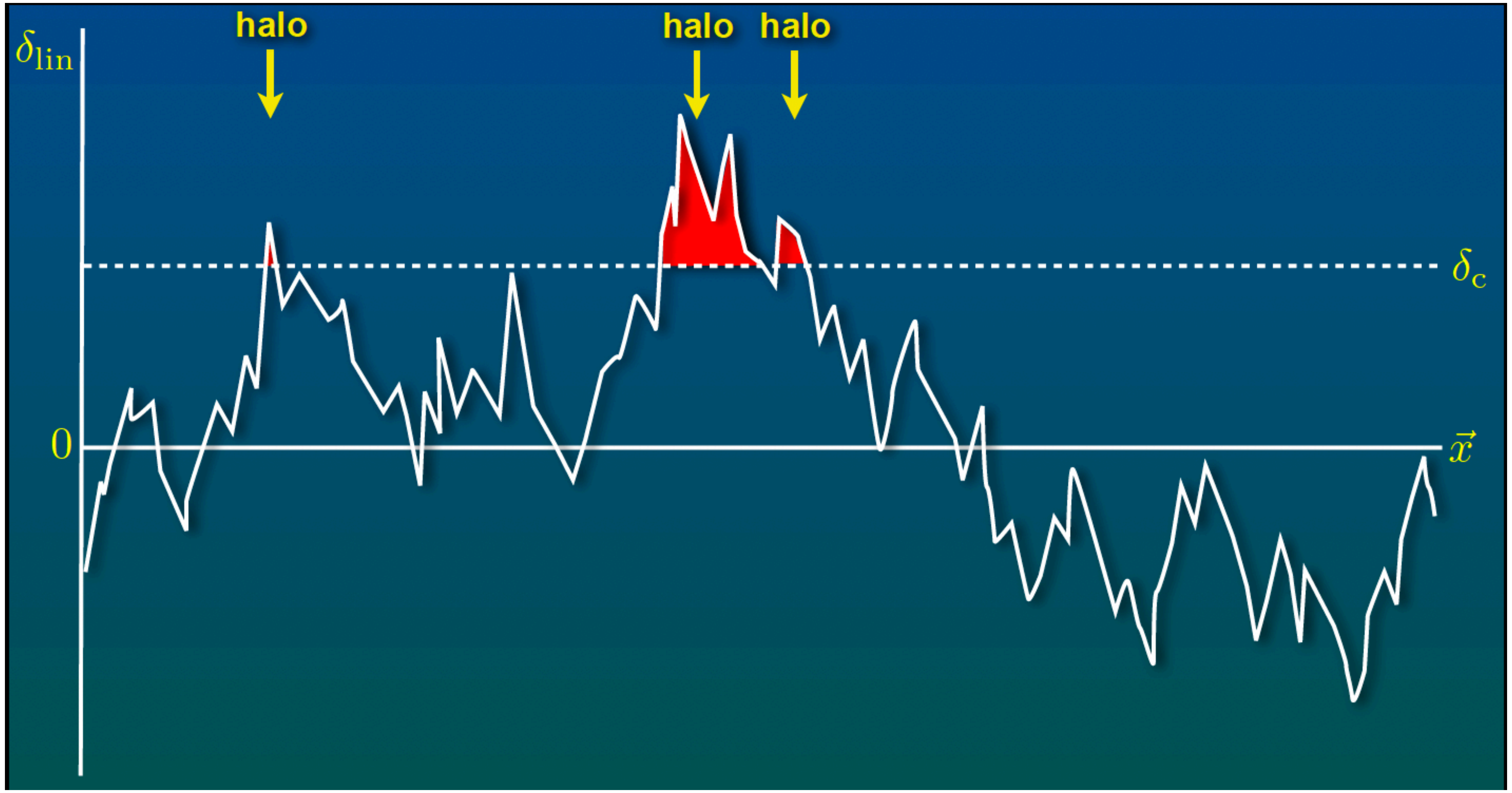
Excursion Set Theory, Structure Formation and Reionization

Florian Stäbler

30.11.23

The Linear Cosmological Density Field

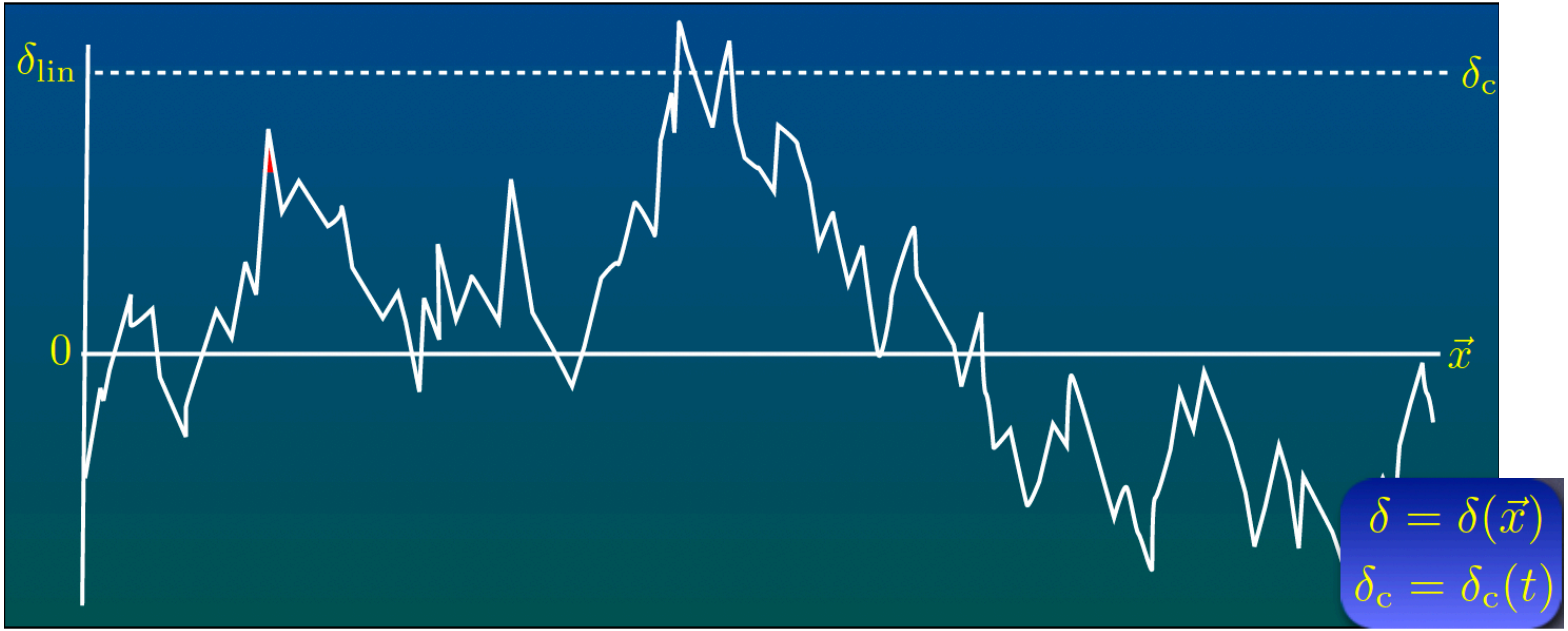
- According to linear theory, the density field evolves as $\delta(\vec{x},t) = D(t)\delta_0(\vec{x})$



- Spherical collapse model: $\delta(\vec{x},t) \geq \delta_c \approx 1,686$ will produced collapsed DMH

The Linear Cosmological Density Field

- Alternatively we can consider a time evolving barrier for fixed initial density
- Regions with $\delta_0(\vec{x}) \geq \delta_c/D(t)$ will collapse at time t



