# Wandering across the edge

## Excursion set theory applied to the modelling of the reionization epoch

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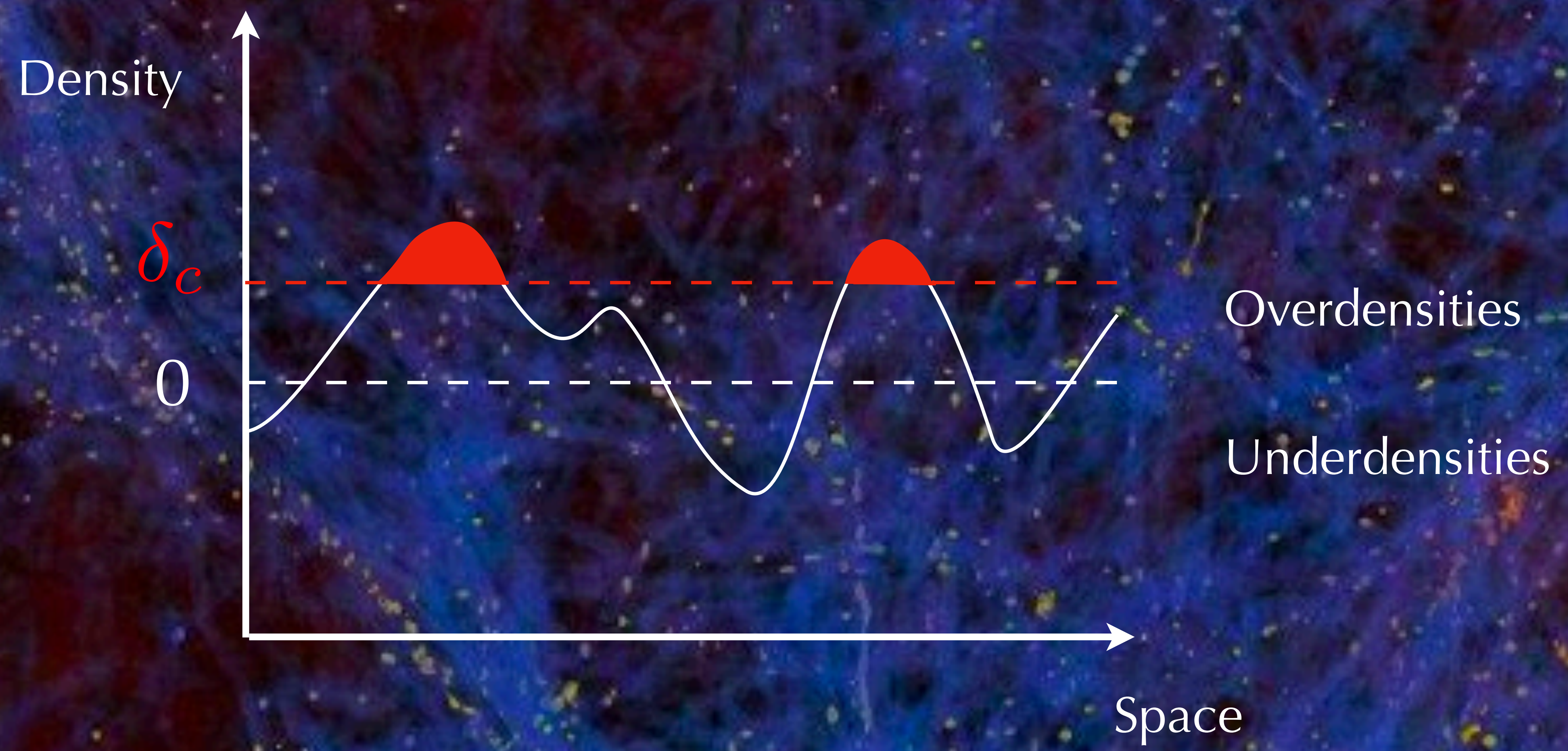
# Models of gravitational instability

**Objective**

Predict the abundance of nonlinear objects in the cosmic density field at a given time and coordinate (mass function).

**Main idea**

These objects form in regions where the density (or a combination of density and variance) exceeds a given threshold.




Linear threshold for spherical collapse:

$\delta_c \simeq 1.686$

# Useful definitions

Density contrast

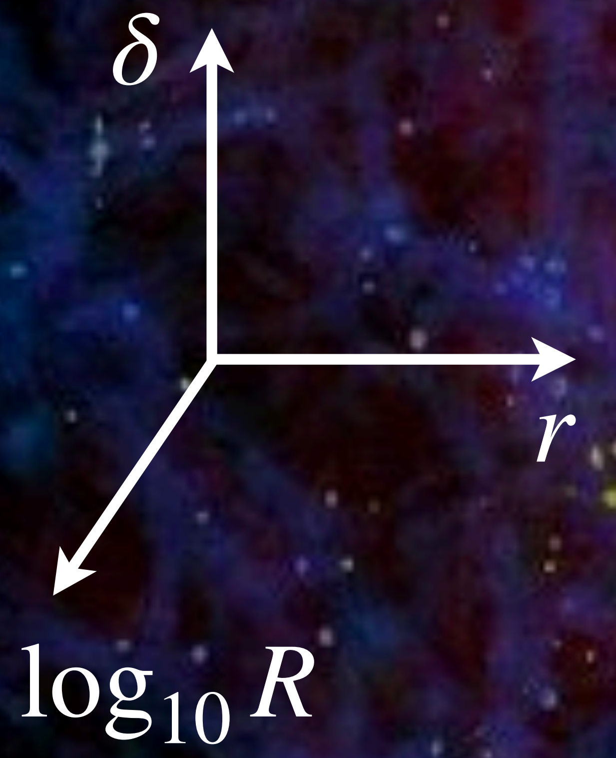
$$\delta(\vec{r}) := \frac{\rho_m(\vec{r}) - \bar{\rho}_m}{\bar{\rho}_m}$$

