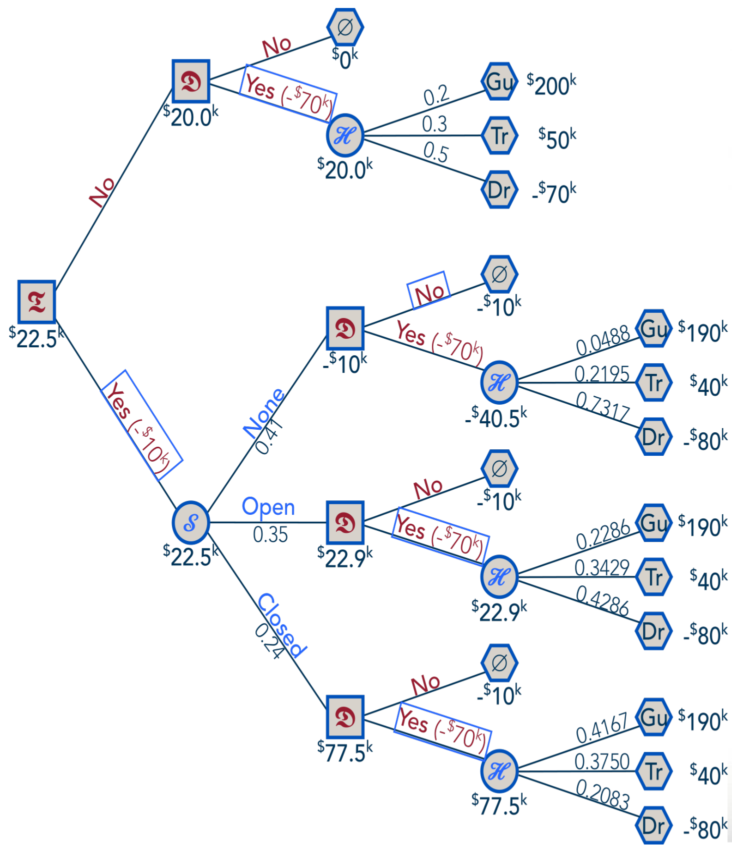
This case study appears multiple in the Theory-of-Evidence literature to illustrate its application to decision analysis. An oil-drilling illustration we decide whether to "test" with a geophysical survey, and then whether to drill.

Decision-Tree Representation



Case-Study Description

3. DECISION ANALYSIS

In the preceding section we have defined the concept of an expected utility interval for belief functions and we have shown that it bounds the expected utility that would be obtained with any probability distribution consistent with that belief function. Furthermore, we have proposed a parameter (the probability that residual ambiguity will be decided in our behalf) that can be used as the basis for computing a unique expected utility when the available evidence warrants only bounds on that expected utility. In this section we will show how the expected utility interval can be used to generalize probabilistic decision analysis.

Decision analysis was first developed as a means by which one could organize and systematize one's thinking when confronted with an important and difficult choice (Howard, 1966; Raiffa, 1970). Its formal basis has made it adaptable as a computational procedure by which computer programs can choose actions when provided with all relevant information. Simply stated, the analysis of a decision problem under uncertainty entails the following steps:

- List the viable options available for gathering information, for experimentation, and for action.
- List the events that may possibly occur.
- Arrange the information you may acquire and the choices you may make in chronological order.
- Decide the value to you of the consequences that result from the various courses of action open to you.
- Judge the chances that any particular uncertain event will occur.

3.1. Decision analysis using probabilities

First we will illustrate the use of decision analysis on a problem that can be represented with probabilities to acquaint the reader with the method and terminology.

Example 4 (Oil Drilling # 1): A wildcatter must decide whether or not to drill for oil. He is uncertain whether the hole will be dry, have a trickle of oil, or be a gusher. Drilling a hole costs $70,000. The payoffs for hitting a gusher, a trickle, or a dry hole are $270,000, $120,000, and $0, respectively. At a cost of $10,000 the wildcatter could take seismic soundings that would help determine the underlying geologic structure. The soundings will determine whether the terrain has no structure, open structure, or closed structure. The

experts have provided us with the joint probabilities shown in Table 13.5. We are to determine the optimal strategy for experimentation and action (Lapin, 1981).

