Chapter 6
Writing a Paper

The title of a book is its special tag, its distinguishing label. The choice of words is limited only by the number of words in the English language, yet how few of them show up in titles.

— ELSIE MYERS STAINTON, *A Bag for Editors* (1977)

Go straight to the point, rather than begin with an historical reference or resounding banality ('There is much research interest at present in the biochemistry of memory').

— BERNARD DIXON, *Sciwrite* (1973)

To write a reference, you must have the work you're referring to in front of you. Do not rely on your memory. Do not rely on your memory. Just in case the idea ever occurred to you, do not rely on your memory.

— MARY-CLAIRE VAN LEUNEN, *A Handbook for Scholars* (1992)

It was said of Jordan's writings that if he had 4 things on the same footing as $(a,\ b,\ c,\ d)$ *they would appear as* $a,\ M_3',\ \epsilon_2,\ \Pi_{1,2}''$.

— J. E. LITTLEWOOD, *Littlewood's Miscellany*[9] (1986)

May all writers learn the art (it is not easy) of preparing an abstract containing the essential information *in their compositions.*

— KENNETH K. LANDES, *A Scrutiny of the Abstract. II* (1966)

[9]See [34].

We write scientific papers to communicate our ideas and discoveries. Our papers compete for the readers' attention in journals, conference proceedings and other outlets for scholarly writing. If we can produce well-organized papers that are expressed in clear, concise English, and that convey our enthusiasm and are accessible to people outside our particular speciality, then our papers stand a better than average chance of being read and being referenced. It is generally accepted that the standard of scientific writing is not high[10] [63], [71], [191], [299], so a well-written paper will stand out from the crowd.

In this chapter I examine issues specific to writing a paper, as opposed to the general principles discussed in the previous chapters. Much of the chapter is applicable to writing a thesis (see also Chapter 9), book, or review. After considering the general issues of audience and organization I explore the building blocks of a research paper, from the title to the reference list.

6.1. Audience

Your first task in writing a paper is to determine the audience. You need to identify a typical reader and decide the breadth of your intended readership. Your paper might be written for a mathematics research journal, an undergraduate mathematics magazine, or a book for pre-university students. The formality of the prose and mathematical developments will need to be different in each of these cases. For research journals, at one extreme, you might be writing a very technical paper that builds upon earlier work in a difficult area, and you might be addressing yourself only to experts in that area. In this case it may not be necessary to give much motivation, to put the work in context, or to give a thorough summary and explanation of previous results. At the other extreme, as when you are writing a survey paper, you do need to motivate the topic, relate it to other areas, and explain and unify the work you are surveying. The requirements set forth in the "Guidelines for Authors" for the journal *SIAM Review* (prior to 1998) are even more specific:

> In their introductory sections, all papers must be accessible to the full breadth of SIAM's membership through the motivation, formulation, and exposition of basic ideas. The importance and intellectual excitement of the subject of the paper must

[10] In 1985, the editor of the British Medical Journal said "The 5000 or so articles we see at the BMJ every year are mostly dreadfully written, with numerous faults in English and overall construction" [180, p. 232].

> be plainly evident to the reader. Abstraction and specialization must illuminate rather than obscure. The primary threads of the intellectual fabric of the paper can never be hidden by jargon, notation, or technical detail.

The goals described in the last three sentences are worth striving for, whatever your audience.

The audience will determine the particular slant of your paper. A paper about Toeplitz matrices for engineers would normally phrase properties and results in terms of the physical problems in which these matrices arise, whereas for an audience of pure mathematicians it would be acceptable to consider the matrices in isolation from the application.

The language you use will depend on the audience. For example, whereas for linear algebraists I would write about the least squares problem $\min_x \|Ax - b\|_2$ with its least squares solution x, for statisticians I would translate this to the linear regression problem $\min_b \|Xb - y\|_2$ and the least squares estimate $\widehat{b}$ of the regression coefficients. Failing to use the notation accepted in a given field can cause confusion and can make your work impenetrable to the intended readership.

A good question to ask yourself is why a member of your intended audience should want to read your paper. If the paper is well focused you will find it easy to answer this question. If you cannot find an answer, consider altering the aims of the paper, or doing further work before continuing with the writing.

Whatever your audience, it is worth keeping in mind the words of Ivars Peterson, the editor of *Science News* [223]:

> The format of most journal papers seems to conspire against the broad communication of new mathematical ideas .... The titles, abstracts and introductions of many mathematical papers say: "Outsiders keep out! This is of interest only to those few already in the know."

With a little effort it should be possible to make your work at least partially understandable to non-experts.

6.2. Organization and Structure

At an early point in the writing of your paper you need to think about its high-level organization. It is a good idea to rank your contributions, to identify the most important. This will help you to decide where to put the emphasis and how to present the work, and it will also help you to write the title and abstract.

At the outset you should have some idea of the length of the paper. The larger it is the more important it is that it be well organized. However, it can be harder to write a good short paper than a good long one, for it is difficult to be simultaneously thorough, lucid and concise.

You need to decide in what order to treat topics and how to present results. You should aim to minimize the length by avoiding repetition; to obtain general results that provide others as special cases (even if the latter are more interesting); and to emphasize similarities and differences between separate analyses.

In addition to considering the first-time reader, you must think about the reader who returns to the paper some time after first studying it. This reader will want to skim through the paper to check particular details. Ideally, then, your paper should not only be easy to read through from beginning to end, but should also function as a reference document, with key definitions, equations and results clearly displayed and easy to find.

6.3. Title

According to Kerkut [150], for every person who reads the whole text of a scientific paper, five hundred read only the title. This statistic emphasizes the importance of the title. The title should give a terse description of the content, to help someone carefully scanning a journal contents page or a reference list decide whether to read the abstract or the paper itself. Ideally, it should also be catchy enough to attract the attention of a browser. Achieving a balance between these two aims is the key to writing an effective title. Kelley [147] mentions that he published an abstract with the title "A Decomposition of Compact Continua and Related Results on Fixed Sets under Continuous Mappings". After Paul Halmos suggested to him that it is not a good idea to put the whole paper into the title, he changed it to "Simple Links and Fixed Sets" for the published paper.

A note on the interpolation problem is too vague a title: what is the breakthrough heralded by *A note*, and which *interpolation problem* is under discussion? Similarly, *Approximation by cubic splines* is too vague (except for a thorough survey of the topic), since the approximation problem being addressed is not clear. Here are some examples of real titles, with my comments.

- Computing the eigenvalues and eigenvectors of symmetric arrowhead matrices [D. P. O'Leary and G. W. Stewart. *J. Comp. Phys.*, 90:497–505, 1990]. This is a lively and informative title. It is good to have action words in the title, such as *computing* or *estimating*.