Smoothing/filtering on scale *R*

$$\delta(\vec{r}, R) := \int \mathrm{d}^3 r' W(|\vec{r}\,' - \vec{r}|, R)\, \delta(\vec{r}\,')$$

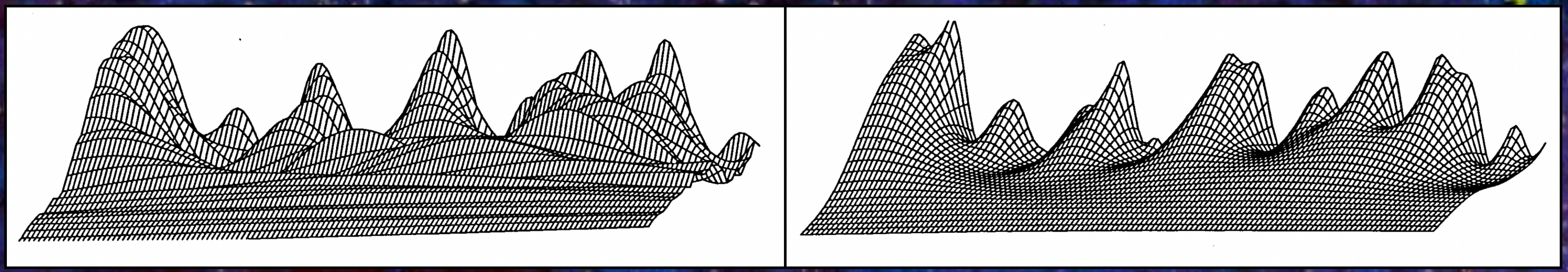
Variance of the smoothed field

$$S(R) = \int \frac{dk}{(2\pi)^2} k^2 P(k) |W(k, R)|^2$$




Sharp *k*-space

Gaussian



Reproduced from Bond et al. (1991)

# Peaks of Gaussian random fields

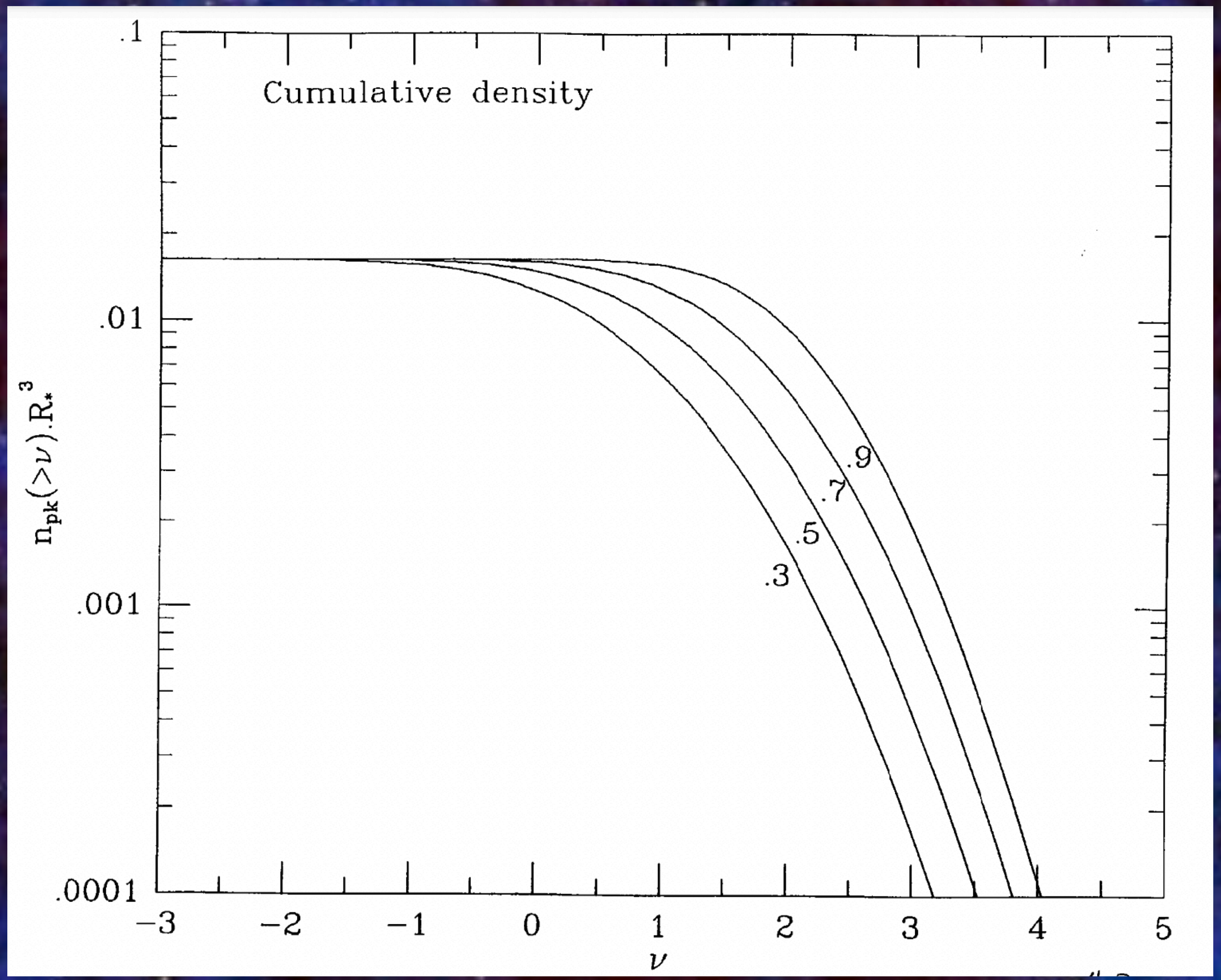
Bardeen et al. (1986)

