Quantum Circuit Simulation of Charge-Spin Separation in 1D Hubbard Model

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Introduction

- Spin-charge separation is a phenomenon observed in 1D strongly correlated systems.
- Luttinger's Liquid theory describes the low energy excitations in 1D electron gas as of electrons into *Spinons* and *Holons*.

System

$$\mathcal{H} = -J \sum_{j=1}^{L-1} \sum_{\nu=\uparrow,\downarrow} c_{j,\nu}^{\dagger} c_{j+1,\nu} + h.c. + U \sum_{j=1}^{L} n_{j,\uparrow} n_{j,\downarrow} + \sum_{j=1}^{L} \sum_{\nu=\uparrow,\downarrow} \epsilon_{j,\nu} n_{j,\nu}$$

- 1D Fermi-Hubbard model on L lattice sites with open-boundary conditions.
  - $\epsilon_{j,\nu}$ represents spin-dependent local potentials

$$\rho_j^{\pm} = \langle n_{j,\uparrow} \rangle \pm \langle n_{j,\downarrow} \rangle$$

- Charge and spin densities are defined as the sum and difference of the spin-up and down particle densities respectively.

Motivation

- Transport of electrons through nanowires is subject to spin-charge separation
- Advancements towards spintronics
- High T_c superconductivity

Google Quantum AI Experiment

- Simulated Hubbard model on a programmable superconducting quantum processor.
  - In highly excited regime
  - Trapping potentials were abruptly removed
  - The on-site interactions were suddenly activated
- Introduced accurate gate calibration procedure fast enough to capture temporal drifts of the gate parameters.

Compare with Numerical Simulations

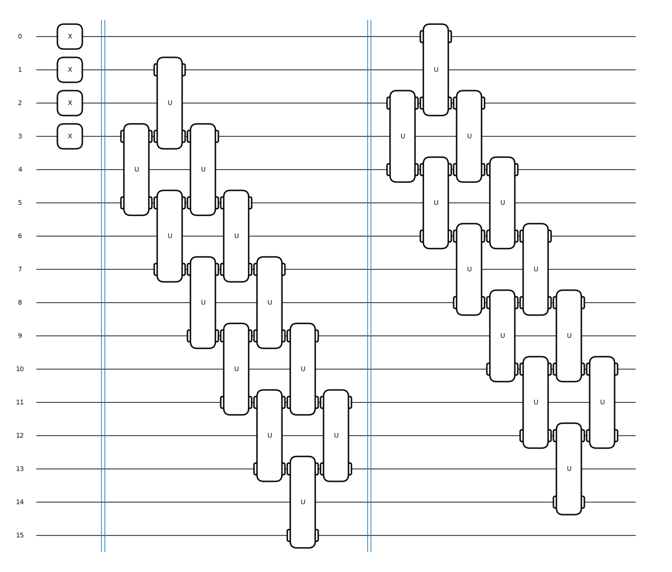
- For comparison, *OpenFermion-FQE*, a specialized quantum emulator for simulating Fermionic many-body problems was used.
- We have recreated the results of numerical simulation using another package *Pennylane*.

Initial State

- Exact Diagonalization
  - Ground state is obtained by diagonalizing the 4 particle non interacting Hamiltonian.
- Can be implemented using Givens Network
  - Initial state prepared is in one particle basis.
  - Converted to a state in two mode fermionic basis by performing a basis transformation using Given's rotation.

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Given's Rotation Circuit



Trotterization

- Trotter's Product

$$|\psi(t)\rangle \approx \left(\prod_j e^{-ia_j\mathcal{H}_j dt}\right)^n |\psi(0)\rangle$$

  - Time independent Hamiltonian H can be written as a weighted sum of Pauli terms which may or may not commute.

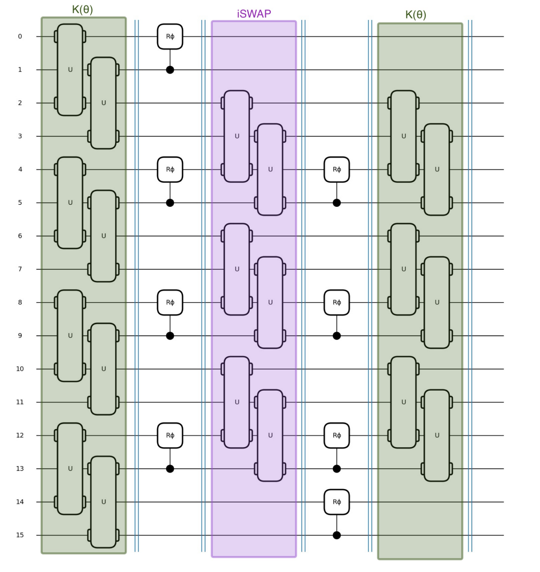
$$\mathcal{H} = \sum_j a_j h_j$$

  - Product of matrix exponentials is a good approximation for the exponent of the sum.

