Search

- Concepts: state space and search tree
- Basic search
  - finding a goal node
- Local search
  - finding the ”fittest” node
- Shortest Path
  - Dynamic Programming
  - Heuristic search

Best-first search (cost, not fitness)

- Each node n has a cost value f(n).
- Lower is better! (don’t confuse with fitness)
- **Frontier** is sorted best-first.

Algorithm A

Best-first search, using the following cost function:

f(n) [estimated shortest path from start to goal via n]

=

g(n) [length of shortest path to n found so far]

+

h(n) [**heuristic**: estimated distance from n to goal]

Compare

f(n) [<u>estimated</u> shortest path from start to goal via n]

=

g(n) [length of shortest path to n <u>found so far</u>]

+

h(n) [**heuristic**: <u>estimated</u> distance from n to goal

f*(n) [<u>actual</u> shortest distance from start to goal via n]

=

g*(n) [length of <u>actual</u> shortest path to n]

+

h*(n) [<u>actual</u> shortest path from n to goal]

Algorithm A* [3.5.2]

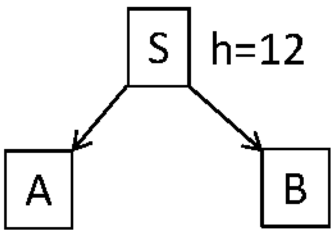
Best-first search where $f(n) = g(n) + h(n)$ and:

- h is optimistic:
  - $h(n) \leq h^*(n)$ for all nodes n.
- h is monotonic
  - $h(n) \leq h(n') + c(n,n')$, for all nodes, n, and all successors n′of n.

A* guarantees that when we have visited thegoal, we have found the shortest path.

- A* using tree-search requires optimism.
- A* using graph-search and therefore not revisiting visited nodes requires monotonicity (which implies optimism).

Example

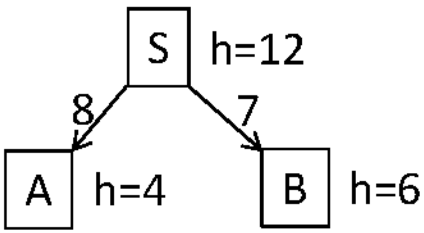


Frontier

S(12)

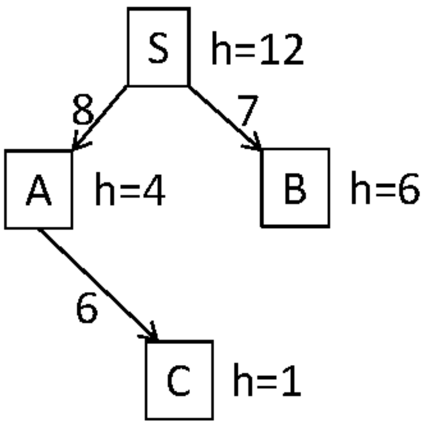
Visited

Example



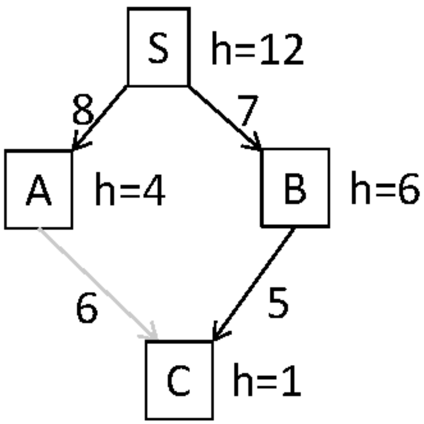
Frontier	Visited
S(12)	
A(12) B(13)	S(12)

Example



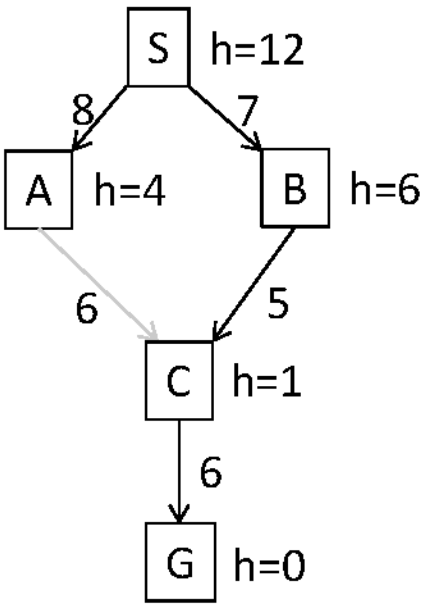
Frontier	Visited
S(12)	
A(12) B(13)	S(12)
B(13) C(15)	S(12) A(12)