## Ingredients

- Gaussian-distributed density field
- Filter function
- Selection function

## Recipe

1. Choose a threshold $\nu = \dfrac{\delta_c}{\sigma(R)}$
2. Identify the peaks that surpass it
3. Weight this number via the selection function




Reproduced from Bardeen et al. (1986)

Cumulative number density of peaks above a given threshold for different shapes of the power spectrum

Cloud-in-cloud issue: how can we include substructures?

# Excursion set theory

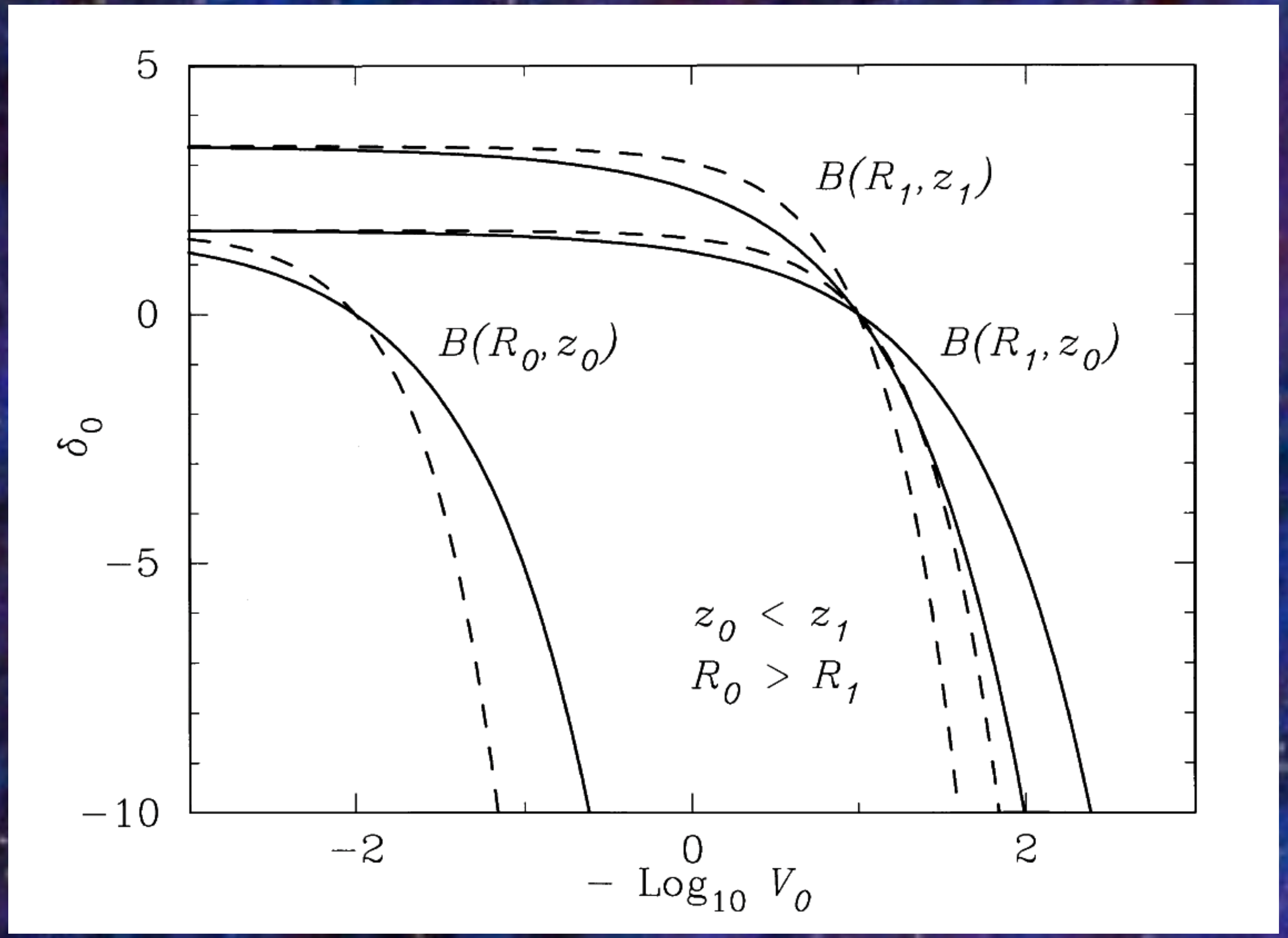
Original formulation Press & Schechter (1974)
Extension by Bond et al. (1991) and Sheth (1998)

<u>Change perspective</u>:

- Consider all points in space and not only the ones around maxima
- Treat the variance of the field and not the threshold as the reference stochastic variable

## Ingredients

- Initial density field with $|\delta| \ll 1$
- Filter function
- Threshold for collapse (barrier), not necessarily a constant




Reproduced from Sheth (1998)

# Excursion set theory

Bond et al. (1991)
Sheth (1998)