- How and how not to check Gaussian quadrature formulae [Walter Gautschi. *BIT*, 23:209–216, 1983]. "How to" titles immediately arouse the reader's interest.

- Gaussian elimination is not optimal [V. Strassen. *Numer. Math.*, 13: 354–356, 1969]. If you can summarize your paper in a short sentence, that sentence may make an excellent title.

- How near is a stable matrix to an unstable matrix? [Charles F. Van Loan. In *Linear Algebra and Its Role in Systems Theory*, B. N. Datta, editor, volume 47 of *Contemporary Math.*, Amer. Math. Soc., 1985, pages 465–478]. A title that asks a question is direct and enticing.

- *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity* [D. A. Belsley, E. Kuh, and R. E. Welsch. Wiley, New York, 1980]. This title has the classic form "general statement followed by colon and more specific information". Dillon [70] defines such a title to have "titular colonicity" and suggests that "To achieve scholarly publication, a research title should be divided by a colon into shorter and longer pre- and postcolonic clauses, respectively, the whole not to fall below a threshhold [sic] of 15–20 words minimum." This paper appears in the August (not the April) issue of *American Psychologist* and appears not to be a spoof.

- ALGOL 68 with fewer tears [Charles H. Lindsey. *Computer Journal*, 15:176–188, 1972]. This is an excellent title—it announces that the paper is about the programming language ALGOL 68 and whets the reader's appetite for a demystifying explanation. This paper has the distinction of being a syntactically valid ALGOL 68 program!

- Nineteen dubious ways to compute the exponential of a matrix [Cleve B. Moler and Charles F. Van Loan. *SIAM Review*, 20(4):801–836, 1978]. A classic paper in numerical analysis, with a memorable title.

- Can you count on your calculator? [W. Kahan and B. N. Parlett. Memorandum No. UCB/ERL M77/21, Electronics Research Laboratory, College of Engineering, University of California, Berkeley, April 1977]. Puns rarely make their way into titles, but this one is effective.

- Performing armchair roundoff analyses of statistical algorithms [Webb Miller. *Comm. Statist. Simulation Comput.*, B7(3):243–255, 1978]. An otherwise drab title transformed by the word *armchair*. It suggests a gentle approach to the error analysis.

- Tricks or treats with the Hilbert matrix [Man-Duen Choi. *Amer. Math. Monthly*, 90:301–312, 1983]. This is another attractive and imaginative title.

- Can one hear the shape of a drum? [Marc Kac. *Amer. Math. Monthly*, 73(4, Part II):1–23, 1966]. The meaning of this attention-grabbing title is explained on the third page of Kac's expository paper:

> You can now see that the "drum" of my title is more like a tambourine (which is really a membrane) and that stripped of picturesque language the problem is whether we can determine Ω if we know all the eigenvalues of the eigenvalue problem
>
> $$\begin{aligned} \tfrac{1}{2} \nabla^2 U + \lambda U = 0 \quad &\text{in} \quad \Omega, \\ U = 0 \quad &\text{on} \quad \Gamma. \end{aligned}$$

  This paper won its author the Chauvenet Prize and the Lester R. Ford Award (see Appendix E). Kac's question is answered negatively in "One cannot hear the shape of a drum" [Carolyn Gordon, David L. Webb, and Scott Wolpert. *Bull. Amer. Math. Soc.*, 27(1):134–138, 1992], and his "hearing" terminology is used in "You can not hear the mass of a homology class" [Dennis DeTurck, Herman Gluck, Carolyn Gordon, and David Webb. *Comment. Math. Helvetici*, 64:589–617, 1989].

- The perfidious polynomial [James H. Wilkinson. In *Studies in Numerical Analysis*, G. H. Golub, editor, volume 24 of *Studies in Mathematics*, Mathematical Association of America, Washington, D.C., 1984, pages 1–28]. A delightful alliteration for a paper that explains why numerical computations with polynomials can be treacherous. This paper won its author the Chauvenet Prize (see Appendix E).

- Fingers or fists? (The choice of decimal or binary representation) [W. Buchholz. *Comm. ACM*, 2(12):3–11, 1959]. An analogy and an alliteration combine here to make an appealing title.

See Appendix E for many more examples of good titles.

A few years ago I submitted for publication a manuscript titled "Least Squares Approximation of a Symmetric Matrix". A referee objected to the title because it does not fully define the problem, so I changed it to "The Symmetric Procrustes Problem", which is even less informative if you are not familiar with Procrustes problems, but is perhaps more intriguing.

Unless the title is short it will have to be broken over two or more lines at the head of the paper (or on the front cover of a technical report). Rules of thumb are that a phrase should not be split between lines, a line should not start with a weak word such as a conjunction, and the lines should not differ too much in length. If the title is to be capitalized then all words except articles, short prepositions and conjunctions should be capitalized. Here are some examples, the first of each pair being preferable. The quotes at the beginning of each chapter provide further examples of the choice of line breaks.

On Real Matrices with
Positive Definite Symmetric Component

On Real Matrices with Positive
Definite Symmetric Component

An Iteration Method for the
Solution of the Eigenvalue Problem of
Linear Differential and Integral Operators

An Iteration Method for the Solution of
the Eigenvalue Problem of
Linear Differential and Integral Operators

Numerically Stable Parallel Algorithms
for Interpolation

Numerically Stable Parallel Algorithms for
Interpolation

In 1851 Sylvester published a paper with the title "Explanation of the Coincidence of a Theorem Given by Mr Sylvester in the December Number of This Journal, with One Stated by Professor Donkin in the June Number of the Same" [268]. Thankfully, titles are generally shorter nowadays.

6.4. Author List

In 1940, over 90% of papers reviewed in *Mathematical Reviews* had one author [120]. Today that figure is about 50%, showing that the proportion of jointly authored works in mathematics has increased greatly.