Table 13.5: Probabilities for Oil Drilling # 1.

State	No struct	Open	Closed	Marginal
Dry	0.30	0.15	0.05	0.50
Trickle	0.09	0.12	0.09	0.30
Gusher	0.02	0.08	0.10	0.20
Marginal	0.41	0.35	0.24	1.00

In decision analysis, a decision tree is constructed that captures the chronological order of actions and events (Lapin, 1981; La Valle, 1970). A square is used to represent a decision to be made, and its branches are labeled with the alternative choices. A circle is used to represent a chance node, and its branches are labeled with the conditional probability of each event, given that the choices and events along the path leading to the node have occurred.

To compute the best strategy, the tree is evaluated from its leaves toward its root.

- The value of a leaf node is the utility of the state of nature it represents.
- The value of a chance node is the expected utility of the probability distribution represented by its branches as computed using (1).
- The value of a choice node is the maximum of the utilities of each of its sons. The best choice for the node is denoted by the branch leading to the son with the greatest utility. Ties are broken arbitrarily.

This procedure is repeated until the root node has been evaluated. The value of the root node is the expected utility of the decision problem; the branches corresponding to the maximal value at each choice node give the best *strategy* to follow (i.e., choices to make in each situation).

The evaluated decision tree for the oil drilling example is portrayed in Figure 13.4. It can be seen that the expected value is $22,500 and that the best strategy is to take seismic soundings, to drill for oil if the soundings indicate open or closed structure, and not to drill if the soundings indicate no structure.

T. M. Strat, "Decision analysis using belief functions," *Advances in the Dempster-Shafer Theory of Evidence*, Wiley, 1994.

Marginalizing the decision tree leaf-node conditional probabilities leads to these outcome-event quantiles.

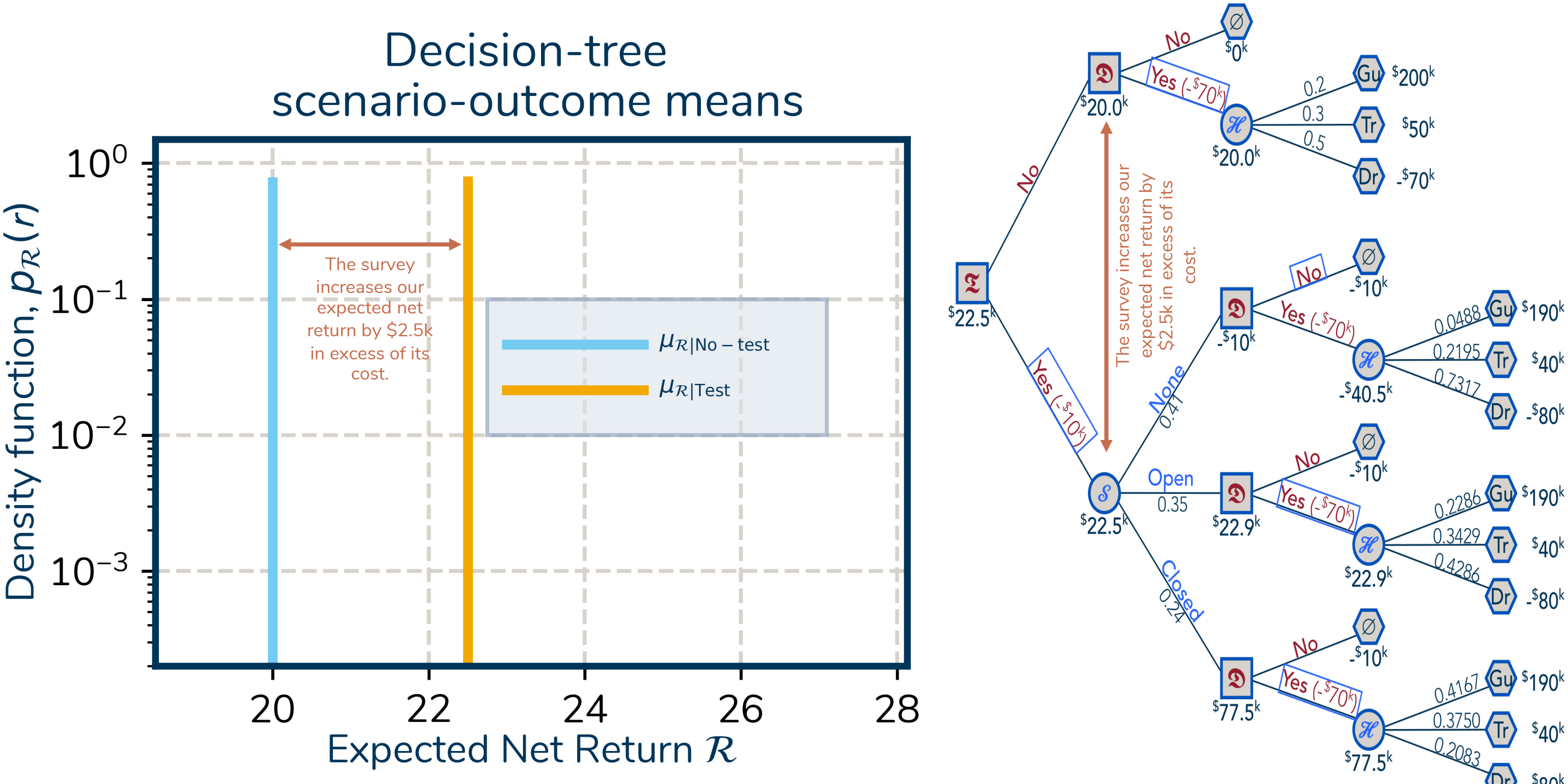
Outcome-scenario probabilities conditioned on information from geophysical survey.

