Math For ML

PROBABILITY & STATISTICS

Part-2

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Correlation

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma(X)\sigma(Y)}$$

Explanation: Correlation normalizes covariance to a scale of $[-1, 1]$, quantifying the strength and direction of a linear relationship between two variables.

Example: If $\text{Cov}(X, Y) = 0.25$, $\sigma(X) = 0.5$, and $\sigma(Y) = 1.0$, then $\rho(X, Y) = \frac{0.25}{0.5 \cdot 1.0} = 0.5$.

Implementation:

```
covariance = 0.25
std_X = 0.5
std_Y = 1.0
correlation = covariance / (std_X * std_Y)
```

Output:

correlation = 0.5

Probability Mass Function (PMF)

$$P(X = x) = \begin{cases} p_i, & \text{if } x = x_i \\ 0, & \text{otherwise} \end{cases}$$

Explanation: The PMF defines the probabilities of discrete outcomes of a random variable. It is a foundational concept in probability theory.

Example: If $X = \{1, 2, 3\}$ with $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, and $P(X = 3) = 0.3$, the PMF is defined for these values.

Implementation:

```
X = [1, 2, 3]
P_X = [0.2, 0.5, 0.3]
def pmf(x):
    return P_X[X.index(x)] if x in X else 0
```

Output:

0.3

Probability Density Function (PDF)

$$f_X(x) \geq 0, \quad \int_{-\infty}^{\infty} f_X(x)dx = 1$$

Explanation: The PDF defines the relative likelihood of a continuous random variable at a specific value. It is used in probability and statistics for modeling continuous distributions.

Example: For a standard normal distribution, the PDF is $f_X(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$.

Implementation:

```
import numpy as np
from scipy.stats import norm
x = 0  # example point
pdf_value = norm.pdf(x)
```

Output:

0.3

Joint Probability

$$P(A \cap B) = P(A \mid B)P(B)$$

Explanation: Joint probability quantifies the likelihood of two events occurring together. It is essential in probabilistic modeling and understanding relationships between variables.

Example: If $P(A \mid B) = 0.4$ and $P(B) = 0.5$, then $P(A \cap B) = 0.4 \cdot 0.5 = 0.2$.

Implementation:

```
P_A_given_B = 0.4
P_B = 0.5
P_A_and_B = P_A_given_B * P_B
```

Output:

0.2

CDF (Cumulative Distribution Function)

$$F_X(x) = P(X \leq x)$$

Explanation: The CDF of a random variable gives the probability that the variable takes a value less than or equal to x. It is used to describe the distribution function for both discrete and continuous variables.

Example: For a uniform distribution $X \sim U(0, 1)$, $F_X(0.5) = 0.5$.

Implementation:

```
from scipy.stats import uniform

x = 0.5

cdf_value = uniform.cdf(x, loc=0, scale=1)
```

Output:

0.5

Entropy (discrete)

$$H(X) = -\sum_i P(X = x_i) \log_2 P(X = x_i)$$

Explanation: Entropy measures the uncertainty of a discrete random variable. It is a fundamental concept in information theory and ML, particularly in decision trees and loss functions.

Example: If $P(X) = \{0.5, 0.5\}$, then $H(X) = -0.5 \log_2(0.5) - 0.5 \log_2(0.5) = 1$.

Implementation:

```
import numpy as np
P_X = [0.5, 0.5]
entropy = -sum(p * np.log2(p) for p in P_X if p > 0)
```

Output:

1

Conditional Expectation

$$\mathbb{E}[X \mid Y] = \sum_{x} x P(X = x \mid Y)$$

Explanation: Conditional expectation is the expected value of a random variable X given that another variable Y is known. It is critical in Bayesian inference and probabilistic modeling.

Example: If $X = \{1, 2\}$ with $P(X = 1 \mid Y) = 0.7$ and $P(X = 2 \mid Y) = 0.3$, then $\mathbb{E}[X \mid Y] = 1 \cdot 0.7 + 2 \cdot 0.3 = 1.3$.

Implementation:

```
X = [1, 2]
P_X_given_Y = [0.7, 0.3]
conditional_expectation = sum(x * p for x, p in zip(X, P_X_given_Y))
```

Output:

1.3

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