Chapter 4
English Usage

There's almost no more beautiful sight than a simple declarative sentence.
— WILLIAM ZINSSER, *Writing with a Word Processor* (1983)

Quite aside from format and style,
mathematical writing is supposed to say something.
Put another way: the number of ideas divided by the
number of pages is supposed to be positive.
— J. L. KELLEY, *Writing Mathematics* (1991)

Let us not deceive ourselves.
"There is no God but Allah" is a more gripping sentence than
Mohammed (also Mahomet, Muhammad; 570?–632) asserted a doctrine of
unqualified monotheism (suras 8, 22, 33–37, 89, 91, Koran).
— MARY-CLAIRE VAN LEUNEN, *A Handbook for Scholars* (1992)

I am about to—or I am going to—die;
either expression is used.
— DOMINIQUE BONHOURS[4] (on his deathbed)

One should not aim at being possible to understand,
but at being impossible to misunderstand.
— QUINTILLIAN

[4]Quoted in [42, p. 146].

In this chapter I discuss aspects of English usage that are particularly relevant to mathematical writing. You should keep three things in mind as you read this chapter. First, on many matters of English usage rules have exceptions, and, moreover, not all authorities agree on the rules. I have consulted several usage guides (those described in §2.2) and have tried to give a view that reflects usage in writing today. Second, about half the topics discussed here are not peculiar to the English language, but simply reflect common sense in writing. Third, many of the points mentioned are not vitally important when taken in isolation. But, as van Leunen explains (quoted in [164, p. 97]),

> Tone is important, and tone consists entirely of making these tiny, tiny choices. If you make enough of them wrong ... then you won't get your maximum readership. The reader who has to read the stuff will go on reading it, but with less attention, less commitment than you want. And the reader who doesn't have to read will stop.

4.1. A or An?

Whether *a* or *an* should precede a noun depends on how the first syllable is pronounced: *a* is used if the first syllable begins with a consonant sound and *an* if it begins with a vowel sound. For this rule, the initial "yew" sound in the words university and European is regarded as a consonant sound: thus "a university", "a European". For words beginning with h, *a* is used unless the h is not sounded. The only words in this last category are *heir*, *honest*, *honour* (US *honor*), *hour* and their derivatives.

The question "*a* or *an*?" most frequently arises with acronyms, abbreviations and proper nouns. An easy example is "an NP-complete problem". In the world of mathematical software, given that the usual pronunciation of LAPACK is l-a-pack, and that of NAG is nag, we would write "an LAPACK routine" but "a NAG library routine".

4.2. Abbreviations

One school of thought says that the use of the abbreviations *e.g.* and *i.e.* is bad style, and that *for example* and *that is* make for a smoother flowing sentence. In any case, *i.e.* and *that is* should usually be followed by a comma, and all four forms should be preceded by a comma. When using the abbreviations you should type *e.g.* and *i.e.*, not *eg.* or *ie.*, since the abbreviations represent two words (the Latin *exempli gratia* and *id est*). Note

that in the following sentence *i.e.* should be deleted: "The most expensive method, i.e. Newton's method, converges quadratically."

The less frequently used *cf.* has only one full stop (a "period" in American English), because it is an abbreviation of a single word: the Latin *confer*, meaning compare. Often, cf. is used incorrectly in the sense of "see", as in "cf. [6] for a discussion". The abbreviation *et al.* is short for *et alia*, so it needs only one full stop.

The abbreviation *N.B.* (of the Latin *nota bene*) is not often used in technical writing, probably because it has to appear at the beginning of a sentence and is somewhat inelegant. You can usually find a better way to give the desired emphasis.

The abbreviation *iff*, although handy in notes, is usually spelled out as *if and only if.*

The normal practice when introducing a nonstandard abbreviation or acronym is to spell out the word or phrase in full on its first occurrence and place the abbreviation immediately after it in parentheses. Thereafter the abbreviation is used. Example:

> Gaussian elimination (GE) is a method for solving a system of n linear equations in n unknowns. GE has a long history; a variant of it for solving systems of three equations in three unknowns appears in the classic Chinese work "Chiu-Chang Suan-Shu", written around 250 B.C.

4.3. Absolute Words

Certain adjectives have an absolute meaning and cannot be qualified by words such as less, quite, rather and very. For example, it is wrong to write *most unique* (replace by *unique*, or perhaps *most unusual*), *absolutely essential*, *more ideal*, or *quite impossible.* However, *essentially unique* is an acceptable term in mathematical writing: it means unique up to some known transformations. Many other words are frequently used with an absolute sense but can be modified (although for some words this usage is open to criticism); example phrases are "convergence is *almost certain*", "a *very complete* survey", "the *most obvious* advantage", and "the function is differentiable *almost everywhere.*"

4.4. Active versus Passive

Prefer the active to the passive voice (prefer "X did Y" to "Y was done by X"). The active voice adds life and movement to writing, whereas too

much of the passive voice weakens the communication between writer and reader.

Passive: The answer was provided to sixteen decimal places by Gaussian elimination.

Active: Gaussian elimination gave the answer to sixteen decimal places.

Passive: The failure of Newton's method to converge is attributed to the fact that the Jacobian is singular at the solution.

