week 2 HD tutor

2b) recap

To constract no more than one '1', at least two '0'.

This string could be summarized by $\{0^{n.1.0}m\}$. The DFA solution bases on the number of zeroes from the beginning of string.

Hence the DFA have 3 main cases - no '0's before '1' - exactly one '0' before '1' - at least two '0' before '1', including no '1' (infinite '0')s

Hence we can construct an inductive proof as follows,

Prove

There has to be 3 cases - Let 'f' denote the depth of the '1'-erecting state from entry. - If f(w) = 1 iff no 0s - If f(w) = 2 iff exactly one 0s - If f(w) = 3 iff at least two 0s, including infinity

- Base case: $w = \epsilon$
  - $f(w) = 1$, stayed at first node with depth 1. Hence this case is true
- Inductive case:
  - Assume true for some $x \in Epsilon^*$
  - Prove for $xa \in Epsilon$
  - Case now splits: ____
    - Case 1: f(x) = 1, no '0's before '1'.
    - DFA moves takes the bottom most path from starting state
    - The distance to the accepting state is 2
    - The path to accepting state comprises only of 0
    - 'a' := 0 from here on since xa already comprise of one '1'
    - f(xa) remains 1.
    - hence $xa \in Epsilon^*$. ____
    - Case 2: f(x) = 2, exactly one '0' before '1'.
    - DFA moves takes the upper path to '0', then takes '1'
    - The distance to the accepting state is 1
    - Similarly to Case 1 here,
    - The path to accepting state comprises only of 0
    - 'a' := 0 from here on since xa already comprise of one '1'
    - f(xa) remains 2.
    - hence $xa \in Epsilon^*$. ____
    - Case 3: f(x) = 3, at least two '0's before '1'.
    - DFA moves takes the bottom most path from starting state
    - The distance to the accepting state is 0, not including loops
    - the path to accepting state can take either 0 or 1.

    - $a \in 0, 1$ since xa has not hit the limit of one '1',
    - f(xa) remains 3
    - hence $xa \in Epsilon^*$.
- Hence by structural induction, the DFA constructed for 2 b) is correct. QED