Regular Language

A language L is **regular** if $L = L(A)$ for some DFA A.

Alternative:

- Language accepted by an ($\epsilon-$ NFA)

Regular languages are closed under

- Determinism / Non-determinism
- Complement
- Intersection
- Union, set and symmetric differences
- Regular ops: Union, Concat, Kleene *
- Odd and Rev $(\cdot)$
- Cone ops: homomorphism, inverse homomorphism, intersection
- ... (Other abstract alg defined operation)

Regular languages are limited to

- Finite malloc
- Finite input string
- Yes/No outputs

> [!WARNING] Reading groupd in TCS (ATTCS): Email Barry Schedule: - 11 - 1 Tue - wk 4-6 Elec G03

Pumping Lemma

A regular language L can be broken into three substring A, B and C, where B will be a repeated string over some periodic string T

Example

Prove that $L = \{0^{2n} : n \in \mathbb{N}\}$ is not regular

Solution

Consider w = 0

Myhill-Nerode Thm.

Predefinition

For $L \in E^*$. If there exists a $z \in E^*$ s.t. $xz \in L$ and $yz \notin L$, we call x, y *distinguishable* by L. We write $x \equiv_L y$ if x, y are **NOT** *distinguishable* by L

Theorem:

- $L \subseteq E^*$ is regular iff the index of L is finite;
- Moreover, the index is the *minimum number of states* of a DFA that accepts L

Sketch of proof

- Fwd: Let $A = (Q, E, \delta, q_0, F)$ be a DFA with $L(A) = L$. Show that $|Q| \geq$ index of L
- Bwd: Assume the index of L is finite, define DFA

  $$A_L = (\{[w]_L | w \in E^*\}, E, \delta_L, [\epsilon]_L, F_L)$$

  where $\delta_L([w]_L, a) = [wa]_L$ and $F_L = \{[w]_L | w \in L\}$, show that $L(A_L) = L$.

Example Proof:

Strategy:

- $Q = \{[w] : w \in E^*\}$
- Start from $q_0 = []\epsilon]$
- $F = [w] : w \in L$
- $L(A) = \{w : w \in L\} \cup \varphi$

Using Myhill-Nerode to prove irregularity

Example

$$L = \{0^n \cdot 1^n : n \in \mathbb{N}\}$$

Let:

- $w_0 = 0^0 = \epsilon$
- $w_1 = 0^1 = 0$
- $w_2 = 0^2 = 00$
- $\ldots$
- $w_i = 0^i$

So the context is $V_i j = i$. However, it is clear that i is not finite

By MN, L is not regular. *More examples of substring on pg.28*