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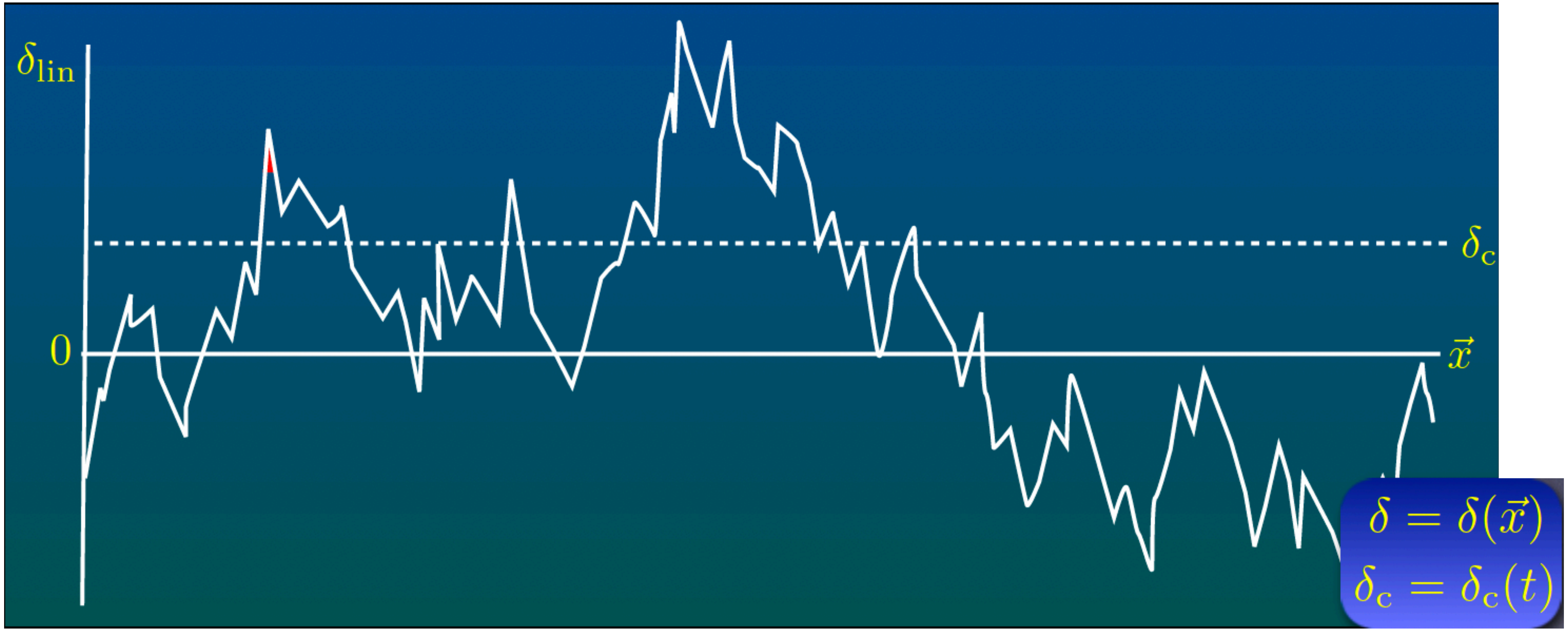

Smoothing

- Let us consider a density smoothed with a window function or ‚filter' $W(\vec{x}, R)$
- The density field smoothed on some length scale R is

$$\boxed{\delta(\vec{x}, R) = \int d^3\vec{x}' \; W(\vec{x} - \vec{x'})\delta(\vec{x'})}$$

- We can assign a mass to the scale $M = \gamma_f \bar{\rho} R^3$

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Top Hat Filter: $\gamma_f = 4\pi/3$

$$W(\vec{x}; R) = \begin{cases} \frac{3}{4\pi R^3} & r \leq R \\ 0 & r > R \end{cases}$$

$$\widetilde{W}(kR) = \frac{3}{(kR)^3}\left[\sin(kR) - (kR)\,\cos(kR)\right]$$

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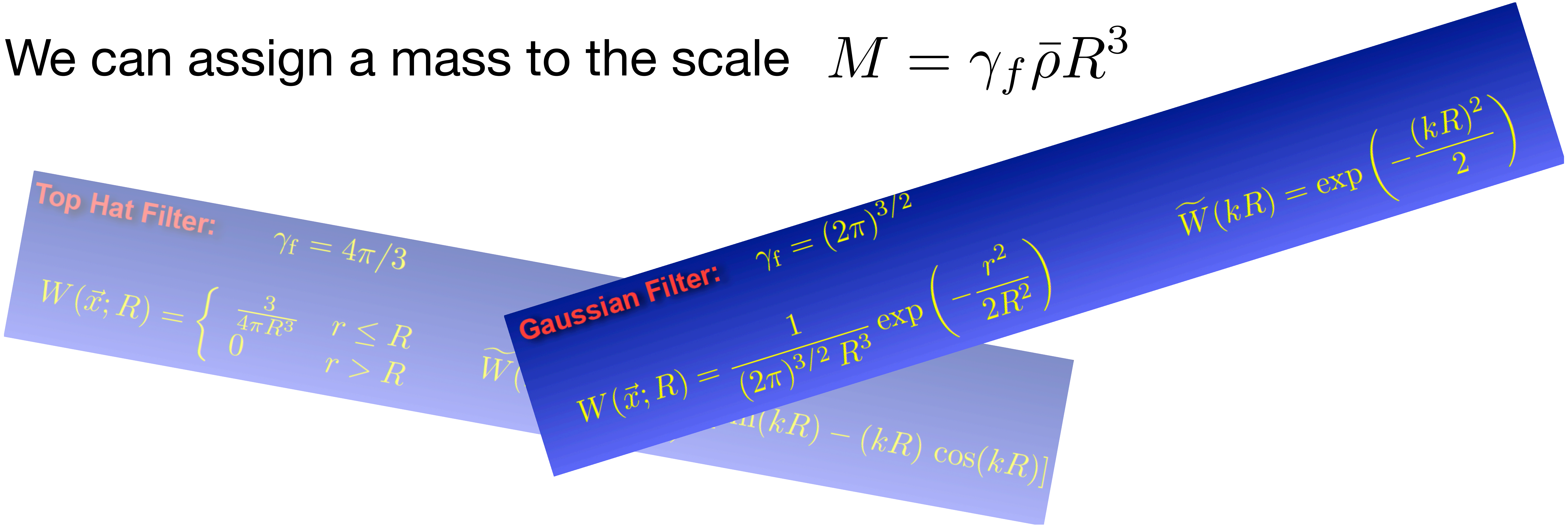

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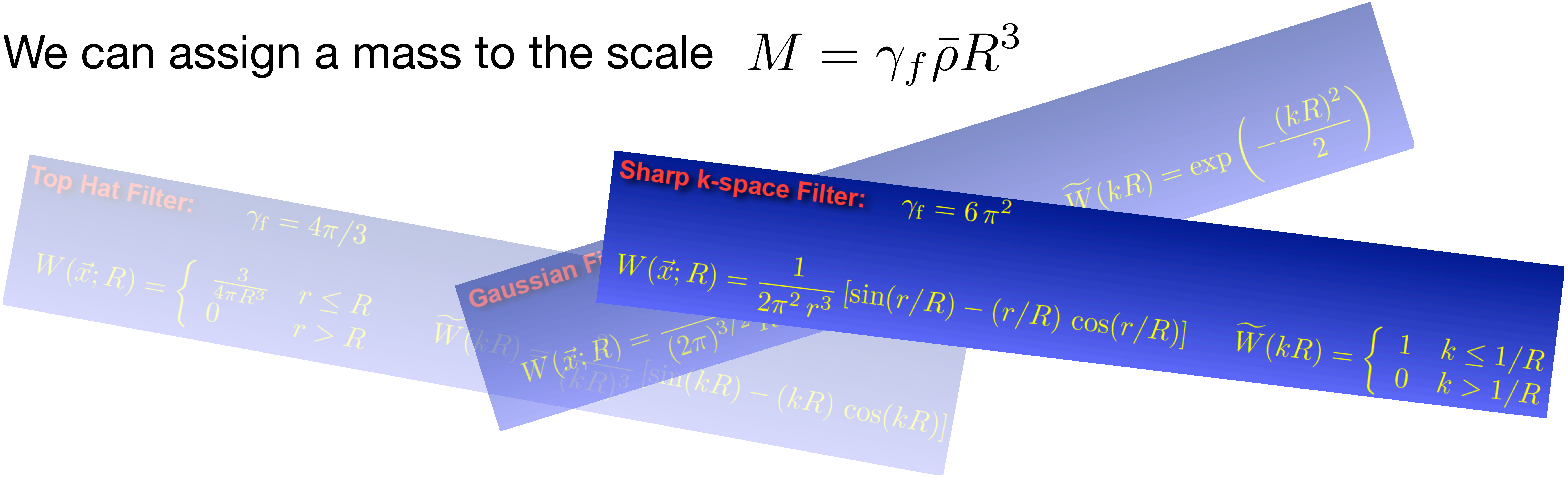

Assigning Halo Mass to Collapsed Regions

- It is clear that we can label a filter using R or M
- If $\delta(\vec{x})$ is a gaussian field we have that

$$\boxed{\mathcal{P}(\delta_M)d\delta_M = \frac{1}{\sqrt{2\pi}\sigma_M} \exp\left[-\frac{\delta_M^2}{2\sigma_M^2}\right] d\delta_M}$$

$$\delta_M = \delta(\vec{x}, M)$$
$$\sigma_M = \sigma(M)$$

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- How can we assign mass to virialized dark matter halos ($\delta > \delta_c$)?
- Idea: Locations where $\delta_M > \delta_c$ are the locations where, at time t , a halo of mass M condenses out of the evolving density field....