Active: Newton's method fails to converge because the Jacobian is singular at the solution.

Passive: A numerical example is now given to illustrate the above result.

Active: We give a numerical example to illustrate this result, *or* The following numerical example illustrates this result.

The second example in the following trio illustrates a further degree of abstraction in which a verb is replaced by an abstract noun modified by another verb.

Passive: The optimality of y was verified by checking that the Hessian matrix was positive definite.

Passive and indirect: Verification of the optimality of y was achieved by checking that the Hessian matrix was positive definite.

Active: We verified the optimality of y by checking that the Hessian matrix was positive definite.

Other "was" phrases that can often be removed by rewriting are "was performed", "was experienced", "was carried out", "was conducted" and "was accomplished".

The passive voice has an important role to play, however. It adds variety, may be needed to put emphasis on a certain part of a sentence, and may be the only choice if specific information required for an active variant is unknown or inappropriate to mention. Examples where the passive voice allows the desired emphasis are "An ingenious proof of this conjecture was constructed by C. L. Ever", and (from the writings of Halmos [245, p. 96]) "The subjects just given honorable mention, as well as the three actually discussed in detail, have been receiving serious research attention in the course of the last twenty years." The passive voice is also useful for euphemistic effect, allowing the clumsy experimenter to say "The specimen was accidentally strained during mounting" instead of "I dropped the specimen on the floor."

The Ten Commandments of Good Writing

1. Each pronoun should agree with their antecedent.
2. Just between you and I, case is important.
3. A preposition is a poor word to end a sentence with.
4. Verbs has to agree with their subject.
5. Don't use no double negatives.
6. Remember to never split an infinitive.
7. When dangling, don't use participles.
8. Join clauses good, like a conjunction should.
9. Don't write a run-on sentence it is difficult when you got to punctuate it so it makes sense when the reader reads what you wrote.
10. About sentence fragments.

Reprinted, with permission, from *How to Write and Publish a Scientific Paper* [68].

4.5. Adjective and Adverb Abuse

Use an adjective or adverb only if it earns its place. The adjectives or adverbs *very*, *rather*, *quite*, *nice* and *interesting* should be used with caution in technical writing, as they are imprecise. For example, in "The proof is very easy" and "This inequality is quite important" the adverbs are best omitted. Examples of acceptable usage are "These results are very similar to those of Smith" and "This bound can be very weak." *Interesting* is an overworked adjective that can often be avoided. For example, in the sentence "It is interesting to re-prove this result using Laplace transforms", *instructive* is probably the intended word.

Try to avoid using nouns as adjectives. "The method for iteration parameter estimation" can be expressed more clearly as "The method for estimating iteration parameters." While proper nouns are often used as adjectives in speech ("this sequence is Cauchy", "that matrix is Toeplitz"), such usage in formal writing is best avoided (write "this is a Cauchy sequence", "that is a Toeplitz matrix"). Similarly, write "Euler's method is unstable" instead of "Euler is unstable."

An *adverb* that is overworked in mathematical writing is *essentially*. Dictionaries define it to mean necessarily or fundamentally, but it is often used with a vague sense meaning "almost, but not quite". Before using the word, consider whether you can be more precise.

Bad: Beltrami (1873) essentially derived the singular value decomposition.

Good: Beltrami (1873) derived the singular value decomposition for square, nonsingular matrices.

A valid use of *essentially* is in the expression "essentially the same as", which by convention in scientific writing means "the same, except for minor details".

4.6. -al and -age

Certain words that can be extended with an -al or -age ending are often misused in the extended form. The suffix tends to give a more abstract meaning, which makes it more difficult to use the word correctly. For example, *usage* means a manner of using, so correct *usage* is illustrated by "in the original usage the conjugate gradient method was not preconditioned" and "the use of Euler's method is not recommended for stiff differential equations."

An example where an -al ending is used incorrectly is "the most pragmatical opinion is the one expressed by the term's inventor", in which the third word should be "pragmatic".

4.7. Ambiguous "This" and "It"

A requirement of good writing is to make clear to the reader, at all times, what is the entity under discussion. *This* phrases such as "This is a consequence of Theorem 2" should be used with caution as they can force the reader to backtrack to find what "this" refers to. Often it helps to insert an appropriate noun after *this*. *It* can also be ambiguous: in the sentence "Condition 3 is not satisfied for the steepest descent method, which is why we do not consider it further" we cannot tell whether it is the condition or the method that is not being pursued.

4.8. British versus American Spelling

In my opinion (as a Briton) it makes little difference whether you use British or American (US) spellings, as long as you are consistent within a given

piece of writing. For some journals, copy editors will convert a manuscript to one or the other form of spelling. Major dictionaries give both spellings. I find it natural to use the spelling of the country in which I am working at the time! See §5.9 for some examples of the different spellings.

4.9. Capitalization

Words that are derived from a person's name inherit the capitalization. Thus: Gaussian elimination, Hamiltonian system, Hermitian matrix, Jacobian matrix, Lagrangian function, Euler's method, and so on. The incorrect form "hermitian" is sometimes seen. The Lax Equivalence Theorem is quite different from a lax equivalence theorem! Some (but not all) dictionaries list "abelian" with a small "a", showing that eponymous adjectives can gradually become accepted in uncapitalized form.