## Recipe

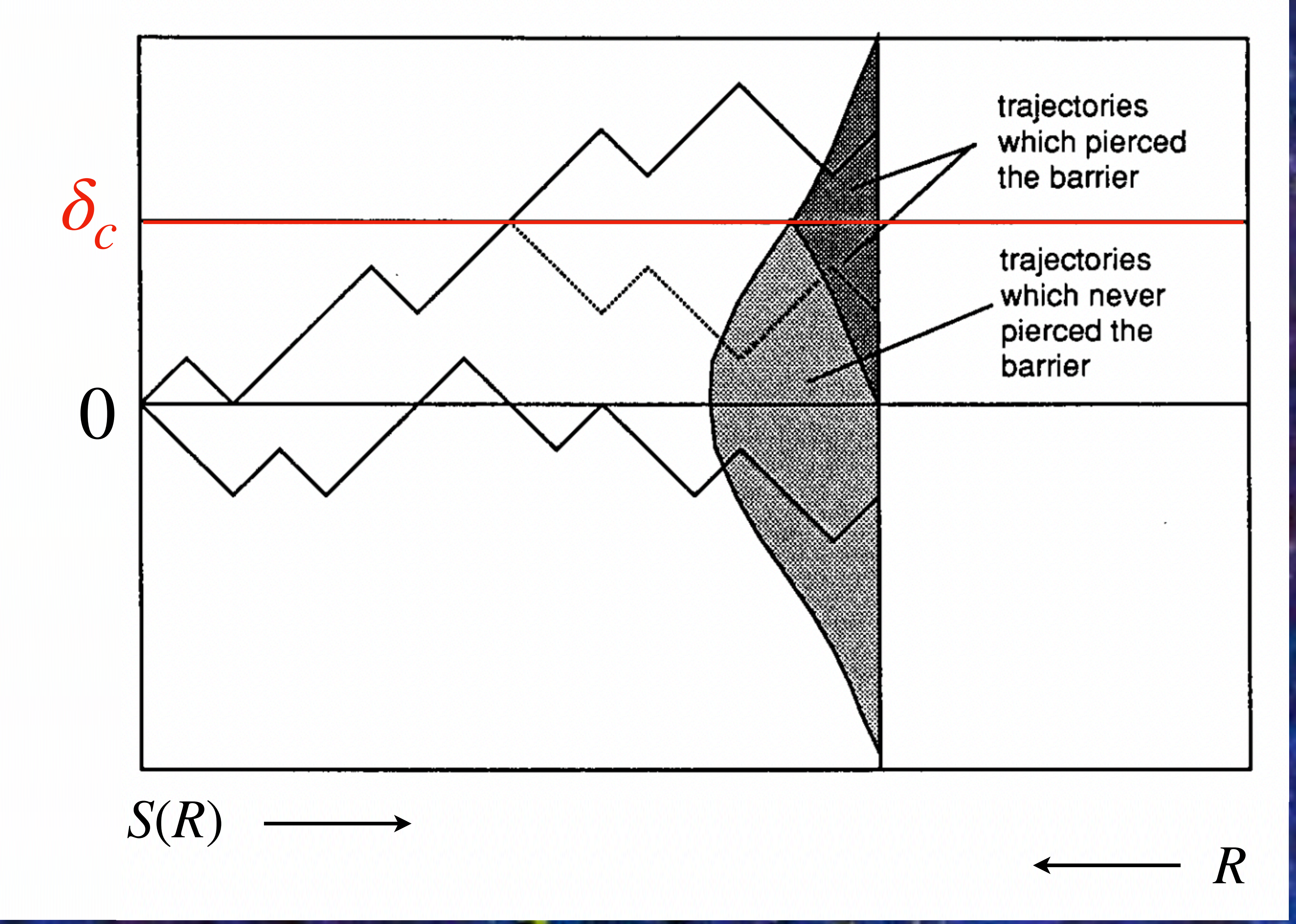
1. Choose a random position in the initial field
2. Compute $\delta(\vec{r}, R)$ while decreasing $R$ (trajectory)
3. Select the largest value of $R$ where the barrier is crossed
4. Associate a mass $M = \dfrac{4\pi}{3}\bar{\rho}\,R^3\,(1+\delta)$ to this trajectory
5. Repeat this procedure for an ensemble of trajectories

↓

The fraction of trajectories that have crossed the barrier at $R$ represents the fraction of mass in regions with mass $M$ and size $R$ (at a given redshift)

↓

Obtain the mass function assuming spherical symmetry




Adapted from Bond et al. (1991)

# Excursion set theory

Bond et al. (1991)
Sheth (1998)