➡ Calculate the number density of peaks in the smoothed density field, i.e.,

$$\boxed{n(> M) = n_{\rm pk}(\delta_M)}$$

Peak Formalism & Cloud-in-Cloud Problem

This idea was explored in a seminal paper by Bardeen et al. (1986), known as "BBKS".

number density, clustering properties, shapes and density profiles of peaks, as function of the peak height

THE STATISTICS OF PEAKS OF GAUSSIAN RANDOM FIELDS

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ABSTRACT

Cosmological density fluctuations are often assumed to be Gaussian random fields. The local maxima of such fields are obvious sites for the formation of nonlinear structures. The statistical properties of the peaks can be used to predict the abundances and clustering properties of objects of various types. In this paper, we derive (1) the number density of peaks of various heights $\nu\sigma_0$ above the rms σ_0; (2) the factor by which the peak density is enhanced in large-scale overdense regions; (3) the n-point peak-peak correlation function in the limit that the peaks are well separated, with special emphasis on the two- and three-point correlations; and (4) the density profiles centered on peaks. To illustrate the predictive power of this semianalytic approach, we apply our formulae to structure formation in the adiabatic and isocurvature $\Omega = 1$ cold dark matter (CDM) models. We assume bright galaxies form only at those peaks in the density field (smoothed on a galactic scale) that are above some global threshold height $\nu_t \approx 3$ fixed by normalizing to the galaxy number density. We find, for example, that the shapes of the peak-peak two- and three-point correlation functions for the adiabatic CDM model agree well with observations before any dynamical evolution, just due to the propensity of the peaks to be clustered in the initial conditions. Only moderate dynamical evolution is required to bring the amplitude of the correlations up to the observed level. The corresponding redshift of galaxy formation z_g in the isocurvature model is too recent ($z_g \approx 0$) for this model to be viable. Even for the adiabatic models $z_g \approx$ 3–4 is predicted. We show that the mass-per-peak ratio in clusters, and thus presumably the cluster mass-to-light ratio, is substantially lower than in the ambient medium, alleviating the Ω problem. We also confirm that the smoothed density profiles of collapsing structures of height $\sim \nu_t$ are inherently triaxial.

Subject headings: early universe — galaxies: clustering — galaxies: formation

Peak Formalism & Cloud-in-Cloud Problem

- The identification is problematic:

peaks in δ_M ⟺ halos with mass $> M$

- Consider same density field associated with two scales $M_2 > M_1$
- Let δm be a mass associated with a peak in δ_{M_1} and δ_{M_2} Is it halo M_1 or M_2 ?
    - If $\delta_{M_1} > \delta_{M_2}$, δm is part of M_1 at time t_1 and part of M_2 at $t_2 > t_1$
    - If $\delta_{M_2} > \delta_{M_1}$, δm can never be a part of M_1

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Peak Formalism & Cloud-in-Cloud Problem

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peaks in δ_M $\Leftrightarrow$ halos with mass $> M$

- Cloud-in-cloud problem: Exclude peaks that are part of a higher peak when smoothed with a larger filter!

- Less rigorous but more famous Press-Schechter postulate:
  - Prob. of $\delta_M > \delta_c$ = mass fraction at time t in halos $> M$

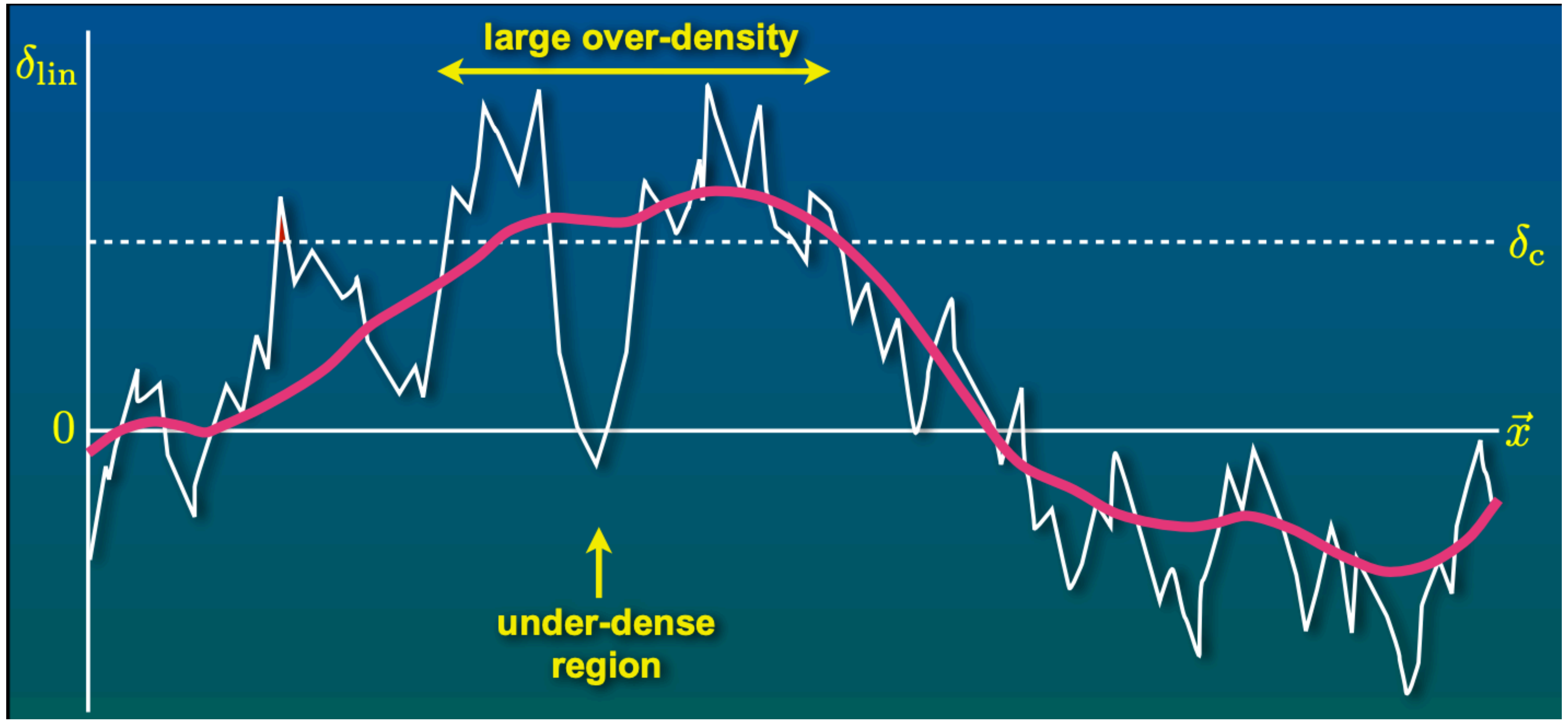
$$\mathcal{P}(\delta_M > \delta_c) = \frac{1}{\sqrt{2\pi}\sigma_M} \int_{\delta_c}^{\infty} d\delta_M \, \exp\left[-\frac{\delta_M^2}{2\sigma_M}\right] = \frac{1}{2}\mathrm{erfc}\left[\frac{\delta_c}{2\sigma_M}\right]$$

Fudge factor

- Only half of the matter is collapsed in dark matter halos

$$\lim_{M \to 0} \sigma_M = \infty \quad \rightarrow \quad \frac{1}{2}\mathrm{erfc}\left[0\right] = \frac{1}{2}$$