There does not seem to be a standard rule for when to capitalize the word following a colon. Bernstein [28] and Knuth [164, p. 11] both suggest the useful convention of capitalizing when the phrase following the colon is a full sentence.

4.10. Common Misspellings or Confusions

The errors shown in Table 4.1 seem to be common in mathematical writing. The misspellings marked with an asterisk are genuine words, but have different meanings from the corresponding words in the left column.

One Web page I visited describes "seperate" as the most common misspelling on the Internet and lose/loose as the second most common. Using the Web search engine Alta Vista I found one occurrence of "seperate" for every 24 occurrences of "separate".

The journal *Physical Review Letters* started spelling *Lagrangian* as *Lagrangean* in mid 1985, a change which is incorrect according to most dictionaries. Mermin, a Cornell physicist, spotted the switch and wrote an article criticizing it [201]. The journal has now reverted to *Lagrangian*.

According to McIlroy [199], on most days at Bell Laboratories someone misspells the word *accommodate*, in one of seven incorrect ways.

4.11. Consistency

It is important to be consistent. Errors of consistency often go unnoticed, but can be puzzling to the reader. Don't refer to $\ker(A)$ as the null-space, or $\text{null}(A)$ as the kernel—stick to matching synonyms. If you use the term "Cholesky factorization", don't say the "Cholesky decomposition" in the

Table 4.1. Common errors. Asterisk denotes a genuine word.

Correct/Intended	Misspelling
analogous	analagous
criterion	criteria*
dependent	dependant*
discrete	discreet*
Frobenius	Frobenious
idiosyncrasy	idiosyncracy
in practice	in practise
led (past tense of lead)	lead*
lose	loose* (very common)
phenomenon	phenomena* (plural of phenomenon)
preceding	preceeding
principle	principal*
propagation	propogation
referring	refering
Riccati	Ricatti
separate	seperate
supersede[a]	supercede
zeros	zeroes[b]

[a]The only English word ending in -sede.

[b]Some, but not all, dictionaries give the -oes ending as an alternative spelling for the plural noun.

same work. If words have alternative spellings, stick to one: don't use both orthogonalise and orthogonalize, and if you use orthogonalize also use, for example, optimize, not optimise. But note that not all -ise words can be spelt with -ize; examples are listed in §5.9.

4.12. Contractions

Contractions such as *it's*, *let's*, *can't* and *don't* are not used in formal works, but are acceptable in popular articles if used sparingly. Note the distinction between the contraction *it's* (short for *it is*) and the possessive *its*: "It's raining", "A matrix is singular if its determinant is zero." One editor comments that the two most frequent errors she encounters are the use of *it's* for *its* and incorrect punctuation surrounding *however* [286, p. 39].

4.13. Dangling Participle

What is wrong with the following sentence?

> Substituting (3) into (7), the integral becomes $\pi^2/9$.

This sentence suggests that the integral makes the substitution. The error is that the intended subject ("we") of the participle *substituting* is not present in the sentence. Rewritten and unambiguous versions are

> Substituting (3) into (7), we find that the integral is $\pi^2/9$.
> When (3) is substituted into (7), the integral becomes $\pi^2/9$.

A similar example that is less obviously wrong is

> When deriving parallel algorithms, the model of computation must be considered carefully.

Dangling participles are usually not ambiguous when read in context, but they can be distracting:

> A bug was found in the program using random test data.

Here is another example of a different type:

> Being stiff, the Runge-Kutta routine required a large amount of CPU time to solve the differential equation.

Here, the problem is that the noun immediately following "being" is not the one to which this participle refers. There are several ways to rewrite the sentence. One that preserves the emphasis is

> Because the differential equation is stiff, the Runge-Kutta routine required a large amount of CPU time to solve it.

Certain participial constructions are idiomatic and hence are regarded as acceptable:

> Assuming $G(x^*)$ is positive definite, x^* is a minimum point for F.
> Considering the large dimension of the problem, convergence was obtained in remarkably few iterations.
> Strictly speaking, the bound holds only for $n\epsilon < 1$.

4.14. Distinctions

Affect, Effect. *Affect* is a verb meaning to produce a change. *Effect* is a noun meaning the result of a change. Examples: "Multiple roots affect the convergence rate of Newton's method", "One effect of multiple roots is to reduce the convergence rate of Newton's method to linear." Effect is also a verb meaning to bring about (as in "to effect a change"), but in this form it is rarely needed in mathematical writing.

Alternate, Alternative. *Alternate* implies changing repeatedly from one thing to another. An *alternative* is one of several options. Compare "While writing his thesis the student alternated between elation and misery", with "The first attempt to prove the theorem failed, so an alternative method of proof was tried."

Compare with, Compare to. *Compare with* analyses similarities and differences between two things, whereas *compare to* states a resemblance between them. Examples: "We now compare Method A with Method B", "Shakespeare compared the world to a stage", "Shall I compare thee to a summer's day?" As Bryson [41] explains, "Unless you are writing poetry or love letters, *compare with* is usually the expression you want." *Compare and* is an alternative to *compare with*: "We now compare Method A and Method B" or, better, "We now compare Methods A and B."

