppliers are different companies ?**
2. **Is the price equal to the cost of the marginal player ?**
3. **How to interpret the dual problem ?**
4. **Try to prove the question 3** by using the dual formulation and the complementary slackness, i.e., :

   Flow[i][j] > 0 $\leftrightarrow$ the corresponding dual constraint is binding