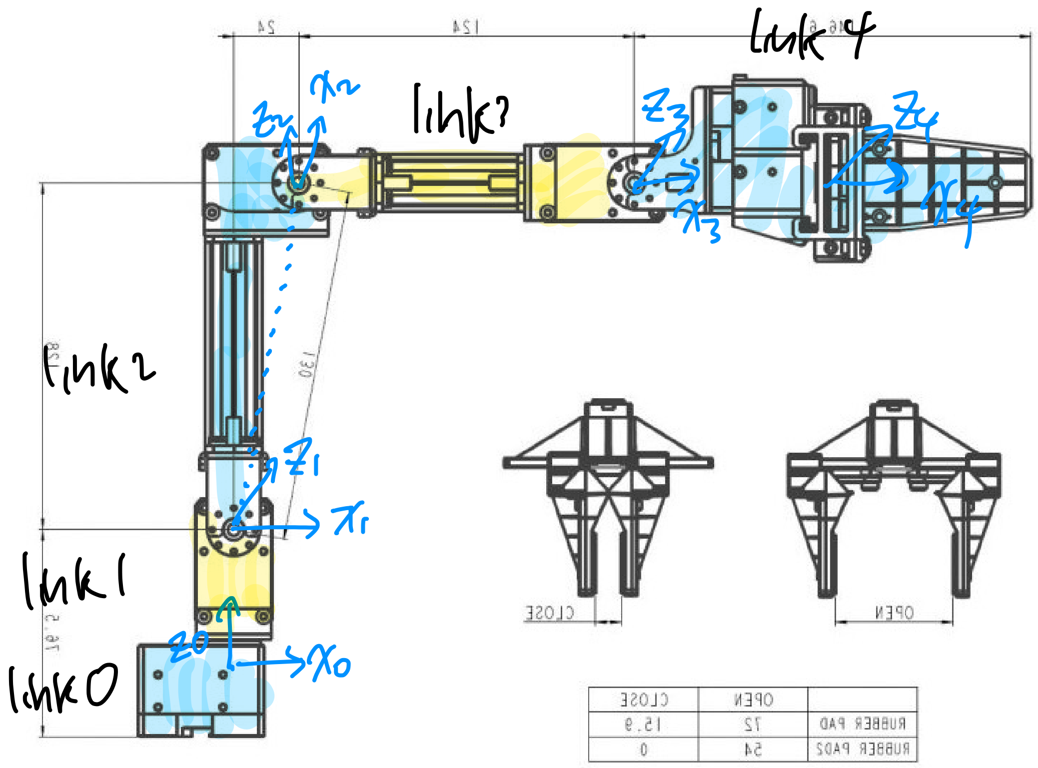
2022-02-27



D-H parameter

Link#	α_i	a_i	d_i	θ_i
1	$-90°$	0	59.5	θ_1
2	0	130	0	θ_2
3	0	124	0	θ_3
4	$90°$	81.7	0	θ_4

(deg, mm from URDF)

Eq'n of Motion $\Rightarrow \sum_{j=1}^{n} H_{ij}\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}\dot{q}_j\dot{q}_k + G_i = \tau_i$

$$H(q) = \sum_{i=1}^{n}\left(m_i J_L^{(i)T} J_L^{(i)} + J_A^{(i)T} I_i J_A^{(i)}\right)$$

$$h_{ijk} = \frac{\partial H_{ij}}{\partial q_k} - \frac{1}{2}\frac{\partial H_{jk}}{\partial q_i} \qquad G_i = -\sum_{j=1}^{n} m_j \underline{g}^T J_{Li}^{(j)} \qquad \underline{g} = \begin{bmatrix} 0 \\ 0 \\ -9.81 \end{bmatrix}$$

from URDF

$$m_1 = 9.8406837e^{-2} \quad I_1 = \begin{bmatrix} 3.4543422e^{-5} & -1.6031095e^{-8} & -3.8375155e^{-7} \\ & 3.2689329e^{-5} & 2.8571935e^{-8} \\ & & 1.8850320e^{-5} \end{bmatrix}$$

$$m_2 = 1.3850917e^{-1} \quad I_2 = \begin{bmatrix} 3.3055381e^{-4} & -9.7940978e^{-8} & -3.8505711e^{-5} \\ & 3.4290447e^{-4} & -1.5717516e^{-6} \\ & & 6.0346498e^{-5} \end{bmatrix}$$

$$m_3 = 1.3274562e^{-1} \quad I_3 = \begin{bmatrix} 3.0654178e^{-5} & -1.2764155e^{-6} & -2.6874417e^{-7} \\ & 2.4230292e^{-4} & 1.1559550e^{-8} \\ & & 2.5155057e^{-4} \end{bmatrix}$$

