Week 3 HW, distinction tasks

a)

We can construct an NFA to accept this language trivially as follows,

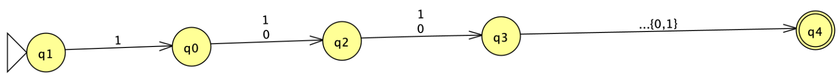

Figure 1: image

The intuition comes from that all states after the occurence of n-th symbol has to be constant, hence implemented by the NFA. By counting, this results in an NFA with exactly $n + 1$ states that can describe L_n.

b)

Recall Myhill-Nerode Theorem. For a regular language L, for all i, j, substring $u_i, u_j \in L$ is *indistinguishable* by L iff,

$$\{u_i \cdot w_{ij},\ u_j \cdot w_{ij}\} \subseteq L$$

, where $w_{ij} \in \Sigma \notin L$. Vice versa, *distinguishable* if that w_{ij} appended makes either u_i or u_j in L but not both.

The number of equivalence classes of L is the number of (u_i, u_j) pairs that are *distinguishable* above, also the number of minimum states the accepting DFA of L must hold.

Given that the regex for $L_n = \Sigma^* \cdot 1 \cdot \Sigma^{n-1}$. Define $u_m := \Sigma^* \cdot 1 \cdot Sigma^{i-1}$, since w_{ij} cannot be in L. We can then, for $i \neq j$,

$$\begin{aligned} u_i \cdot w_{ij} &= \Sigma^* \cdot 1 \cdot \Sigma^{n-1} \in L; \\ u_j \cdot w_{ij} &= \Sigma^* \cdot 1 \cdot \Sigma^{n-i+j-1} \notin L; \\ w_{ij} &= \Sigma^{n-i} \end{aligned}$$

For all $i, j \in \{1, 2, ..., n\}, i \neq j$. The number of i, j pair at i is equal to C_i^n (abbreviation for n choose i).

Hence for all i listed above, we have $C_1^n + C_2^n + ... + C_n^n = 2^n$, a combinatorial identity.

Hence the index of L is 2^n, meaning that the number of states of an L-accepting DFA cannot be less than 2^n.