	$\mathfrak{t}$ = Test; $\mathfrak{s}$ ≠ None; $\mathfrak{h}$ = Dr	$\mathfrak{t}$ = Test; $\mathfrak{s}$ = None	$\mathfrak{t}$ = Test; $\mathfrak{s}$ ≠ None; $\mathfrak{h}$ = Tr	$\mathfrak{t}$ = Test; $\mathfrak{s}$ ≠ None; $\mathfrak{h}$ = Gu
Expected Net Return, $\mathcal{R}$	-80.000000	-10.000000	40.000000	190.000000
Event Probability, $P(r = \mathscr{R})$	0.200002	0.410000	0.210015	0.180018
Quantile Proability, $P(r \leq \mathscr{R})$	0.195117	0.595103	0.799989	0.975610

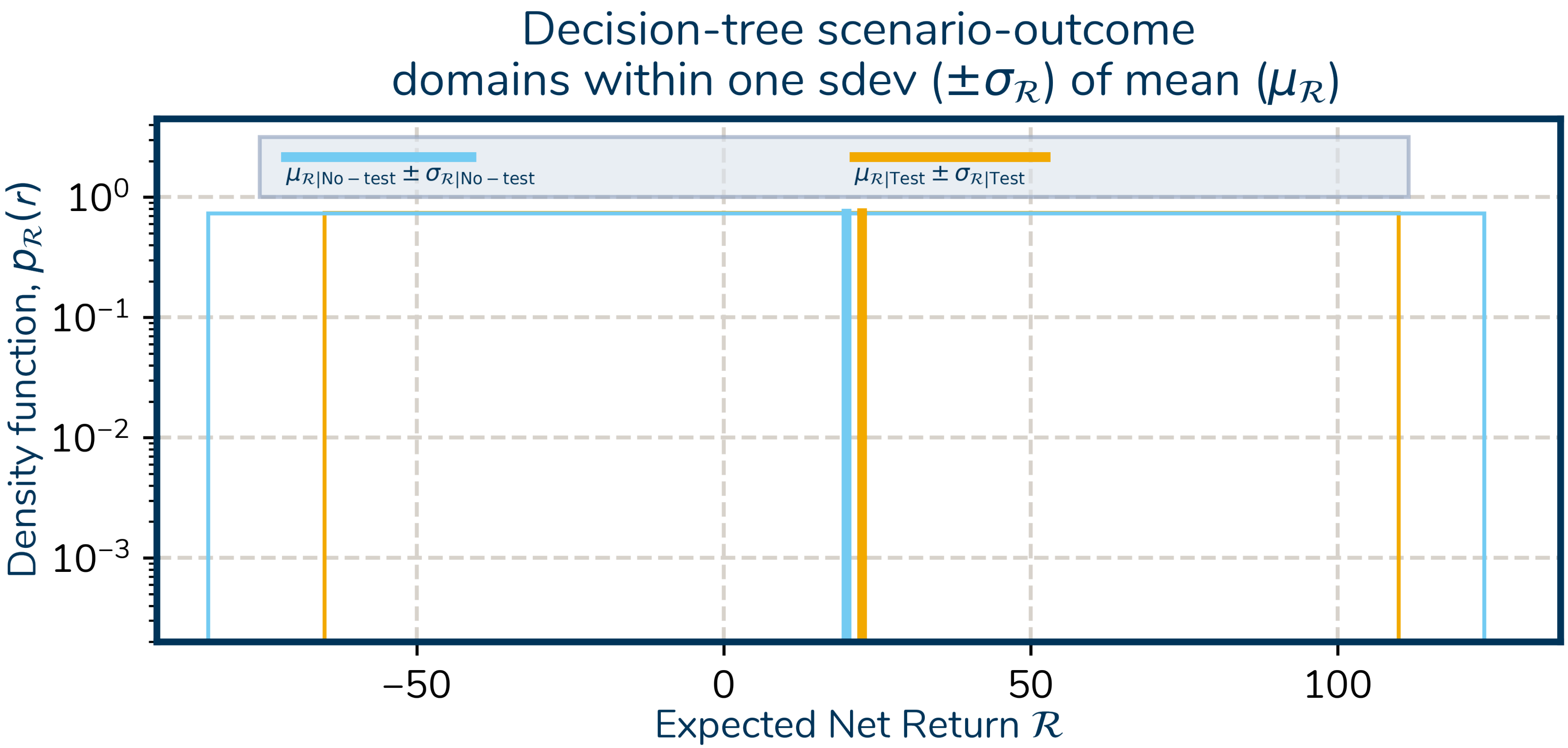
Outcome-scenario probabilities without information from the geophysical survey.

	$\mathfrak{t}$ ≠ Test; $\mathfrak{h}$ = Dr	$\mathfrak{t}$ ≠ Test; $\mathfrak{h}$ = Tr	$\mathfrak{t}$ ≠ Test; $\mathfrak{h}$ = Gu
Expected Net Return, $\mathcal{R}$	-70.000000	50.000000	200.00000
Event Probability, $P(r = \mathscr{R})$	0.500000	0.300000	0.20000
Quantile Proability, $P(r \leq \mathscr{R})$	0.487805	0.780488	0.97561

