

Towards explicit calculation of terms in the Bias-Variance Decomposition (BVD).



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<https://hamlett-neil-ur.github.io/>

The **Bias-Variance Decomposition*** characterizes ML uncertainty.

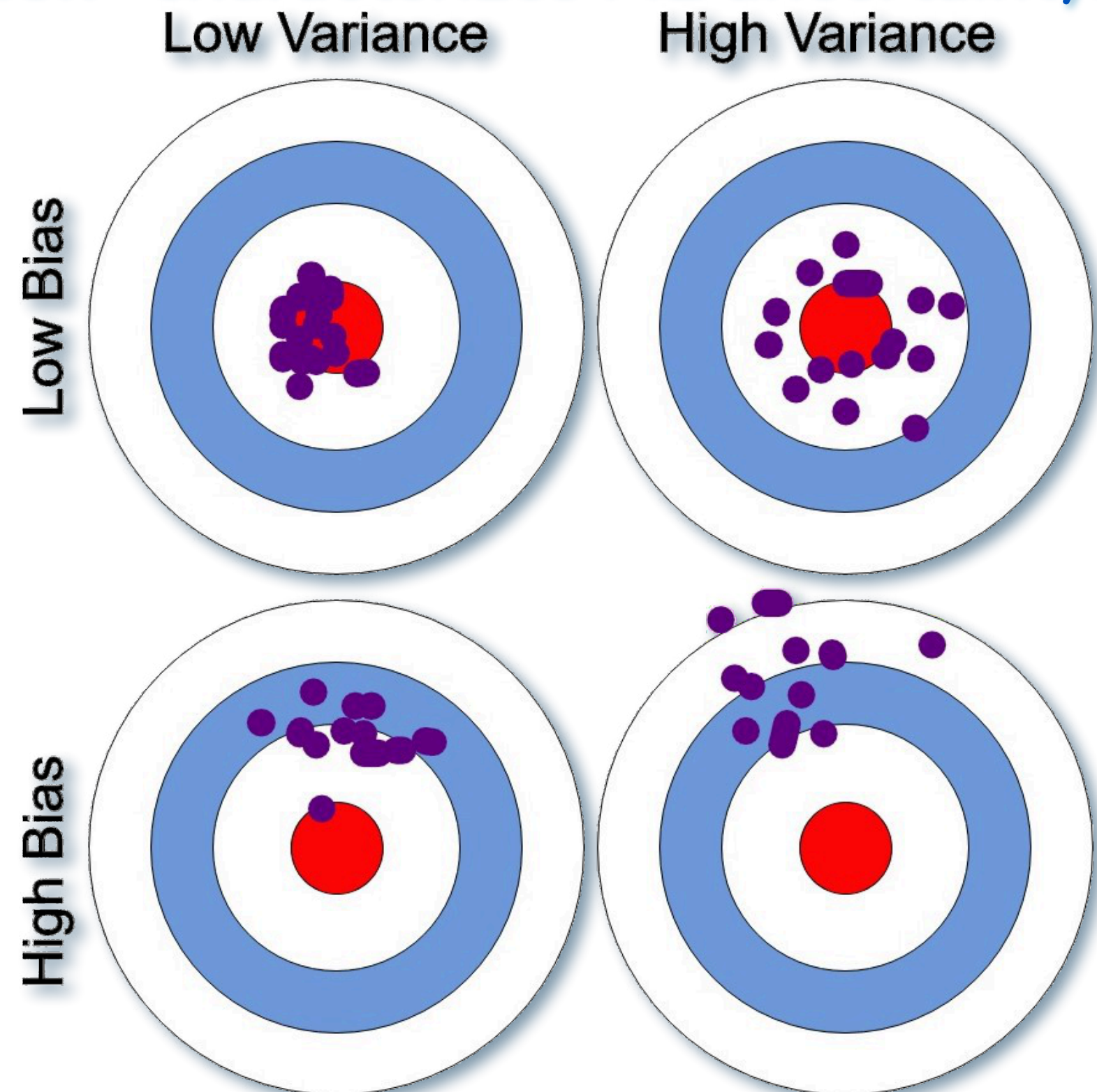
Irreducible Error. The extent to which variation in the data defies explanation **by any model**.

Bias. The extent to which **a given model** fails to explain the phenomenologically-based structure of the data.

Variance. The extent to which **a given model** explains the “structure” of the idiosyncratic error of a given sample. (Overfitting.)

$$\begin{aligned} Err(x_0) &= \sigma_\epsilon^2 \\ &+ \left[\mathcal{E} \hat{f}(x_0) - f(x_0) \right]^2 \\ &+ \left[\hat{f}(x_0) - \mathcal{E} \hat{f}(x_0) \right]^2 \end{aligned}$$

* T. Hastie, R. Tibshirian, J. Friedman, *Elements of Statistical Learning*, Springer, 2009, <https://bityl.co/5qZD>.