There are no hard and fast rules about the order in which the authors of a multiply authored paper are listed. Sometimes the person who did the greatest part of the work is listed first. Sometimes the academically senior person is listed first. In some disciplines and institutions the senior person

The Spotlight Factor

It is the custom in the theoretical computer science community to order authors alphabetically. In a tongue-in-cheek article, Tompa [273] defines the *spotlight factor* of the first author of a paper in which the k authors are listed alphabetically to be the probability that if $k-1$ coauthors are chosen independently at random they will all have surnames later in the alphabet than the first author. According to Tompa, the best (smallest) spotlight factor of 0.0255 in theoretical computer science belongs to Santoro, for his paper "Geometric containment and vector dominance" [Nicola Santoro, Jeffrey B. Sidney, Stuart J. Sidney, and Jorge Urrutia. *Theoretical Computer Science*, 53:345–352, 1987]. This value is calculated as

$$(1-.\text{santoro})^3 = \left(1 - \left(\frac{19}{27} + \frac{1}{27^2} + \frac{14}{27^3} + \frac{20}{27^4} + \frac{16}{27^5} + \frac{18}{27^6} + \frac{16}{27^7}\right)\right)^3,$$

where $a = 1, \ldots, z = 26$ and blanks or punctuation are represented by zero. By comparison, of those publications in the bibliography of this book, the best spotlight factors are the 0.1829 of O'Leary [211], the 0.2679 of Strunk [263] and the 0.3275 of Knuth [164].

(typically the director of a laboratory) is listed last. Perhaps most often, the authors are listed alphabetically, which is the practice I favour.

In their book *Computer Architecture* [137], Hennessy and Patterson adopt an unusual solution to the problem of deciding on author ordering: they vary the ordering in the book and in advertisements, even alternating the order when they reference the book! They comment that "This reflects the true collaborative nature of this book ... We could think of no fair way to reflect this genuine cooperation other than to hide in ambiguity—a practice that may help some authors but confuses librarians."

Being the first-named author is advantageous because the first name is easier to find in a reference list and the paper will be associated solely with that name in citations of the form "Smith et al. (1992)". Also, some citation services ignore all names after the first (see §14.3).

Make sure that you use precisely the same name on each of your publications. I declare myself as Nicholas J. Higham, but not N. J. Higham, N. Higham or Nick Higham. If you vary the name, your publications may not all be grouped together in bibliographic lists and indexes and there may be confusion over whether the different forms represent the same person.

Whether to spell out your first name(s) is a matter of personal preference. Chinese and Japanese authors need to decide whether to Westernize their name by putting the surname (family name) after the Christian (given) name, or to maintain the traditional ordering of surname first.

6.5. Date

Always date your work. If you give an unpublished paper to others they will want to know when it was written. You may not be able to distinguish different drafts if they are not dated. The date is usually placed on a line by itself after the author's name, or in a footnote. Spell the month out, rather than using the form "xx-yy-zz", since European and American authors interpret xx and yy in the opposite senses.

6.6. Abstract

The purpose of the abstract is to summarize the contents of the paper. It should do so in enough detail to enable the reader to decide whether to read the whole paper. The reader should not have to refer to the paper to understand the abstract.

Frequently, authors build an abstract from sentences in the first section of the paper. This is not advisable. The abstract is a mini-paper, and should be designed for its special purpose. This usually means writing the abstract from scratch once the paper is written. The shorter it is, the better, subject to the constraint of it being sufficiently informative. Most abstracts occupy one paragraph. Many mathematics journals state a maximum size for the abstract, usually between 200 and 300 words.

Some specific suggestions are as follows.

- Avoid mathematical equations in the abstract if possible, particularly displayed equations. One reason is that equations may cause difficulties for the review services that publish abstracts (though not those, such as *Mathematical Reviews*, that use TEX).

- Do not cite references by number in the abstract, since the list of references will not usually accompany the abstract in the review journals. If a paper must be mentioned, spell it out in full:

  An algorithm given by Boyd [*Linear Algebra and Appl.*, 9:95–101, 1974] is extended to mixed subordinate matrix norms.

- Try to make the abstract easy to understand for those whose first language is not English. Also, keep in mind that the abstract may

be translated into another language, for a foreign review journal, for example.

- Some journals disallow the word *we* in abstracts, preferring the passive voice (usually necessitating *it*). If you are writing for such a journal it pays to adopt the required voice; while a copy editor will convert as necessary, the conversion may lessen the impact of your sentences.
- Obviously, the abstract should not make claims that are not justified in the paper. Yet this does happen—possibly when the abstract is written before the paper is complete and is not properly revised.
- The abstract should give some indication of the conclusions of the paper. An abstract that ends "A numerical comparison of these methods is presented" and does not mention the findings of the comparison is uninformative.
- Your abstract should lay claim to some new results, unless the paper is a survey. Otherwise, if you submit the paper to a research journal, you are making it easy for a referee to recommend rejection.
- Try not to start the abstract with the common but unnecessary phrases "In this paper" or "This paper". Some journals make this request in their instructions to authors.

The suggestions above are particularly relevant for an abstract that is submitted to a conference and appears in a conference programme. Such an abstract will be read and judged in isolation from the paper, so it is vital for it to create a strong impression in isolation.

An intriguing opening paragraph of an abstract is the one by Knuth (1979) in [158]:

ABSTRACT. Mathematics books and journals do not look as beautiful as they used to. It is not that their mathematical content is unsatisfactory, rather that the old and well-developed traditions of typesetting have become too expensive. Fortunately, it now appears that mathematics itself can be used to solve this problem.

If inspiration fails you, you could always use the following generic abstract from [246]:

> ABSTRACT. After a crisp, cogent analysis of the problem, the author brilliantly cuts to the heart of the question with incisive simplifications. These soon reduce the original complex edifice to a [s]mouldering pile of dusty rubble.

6.7. Key Words and Subject Classifications

Some journals list key words supplied by the author, usually after the abstract. The number of key words is usually ten or less. Since the key words may be used in computer searches, you should try to anticipate words for which a reader might search and make them specific enough to give a good indication of the paper's content.

Some journals also require subject classifications. The AMS Mathematics Subject Classifications (1991) divide mathematics into 61 sections with numbers between 0 and 94, which are further divided into many subsections. For example, section 65 covers numerical analysis and has 106 subsections; subsection 65F05 covers direct methods for solving linear systems while subsection 65B10 covers summation of series. The classifications are listed in the *Annual Subject Index* of *Mathematical Reviews* and can be downloaded from the American Mathematical Society's e-MATH service (see §14.1).

Other classification schemes exist, such as the one for computer science from the journal *Computing Reviews*. The Computing Reviews Classification System (1987) is a four-level tree that has three numbered levels and an unnumbered level of descriptors. The top level consists of eleven nodes, denoted by letters A (General Literature) to K (Computing Milieux). An example of a category is

> G.1.3 [Numerical Analysis]: Numerical Linear Algebra—sparse and very large systems.

This specifies the Numerical Linear Algebra node of the Numerical Analysis area under G (Mathematics of Computation), and "sparse and very large systems" is one of several descriptors for G.1.3 listed in the definition of the classification scheme.