Compose, Comprise, Constitute. *Compose* means to make up, *comprise* means to consist of. "Comprised of" is always incorrect. Thus, "the course is composed of three topics" or "the course comprises three topics", but not "the course is comprised of three topics." *Constitute* is a transitive verb used in the reverse sense: "these three topics constitute the course."

Due to, Owing to. These two expressions are not interchangeable, though writers frequently use *due to* in place of *owing to*. Use *due to* where you could use "caused by", or "attributable to"; use *owing to* where you could use "because of". Thus "The instability is due to a rank deficient submatrix" but "Owing to a rank deficient submatrix the computed result was inaccurate."

Fewer, Less. *Less* refers to quantity, amount or size, *fewer* to number. Thus "the zeros of $f(x)$ are less than those of $g(x)$" means that if x is a zero of f and y a zero of g then $x < y$, whereas "the zeros of $f(x)$ are fewer than those of $g(x)$" means that g has more zeros than f. Bryson [41] states the rule of thumb that *less* should be used with singular nouns and *fewer* with plural nouns: less research, less computation, fewer graduates, fewer papers.

Practice, Practise. In British English, *practice* is the noun and *practise* the verb (as with advice and advise). Thus "in practice", "practice

session", "practise the technique", "practised speaker". But in American English both verb and noun are spelt *practice.*

Which, That. A "wicked which"[5] is an instance of the word *which* that should be *that* (example: replace the word before *should*, earlier in this sentence, by *which*). The rule is that *that* defines and restricts, whereas *which* informs and does not restrict.[6] (Mathematicians should be good at spotting definitions.) Note the difference between the following two examples.

"Consider the Pei matrix, which is positive definite." We are being told additional information about the Pei matrix: that it is positive definite.

"Consider the Pei matrix that is positive definite." Now we are being asked to focus on a particular Pei matrix from among several: the one that is positive definite.

A useful guide is that which-clauses are surrounded by commas, or preceded by a comma if at the end of a sentence. If you're not sure whether to use *which* or *that*, see whether your sentence looks right with commas around the relevant clause. Sometimes it pays to introduce a wicked which to avoid ugly repetition, as has been done in the sentence "This approach is similar to that which we used in our earlier paper" (though "the one we used" is better). A rule of thumb discussed in [164, pp. 96–97] is to replace *which* by *that* whenever it sounds right to do so.

4.15. Elegant Variation

Elegant variation is defined by the Fowlers [84] as "substitution of one word for another for the sake of variety". It is a tempting way to avoid repetition, but is often unnecessary and can introduce ambiguity. Consider the sentence "The eigenvalue estimate from Gershgorin's theorem is a crude bound, but it is easy to compute." Does Gershgorin's theorem yield an estimate or a strict bound? We cannot tell from the sentence. In fact, the answer is that it can yield either, depending on how you interpret the theorem. A rewrite of the sentence (with knowledge of the theorem) produces "The eigenvalue inclusion regions provided by Gershgorin's theorem are crude, but easy to evaluate", where *inclusion regions* can be replaced by *estimates* or *bounds*, depending on the desired emphasis.

The opposite of elegant variation is when the same word is repeated in different forms or with different meanings. Here are two examples.

> The performance is impressive and gives the impression that

[5] A term coined by Knuth [164].

[6] Some authorities permit *which* to be used in a defining clause (e.g., Gowers [115]), but, as Bryson [41] puts it, "the practice is on the whole better avoided."

> the blocksize is nearly optimal. [impressive, impression]
> In the remainder of this chapter we examine the remainder in Euler's summation formula. [remainder, remainder]

Such echoing of words is distracting and is easily avoided by choosing a synonym for one of them.

4.16. Enumeration

Consider the extract

> The Basic Linear Algebra Subprograms (BLAS) have several advantages. They
>
> - Lead to shorter and clearer codes.
> - Improve modularity.
> - Machine dependent optimizations can be confined to the BLAS, aiding portability, and
> - Tuned BLAS have been provided by several manufacturers.

This explanation reads badly because the entries in the list are not grammatically parallel: the preceding "they" applies only to the first two entries of the list, and the third entry is not a complete sentence, unlike the others. This is an example of bastard enumeration, so-named by Fowler [83, p. 28], who explains that in an enumeration "there must be nothing common to two or more of the items without being common to all."

4.17. False If

The if–then construct is a vital tool in expressing technical arguments, but it is sometimes used incorrectly. Consider the sentence

> If we wish to compare the solutions of $f - \lambda k(f) = 0$ and $f_n - \lambda k_n(f_n) = 0$, then Jones shows that for a wide class of nonlinear $k(f)$, $\|f - f_n\| \leq c(\lambda)\|k_n(f) - k(f)\|$.

Jones's demonstration is independent of whether or not we wish to compare solutions, so the *if* is misleading: it falsely heralds a logical condition. False ifs can always be removed by rewriting:

> To compare the solutions of $f - \lambda k(f) = 0$ and $f_n - \lambda k_n(f_n) = 0$, we can use Jones's result that for a wide class of nonlinear $k(f)$, $\|f - f_n\| \leq c(\lambda)\|k_n(f) - k(f)\|$.