$m_4 = 1.4321573e^{-1} + 2 \times 1e^{-3}$

$$I_4 = \begin{bmatrix} 8.0870799e^{-5} & 0.0 & -1.0157896e^{-6} \\ & 7.5980465e^{-5} & 0.0 \\ & & 9.3127351e^{-5} \end{bmatrix}$$

for rev. Joint $\begin{bmatrix} J_{Li} \\ J_{Ai} \end{bmatrix} = \begin{bmatrix} \underline{b}_{i-1} \times r_{i-1,e} \\ \underline{b}_{i-1} \end{bmatrix}$

$$R_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & -1 & 0 \end{bmatrix}, \quad R_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{b}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \underline{b}_1 = R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \\ 0 \end{bmatrix}$$

$$\underline{b}_2 = R_1^0 R_2^1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 \cdot \cos\theta_2 & -\cos\theta_1 \sin\theta_2 & -\sin\theta_1 \\ \sin\theta_1 \cdot \cos\theta_2 & -\sin\theta_1 \sin\theta_2 & \cos\theta_1 \\ -\sin\theta_2 & -\cos\theta_2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \\ 0 \end{bmatrix}$$

$$\underline{b}_3 = R_1^0 R_2^1 R_3^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \\ 0 \end{bmatrix}$$

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & a_4\cos\theta_4 \\ \sin\theta_4 & 0 & -\cos\theta_4 & a_4\sin\theta_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & a_4\cos\theta_4 \\ \sin\theta_4 & 0 & -\cos\theta_4 & a_4\sin\theta_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X_{i-1,e} = \overbrace{A_1^0 \cdots A_n^{n-1}\bar{X}}^{X_{0,e}} - \overbrace{A_1^0 \cdots A_{i-1}^{i-2}\bar{X}}^{X_{0,i-1}}, \quad \bar{X} = [0\ 0\ 0\ 1]^T$$

$$X_{0,e} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_4\cos\theta_4 \\ a_4\sin\theta_4 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & -\sin\theta_1 & a_2\cos\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & \cos\theta_1 & a_2\sin\theta_1\cos\theta_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & -a_2\sin\theta_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{bmatrix} a_4(\overbrace{\cos\theta_3\cos\theta_4 - \sin\theta_3\sin\theta_4}^{\cos(\theta_3+\theta_4)}) + a_3\cos\theta_3 \\ a_4(\overbrace{\sin\theta_3\cos\theta_4 + \cos\theta_3\sin\theta_4}^{\sin(\theta_3+\theta_4)}) + a_3\sin\theta_3 \\ 0 \\ 1 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ \\ \\ \end{array}$$

$$= \begin{bmatrix} (a_2 + a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\cos\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\cos\theta_1\sin\theta_2 \\ (a_2 + a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\sin\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\sin\theta_1\sin\theta_2 \\ -a_4\sin(\theta_2+\theta_3+\theta_4) - a_3\sin(\theta_2+\theta_3) - a_2\sin\theta_2 + d_1 \\ 1 \end{bmatrix} = r_{0,e}$$

$$X_{0,3} = \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & -\sin\theta_1 & a_2\cos\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & \cos\theta_1 & a_2\sin\theta_1\cos\theta_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & -a_2\sin\theta_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_3\cos\theta_3 \\ a_3\sin\theta_3 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (a_2 + a_3\cos\theta_3)\cos\theta_1\cos\theta_2 - a_3\sin\theta_3\cos\theta_1\sin\theta_2 \\ (a_2 + a_3\cos\theta_3)\sin\theta_1\cos\theta_2 - a_3\sin\theta_3\sin\theta_1\sin\theta_2 \\ -a_3\sin(\theta_2+\theta_3) - a_2\sin\theta_2 + d_1 \\ 1 \end{bmatrix}$$