PS: „Only initially overdense regions in collapsed objects“



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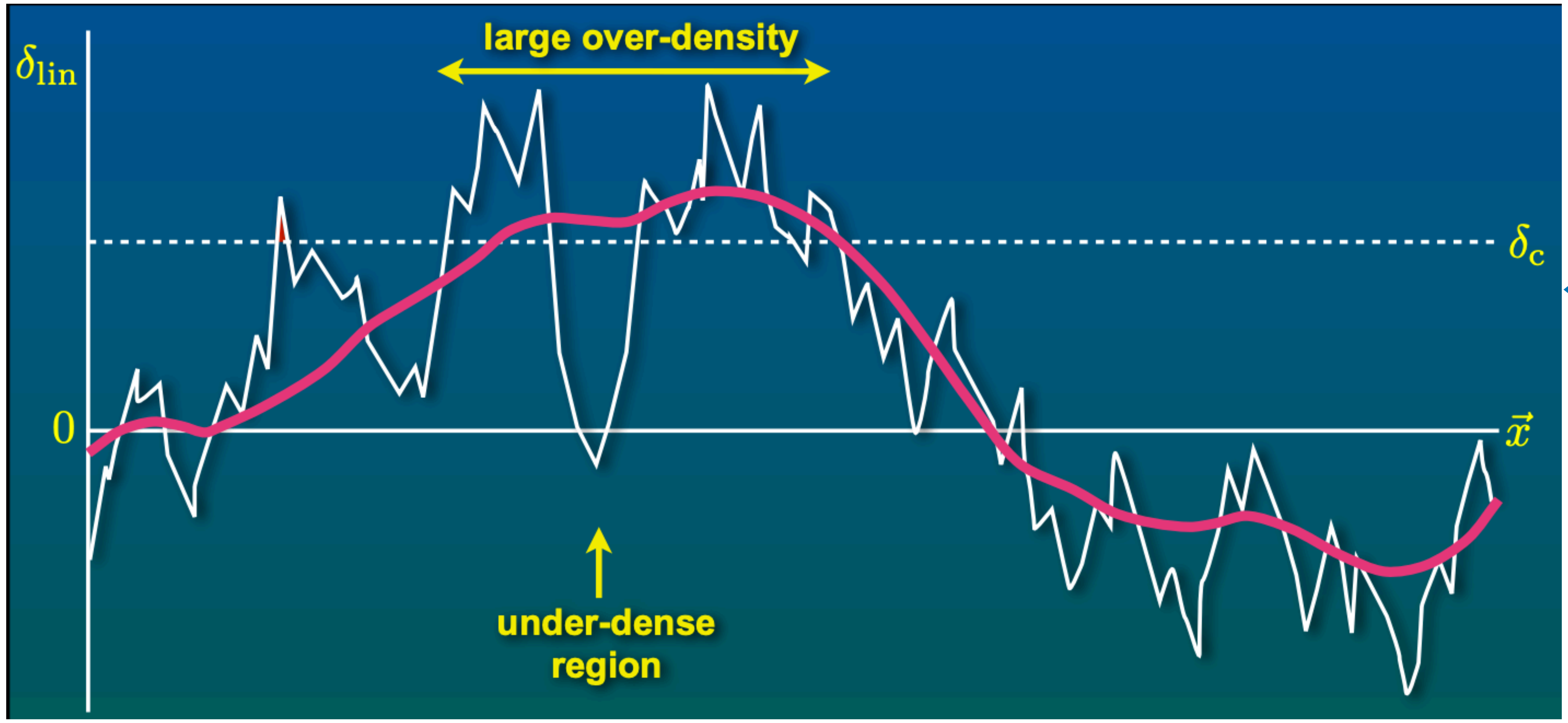

Underdense regions can be enclosed within larger overdense regions

PS: „OK let’s multiply by 2“

- From the course:

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c}{\sigma}\left|\frac{d\ln\sigma}{d\ln M}\right|\exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$$

Excursion Set Approach - EPS formalism

Filter $W(\vec{x}, R)$ is sharp k-space, density fluctuations are markovian random walk

EXCURSION SET MASS FUNCTIONS FOR HIERARCHICAL GAUSSIAN FLUCTUATIONS

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Received 1990 July 23; accepted 1990 December 28

ABSTRACT

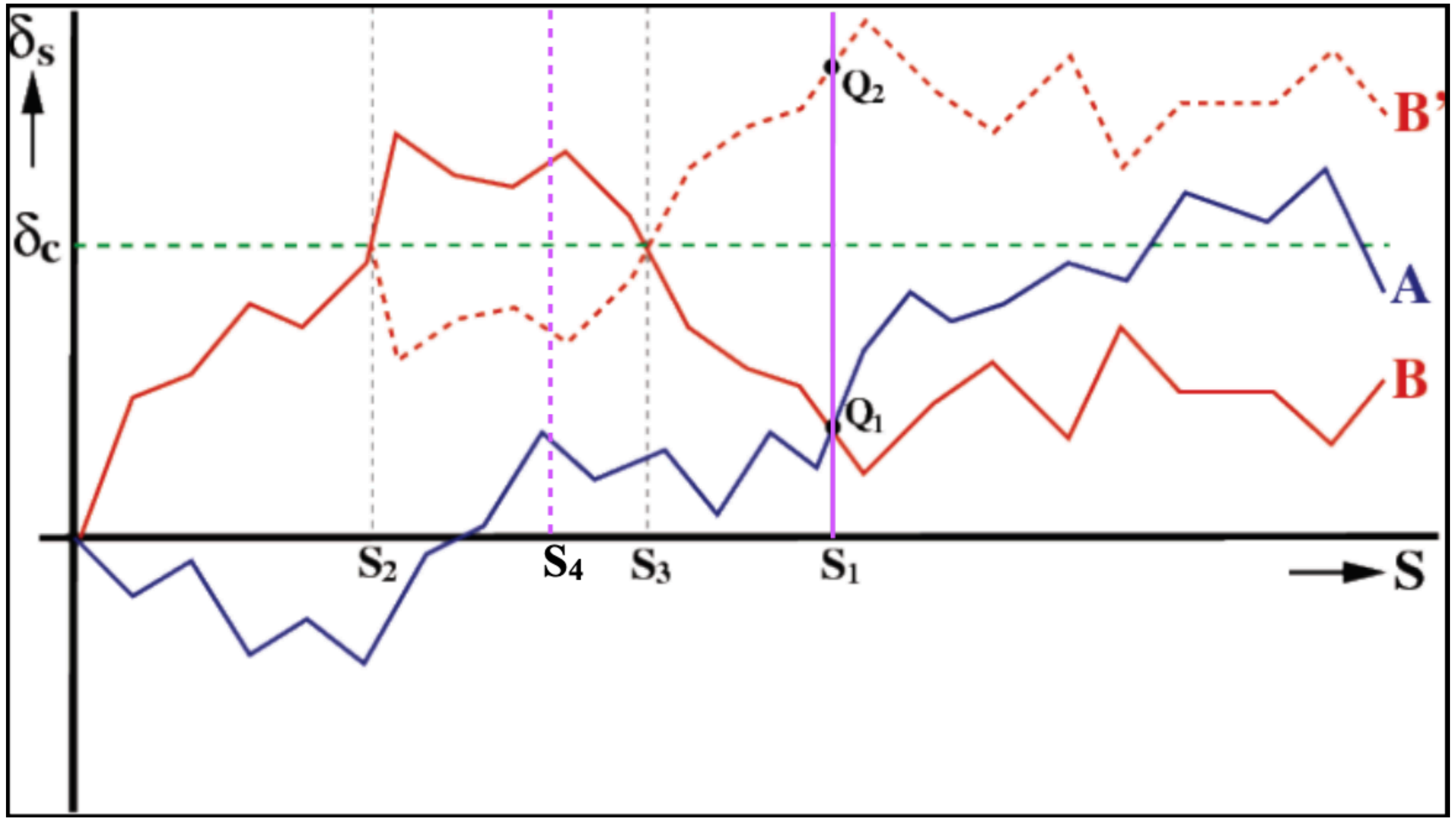
Most schemes for determining the mass function of virialized objects from the statistics of the initial density perturbation field suffer from the "cloud-in-cloud" problem of miscounting the number of low-mass clumps, many of which would have been subsumed into larger objects. We propose a solution based on the theory of the excursion sets of $F(\boldsymbol{r}, R_f)$, the four-dimensional initial density perturbation field smoothed with a continuous hierarchy of filters of radii R_f. We identify the mass fraction of matter in virialized objects with mass greater than M with the fraction of space in which the initial density contrast lies above a critical overdensity when smoothed on some filter of radius greater than or equal to $R_f(M)$. The differential mass function is then given by the rate of first upcrossings of the critical overdensity level as one decreases R_f at constant position $\boldsymbol{r}$. The shape of the mass function depends on the choice of filter function. The simplest case is "sharp k-space" filtering, in which the field performs a Brownian random walk as the resolution changes. The first upcrossing rate can be calculated analytically and results in a mass function identical to the formula of Press and Schechter—complete with their normalizing "fudge factor" of 2. For general filters (e.g., Gaussian or "top hat") no analogous analytical result seems possible, though we derive useful analytical upper and lower bounds. For these cases, the mass function can be calculated by generating an ensemble of field trajectories numerically. We compare the results of these calculations with group catalogs found from N-body simulations. Compared to the sharp k-space result, less spatially extended filter functions give fewer large-mass and more small-mass objects. Over the limited mass range probed by the N-body simulations, these differences in the predicted abundances are less than a factor of 2 and span the values found in the simulations. Thus the mass functions for sharp k-space and more general filtering all fit the N-body results reasonably well. None of the filter functions is particularly successful in identifying the particles which form low-mass groups in the N-body simulations, illustrating the limitations of the excursion set approach. We have extended these calculations to compute the evolution of the mass function in regions that are constrained to lie within clusters or underdensities at the present epoch. These predictions agree well with N-body results, although the sharp k-space result is slightly preferred over the Gaussian or top hat results.