Example



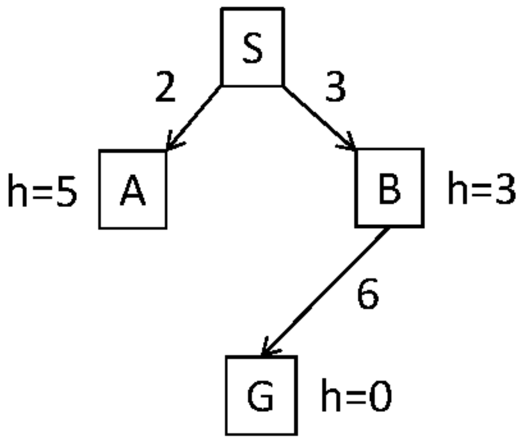
Frontier	Visited
S(12)	
A(12) B(13)	S(12)
B(13) C(15)	S(12) A(12)
C(13)	S(12) A(12) B(13)

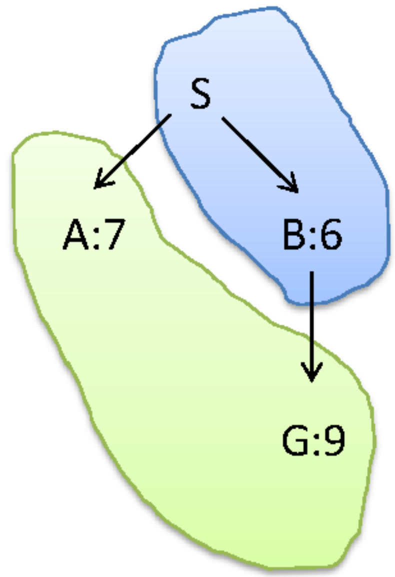
Example



Frontier	Visited
S(12)	
A(12) B(13)	S(12)
B(13) C(15)	S(12) A(12)
C(13)	S(12) A(12) B(13)
G(18)	S(12) A(12) B(17)
	G(18) ...

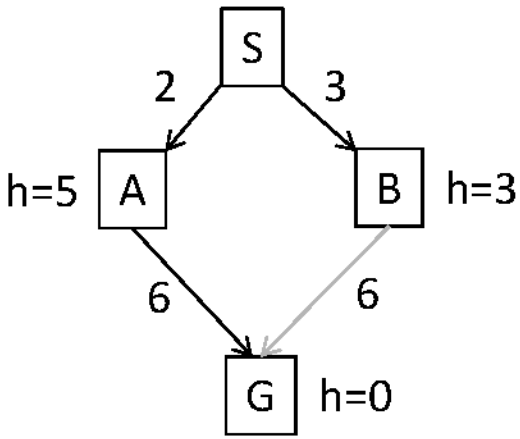
Observe: visit the goal node!

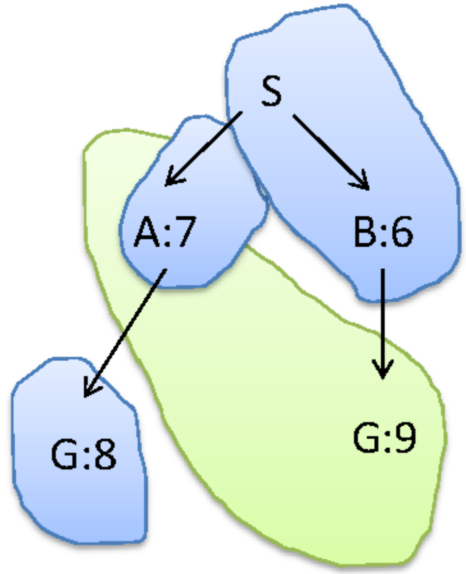




We have found the goal G(9), but need to visit A(7) first.