A more confusing example is

> If we assume that rational fractions behave like almost all real numbers, a theorem of Khintchine states that the sum of the first k partial quotients will be approximately $k \log_2 k$.

The *if* appears to be a false one, because the statement of Khintchine's theorem must be independent of what the writer assumes. In fact, with knowledge of the theorem, it can be seen that the main error is in the word "states". If we change "states" to "implies" and delete "we assume that", then the sentence is correct.

The next example is an unnecessary *if*, rather than a false *if*.

> We show that if $\widehat{x}$ is the computed solution to $Lx = b$ then $(L + \Delta L)\widehat{x} = b$, where $\|\Delta L\| \leq \alpha(n)\epsilon\|L\|$.

This type of construction is acceptable if used sparingly. I prefer

> We show that the computed solution $\widehat{x}$ to $Lx = b$ satisfies $(L + \Delta L)\widehat{x} = b$, where $\|\Delta L\| \leq \alpha(n)\epsilon\|L\|$.

4.18. Hyphenation

As Turabian [278, p. 44] notes, the trend is not to hyphenate compound words beginning with prefixes such as multi, pre, post, non, pseudo and semi. In mathematical writing it is common to write *nonsingular*, *semidefinite* (but *semi-infinite* to avoid a double i) and *pseudorandom*. However, a hyphen is retained before a proper noun, as in *non-Euclidean*. In deciding whether to hyphenate or to combine two words as one, it is worth bearing in mind that the hyphenated form tends to be easier to read because the prefix can be seen at a glance. Readers whose first language is not English may appreciate the hyphenated form.

Compound words involving *ill* and *well* occur frequently in mathematical writing and opinions differ about their hyphenation. *The Chicago Manual of Style* [58] recommends hyphenation when a compound with *ill*, *well*, *better*, *best*, *little*, *lesser*, etc., appears before a noun, unless the compound is itself modified. The purpose of this hyphenation rule is to avoid ambiguity. Examples:

> This is an ill-posed problem *but* This problem is ill posed.
> The well-known theorem *but* The theorem is well known.
> An ill-conditioned function *but* A very ill conditioned function.
> The second-order term has a constant 2 *but* This term is of second order.

The second example is widely accepted, but many writers always hyphenate compounds involving *ill*, such as ill-conditioned, and it is hard to argue against this practice. There are some common phrases that some writers hyphenate and others do not. An example is *floating point arithmetic* (*floating-point arithmetic*).

In the phrase "we use the 1, 2 and ∞-norms", a *suspensive hyphen* is required after "1" and "2" since they are prefixes to "norm" and we need to show that they are to be linked to this word. Thus the phrase should be rewritten "we use the 1-, 2- and ∞-norms".

Notice the hyphen in the title of Halmos's best-seller *Finite-Dimensional Vector Spaces* [122]. Halmos explains in [128, p. 146] that in the original 1942 edition the hyphen was omitted, but it was added for the 1958 edition.

4.19. Linking Words

If a piece of writing is to read well there must be no abrupt changes in mood or direction from sentence to sentence within a paragraph (unless such changes are used deliberately for effect). One way to achieve a smooth flow is to use linking words. Notice how the following paragraph would be improved by adding "In particular" to the start of the second sentence and "Furthermore" to the start of the third.

> Once we move from a convex program to a general nonlinear program, matters become far more complicated. Certain topological assumptions are required to avoid pathological cases. The results apply only in a neighbourhood of a constrained minimizer, and involve convergence of subsequences of global minimizers of the barrier function.

Of course, a sequence of sentences of the form "adverb, fact" quickly becomes tiresome, so linking words should not be overused.

Here is a list of linking words and phrases, arranged according to sense. For examples of use see §5.8.

combinations. also, and, as well as, besides, both, furthermore, in addition to, likewise, moreover, similarly.

implications or explanations. as, because, conversely, due to, for example, given, in other words, in particular, in view of, it follows that, otherwise, owing to, since, specifically, that is, thus, unlike.

modifications and restrictions. although, alternatively, but, despite, except, however, in contrast, in spite of, nevertheless, of course, on the contrary, on the other hand, though, unfortunately, whereas, yet.

emphasis. actually, certainly, clearly, in fact, indeed, obviously, surely.

consequences. accordingly, consequently, hence, therefore, thus.

4.20. Misused and Ambiguous Words

Both. A common misuse of *both* is illustrated by "In Gaussian elimination we can order the inner loops 'ij' or 'ji'. Both orderings are equivalent, mathematically." *Both* means "the two together" and is redundant when the sentence already carries this implication, as in this example. It would also be incorrect to say "Both orderings produce identical results." Correct versions are "These orderings are equivalent, mathematically", "Both orderings yield the same result", or "The two orderings produce identical results." Another common misuse of *both* is misplacement when it is used with prepositional phrases. For example,

> Incorrect: "Solutions are found both in the left and right quadrants." (*Both* is followed by a preposition, *in the left*, but *and* is followed by a noun.)
>
> Correct: "'Solutions are found both in the left and in the right quadrants." (Prepositional phrases follow *both* and *and*.)
>
> Correct: "'Solutions are found in both the left and the right quadrants." (Nouns follow *both* and *and*.)