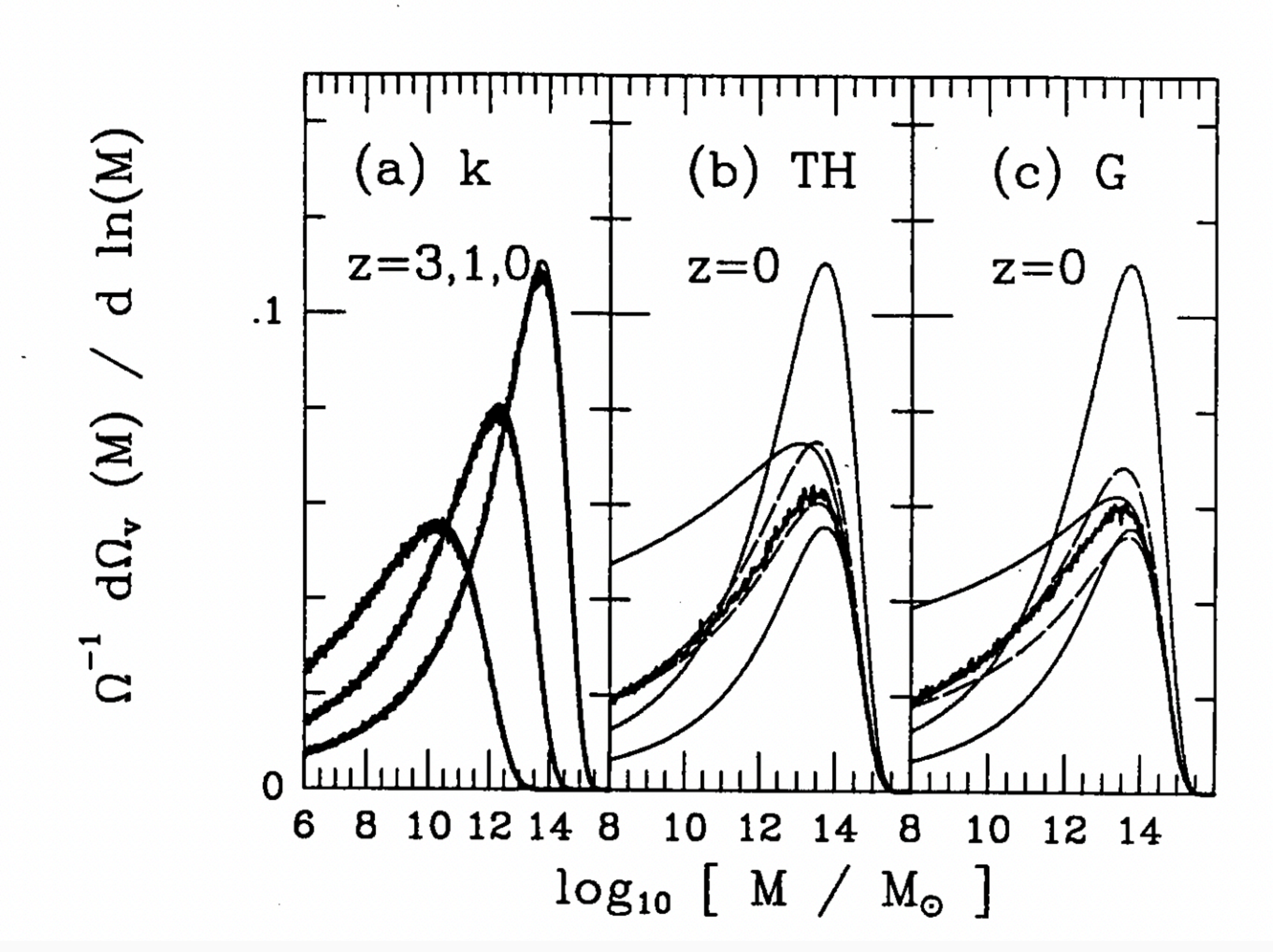
Analytical result for sharp *k*-space filter and constant barrier:

- The trajectories follow Brownian random walks
- The distribution of particle positions obeys a diffusion equation

$$\frac{\partial \Pi}{\partial S} = \frac{1}{2}\frac{\partial^2 \Pi}{\partial \delta^2}$$

- This yields the mass function

$$\frac{\mathrm{d}n}{\mathrm{d}M} = \sqrt{\frac{2}{\pi}}\frac{\delta_c}{S}\left|\frac{\mathrm{d}S}{\mathrm{d}M}\right|\exp\left(-\frac{\delta_c^2}{2S}\right)$$




Reproduced from Bond et al. (1991)

Need Monte Carlo approach to determine trajectory distributions for other filters and barriers!

# Application to reionization

Furlanetto et al. (2004)
Furlanetto et al. (2005)