Subject headings: cosmology — galaxies: clustering — numerical methods

Use mass variance instead of mass

$S = \sigma(M)$

Excursion Set Approach - EPS formalism



Random walk explains fudge factor

Necessary upcrossing
→ all mass in some halo

EPS: Fraction of trajectories with a first upcrossing of the barrier $\delta_S \geq \delta_c(t)$ at $S > \sigma(M_1)$ is equal to the mass fraction t that at time resides in haloes with masses $M < M_1$

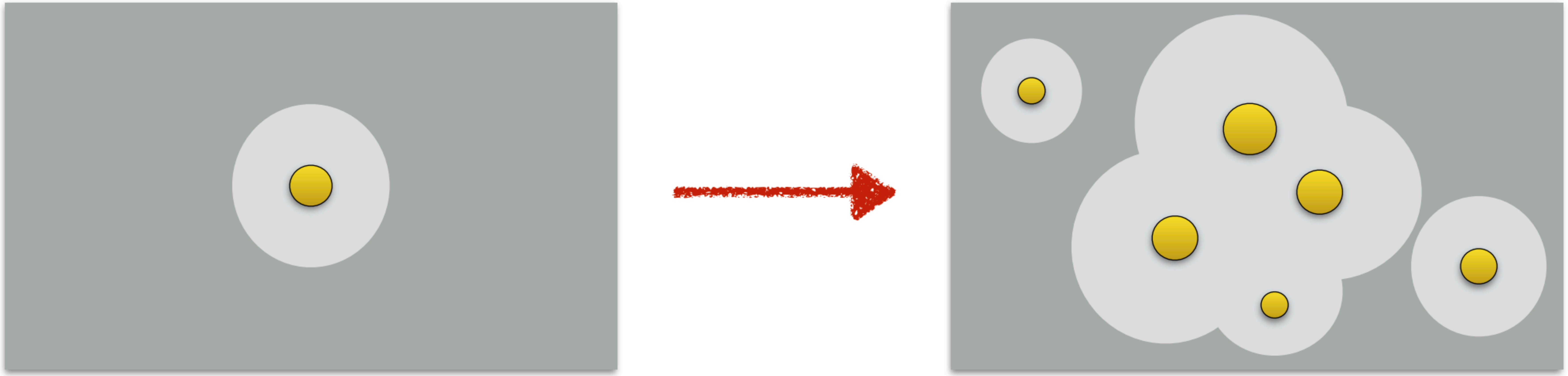
Analytical Models of HII Bubbles

- Prediction of 21cm signal: Radiative transfer of ionising photons through IGM
  - → Computationally expensive

Analytical Models of HII Bubbles

- Prediction of 21cm signal: Radiative transfer of ionising photons through IGM
  - ➔ Computationally expensive
- Bubbles are basic unit of ionisation field
  - Halo Model : All dark matter is in halos
  - Bubble Model : All HI is outside bubbles

Photon source switches on and ionises a region around itself



HII Bubbles

Dark matter halos larger than $M_{\min}$ contain an ionising source

A source in a halo of mass M ionises an HI mass ζM

ζ uncertain physics

THE GROWTH OF H II REGIONS DURING REIONIZATION

STEVEN R. FURLANETTO[1], MATIAS ZALDARRIAGA[2,3], & LARS HERNQUIST[2]

Draft version February 2, 2008

ABSTRACT

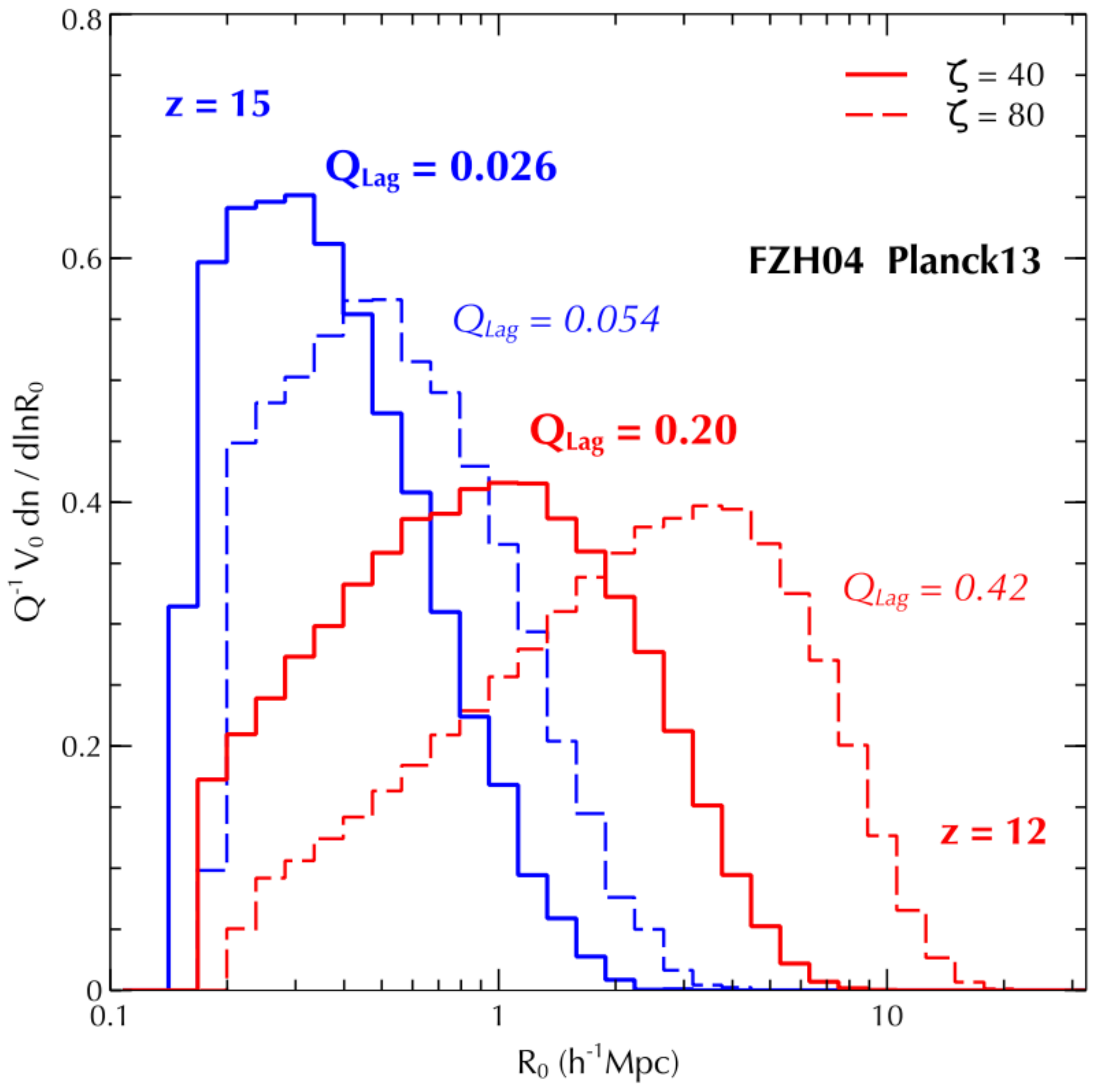
Recently, there has been a great deal of interest in understanding the reionization of hydrogen in the intergalactic medium (IGM). One of the major outstanding questions is how this event proceeds on large scales. Motivated by numerical simulations, we develop a model for the growth of H II regions during the reionization era. We associate ionized regions with large-scale density fluctuations and use the excursion set formalism to model the resulting size distribution. We then consider ways in which to characterize the morphology of ionized regions. We show how to construct the power spectrum of fluctuations in the neutral hydrogen field. The power spectrum contains definite features from the H II regions which should be observable with the next generation of low-frequency radio telescopes through surveys of redshifted 21 cm emission from the reionization era. Finally, we also consider statistical descriptions beyond the power spectrum and show that our model of reionization qualitatively changes the distribution of neutral gas in the IGM.