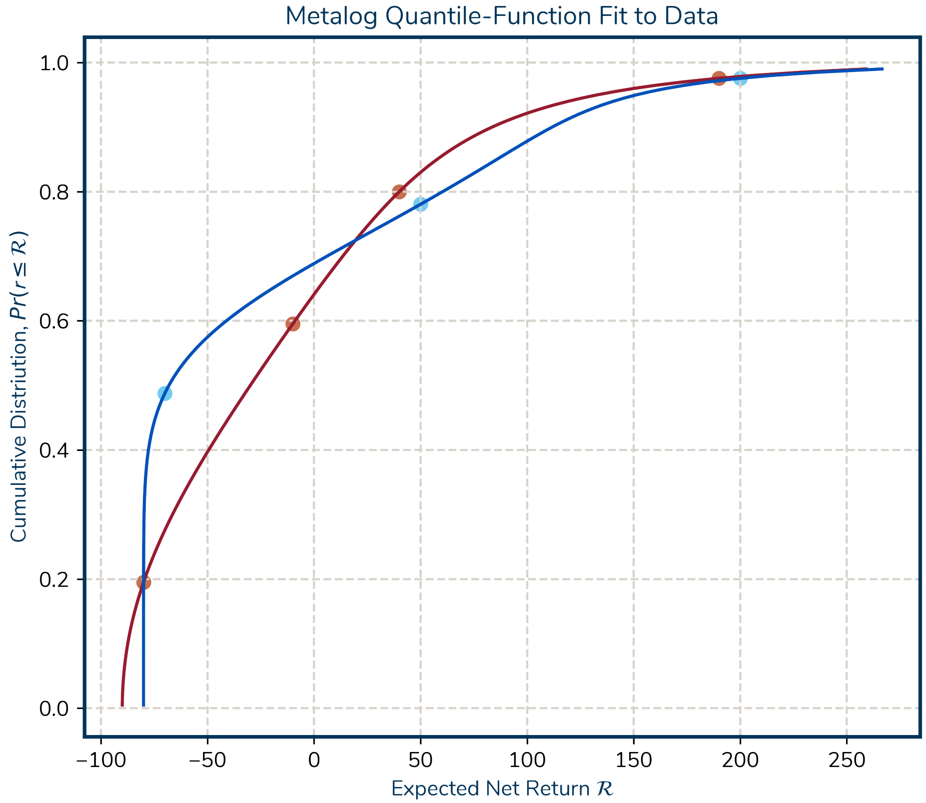
The case illustrates that incurring the cost of a geophysical survey adds information that increases the expected net return. The increase exceeds the cost of the survey. We should therefore do the survey.