Operators Required for Trotterization

- $$K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -i\ \sin\theta & 0 \\ 0 & -i\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
- $$iSWAP = K(-\frac{\pi}{2})$$
- $$CPhase = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{pmatrix}$$

Trotterization Step Circuit



Measurement

- Jordan-Wigner Transformation

$$a_0^\dagger = \left(\frac{X_0 - iY_0}{2}\right), \ldots, a_n^\dagger = Z_0 \otimes Z_1 \otimes \ldots \otimes Z_{n-1} \otimes \left(\frac{X_n - iY_n}{2}\right)$$

$$a_0 = \left(\frac{X_0 + iY_0}{2}\right), \ldots, a_n = Z_0 \otimes Z_1 \otimes \ldots \otimes Z_{n-1} \otimes \left(\frac{X_n + iY_n}{2}\right)$$

- The number operator transforms to

$$n_j = \frac{\mathbb{1} - Z_j}{2}$$

Charge and Spin Density for U = 0

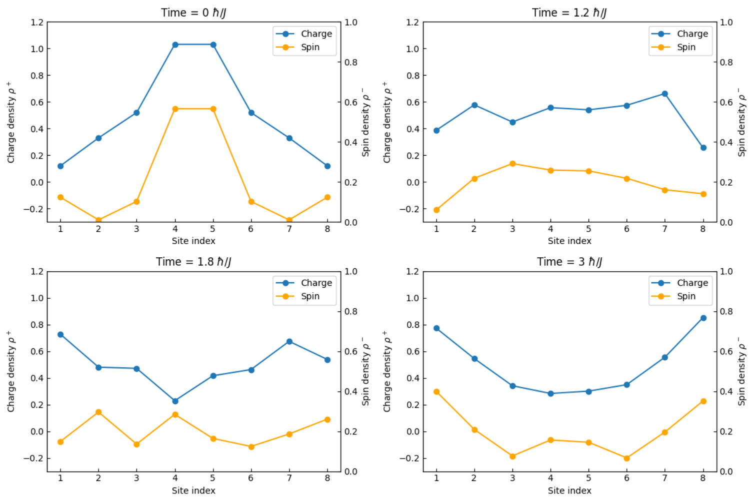


Figure: Evolution of charge and spin density at each site

Charge and Spin Density for U = 3

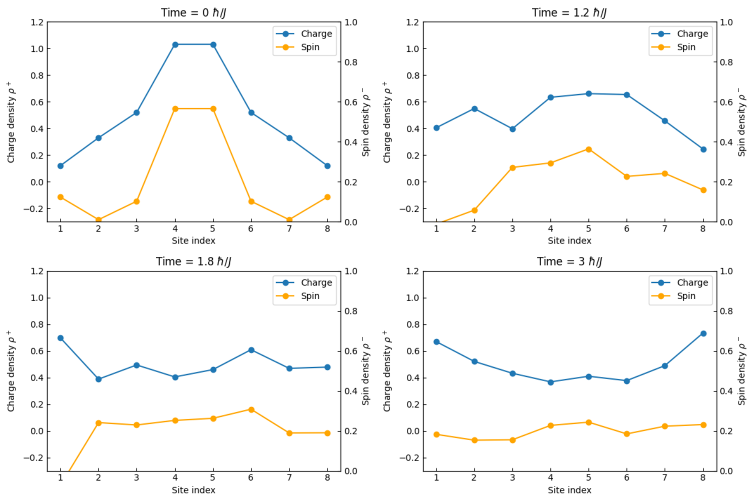


Figure: Evolution of charge and spin densities at each site

Charge and Spin Spread

