ECE 657 Assignment 2 Hammad Rehman (20293390)

1) (1)

$$w(n+1) = w(n) - \mu_w \frac{\partial}{\partial w} J(n) \Big|_{w=w(n)}$$

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$$J(n) = \frac{1}{2}\left[y_d(n) - \sum_{k=1}^{N} w_k(n)\, Q\{x(n), c_k(n), \sigma_k(n)\} \right]^2$$

$$\frac{dJ}{dw}\Big|_{w=w(n)} = \Big[\underbrace{y_d(n) - \sum_{k=1}^{N} w_k(n)\, Q\{\ \}}_{e(n)}\Big] \left(-\sum_{k=1}^{N} Q\{\ \} \right)$$

$$\sum Q_k = Q(x(n), c_1(n), \sigma_1(n)) + Q(x(n), c_2(n), \sigma_2(n)) + \ldots$$
$$Q(x(n), c_k(n), \sigma_k(n)) \Rightarrow$$ replace with Transpose matrix to allow dot product

$$= -[e(n)] \cdot \Psi(n)$$

so $$\underline{w(n+1) = w(n) + \mu_w [e(n)] \cdot \Psi(n)}$$

$$C_k(n+1) = C_k(n) - \mu_c \frac{\partial}{\partial C_k} \boxed{J(n) \Big|_{C_k = C_k(n)}}$$

$$\left[y_d(n) - \underbrace{\sum_1^N w_k \exp\left(\frac{-\|x(n) - C_k(n)\|^2}{2\sigma_k^2(n)} \right)}_{\to z_{\,C_k = C_k(n)}} \right] \left[\frac{dz}{dC_k} \right]$$

$$\frac{dz}{dC_k} = -\frac{2 w_k(n) \left(-\|x(n) - C_k(n)\| \right)}{2\sigma_k^2(n)} \exp\left[\frac{-\|x(n) - C_k(n)\|^2}{2\sigma_k^2(n)} \right] \to = \{ \} \text{ from earlier}$$

$$= -\frac{\cancel{2} w_k(n) \left(\|x(n) - C_k(n)\| \right)}{\sigma_k^2(n)} Q\{\}$$

$$\left[y_d(n) - \underbrace{\sum_1^N w_k Q\{\}}_{y(n)} \right] \cdot -2 w_k(n) \frac{\|x(n) - C_k(n)\|}{2\sigma_k^2(n)} Q\{\}$$

$$= -2 [e(n)] \frac{w_k(n)}{2\sigma_k^2(n)} Q\{\} \, [x(n) - C_k(n)]$$

$$C_k(n+1) = C_k(n) + \frac{\cancel{2} \mu_c \, e(n) w_k(n)}{\cancel{2} \sigma_k^2(n)} Q\{\} \, [x(n) - C_k(n)]$$

$$\sigma_k(n+1) = \sigma_k(n) - \mu_\sigma \frac{\partial}{\partial \sigma_k} J(n) \Big|_{\sigma_k = \sigma_k(n)}$$

$$\left[y_d(n) - w_k(n) \exp\left(\frac{-\|x(n) - c_k(n)\|^2}{2\sigma_k^2(n)} \right) \right]^2 \left(\frac{dz}{d\sigma_k} \right)$$

$$\frac{dz}{d\sigma_k} = -\frac{-2 w_k(n)}{2\sigma_k^3(n)} \exp\left(\frac{-\|x(n) - c_k(n)\|^2}{2\sigma_k^2(n)} \right) \left(-\|x(n) - c_k(n)\|^2 \right)$$

$$= \frac{-2 w_k(n)}{2\sigma_k^3(n)} \left(\|x(n) - c_k(n)\|^2 \right) \, Q\{\ \}$$

$$-2 \left[y_d(n) - y(n) \right]^{-e(n)} \frac{w_k(n)}{2\sigma_k^3(n)} \, Q\{\ \} \left(\|x(n) - c_k(n)\| \right)^2$$

$$\sigma_k(n+1) = \sigma_k(n) + \cancel{2}\mu_\sigma \frac{e_n w_k(n)}{\cancel{2}\sigma_k^3(n)} \, Q\{\ \} \left(\|x(n) - c_k(n)\|^2 \right)$$