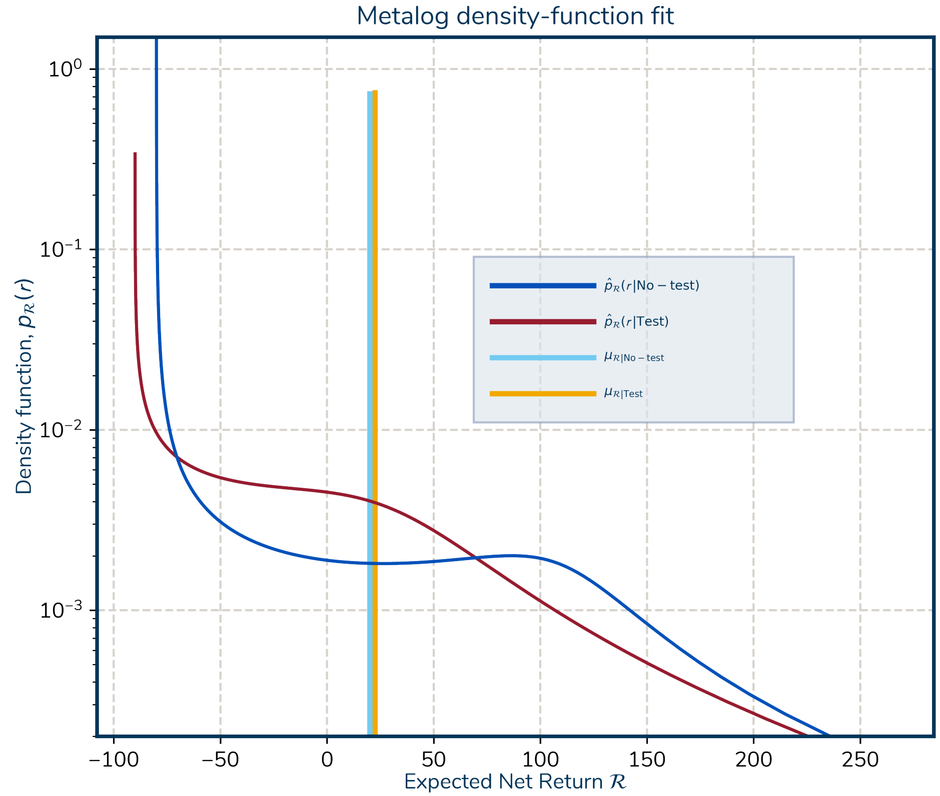


When however you look at the standard deviations of the event probabilities (from tables on slide 2), you find that the differences between the expected outcomes might not be significantly different. Information introduced from the geophysical survey reduces the variance of the outcome results ($\sigma_{\mathcal{R}}$). The $\mu_{\mathcal{R}} \pm \sigma_{\mathcal{R}}$ regions however mostly overlap.

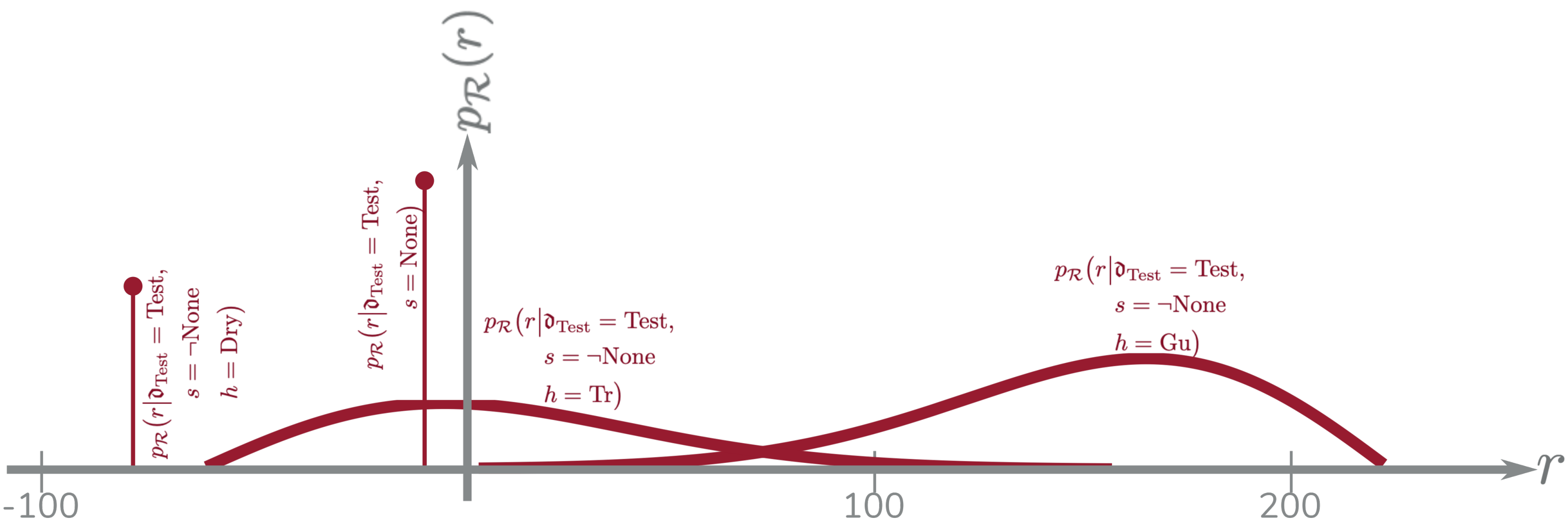


Following Keelin (2016), we can fit the quantile probabilities from our outcomes to metalog distributions. The resulting density functions exhibit pronounced skewness due to high probabilities of adverse outcomes near the left-hand limits.