$$\kappa_{\mu}^{\pm} = \sum_{j} |j - \frac{L+1}{2}| \rho_{\mu,j}^{\pm}$$

Charge and Spin Spreads

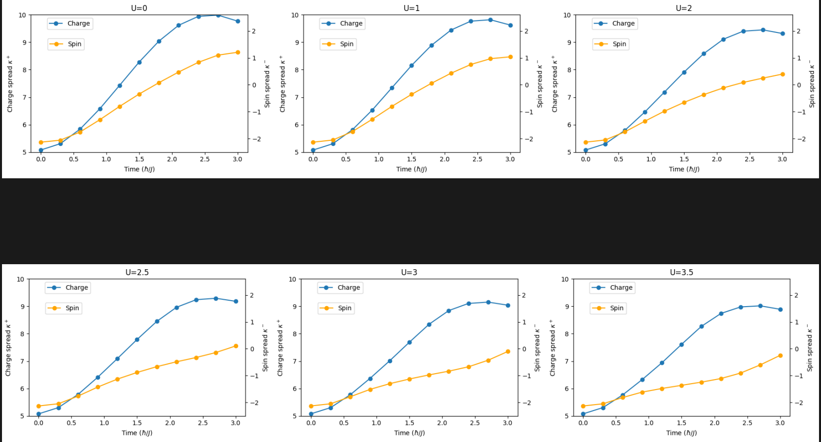


Figure: Time evolution of charge and spin spreads with varying U

More on Charge & Spin Density Waves

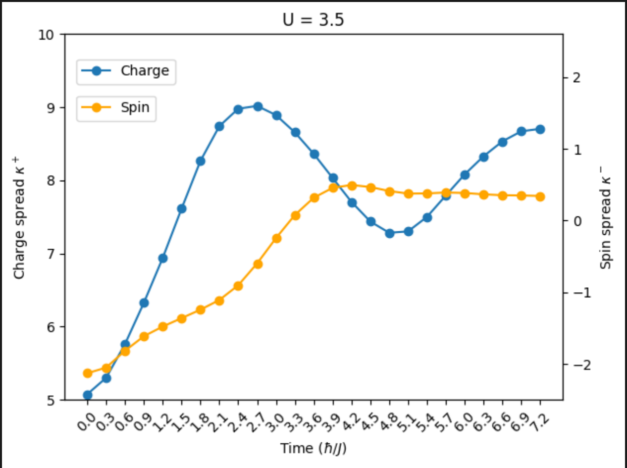


Figure: Damped oscillation like behaviour in spin and charge density waves

Rate of Change of Spreads

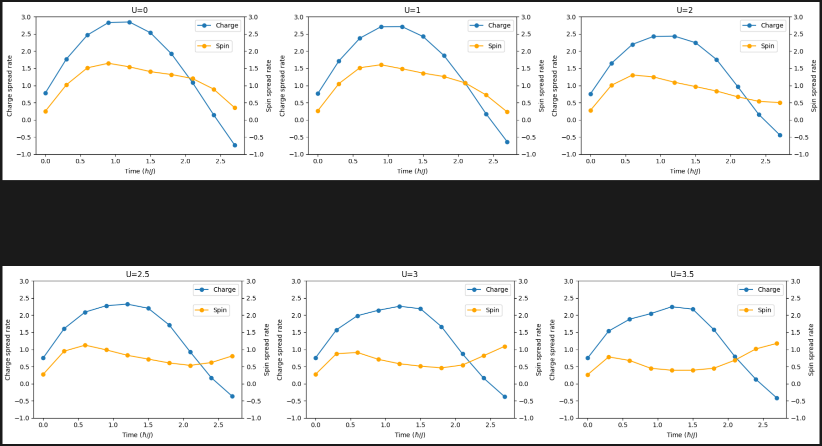


Figure: Numerical derivative of charge and spin spread with respect to evolution time

Mott Insulator Regime: 8 electron system

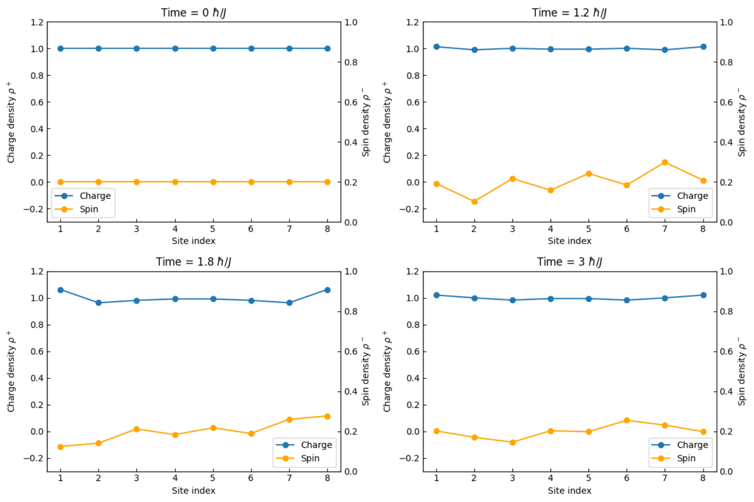


Figure: Evolution of charge and spin densities for 8 electron system

Sources of Errors

- During the calculation of the initial state some terms were truncated to 0 due to machine precision.
- First order Trotter formula is used with error $\mathcal{O}(t^2/n)$.
- CPhase gate is not a Clifford gate and hence not simulable to a good precision.
- Did not include $V\sum_{j}^{L-1}\sum_{\nu=\uparrow,\downarrow} n_{j,\nu}n_{j+1,\nu}$ in $\mathcal{H}$.