Like. Consider the sentence

> Solving triangular systems is such a common operation that it has been standardized as a subroutine, along with many other common linear algebra operations like matrix multiplication.

The word *like* incorrectly limits the choice of linear algebra operations rather than serving as an example. Replacing *like* by *such as* conveys the intended meaning. The correct use of *like* is illustrated by "The Schulz iteration is quadratically convergent, like the Newton iteration."

Problem. An overused and, at times, ambiguous word in mathematical writing is *problem*, which can refer to both the focus of a piece of work and the difficulties encountered in carrying out the work. Sentences such as "In solving this problem we encountered a number of problems" can be avoided by using a synonym for the second occurrence of "problem", or rewriting. The sentence "We describe the special problems arising when solving stiff differential equations" is ambiguous: "problems" could refer to

classes of sub-problems produced by the solution process (such as nonlinear equations), or particular difficulties faced when solving the differential equations. Again, a rewrite is necessary.

Reason. In the phrase "the reason ... is because" the word *because* is redundant, since it means "for the reason that". Therefore in the sentence "The reason for the slow convergence is because α is a double root" *because* should be replaced by *that.* Similarly, in the phrase "the reason why" the word *why* can often be omitted. Thus "The reason why this question is important is that" is better written as "The reason this question is important is that" or "This question is important because".

Significant. Be careful if you use the word *significant* in mathematical writing, because to some readers it is synonymous with *statistically significant*, which carries a precise statistical meaning.

Try and, try to. The expression *try and* is frequently used in spoken English, but it is colloquial and should be replaced by *try to* in written English. Thus in the sentence "We sum the numbers in ascending order to try and minimize the effect of rounding errors" *try and* should be replaced by *try to.*

4.21. Numbers

Small integers should be spelled out when used as adjectives ("The three lemmas"), but not when used as names or numbers ("The median age is 43" or "This follows from Theorem 3"). The number 1 is a special case, for often "one" or "unity" reads equally well or better: "z has modulus one".

4.22. Omit These Words?

Here are some words and phrases whose omission often improves a sentence:

actually, very, really, currently, in fact, thing, without doubt.

The phrase "we have" can often be omitted. "Hence we have $x = 0$" should be replaced by "Hence $x = 0$." "Hence we have the following theorem" should be deleted or replaced by a sentence conveying some information (e.g., "Hence we have proved the following theorem").

4.23. Paragraphs

A standard device for making text more appetizing is to break it into small paragraphs, as is done in newspapers. The short paragraph principle is worth following in technical writing, where the complexity of the ideas

makes it more important than usual for the reader to be given frequent rests. Furthermore, long paragraphs tend to give a page a heavy image that can be a visual deterrent to the potential reader. A mix of different lengths is best. Ideally, each paragraph contains a main idea or thought that separates it from its neighbours. A long paragraph that is hard to break may be indicative of convoluted thinking.

The best writers occasionally slip in one-sentence paragraphs.

4.24. Punctuation

Much can be said about punctuation, and for thorough treatments of the topic I refer the reader to the references mentioned in Chapter 2. It is worth keeping in mind Carey's explanation [52] that "the main function of punctuation is *to make perfectly clear the construction* of the written words." A few common mistakes and difficulties deserve mention here.

- In "This result is well known, see [9]" the comma should be a semicolon, which conveys a slightly longer pause. Even better is to say "This result is well known [9]." A common mistake is the let-comma-then construction: "Let x^* be a local maximum of $F(x)$, then a Taylor series expansion gives ...." The comma should be a full stop.

  Similarly, the comma should be a semicolon, or even a full stop, in this sentence: "This bound has the disadvantage that it uses a norm of X, moreover the multiplicative constant can be large when X is not a normal matrix." These errors are called "comma splices" by Gordon [111].

- In this sentence the semicolon should be a comma: "The secant method can also be used; its lack of need for derivatives being an advantage." A rough guide is to use a semicolon only where you could also use a full stop.

- In the sentence "If we use iterative refinement solutions are computed to higher accuracy", a comma is needed after "refinement", otherwise the reader may take "iterative refinement" as modifying "solutions". Another example where a comma is needed to avoid ambiguity is the sentence "However, the singularity can be removed by a change of variable."

- In sentences such as

  > Fortran 77 contains the floating point data types real, complex and double precision.
  > The output can be rotated, stretched, reduced or magnified.

we have a choice of whether or not to place a comma (called a serial comma) before the *and* or *or* that precedes the final element of the list. In some sentences a serial comma is needed to avoid ambiguity, as in the sentence "A dictionary is used to check spelling, shades of meaning, and usage", where the absence of the comma makes "shades" modify usage. Opinion differs on whether a serial comma should always be used. It is a matter of personal preference. The house styles of some publishers require serial commas.

If a list contains commas within the items, ambiguity can be avoided by using a semicolon as the list separator. Example:

> The test collection includes matrices with known inverses or known eigenvalues; ill-conditioned or rank deficient matrices; and symmetric, positive definite, orthogonal, defective, involutary, and totally positive matrices.