Subject headings: cosmology: theory – intergalactic medium – diffuse radiation

Bubble size distribution
Bubble spatial distribution

halo mass function from halos

Excursion set bubbles



Mesinger & Furlanetto (2007); Mesinger+ (2010)

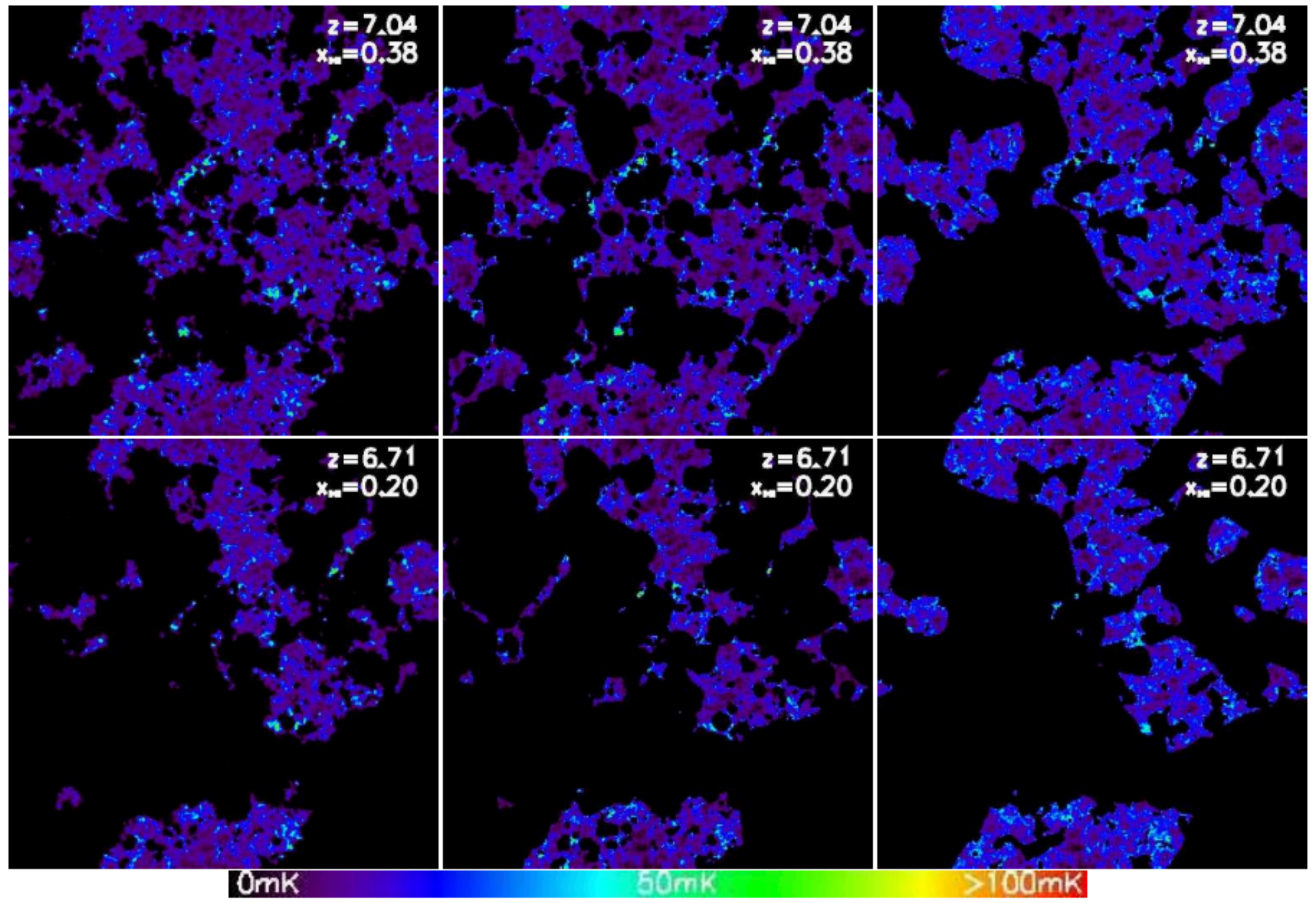
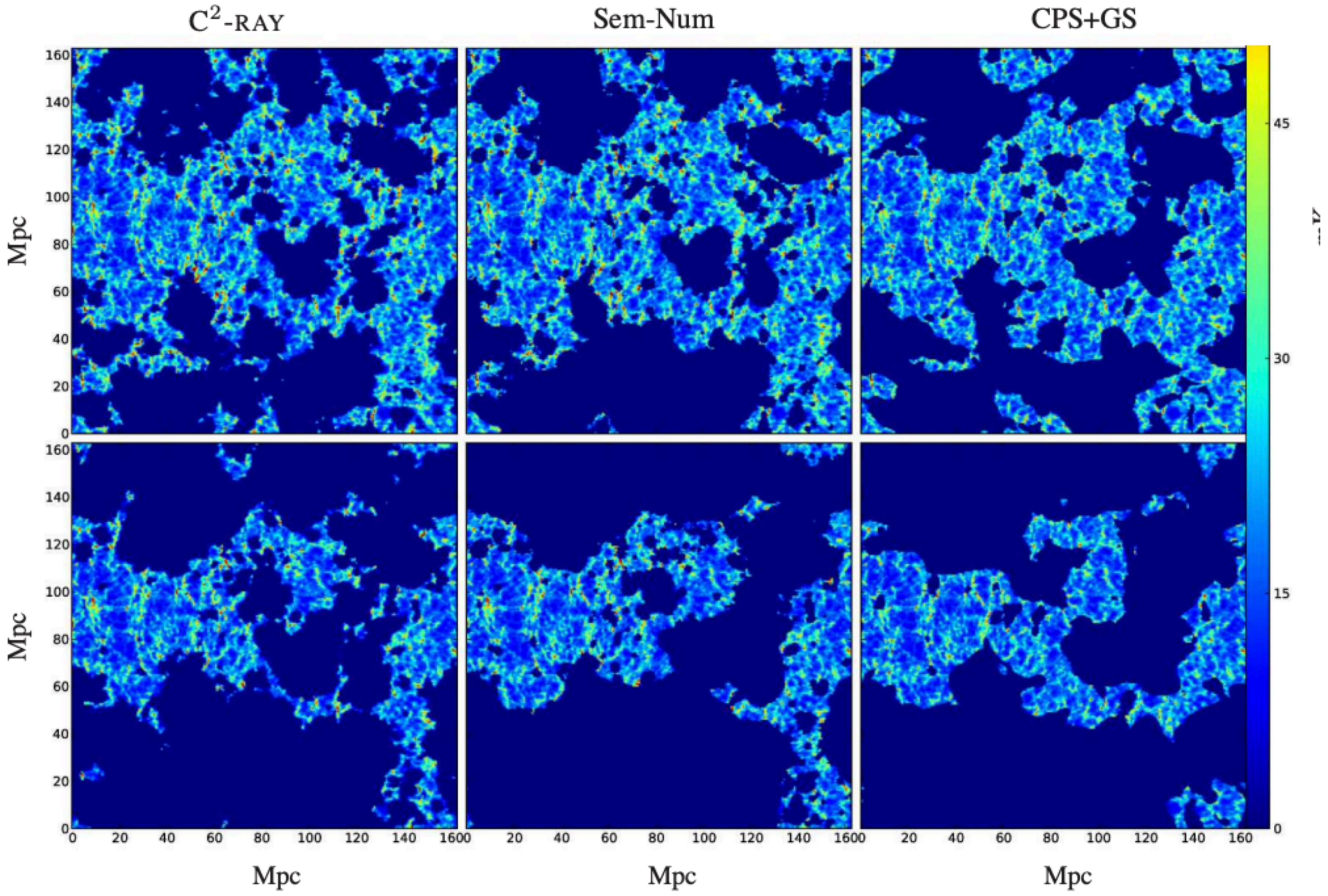


Figure 7. δT_b maps. The slices are generated from the hydrodynamic simulation, DexM (MF07), and 21cmFAST, left to right columns. All slices are 143 Mpc on a side and 0.56 Mpc thick, and correspond to $(z, \bar{x}_{\rm HI})$ = (9.00, 0.86), (7.73, 0.65), (7.04, 0.38), and (6.71, 0.20), top to bottom.