Conclusion

- Separation between charge and spin spreads increases as U increases.
- Maximum value of charge spread rate almost remains the same as U increases but the maximum of spin spread rate decreases.
- Charge spread follows a damped oscillation-like behavior.
- In the Mott insulator regime the charge degree of freedom is frozen while the spin degree of freedom fluctuates.

All code can be found here: https://github.com/hammingheavy/QPM_TermPaper

References

- arXiv:2010.07965 [quant-ph], Observation of separated dynamics of charge and spin in the Fermi-Hubbard model, Google AI Quantum and collaborators.
- A. M. Childs and Y. Su, "Nearly Optimal Lattice Simulation by Product Formulas," Physical Review Letters 123, 050503 (2019).
- P.-L. Dallaire-Demers, M. Stęchły, J. F. Gonthier, N. T. Bashige, J. Romero, and Y. Cao, "An application benchmark for fermionic quantum simulations," arXiv:2003.01862 (2020).
- Y. Jompol, C. J. B. Ford, J. P. Griffiths, I. Farrer, G. a. C. Jones, D. Anderson, D. A. Ritchie, T. W. Silk, and A. J. Schofield, "Probing Spin-Charge Separation in a Tomonaga-Luttinger Liquid," Science 325, 597 (2009)

Appendix

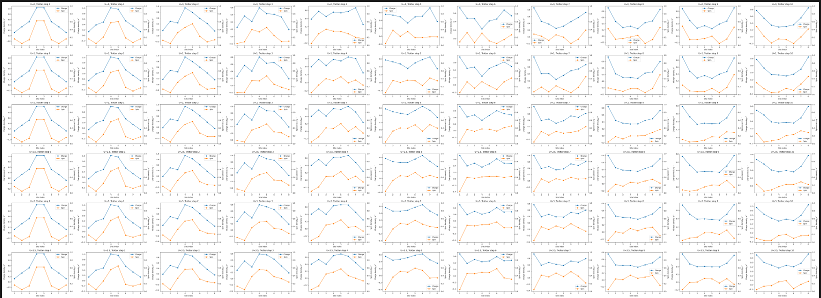


Figure: Evolution of system at different values of U for 10 trotter steps

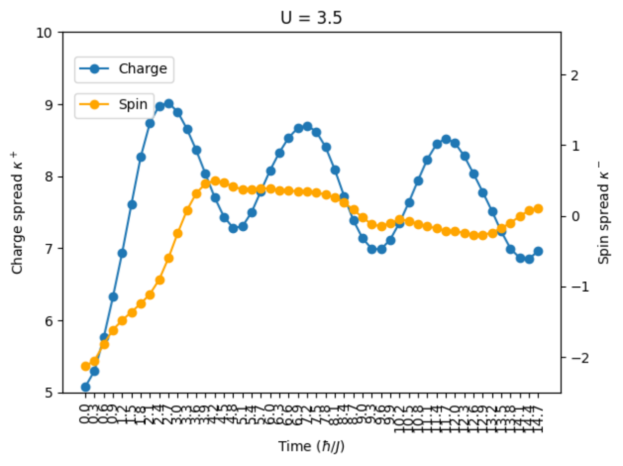


Figure: Damped oscillation like behaviour in spin and charge density waves

Results From Paper

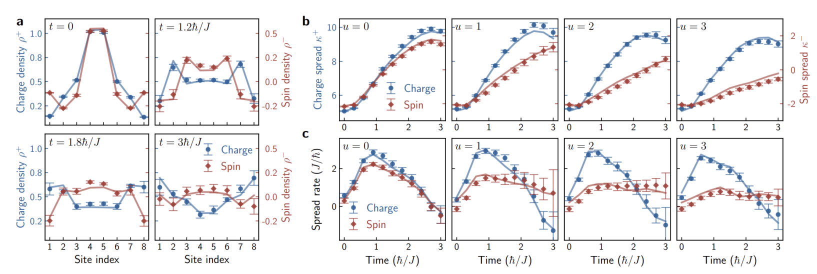


Figure: Plots in the referenced paper used to compare our results