- The exclamation mark should be used with extreme caution in technical writing. If you are tempted to exclaim, read "!" as "shriek"; nine times out of ten you will decide a full stop is adequate. An example of correct usage is, from [217, p. 46], "When A is tridiagonal the computation of $A^{-1}u$ costs little more than the computation of Au!" The exclamation mark could be omitted, but then the reader might not realize that this is a surprising fact. Another example is, from [159, p. 42], "The chi-square table tells us, in fact, that V_2 *is much too low*: the observed values are so close to the expected values, we cannot consider the result to be random!"

- In the US, standard practice is to surround quotes by double quotation marks, with single quotation marks being used for a quote within a quote. In the UK, the reverse practice is generally used. The placement of final punctuation marks also differs: in US usage, the final punctuation is placed inside the closing quotation marks (except for "!" and "?" when they are not part of the quotation), while in UK usage it goes outside (except for "!" and "?" when they are part of the quotation). In this book, for quotations that end sentences, the end of sentence period appears outside quotation marks unless the quotation is itself a valid sentence.

- An apostrophe denotes possession for nouns (*the proof's length*) but not for pronouns (*this book is yours*). An apostrophe is used with the plural of letters and of words when the words are used without regard to their meaning: "there are three l's in the word parallel", "his prose contains too many however's." For plurals of numbers the

apostrophe can be omitted: "a random matrix of 0s and 1s". For plurals of mathematical symbols or expressions the apostrophe can again be omitted provided there is no ambiguity: "these fs are all continuous", "likewise for the Z_ks", "these δs are all of order 10^{-8}."

4.25. Say It Better, Think It Gooder

The title of this section combines the titles of two papers by George Piranian [227] and Paul Halmos [126] that appeared in *The Mathematical Intelligencer* in 1982. As Gillman explains in [105], "George said good English is important. Paul said, what do you mean, good English is important?—good mathematics is more important. They are both right." While correct English usage is important, it must not be allowed to deflect you from the language-independent tasks of planning and organizing your writing.

4.26. Saying What You Mean

In technical writing you need to take great care to say what you mean. A hastily constructed sentence can have a meaning very different from the one intended. In a book review in *SIAM Review* [vol. 34, 1992, pp. 330–331] the reviewer quotes the statement "According to Theorem 1.1, a single trajectory $X(t, x)$ passes through almost every point in phase space ...." The book's author meant to say that for every point in phase space there is a unique trajectory that goes through it.

4.27. Sentence Opening

Try not to begin a sentence with *there is* or *there are*. These forms of the verb *be* make a weak start to a sentence, because they delay the appearance of the main verb (see the quote by Dixon on page 77). Sometimes these phrases can simply be deleted, as in the sentence "There are several methods that are applicable" ("Several methods are applicable"). Also worth avoiding, if possible, are "It is" openers, such as "It is clear that" and "It is interesting to note that". If you can find alternative wordings, your writing will be more fresh and lively.

4.28. Simplification

Each word or phrase in the left column below can (or, if marked with an asterisk, should) be replaced by the corresponding one in the right column. This is not an exhaustive list (see [69, Appendix 4] or [14] for many

Table 4.2. The origins of some synonyms.

Anglo-Saxon	French	Latin
ask	question	interrogate
rise	mount	ascend
good	marvellous	superior
show	establish	exhibit, demonstrate
need	requirement	necessity
think	ponder	cogitate

more examples) but comprises periphrastic phrases that I have spotted in mathematical writing.

by means of	by
conduct an investigation	investigate
due to the fact that*	since/because
firstly	first
in a position to	can
in order to	to
in the case that*	if
in the course of	during
in the event that*	if
in the first place*	first
it is apparent that	apparently
it may happen that	there may/might
most unique*	unique
take into consideration	consider
that is to say	in other words/that is
to begin with*	to begin

4.29. Synonym Selection

English is one of the most synonym-rich languages, thanks to the words it has adopted from other languages, and each member of a set of synonyms can have a different tone and shade of meaning. A dictionary and a thesaurus are vital aids to choosing the right word (see §2.1). When you have a choice of words, use a short, concrete one in preference to a long or abstract one. Often, this means using an Anglo-Saxon word instead of one of French or Latin origin, as Table 4.2 illustrates.

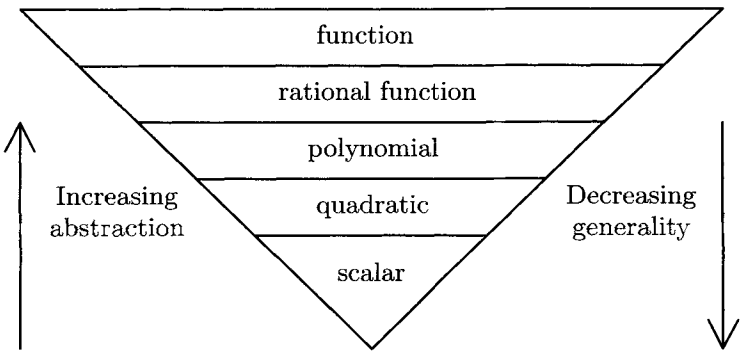

Figure 4.1. Word pyramid.