<u>Objective</u>: calculate the size distribution of ionized bubbles in the IGM

## Previous approach

Model in terms of Strömgren spheres around individual galaxies

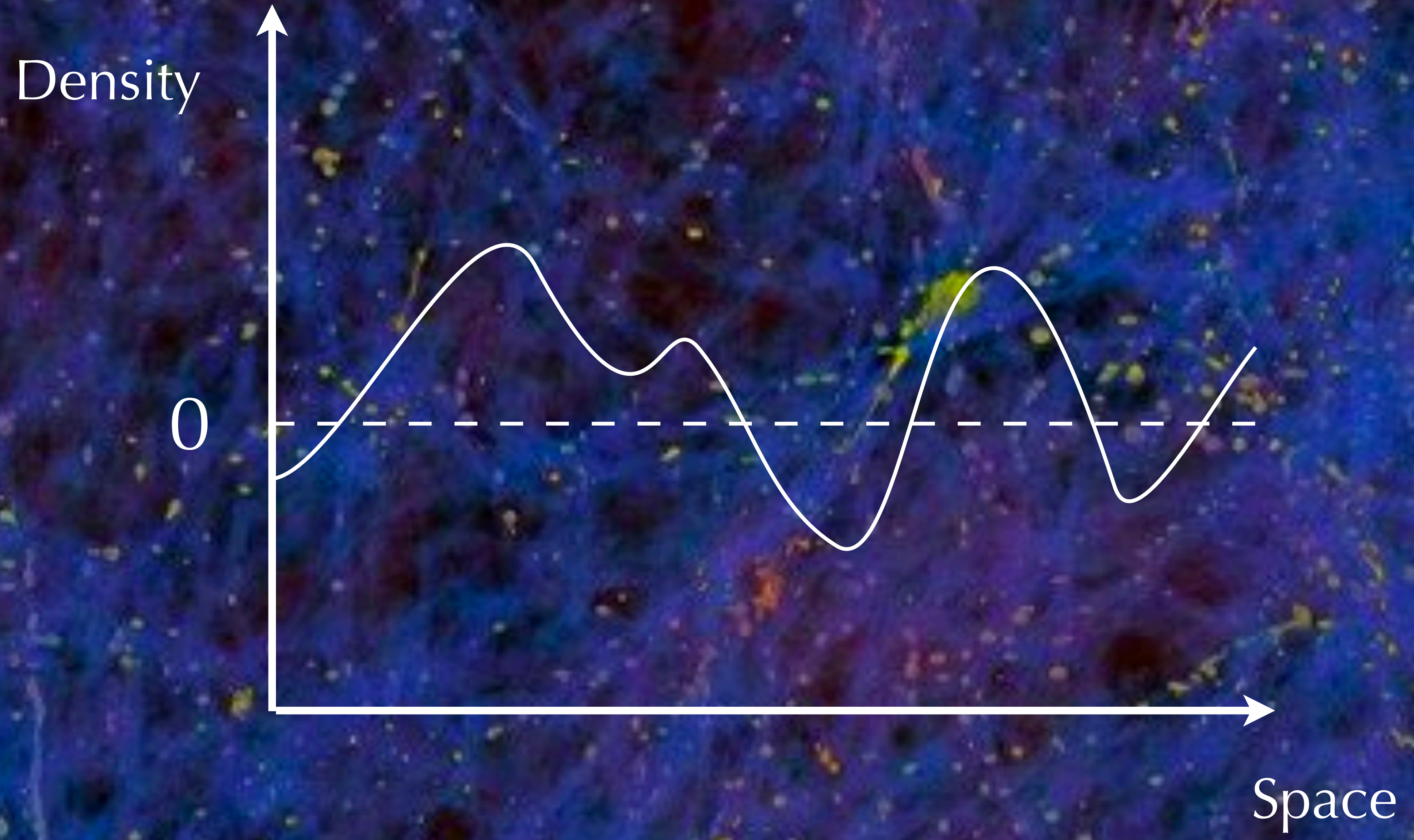
Several small bubbles around sources

vs

Simulations show an inhomogeneous picture

## Excursion set approach

Takes into account the large-scale density fluctuations



# Application to reionization

Furlanetto et al. (2004)
Furlanetto et al. (2005)

<u>Main idea</u>: ask whether a region of given mass $m$ is ionized or not

## Ingredients

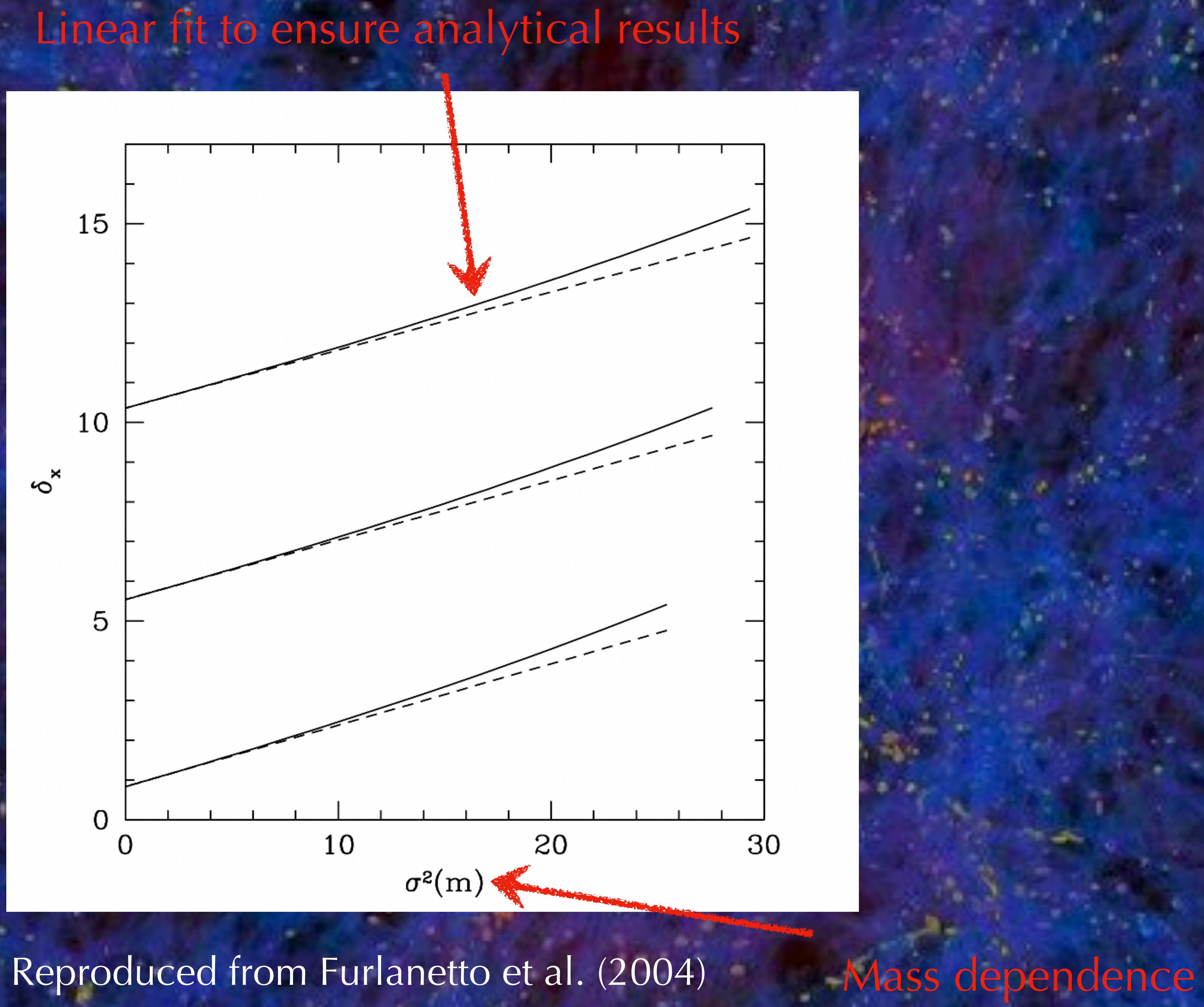
- <u>Two input parameters</u>:

