# Week 3 HW, distinction tasks

### a)

A stated properties of regular language is, finite unions of regular languages are also regular. However countable unions can be infinite. A trivial antiexample that disproves countably infinite union are closed can be as follows.

Consider a series of finite (regular) languages $L_i = 0^i \cdot 1^i$. For $L = 0^j 1^j \forall j \in \mathbb{N}$, this is not regular. Hence we can take $w = 0^p 1^p \in L$. By the Pumping Lemma, we can divide the string into $xyz$. Notice that for the following three cases,

- If $y$ contains 0, then $|xy| > p$. This dissatisfies the first condition of PL.
- $y$ cannot be epsilon due to the second condition of PL.
- If $y = a^m$ where $m < p$, then $xyyz \notin L$, violates third condition of PL.

Hence the general $L$ described above is not pumpable by the Pumping Lemma, therefore $L$ is not regular.

Meanwhile, any $L_i$ is trivial to represent with a NFA with $2 * i + 1$ states and $i$ transitions of 0 and 1 sequentially.

Hence, a countable union of regular languages is not necessarily regular.

### b)

As mentioned above already, the given proof ignores the fact that $k$ is not bounded for the DFA $Q_k, \Sigma, \delta_k, q_k^0, F_k)$. The collection of $q^0$ and $F$ is not bounded, meaning that this DFA conains infinite states, which contradicts its nature. Hence the proof is inconsistent.