6.8. The Introduction

Perhaps the worst way to begin a paper is with a list of notation or definitions, such as *Let G be an abelian group and H be a subgroup of G*, or *Let $\mathcal{F}$ be the complex field $\mathcal{C}$ or the real field $\mathcal{R}$, and let $\mathcal{F}_{m\times n}$ be the linear space of all $m \times n$ matrices over $\mathcal{F}$. If $A \in \mathcal{F}_{m\times n}$ we use A^* to denote the conjugate transpose of A.* It can be argued that the first sentence should be the best in the paper—its job is to entice the uncommitted reader into reading the whole paper. A list of notation will not achieve this aim, but a clear, crisp and imaginative statement may. King [152] gives a slightly different specification for the first sentence: "The first sentence has a dual

function: it must carry some essential information, particularly the problem under consideration, and at the same time gently translate the reader into the body of the article."

Ideally, an introduction is fairly short—say a few hundred words. It should define the problem, explain what the work attempts to do, and outline the plan of attack. Unless there is good reason not to do so, it is advisable to summarize the results achieved. Knowing the problem and the progress made on it, the reader can decide after reading the introduction whether to read the whole paper. You may want to leave the punch-line to the end, but in doing so you risk the reader losing interest before reaching it.

Here is an example of a compelling opening paragraph that strongly motivates the work. It is from Gautschi's paper "How and How Not to Check Gaussian Quadrature Formulae", mentioned on page 81.

> The preparation of this note was prompted by the appearance, in the chemistry literature, of a 16-digit table of a Gaussian quadrature formula for integration with measure $d\lambda(t) = \exp(-t^3/3)\,dt$ on $(0, \infty)$, a table, which we suspected is accurate to only 1–2 decimal digits. How does one go about convincing a chemist, or anybody else for that matter, that his Gaussian quadrature formula is seriously defective?

It can be appropriate to begin with a few definitions if they are needed to state the problem and begin the analysis. As an example, here is the beginning of the paper "Estimating the Largest Eigenvalue of a Positive Definite Matrix" by O'Leary, Stewart and Vandergraft [211].

> Let A be a positive definite matrix of order n with eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0$ corresponding to the orthonormal system of eigenvectors $x_1, x_2, \ldots, x_n$. In some applications, one must obtain an estimate of λ_1 without going to the expense of computing the complete eigensystem of A. A simple technique that is applicable to a variety of problems is the power method.

In the next four sentences the authors define the power method, state that the theory of the method is well understood, and note that convergence of the method can be hindered in two ways, which are then analysed. This introduction is effective because it defines the problem and motivates the analysis without a wasted word, and leads quickly into the heart of the paper. The introduction is not labelled as such: this is a three and a half page paper with no section headings.

A possible way to improve an introduction is to delete the first one or more sentences, which are often unimportant general statements. Try it! As an example, consider the following opening two sentences.

> Polynomials are widely used as approximating functions in many areas of mathematics and they can be expressed in various bases. We consider here how to choose the basis to minimize the error of evaluation in floating point arithmetic.

This opening is not a bad one, but the first sentence is general and unexciting. Under the reasonable assumption that the reader knows the importance of polynomials and need not be reminded, it is better to combine and shorten the two sentences and pose a question:

> In which basis should we express a polynomial to minimize the error of evaluation in floating point arithmetic?

Unless the paper is very short it is advisable to outline its organization towards the end of the introduction. One approach is to write, for each section, a sentence describing its contents. The outline can be introduced by a sentence such as "The outline of this paper is as follows" or "This paper is organized as follows." It is best not simply to list the section titles; instead, give a summary that could only be obtained by reading the sections.

Note that some journals prefer the symbol § to the word section when referring to specific sections: §2.1 is written instead of Section 2.1. The plural of § is §§: "see §§2.1 and 2.2."

6.9. Computational Experiments

Many papers describe computational experiments. These may be done for several reasons: to gain insight into a method, to compare competing methods, to verify theoretical predictions, to tune parameters in algorithms and codes, and to measure the performance of software. To achieve these aims, experiments must be carefully designed and executed. Many decisions have to be made, ranging from what will be the objectives of the experiments to how to measure performance and what test problems to use. One editor of a numerical analysis journal commented that his primary reason for rejecting papers is that the computational experiments are unsound; this underlines the importance of proper design and reporting of experiments.

When you report the results of a computational experiment you should give enough detail to enable the results to be interpreted and the experiment to be reproduced. In particular, where relevant you should state the machine precision (working accuracy) and the type of random numbers used (e.g., normal $(0, 1)$ or uniform $(-1, 1)$). If you wish to display the convergence of a sequence it is usually better to tabulate the errors

rather than the values themselves. Error measures should be normalized. Thus, for an approximate solution $\widehat{x}$ to $Ax = b$, the relative residual $\|b - A\widehat{x}\|/(\|A\|\|\widehat{x}\| + \|b\|)$ is more meaningful than the scale-dependent quantity $\|b - A\widehat{x}\|$. If you are measuring the speed of a numerical algorithm it is important to show that the right answers are being produced (otherwise the algorithm "the answer is 42" is hard to beat).

You may also wish to state the programming language, the version of the compiler used, and the compiler options and optimizations that were selected, as these can all have a significant influence on run-times. In areas where attaining full precision is not the aim (such as the numerical solution of ordinary differential equations), it is appropriate to consider both speed and accuracy (e.g., by plotting cost versus requested accuracy).

One of the difficulties in designing experiments is finding good test problems—ones which reveal extremes of behaviour, cover a wide range of difficulty, are representative of practical problems, and (ideally) have known solutions. In many areas of computational mathematics good test problems have been identified, and several collections of such problems have been published. For example, collections are available in the areas of nonlinear optimization, linear programming, ordinary differential equations and partial differential equations. Several collections of test matrices are available and there is a book devoted entirely to test matrices. For references to test problem collections see [140].

In your conclusions you should make a clear distinction between objective statements and opinions and speculation. It is very tempting to extrapolate from results, but this is dangerous. As you analyse the results you may begin to formulate conclusions that are not fully supported by the data, perhaps because they were not anticipated when the experiments were designed. If so, further experimentation will be needed. When evaluating numerical algorithms I have found that it pays to print out every statistic that could conceivably be of interest; if I decide not to print out a residual or relative error, for example, I often find a need for it later on.

Further guidelines on how to report the results of computational experiments can be found in the journals *ACM Transactions on Mathematical Software* [65] and *Mathematical Programming* [146], and in an article by Bailey [12]. Perhaps the best advice is to read critically the presentation of experimental results in papers in your area of interest—and learn from them.

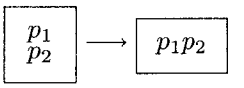
6.10. Tables

Many of the principles that apply to writing apply also to the construction of tables and graphs. However, some particular points should be considered

when designing a table.