    1. Ionizing efficiency $m_{\text{ion}} = \zeta\, m_{\text{gal}}$
    2. Minimum halo mass required to host a source (set through viral temperature)

- <u>Barrier</u>: $f_{\text{coll}} \geq \zeta^{-1}$

    Region must contain enough mass luminous sources to be fully ionized

    - use analytical results from Bond et al. (1991)




Reproduced from Furlanetto et al. (2004)

# Application to reionization

Furlanetto et al. (2004)
Furlanetto et al. (2005)

Main idea: ask whether a region of given mass $m$ is ionized or not

## Recipe

Furlanetto et al. (2004)

1. Choose a point in space and start from $m = \infty$
2. Progressively reduce the mass and compute the smoothed density field
3. At the first barrier crossing, assign the corresponding mass to the trajectory

## Extensions

Furlanetto et al. (2005)

1. Allow $\zeta$ to be a function of galaxy mass
2. Test different formulations for the halo mass function determining the barrier

## Remaining shortcomings

1. Assumption of spherical symmetry
2. Simplified reionization physics

# Application to reionization

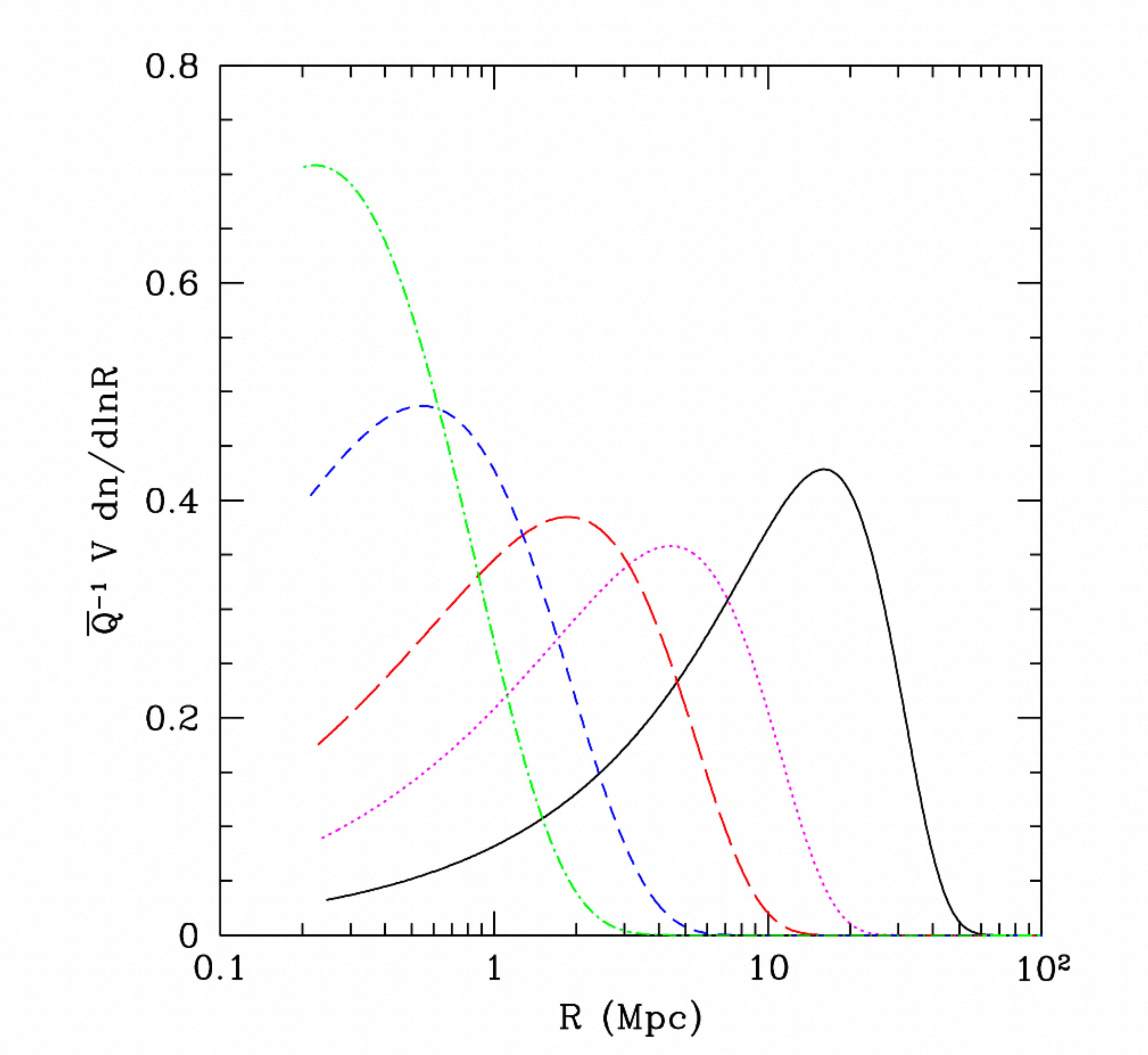
Furlanetto et al. (2004)
Furlanetto et al. (2005)

## Results - bubble size distribution

- Large ionised regions (tens of Mpc)
- Characteristic scale

  interpreted as the typical density fluctuation that is large enough to ionize itself
- Narrowing of the distribution with time

  consequence of the slope of the matter power spectrum




Bubble size distribution at different redshifts for $\zeta = 40$, normalised over the fraction of space occupied by the bubbles.

