Assignment 4

Download and familiarise yourself with the file ForwardEuler.m from Canvas. This assignment has 3 questions.

Question 1

Write a function RungeKutta4, which uses **exactly the same input/output structure** as ForwardEuler.m and which performs time integration of a generic systems of ODEs of the form

$$\begin{aligned} y'(t) &= f(t, y(t)), \qquad t \in (0, T], \\ y(0) &= y_0, \end{aligned}$$

using the following 4th-order Runge-Kutta scheme

$$\begin{aligned} y_{n+1} &= y_n + \frac{1}{6} h \left(k_1 + 2k_2 + 2k_3 + k_4\right) \\ t_{n+1} &= t_n + h \end{aligned}$$

$n = 0, 1, 2, 3, \ldots,$ where h is the stepsize and

$$\begin{aligned} k_1 &= f\left(t_n, y_n\right) \\ k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right) \\ k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right) \\ k_4 &= f\left(t_n + h, y_n + hk_3\right) \end{aligned}$$

Question 2

The solution to this system of ODEs

$$\begin{aligned} x' &= -x \\ y' &= -y \\ x(0) &= 1 \\ y(0) &= 1 \end{aligned}$$

can be found in closed form. Use this system to test ForwardEuler.m and RungeKutta4.m. Provide evidence that the Forward Euler method is an order 1 method, while the Runge Kutta 4 method is an order 4 method.

Question 3

Consider the following system of ODEs

$$x' = y$$
$$y' = -y/2 - \sin x$$

Use the Runge Kutta 4 scheme to plot a phase portrait of the system (you may want to recall what a phase portrait is from the first part of the module). To do this, you can perform time stepping for a range of initial conditions, and plot the corresponding orbits in the (x, y)-plane. How many equilibria do you find in the domain $(x, y) \in [-2\pi, 2\pi] \times [-\pi, \pi]$? Classify these equilibria based on your plots and the data you produced.