Observe: visit the goal node!





Visiting A(7), we find G(8).

When we visit G(8) – we know it's optimal.

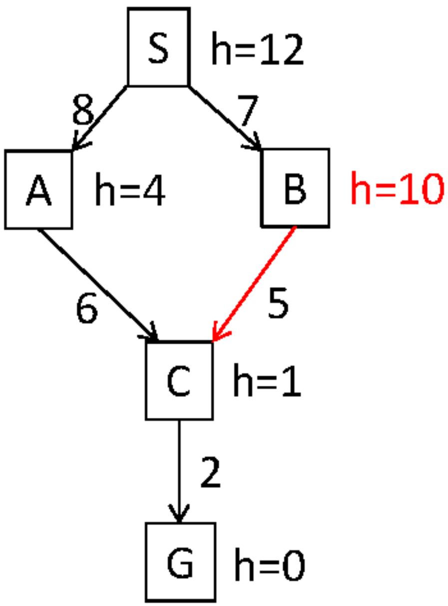
Proof (informal)

$f(G) = g(G)+h(G) = g(G)+0 = g(G)$ = the length of the path to the goal found by A*.

1. Suppose this path is not optimal. Let n be the *first* node on the optimal path that is not visited.
2. Since n is the *first* node on the optimal path not visited, when we visit it from the optimal path $g(n) = g^*(n)$.
3. Since h is optimistic: $h(n) \leq h^*(n)$.
4. Since the found path was not optimal:

   $$f(G) > f^*(G) = g^*(n) + h^*(n) \geq f(n)$$
5. Therefore $f(n) < f(G)$ and n should have been visited before G.

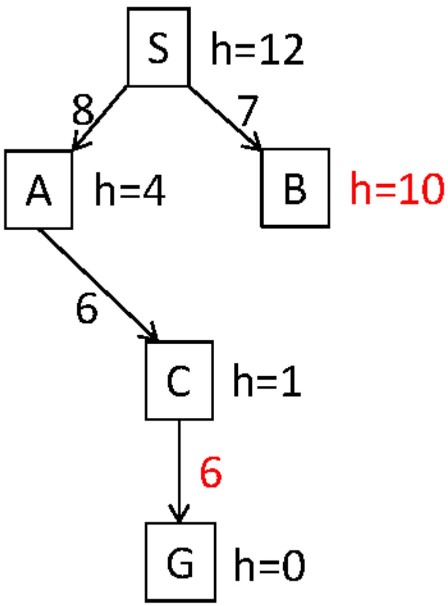
Example (not optimistic)



Frontier	Visited
S(12)	
A(12) B(17)	S(12)
C(15) B(17)	S(12) A(12)
G(16) B(17)	S(12) A(12) C(15)
B(17)	G(16)

The high estimate for B blocks the shortest path.

Example (optimistic, not monotonic)



Frontier	Visited
S(12)	
A(12) B(17)	S(12)
C(15) B(17)	S(12) A(12)
B(17) G(20)	S(12) A(12) C(15)

Selecting the Heuristic

A good estimate is one which is close to the actual value.

- Which nodes will be expanded?
  - If h is monotonic, all nodes where $f(n)=g^*(n)+h(n)<f(G)$
- So if h is higher, fewer nodes are expanded.
- Trade-off:
  - Computing ’smarter’ h can take more time.

Selecting the Heuristic

A good estimate is one which is close to the actual value.

The worst optimistic and monotonic heuristic is $h(n)=0$. This makes A* just look at the cost of getting to nodes and is equivalent to best-first search.

Selecting the Heuristic

A good estimate is one which is close to the actual value.

Getting good estimates is often domain dependent.

But if all edges have cost>=1 then the minimum number of steps between a node and the goal is optimistic and monotonic, and often easy to calculate.

(This might be really useful for project 1! ☺)

Which nodes are expanded?

- If h is monotonic:
  all nodes n with $f(n) = g^*(n)+h(n) < f(G)$
  and some nodes where $f(n)=f(G)$
- If h is higher, fewer nodes are expanded.

- Trade-off:
  computing ”smarter” h takes more time.