Metalog distributions provide a highly-flexible approach to analysis of residual errors from ML-model fits.

Metalog distributions' formulations are based on the quantile function $M_n(y)$ defined as

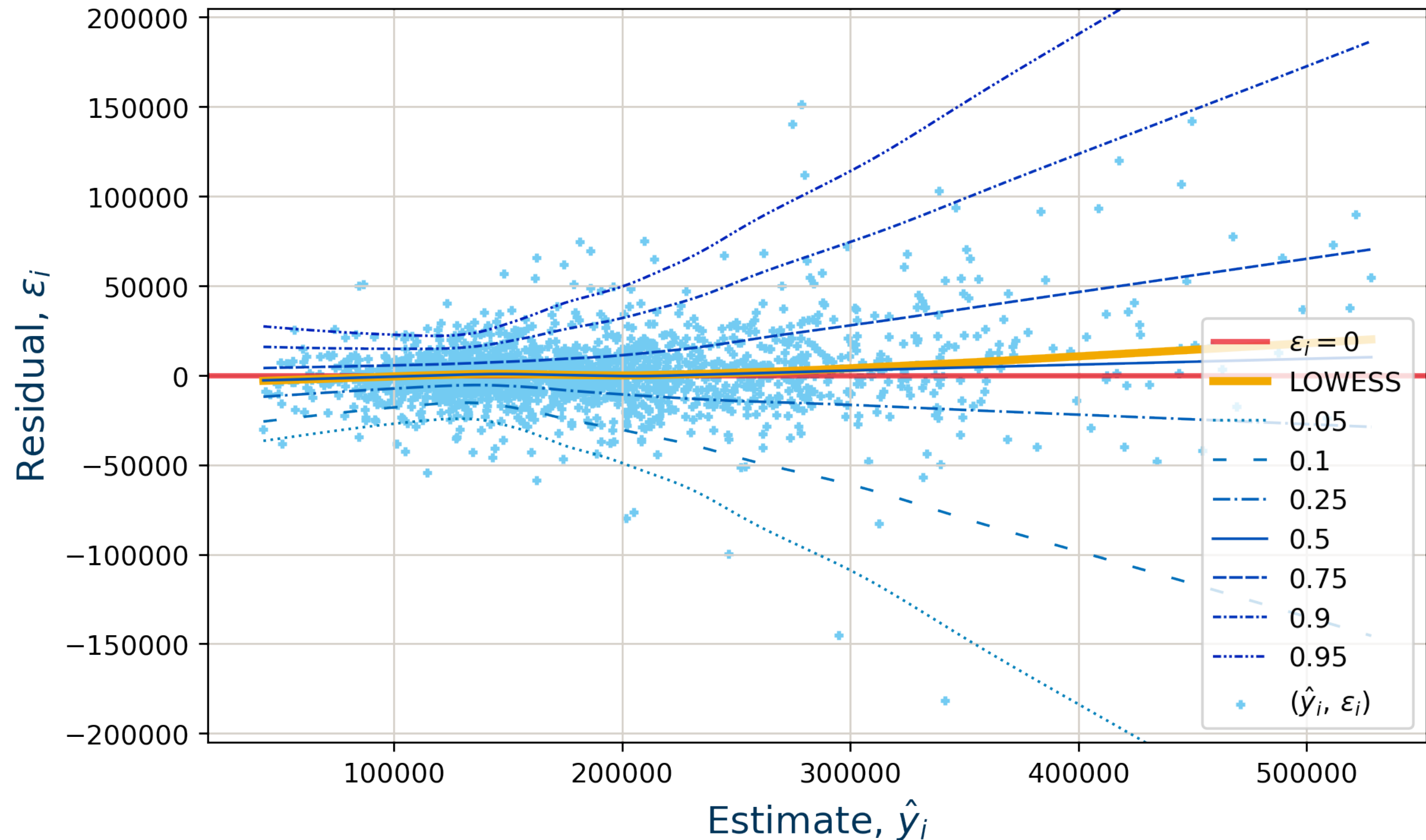
$$x = M_n(y) \Leftrightarrow y = Pr\{\mathcal{X} \leq x\} = P_{\mathcal{X}}(x).$$

The quantile function is estimated using a series

$$\hat{M}_n(y) = \sum_{\nu=1}^{\frac{n}{2}} \left(a_{2\nu-1} \left(y - \frac{1}{2} \right)^{2\nu-1} + a_{2\nu} \left(y - \frac{1}{2} \right)^{2\nu-1} \ln \left(\frac{y}{1-y} \right) \right).$$

* T. W. Keelin, "The metalog distributions", *Decision Analysis*, December 2106, <https://t.ly/WSTG>.

Local metalog fits to ML residuals produce conditional probability distributions.



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