$$X_{0,2} = \begin{bmatrix} a_2\cos\theta_1\cos\theta_2 \\ a_2\sin\theta_1\sin\theta_2 \\ -a_2\sin\theta_2 + d_1 \\ 1 \end{bmatrix}, \quad X_{0,1} = \begin{bmatrix} 0 \\ 0 \\ d_1 \\ 1 \end{bmatrix}$$

$$
r_{0,e} = \begin{bmatrix} (a_2 + a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\cos\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\cos\theta_1\sin\theta_2 \\ (a_2 + a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\sin\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\sin\theta_1\sin\theta_2 \\ -d_4\sin(\theta_2+\theta_3+\theta_4) - a_3\sin(\theta_2+\theta_3) - a_2\sin\theta_2 + d_1 \end{bmatrix}
$$

$$
= \begin{bmatrix} (a_4\cos(\theta_2+\theta_3+\theta_4) + a_3\cos(\theta_2+\theta_3) + a_2\cos\theta_2)\cos\theta_1 \\ (a_4\cos(\theta_2+\theta_3+\theta_4) + a_3\cos(\theta_2+\theta_3) + a_2\cos\theta_2)\sin\theta_1 \\ -a_4\sin(\theta_2+\theta_3+\theta_4) - a_3\sin(\theta_2+\theta_3) - a_2\sin\theta_2 + d_1 \end{bmatrix}
$$

$$
r_{1,e} = \begin{bmatrix} (a_2 + a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\cos\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\cos\theta_1\sin\theta_2 \\ (a_2 + a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\sin\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\sin\theta_1\sin\theta_2 \\ -d_4\sin(\theta_2+\theta_3+\theta_4) - a_3\sin(\theta_2+\theta_3) - a_2\sin\theta_2 \end{bmatrix}
$$

$$
= \begin{bmatrix} (a_4\cos(\theta_2+\theta_3+\theta_4) + a_3\cos(\theta_2+\theta_3) + a_2\cos\theta_2)\cos\theta_1 \\ (a_4\cos(\theta_2+\theta_3+\theta_4) + a_3\cos(\theta_2+\theta_3) + a_2\cos\theta_2)\sin\theta_1 \\ -a_4\sin(\theta_2+\theta_3+\theta_4) - a_3\sin(\theta_2+\theta_3) - a_2\sin\theta_2 \end{bmatrix}
$$

$$
r_{2,e} = \begin{bmatrix} (a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\cos\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\cos\theta_1\sin\theta_2 \\ (a_4\cos(\theta_3+\theta_4) + a_3\cos\theta_3)\sin\theta_1\cos\theta_2 - (a_4\sin(\theta_3+\theta_4) + a_3\sin\theta_3)\sin\theta_1\sin\theta_2 \\ -d_4\sin(\theta_2+\theta_3+\theta_4) - a_3\sin(\theta_2+\theta_3) \end{bmatrix}
$$

$$
= \begin{bmatrix} (a_4\cos(\theta_2+\theta_3+\theta_4) + a_3\cos(\theta_2+\theta_3))\cos\theta_1 \\ (a_4\cos(\theta_2+\theta_3+\theta_4) + a_3\cos(\theta_2+\theta_3))\sin\theta_1 \\ -a_4\sin(\theta_2+\theta_3+\theta_4) - a_3\sin(\theta_2+\theta_3) \end{bmatrix}
$$

$$
r_{3,e} = \begin{bmatrix} a_4\cos(\theta_3+\theta_4)\cos\theta_1\cos\theta_2 - a_4\sin(\theta_3+\theta_4)\cos\theta_1\sin\theta_2 \\ a_4\cos(\theta_3+\theta_4)\sin\theta_1\cos\theta_2 - a_4\sin(\theta_3+\theta_4)\sin\theta_1\sin\theta_2 \\ -d_4\sin(\theta_2+\theta_3+\theta_4) \end{bmatrix}
$$

$$
= \begin{bmatrix} a_4\cos\theta_1\cos(\theta_2+\theta_3+\theta_4) \\ a_4\sin\theta_1\cos(\theta_2+\theta_3+\theta_4) \\ -a_4\sin(\theta_2+\theta_3+\theta_4) \end{bmatrix}
$$