To maximize readability the table design should be as simple as possible. Repetition should be avoided; for example, units of measurement or descriptions common to each entry in a column should go in the column header. Compare Tables 6.1 and 6.2. It is best to minimize the number of rules in the table. Two busy examples are shown in Table 6.3 and Table 6.5. The simplified versions, Table 6.4 and Table 6.6, are surely more aesthetically pleasing. Table 6.3 is taken from [159, p. 366],[11] where it is given without any rules and is still perfectly readable.

It is easier to compare like quantities if they are arranged in columns rather than rows. Research reported by Hartley [132], [134] supports this fact, and Tables 6.7 and 6.8 provide illustration, Table 6.8 being the easier to read. The difference between row and column orientation is more pronounced in complex tables. Of course, the orientation may be determined by space considerations, as a horizontal orientation usually takes less space on the page. If a vertical table is too tall, but is narrow, it can be broken into two pieces side by side:



It is also helpful to put columns or rows that need to be compared next to each other.

Only essential information should be included in a table. Omit data whose presence cannot be justified and state only as many digits as are needed (this number is often surprisingly small). In particular, do not state numerical results to more significant figures than are known for the data. As an example, in Table 6.1 there is no need to quote the timings and speedups to six significant figures, so Table 6.2 gives just one decimal place. Note that there is justification for showing so many digits in Tables 6.3 and 6.4: $\pi(x)$ in this table is the number of primes less than or equal to x, and nearly all the digits of $\pi(10^9)$ are needed to show the error in Riemann's formula. Displaying the first one or two digits of the fractional parts of the approximations emphasizes that the approximations are not integers.

If you need to present a large amount of data in tabular form, consider displaying it in an appendix, to avoid cluttering the main text. You could give smaller tables in the text that summarize the data. Large sets of data are often better displayed as graphs, however, particularly if it is the trends rather than the numerical values that are of interest. Tufte [275,

[11] Donald E. Knuth, *The Art of Computer Programming*, vol. 2, ©1981 by Addison-Wesley Publishing Co. Reprinted by permission of Addison-Wesley Publishing Co., Inc., Reading, MA. The form of the original table is not reproduced exactly here.

Table 6.1. Timings for a parallel algorithm.

# processors	Time	Speedup
$p = 1$	28.352197 secs	—
$p = 4$	7.218812 secs	3.9275
$p = 8$	3.634951 secs	7.7999
$p = 16$	1.929347 secs	14.6952

Table 6.2. Timings for a parallel algorithm.

No. of processors	Time (secs)	Speedup
1	28.4	—
4	7.2	4.0
8	3.6	7.8
16	1.9	14.7

Table 6.3. Approximations to $\pi(x)$.

x	$\pi(x)$	$x/\ln x$	$L(x)$	Riemann's formula
10^3	168	144.8	176.6	168.36
10^6	78498	72382.4	78626.5	78527.40
10^9	50847534	48254942.4	50849233.9	50847455.43

Table 6.4. Approximations to $\pi(x)$.

x	$\pi(x)$	$x/\ln x$	$L(x)$	Riemann's formula
10^3	168	144.8	176.6	168.36
10^6	78498	72382.4	78626.5	78527.40
10^9	50847534	48254942.4	50849233.9	50847455.43

Table 6.5. Results for inverting a lower triangular matrix on a Cray 2.

	Mflops			
n	128	256	512	1024
Method 1 ($n_b = 1$)	95	162	231	283
Method 2 ($n_b = 1$)	114	211	289	330
k variant ($n_b = 1$)	114	157	178	191
Method 1B ($n_b = 64$)	125	246	348	405
Method 2C ($n_b = 64$)	129	269	378	428
k variant ($n_b = 64$)	148	263	344	383

Table 6.6. Mflop rates for inverting a lower triangular matrix on a Cray 2.

	n	128	256	512	1024
Unblocked:	Method 1	95	162	231	283
	Method 2	114	211	289	330
	k variant	114	157	178	191
Blocked:	Method 1B	125	246	348	405
($n_b = 64$)	Method 2C	129	269	378	428
	k variant	148	263	344	383

Table 6.7. SI prefixes (10^{-1}–10^{12}). Row orientation.

Multiple	10^{12}	10^{9}	10^{6}	10^{3}	10^{-1}
Prefix	tera	giga	mega	kilo	deci
Symbol	T	G	M	K	d

Table 6.8. SI prefixes (10^{-1}–10^{12}). Column orientation.

Multiple	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	K
10^{-1}	deci	d

p. 56] advises that tables are usually better than graphs at reporting small data sets of twenty numbers or less.

The caption should be informative and should not merely repeat information contained in the table. Notice the simplification obtained by moving the word "Mflops" from the table to the caption in Table 6.6.

Give a clear reference to the table at an appropriate place in the text—you cannot rely on the reader to refer to the table automatically. It is helpful if you explain the salient features of the table in words. The reader will appreciate this guidance, especially if the table contains a lot of data. However, you should not summarize the whole table—if you do, the table might as well be omitted.

Further Reading

The Chicago Manual of Style [58] devotes a whole chapter to tables and offers much useful advice. Another good reference is *A Manual for Writers* [278, Chap. 6]. Bentley [21, Chap. 10] gives a good example of how to redesign a table. References that discuss the preparation of graphs include Bentley [21, Chaps. 10, 11], Hartley [132], [134], MacGregor [187], [188], and Tufte [275], [276], [277].

6.11. Citations

The two main styles of citation in mathematics journals are by number (as used in this book) and by name and year, which is the Harvard system. Examples of the Harvard system are *These results agree with an existing study of Smith* (1990) and *These results agree with an existing study* (*Smith,* 1990). If more than one paper maps to Smith (1990), the papers are distinguished by appending a letter to the year: Smith (1990a), Smith (1990b), and so on. In the number-only system, the number is usually placed in square brackets, though some styles require it to be superscripted.

The main requirement is that a citation does not intrude upon a sentence. For example, *This method was found* [17] *to be unstable* is better written as *This method was found to be unstable* [17]. There are circumstances, however, where a citation has to be placed part-way through a sentence to convey the correct meaning. A good test for whether a citation is well placed is to see whether the sentence reads properly when the citation is deleted. The style of citation inevitably affects how you phrase sentences, so it is worth checking in advance what style is used by the journal in which you wish to publish. Knuth [164] explains that when his paper "Structured programming with go to statements" [*Computing Surveys*, 6:261–301, 1974] was reprinted in a book, he made numerous changes

to make sentences read well with the citation style used in the book.