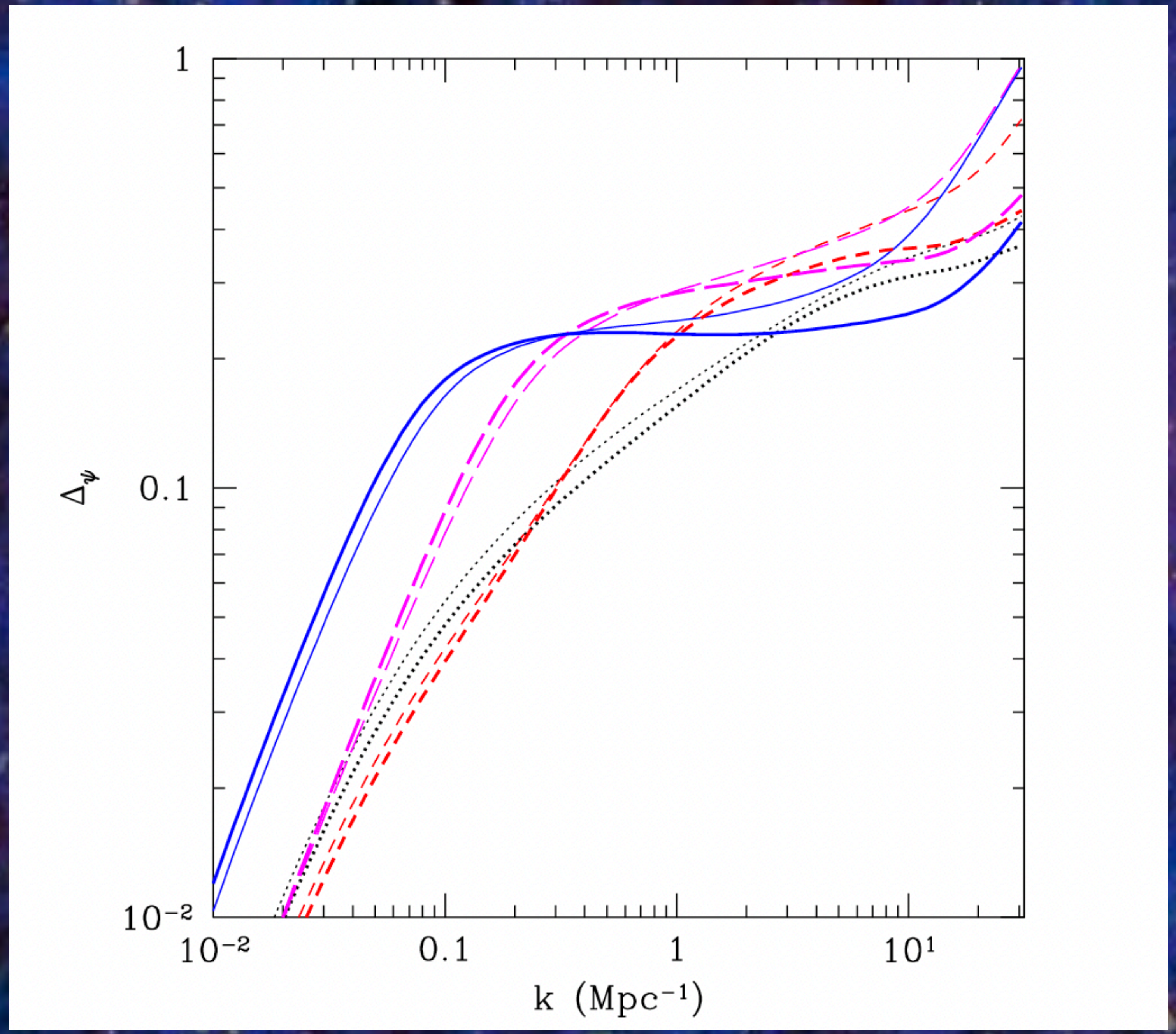
Reproduced from Furlanetto et al. (2004)

# Application to reionization

Furlanetto et al. (2004)
Furlanetto et al. (2005)

## Results - power spectrum

- Strong redshift dependence
- Weak dependence on $\zeta$
- Ionised regions leave strong imprint




Normalised power spectrum of $\psi = x_H(1+\delta)$ for $\zeta = 12$ (thin), $\zeta = 40$ (thick) and different neutral fractions.

Reproduced from Furlanetto et al. (2004)

# Open issues

Excursion set approach violates photon conservation!

Resolution-dependent effect, since caused by the overlap between resolved bubbles and partially ionized regions ⟷ At large scales, the power spectrum depends on the resolution at which ionisation maps are generated

→ Semi-numerical models tracking ionizing photons — Choudhury & Paranjape (2018)

→ Rescale ionising emissivity to account for this effect — Park et al. (2021)

# Outlook

- The excursion set formalism is a powerful semi-analytic tool to determine the mass function of collapsed objects.
- It can be applied to model the bubble size distribution in the reionization epoch.
- This model is in good agreement with simulations.
- However, further refinements are necessary to ensure photon conservation.

*Thank you!*