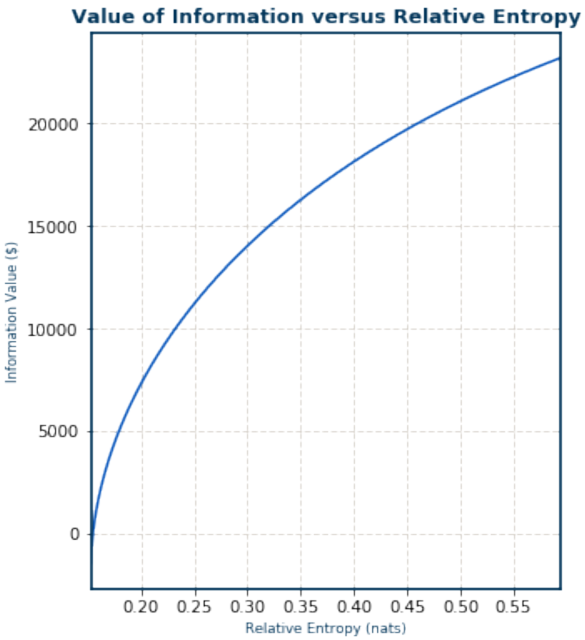


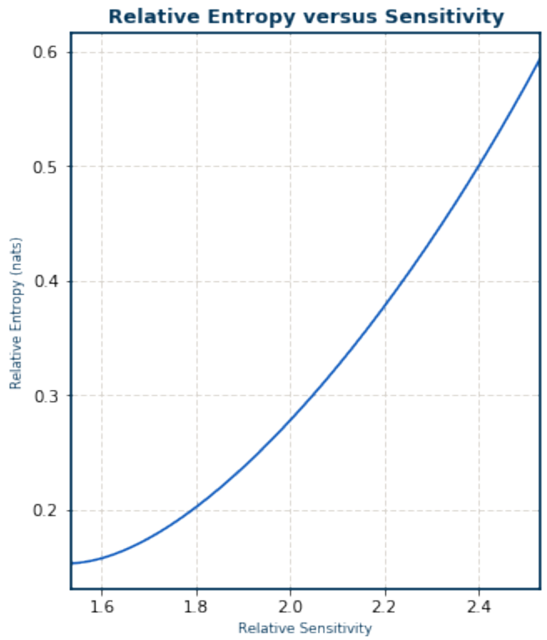


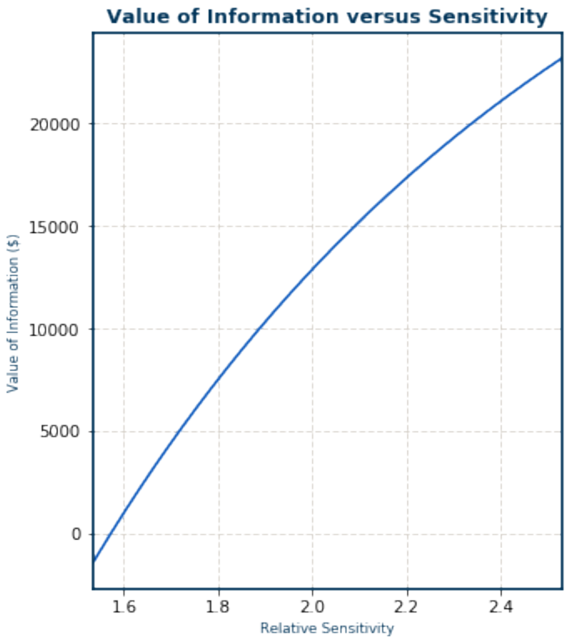
The metalog-distribution fits leave room for improvement. We haven't yet incorporated all of the information we have about the probabilistic structure of our decision. Our probabilities are actually mixtures of distinct event probabilities, each with a unique structure.



The paper to the June 2019 Advances in Decision Analysis conference included a parameter study. These statistical parameters are somewhat opaque. A parameter sweep in terms of more-explicit quantities (e.g., coefficient of variation σ÷μ?) might be more illuminating.







Questions.

1. Are second-order statistics (e.g., standard deviations) commonly used in these types of decision-analysis methods? It doesn't jump out during a skim of Howard's book (https://amzn.to/2PSAErg).
2. This particular scenario — probably stylized, contrived — doesn't seem to provide a compelling degree of separation between decision-driven scenarios. Is this typical?
3. Might another case study occur to you?