houdhury+ (2009); Majumdar+ (2014)



Photon Non-Conservation

Photon number conservation and the large-scale 21 cm power spectrum in semi-numerical models of reionization

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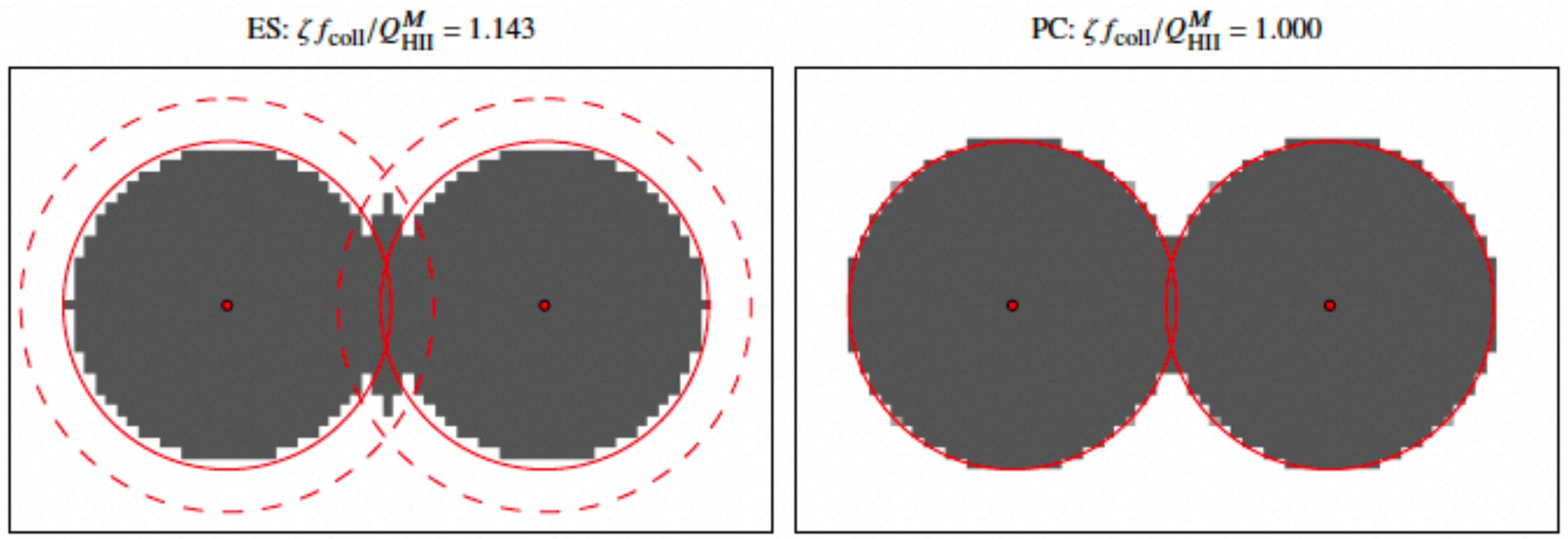
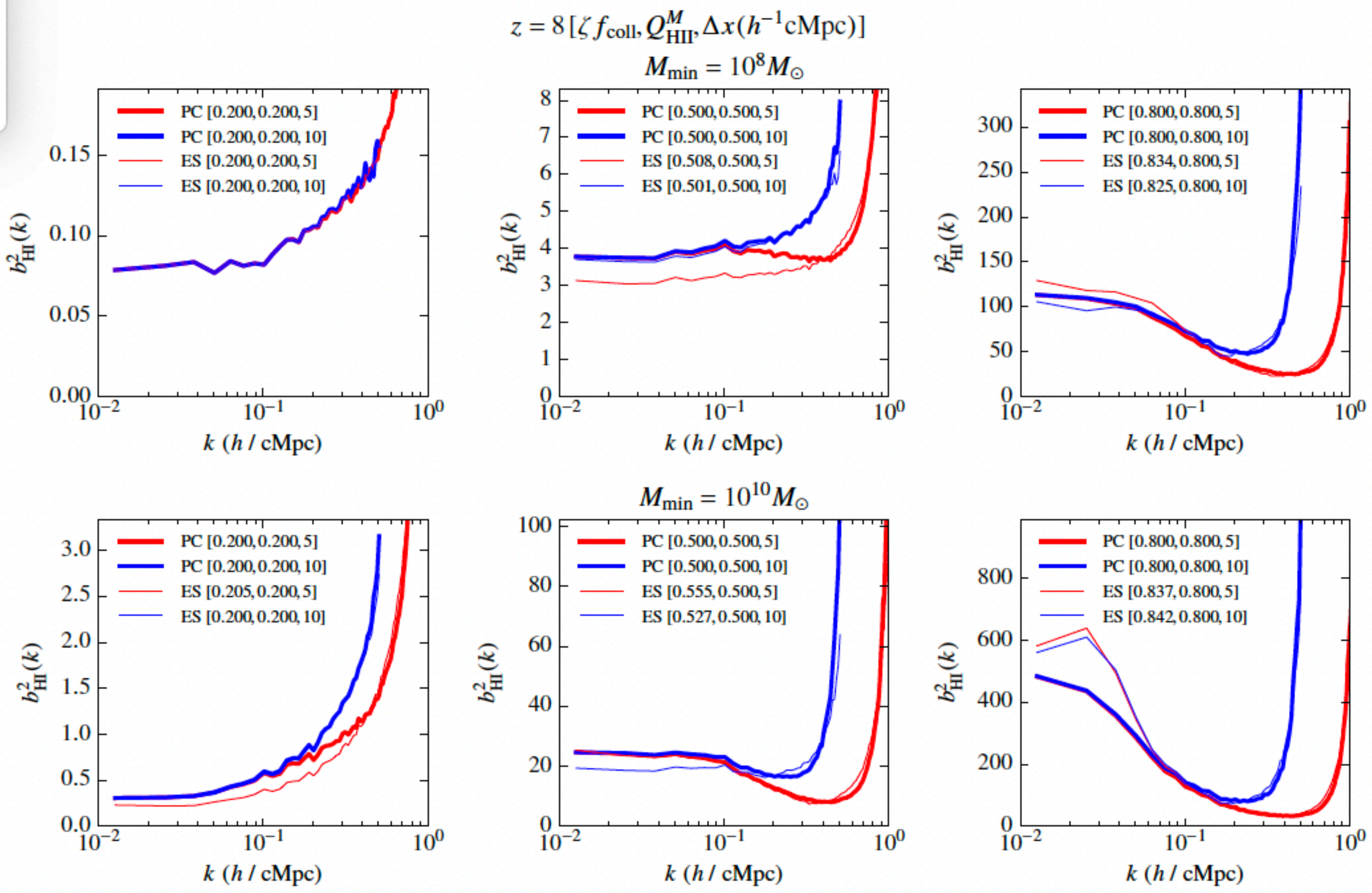


Figure 4. The projected ionization map for a toy model with two identical sources (indicated by the red dots) obtained using the excursion set model with spherical tophat filter *(left panel)* and the photon-conserving model *(right panel)*. The two-dimensional slice has been chosen to lie in the plane containing the two sources. The solid red circles indicate the ionized bubble size for the individual sources. In the left panel, the dashed red circles indicate the bubble size for sources of double the emissivity.

Treat overlapping cells and photon carry over explicitly



Large scale 21cm power spectrum: Dependence on spatial resolution, problem for next generation telescopes

Figure 6. The bias $b^2_{\rm HI}(k) = P_{\rm HI}(k)/P(k)$ of the HI density fluctuations for the photon-conserving model. Different panels and curves are for different resolution and parameter values as indicated in the respective legends. For comparison, we show results obtained using the excursion set model by thin lines.

Bayesian inference

Use Bayesian inference to update the bubble distribution „on the fly“

Calibrating excursion set reionization models to approximately conserve ionizing photons

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[2]ARC Centre of Excellence for All-Sky Astrophysics in 3 Dimensions (ASTRO 3D), University of Melbourne, VIC 3010, Australia
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ABSTRACT
The excursion set reionization framework is widely used, due to its speed and accuracy in reproducing the 3D topology of reionization. However, it is known that it does not conserve photon number. Here, we introduce an efficient, on-the-fly recipe to approximately account for photon conservation. Using a flexible galaxy model shown to reproduce current high-z observables, we quantify the bias in the inferred reionization history and galaxy properties resulting from the non-conservation of ionizing photons. Using a mock 21-cm observation, we perform inference with and without correcting for ionizing photon conservation. We find that ignoring photon conservation results in very modest biases in the inferred galaxy properties, for our fiducial model. The notable exception is in the power-law scaling of the ionizing escape fraction with halo mass, which can be biased from the true value by $\sim 2.4\sigma$ (corresponding to ~ -0.2 in the power-law index). Our scheme is implemented in the public code `21cmFAST`.