When you cite by number, it is good style to incorporate the author's name if the citation is more than just a passing one. As well as saving the reader the trouble of turning to the reference list to find out who you are referring to, this practice has an enlivening effect because of the human interest it introduces. Examples:

> Let $A\Pi = QR$ be a QR factorization with column pivoting [10]. (Passing reference to a textbook for this standard factorization.)
> The rate of convergence is quadratic, as shown by Wilkinson [27]. (Instead of "as shown in [27]".)

The sentence "This question has been addressed by [5]" is logically incorrect and should be modified to "addressed by Jones [5]" or "addressed in [5]".

When you cite several references together it is best to arrange them so that the citation numbers are in increasing order, e.g., "several variations have been developed [2], [7], [13]." Ordering by year of publication serves no purpose when only citation numbers appear in the text. If you want to emphasize the historical progression it is better to add names and years: "variations have been developed by Smith (1974) [13], Hall (1981) [2], and Jones (1985) [7]."

It is important to be aware that the reference list says a lot about a paper. It helps to define the area in which the paper lies and may be used by a reader to judge whether the author is aware of previous work in the area. Some readers look at the reference list immediately after reading the title, and if the references do not look sufficiently familiar, interesting or comprehensive they may decide not to read further. Therefore it is desirable that your reference list contain at least a few of the key papers in the area in which you are writing. Papers should not, however, be cited just for effect. Each citation should serve a purpose within the paper. Note also that if you cite too often (say, for several consecutive sentences) you may give the impression that you lack confidence in what you are saying.

There are several conventions for handling multiple authors using the Harvard system. One such convention is as follows [45], [68, Chap. 12]. For one or two authors, both names are given (e.g., "see Golub and Van Loan (1989)"). If there are three authors, all three are listed in the first citation and subsequent citations replace the second and third names by "et al." (e.g., "see Knuth, Larrabee and Roberts (1989)," then "see Knuth et al. (1989)"). For four or more authors, all citations use the first author with "et al." These conventions can also be used when naming authors in conjunction with the numbered citation system.

If you make significant use of a result from another reference you should give some indication of the difficulty and depth of the result (and give the

author's name). Otherwise, unless readers look up the reference, they will not be able to judge the weight of your contribution.

When you make reference to a specific detail from a book or long paper it helps the reader if your citation includes information that pinpoints the reference, such as a page, section, or theorem number.

For further details on the subtleties of citation consult van Leunen [283].

6.12. Conclusions

If there is a conclusions section (and not every paper needs one) it should not simply repeat earlier sections in the same words. It should offer another viewpoint, discuss limitations of the work, or give suggestions for further research. Often the conclusions are best worked into the introduction or the last section. It is not uncommon to see papers where the conclusions are largely sentences taken from the introduction, such as *We show that X's result can be extended to a larger class ...*; this practice is not recommended.

The conclusions section is a good place to mention further work: to outline open problems and directions for future research and to mention work in progress. Be wary of referring to your "forthcoming paper", for such papers can fail to materialize. A classic example of a justified reference to future work is the following quote from the famous paper[12] by Watson and Crick [290] (*Nature*, April 25, 1953) in which the double helix structure of DNA was proposed:

> It has not escaped our notice that the specific pairing we have postulated immediately suggests a possible copying mechanism for the genetic material.
>
> Full details of the structure, including the conditions assumed in building it, together with a set of co-ordinates for the atoms, will be published elsewhere.

The promise "will be published elsewhere" was fulfilled shortly afterwards, in the May 30, 1953 issue of *Nature*.[13]

6.13. Acknowledgements

Be sure to acknowledge any financial support for your work: grants, fellowships, studentships, sponsorship. A researcher might write "This re-

[12]For some comments by Crick on the writing of this and subsequent papers by Watson and Crick, see [64].

[13]Pyke [231] points out that the words "It has not escaped our notice that" can be removed.

search was supported by the National Science Foundation under grant DCR-1234567"; grant agencies like authors to be this specific. A Ph.D. student supported by the Engineering and Physical Sciences Research Council (UK) might write "This work was supported by an EPSRC Research Studentship." In SIAM journals this type of acknowledgement appears in a footnote on the first page. Other acknowledgements usually appear in a section titled *Acknowledgements* at the end of the paper. (An alternative spelling is *acknowledgments*.)

It is customary to thank anyone who read the manuscript in draft form and offered significant suggestions for improvement (as well as anyone who helped in the research), but not someone who was just doing his or her normal work in helping you (for example, a secretary). The often-used "I would like to thank" can be shortened to "I thank." Note that if you say "I thank X for pointing out an error in the proof of Theorem Y," you are saying that the proof is incorrect; "in an earlier proof" or "in an earlier attempted proof" is what you meant to say.

The concept of anonymous referee sometimes seems to confuse authors when they write acknowledgements. An anonymous referee should not be thanked, as is often the case, for *his* suggestions—it may be a *she*. One author wrote "I thank the anonymous referees, particularly Dr. J. R. Ockendon, for numerous suggestions and for the source of references." Another explained, not realising the two ways in which the sentence can be read, "I would like to thank the unknown referees for their valuable comments."

6.14. Appendix

An appendix contains information that is essential to the paper but does not fit comfortably into the body of the text. The most common use of an appendix is to present detailed analysis that would distract the reader if it were given at the point where the results of the analysis are needed. An appendix can also be used to give computer program listings or detailed numerical results. An appendix should not be used to squeeze inessential information into the paper (though this may be acceptable in a technical report or thesis).

6.15. Reference List

Preparing the reference list can be one of the most tedious aspects of writing a paper, although it is made much easier by appropriate software (see §13.3). The precise format in which references are presented varies among publishers and sometimes among different journals from the same publisher.

Here are four examples.

SIAM journals: J. H. WILKINSON, *Error analysis of floating-point computation*, Numer. Math., 2 (1960), pp. 319–340.

IMA journals: WILKINSON, J. H. 1960 Error analysis of floating-point computation. *Numer. Math.* **2**, 319–340.

Elsevier journals: J. H. Wilkinson, Error analysis of floating-point computation, *Numer. Math.* 2:319–340 (1960).

Springer-Verlag journals: Wilkinson, J. H. (1960): Error analysis of floating-point computation. Numer. Math. **2**, 319–340.

All journals that I am familiar with ask for the use of their own format but will accept other formats and copy edit them as necessary. All publishers have a minimum amount of information that they require for references, as defined in their instructions for authors. It is important to provide all the required information, whatever format you use for the references.

Here are some comments and suggestions on preparing reference lists. For further details I strongly recommend the book by van Leunen [283], but keep in mind that her recommendations may conflict with those of publishers in certain respects.