Don't be afraid of using a long or unusual word if it is just the right one, or if it adds spice to your writing or conveys an interesting image. Acton [3, p. 145] writes "The second term ... is obstreperous." No other word quite so vividly describes the uncontrollable term to which he refers (with the possible exception of *recalcitrant*, another word used by Acton in the same book).

I chose the title "Otiose Symbols" on page 29 in preference to "Unnecessary Symbols" for two reasons. First, the former is shorter and more likely to catch the reader's eye. Second, *otiose* means "serving no practical purpose", so "otiose symbols" has a stronger meaning than "unnecessary symbols".

It helps to think of word pyramids showing levels of abstraction [281], such as the one in Figure 4.1. As Turk and Kirkman [280] point out, "Abstract words are less easy to decode because the reader has to 'scan' all the possibilities subconsciously before deciding on a specific meaning." Generally, it is best to use the least abstract, most specific word possible. This leads to more lively and precise prose. Here are some examples of chains of words in order of increasing specificity:

result–theorem–inequality
statistic–error–relative error
optimum–minimum–global minimum
random–normally distributed–normal (0,1)

4.30. Tense

One of the decisions the writer of a technical paper must make is what tense to use. There are no hard and fast rules, except to be consistent in the use of tense, but I offer some guidelines.

I prefer the present tense to the future tense for referring to later parts of the paper. Thus I write "This is proved in Theorem 3, below" rather than "This will be proved in Theorem 3", and "We discuss these matters in detail in Section 5" rather than "We will discuss these matters in detail." Note that since the latter example does not contain a section reference it could be construed as referring to a future paper; this potential ambiguity is a danger of using the future tense.

The present tense can also be used for backward references, as in "The analysis of Section 1 proves the following ...", though the past tense is more common: "We showed/saw in Section 1 that ...." Tables and figures exist in the present, so the present tense should be used to refer to them: "Table 4 shows that", "Figure 1 displays". Similarly, in a citation where the reference is the subject of the sentence, the present tense is the one to use: "Reference [4] contains the interesting result that ...."

To refer to work in earlier papers either the past or the present tense can be used: "Banach shows that" or "Banach showed that". I use the present tense unless I want to emphasize the historical aspects of the reference, as, for example, when summarizing a number of earlier results in a survey.

A summary or conclusions section should use the past tense when specifying actions: "We have shown that" or "We showed that" rather than "We show that". But if a simple statement of results is given, the present tense is appropriate: "Asymptotic expansions are useful for ...", "Our conclusion is that ...."

In summary, I recommend the rule "if in doubt use the present tense."

4.31. What to Call Yourself

All technical writers face the question of how to refer to themselves in their papers. For example, if you refer to a previous result of yours you may have to choose between

(1) I showed in [3] that
(2) We showed in [3] that
(3) The author showed in [3] that
(4) It was shown in [3] that

Sentence 4 should be avoided because it is in the passive voice, which is unnecessary in this case. Sentence 3 has a formal, stilted air and is again

best avoided. Some authorities on technical writing advise against using "we" to refer to a single author, but are happy for it to be used when there are two or more authors. Nevertheless, in mathematical writing "we" is by far the most common choice of personal pronoun; I frequently use it myself (in papers, but not in this book). "I" creates a more frank, personal contact with the reader and is less commonly used than "we". Zinsser [304, p. 24] suggests "If you aren't allowed to use 'I', at least think 'I' while you write, or write the first draft in the first person and then take the 'I's out. It will warm up your impersonal style."

"We" can be used in the sense of "the reader and I", as in the sentence

> We saw earlier that $f_n \to 0$ as $n \to \infty$.

Whether you choose "I" or "we", you should not mix the two in a single document, except, possibly, when using the "reader and I" form of "we".

Other personal pronouns to consider are "one" and "us". "One", as in "one can show that ..." is often used, but is perhaps best avoided because of its vague, impersonal nature. "Us" is useful in sentences such as

> Let us now see how the results can be extended to non-smooth functions.

where it means "the reader and I". It is generally quite easy to remove both "one" and "us" by phrasing sentences in an alternative form. For example, the sentence above can be rewritten as "How can the results be extended to non-smooth functions? One approach is ...", or "To see how the results can be extended to non-smooth functions, we ...."

4.32. Word Order

The first words of a sentence are usually regarded by the reader as the most important. Therefore, the word order of sentences should be chosen to reflect the desired emphasis. Compare the first sentence of this paragraph with "The reader usually regards the first words of a sentence as the most important", which places the emphasis on the reader rather than "the first words of a sentence".

Reordering the words of a sentence can strengthen it and remove ambiguity. Here is an example of a misplaced *only*.

> Bad: The limit point is only a stationary point when the regularity conditions are satisfied. (Might be taken to mean that the limit point is expected to be more than just a stationary point.)

Good: The limit point is a stationary point only when the regularity conditions are satisfied.

Here is an example of an incorrect *either*.

Bad: Mathematically, either we have an integral equation of the first kind or one of the second kind.

Good: Mathematically, we have an integral equation of either the first kind or the second kind. (Or simply delete *either* from the bad example.)