Key words: cosmology: theory – dark ages, reionization, first stars – diffuse radiation – early Universe – galaxies: high-redshift – intergalactic medium

Bayesian inference

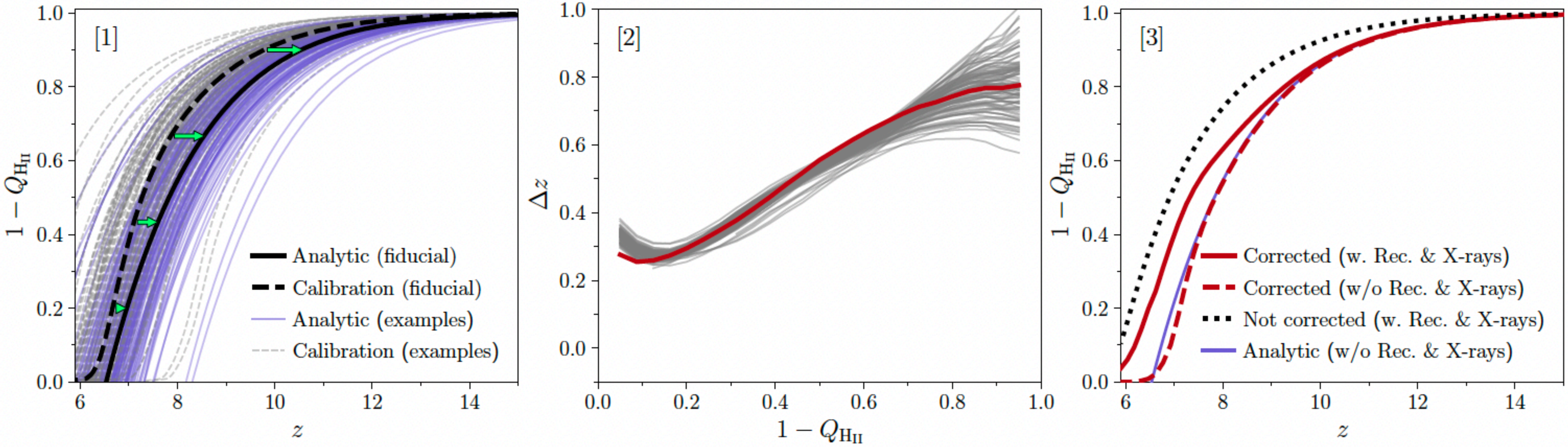
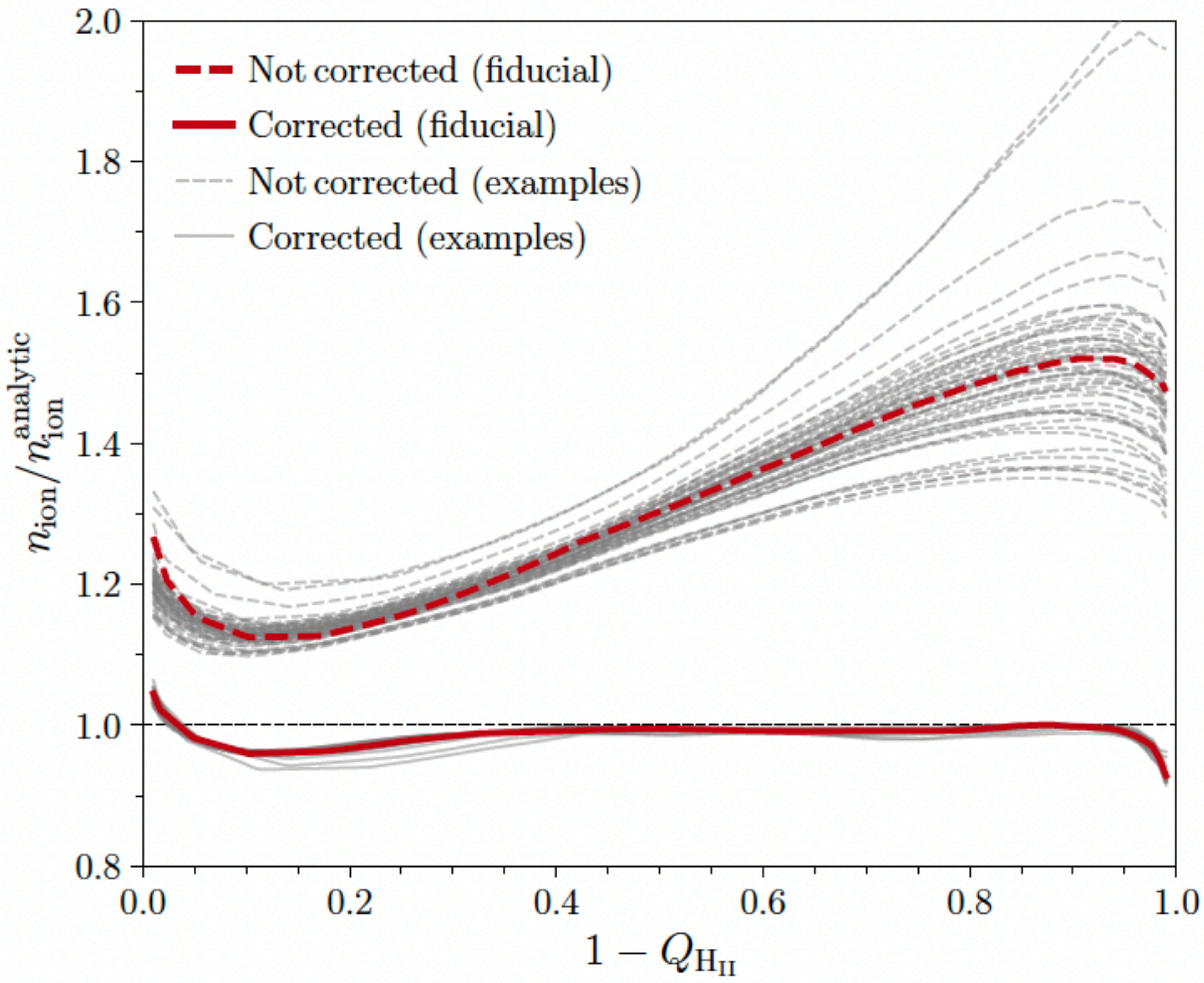


Figure 1. Each panel corresponds to a step of our calibration procedure described in the text. [1]: the EoR history from the analytic expression and the calibration curve (from `21cmFAST`); both include only ionizations by UV sources. Thick lines correspond to the fiducial astrophysical parameters (listed in Table 1). Thin solid (violet) and thin dashed (grey) lines represent analytic EoR histories and calibration curves using 100 different combinations of parameters, which are randomly selected from the MCMC chain (see Sec. 5). Arrows (lime green) illustrate the calibration vector for the fiducial parameter set. [2]: the calibration vector, $\Delta z \equiv z_{\mathrm{analytic}} - z_{\mathrm{calibration}}$ (i.e. Δz corresponds to the size of arrows in panel [1]). Thick solid (red) and thin (grey) lines represent Δz for the fiducial parameter set and the 100 different parameter combinations, respectively. [3]: final EoR histories. The thick solid (red) line represent the corrected EoR history when considering inhomogeneous recombinations and X-rays. The thick dotted line shows the same EoR history, but without photon conservation (i.e. not corrected). We also show the corrected EoR histories when not considering X-rays and inhomogeneous recombinations (thick red dashed line), which is almost identical with the analytic EoR history (thin violet solid line).

Bayesian inference



Summary

- Locations in linear density field where $\delta(\vec{x}, t) \geq \delta_c \approx 1,686$ correspond to collapsed objects (halos)
- Peak vs. PS vs. First Uncrossing
- Excursion sets are Markovian (with sharp-k space filter) $\rightarrow$ analytics
- Analytical models of bubble size distribution for modelling HI brightness temperature fluctuations during EoR.
- Excursion Set approach, pioneered by FZH04.
- FZH04 model does not conserve photon number.
- Can be fixed by explicit carry over of photons or Bayesian inference (approx.)