1. Do not rely on secondary sources to learn the contents of a reference or its bibliographic details—always check the original reference. In studies on the accuracy of citation, the percentage of references containing errors has been found to be as high as 50% [95]. A 1982 paper by Vieira and Messing in the journal *Gene* had been cited correctly 2,212 times up to 1988, but it had also been cited incorrectly 357 times under "Viera"; these errors led to the paper being placed too low in a list of most-cited papers [98]. In another well-documented case in the medical literature, the Czech title "O Úplavici" ("On Dysentery") of an 1887 paper in a Czech medical journal was taken by one writer to be the author's name, and the mistake propagated until it was finally exposed in 1938 [135], [239]. If a secondary source has to be used (perhaps because the reference is unavailable), it is advisable to append to the reference "cited in [ss]", where [ss] is the secondary source.

2. Always provide the full complement of initials of an author, as given in the paper or book you are referencing.

3. Some authors are inconsistent in the name they use in their papers, sometimes omitting a middle initial, for example. In such cases, my

preference is to use the author's full name in the reference list when it is known, to avoid ambiguities such as: Is A. Smith the same author as A. B. Smith?

4. Some sources contain typographical errors or nonstandard usage. Titles should be given unaltered. For example, the title "Van der Monde systems and numerical differentiation" [J. N. Lyness and C. B. Moler, *Numer. Math.*, 8:458–464, 1966] appears to be incorrect because the name is usually written Vandermonde, but it should not be altered (I have occasionally had to reinstate "Van der Monde" in my reference list after a copy editor has changed it). A typographical error in an author's name is rare, but not unknown. It seems reasonable to correct such an error, but to provide some indication of the correction that has been made, such as a note at the end of the reference.

5. Copy bibliographic information of a journal article from the journal pages, not the cover of the journal. The cover sometimes contains typographical errors and you cannot deduce the final page number of the article if the journal puts blank pages between articles or begins articles part-way down a page.

6. Electronic journals do not usually cause any difficulties in referencing, since it is in the journals' interests to make clear how papers should be referenced. For example, the journal *Electronic Transactions on Numerical Analysis* provides papers in PostScript form, and each paper has a clearly defined page range, volume and year; papers are therefore referenced just like those in a traditional journal. It may help readers if a URL for an electronic journal is appended to the reference, but the journal in which you are publishing may delete it to save space.

   It is more difficult to decide how to reference email messages and unpublished documents or programs on the Web. The following suggestions are adapted from those in *Electronic Styles* [178]. I assume that an email address and a URL are both clearly identifiable as such by the `@` and `http`, respectively, so I omit the descriptors "email" and "URL". There are so many different types of item on the Web that no referencing scheme can cover all possibilities.

   (a) A publication available in print and online.

   > Nicholas J. Higham. The Test Matrix Toolbox for MATLAB (version 3.0). Numerical Analysis Report No. 276, Manchester Centre for Computational Mathematics, Manch-

ester, England, Sept. 1995; also available from `ftp://ftp.ma.man.ac.uk/pub/narep/narep276.ps.gz`

(b) A publication available online only.

Melvin E. Page. A Brief Citation Guide for Internet Sources in History and the Humanities (Version 2.1), `http://h-net.msu.edu/~africa/citation.html`, 1996.

(c) A publication on CD-ROM.

A. G. Anderson, Immersed interface methods for the compressible equations. In Proceedings of the Eighth SIAM Conference on Parallel Processing for Scientific Computing (Minneapolis, MN, 1997), CD-ROM, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1997.

(d) A piece of software.

Piet van Oostrum. LaTeX package `fancyhdr`. CTAN archive (e.g., `http://www.tex.ac.uk/tex-archive`), `macros/latex/contrib/supported/fancyhdr`.

(e) An item in a discussion list, digest or newsgroup.

David Hough. Random story. NA Digest, 89 (1), 1989. `na.help@na-net.ornl.gov`, `http://www.netlib.org/index.html`

(f) A standard email message. The title is taken from the `Subject:` line.

Desmond J. Higham (`aas96106@ccsun.strath.ac.uk`). Comments on your paper. Email message to Nicholas J. Higham (`higham@ma.man.ac.uk`), August 18, 1997.

(g) A forwarded email message.

Susan Ciambrano (`ciambran@siam.org`). Reader's comments on HWMS. (Original message A. Reader, Handbook of Writing.) Forwarded email message to Nicholas J. Higham (`higham@ma.man.ac.uk`), October 20, 1995.

7. When you reference a manuscript or technical report that is more than a few months old, check to see if it has appeared in a journal. An author will usually be happy to inform you of its status. If it has appeared, check whether the title has changed. The referees may have asked for a better title, or the copy editor may have added a hyphen, combined a hyphenated pair of words, or changed British spelling to American or vice versa.

8. Take care to respect letters and accented characters from other languages. Examples: Å, å, ß, é, ö, ø, Ø.

9. If you maintain a database of references (for example, in BIBTEX format—see §13.3), it is worth recording full details of a reference, even if not all of them are needed for journal reference lists. For a journal article, record the part (issue) number as well as the volume number; this extra information can speed the process of looking up a reference, especially if the journal issues are unbound. For a technical report, the month of publication is useful to know.

10. Be sure you are using the correct journal name and watch out for journals that change their names. For example, the *SIAM Journal on Scientific and Statistical Computing* (1980–1992) became the *SIAM Journal on Scientific Computing* in 1993.

11. In book titles, van Leunen recommends that a colon be added if it is needed to separate a title from a subtitle, and an awkward comma or colon separating a title from a subtitle should be removed. Thus *On Writing Well An Informal Guide to Writing Nonfiction* (as copied from the title page of [304]) needs a colon added after *Well*, and the colon should be removed from *Interpolation Theory:* 5.

12. Van Leunen recommends simplifying the names of major publishers to the bare bones, so that *John Wiley & Sons* becomes *Wiley*, and *Penguin Books* becomes *Penguin.* She also recommends omitting the city for a major publisher; I usually include it because many journals require it. For obscure publishers it is best to give as complete an address as possible.

13. For a book, the International Standard Book Number (ISBN) is worth recording, as it can be used to search library and publishers' catalogues. (Note, though, that hardback and softback editions of a book usually have different ISBNs.) An ISBN consists of ten digits, arranged in four groups whose size can vary. The first group specifies the language group of the publisher (0, 1 = English speaking countries, 2 = French speaking, 3 = German speaking, etc.). The second group (2–7 digits) identifies the publisher (e.g., Oxford University Press is 19) and the third group (1–6 digits) identifies the particular title. The last digit is a checksum. If the ISBN is expressed as $d_1 d_2 \ldots d_{10}$ then

$$d_{10} = \lceil s/11 \rceil * 11 - s, \quad \text{where} \quad s = \sum_{i=1}^{9} (11 - i) d_i$$

($\lceil x \rceil$ denotes the smallest integer greater than or equal to x). A value $d_{10} = 10$ is written as "X". This book has the ISBN 0-89871-420-6: 89871 identifies SIAM as the publisher and 420 is the book's individual number. An International Standard Serial Number (ISSN) identifies a serial publication such as a journal, yearbook or institutional report. An ISSN has eight digits.

14. The date to quote for a book is the latest copyright date (excluding copyright renewals)—ignore dates of reprinting. Always state the edition number if it is not the first.

15. Make sure that every reference is actually cited in the paper. Some copy editors check this, as you may see from their pencilled marks on the manuscript when you receive the proofs.

16. Most mathematics journals require the reference list to be ordered alphabetically by author. Many science journals order by citation, so that the nth paper to be cited is nth in the reference list.

17. A list of standard abbreviations for mathematics journals can be found in *Mathematical Reviews* (see §14.3).

6.16. Specifics and Deprecated Practices

Capitalization

References to proper nouns should be capitalized: *See Theorem* 1.5, *the proof of Lemma* 3.4 *and the discussion in Section* 6. References to common nouns (generic objects) should not: *Next we prove the major theorems of the paper.*

Dangling Theorem

The term *dangling theorem* [147] (or *hanging theorem* [121]) refers to a construction such as the following one, where a theorem dangles or hangs from the end of a sentence.

> This result is proved in the following
> **Theorem 3.13**. If f is a twice continuously differentiable function ...

Halmos argues that while the practice can be defended, some readers dislike it, and it is not worth risking annoying them for the sake of avoiding the extra word *theorem.*

The following example does not strictly dangle, but is even more irritating.

> **5.1. Accuracy of the Computed Solution.** It depends on the machine precision and the conditioning of the problem.

Section headings stand alone and should not be taken as part of the text. The obvious solution

> **5.1. Accuracy of the Computed Solution.** The accuracy of the computed solution depends on the machine precision and the conditioning of the problem.

is inelegant in its repetition, but this could be avoided by rewriting the sentence or the title.

Footnotes

Footnotes are used sparingly nowadays in mathematical writing, and some journals do not allow them (see page 78 for an example of a footnote). It is bad practice to use them to squeeze more into a sentence than it can happily take. Their correct use is to add a note or comment that would deflect from the main message of the sentence. Donald W. Marquardt, the author of the 92nd most-cited paper in the Science Citation Index 1945–1988 [An algorithm for least-squares estimation of nonlinear parameters. *J. Soc. Indust. Appl. Math.*, 11(2):431–441, 1963], has stated that a critical part of the algorithm he proposed was described in a footnote and has sometimes been overlooked by people who have programmed the algorithm [97].

Numbering Mathematical Objects

Generally, you should number only those equations that are referenced within the text. This avoids the clutter of extraneous equation numbers and focuses the reader's attention on the important equations. Occasionally it is worth numbering key equations that are not referenced but which other authors might want to quote when citing your paper. Except in very short papers it is best to number equations by section rather than globally (equation (2.3) instead of equation (14)), for this makes referenced equations easier to find. The same applies to the numbering of theorems and other mathematical objects. Whether equation numbers appear on the left or the right of the page depends on the journal.

Two possible numbering sequences are illustrated by

> Definition 1, Lemma 1, Theorem 1, Remark 1, Definition 2, Lemma 2, ...
> Definition 1, Lemma 2, Theorem 3, Remark 4, Lemma 5, ...

Opinions differ as to which is the best scheme. The last has the advantage that it makes it easier to locate a particular numbered item, and the equation numbers themselves can even be included in the sequence for complete uniformity. The disadvantage is that the scheme mixes structures of a different character, which makes it difficult to focus on one particular set of structures (say, all the definitions); and on reading Remark 24 (say), the reader may wonder how many previous remarks there have been. A compromise between the two schemes is to number all lemmas, theorems and corollaries in one sequence, and definitions, remarks and so on in another. Some typesetting systems control the numbering of mathematical objects automatically. LaTeX does so, for example, and the numbering sequence for definitions, lemmas, theorems, etc., can be specified by LaTeX commands.

Plagiarism

Plagiarism is the act of publishing borrowed ideas or words as though they are your own. It is a major academic sin. In writing, if you copy a sentence or more you should either place it in quotes and acknowledge the source via a citation, or give an explicit reference such as "As Smith observed ...." In the case of a theorem statement it is acceptable to copy it word for word if you cite the source, but before copying it you should see whether you can improve the wording or make it fit better into your notation and style.

Regarding when to quote and when to paraphrase, van Leunen [283] advises "Quote what is memorable or questionable, strange or witty. Paraphrase the rest." When you wish to paraphrase, it is best to put the source aside, wait a reasonable period, and then rewrite what you want to say in your own words.

If you rework what you yourself have previously published without citing the source, thus passing it off as new, that is self-plagiarism, which is no less a sin than plagiarism.

Plagiarism has led to the downfall of many a career, in academia and elsewhere. Some notable cases are described in Mallon's *Stolen Words* [192] and LaFollette's *Stealing into Print* [169]. The former includes the ironic news, quoted from the *New York Times* of June 6, 1980, that

> Stanford University said today it had learned that its teaching assistant's handbook section on plagiarism had been plagiarized by the University of Oregon. ... Oregon officials apologized and said they would revise their guidebook.

Fraud is another serious malpractice, though apparently and understandably rare in mathematical research. Numerous cases of scientific fraud through history are catalogued in *Betrayers of the Truth* by Broad and Wade [39], while allegations that the psychologist Cyril Burt acted fraudulently are examined carefully in [189].

The Invalid Theorem

Avoid the mistake of calling a theorem into question through sentences such as the following:

> The theorem holds for any continuously differentiable function f. Unfortunately, the theorem is invalid because S is not path connected.

A theorem holds and is valid, by definition. A theorem might be *applicable to any continuously differentiable function* or *its invocation* may be *invalid because S is not path connected.*

"This Paper Proves ..."

In the abstract and introduction it is tempting to use wording such as "this paper proves" or "Section 3 shows" in place of "we prove" or "we show". This usage grates on the ear of some readers, as it is logically incorrect (though "Theorem 2 gives" cannot be criticized). The grating can be avoided by rewriting, but care is required to avoid a succession of sentences beginning "we".