Higgs mechanism (spontaneous breaking of gauge symm.)

Ryder Sec. 8.3

Higgs coupling to $U(1)$ gauge theory

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + D_\mu \phi \, D^\mu \phi^* - V(\phi)$$

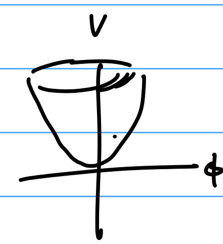
ϕ : Higgs field (complex scalar)

$$D_\mu \phi = (\partial_\mu + i g A_\mu)\phi \qquad \underset{\uparrow \text{ Higgs potential}}{V(\phi)} = \mu \phi\phi^* + \lambda(\phi\phi^*)^2$$

$U(1)$ gauge inv. $\quad \phi(x) \to e^{ig\Lambda(x)}\phi(x) \qquad A_\mu \to A_\mu + \partial_\mu \Lambda(x)$

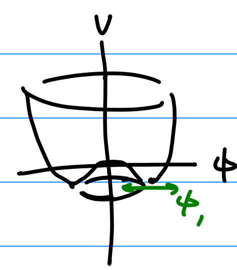
Vacuum : solution of EOM. that minimize $V(\phi)$

$$\left.\begin{aligned} A_\mu &= 0 \\ \phi &= \text{Const.} \end{aligned}\right\} \text{ up to gauge transf.}$$



$$V(\phi) = \text{minimum} \Rightarrow \begin{cases} \mu > 0 & \phi = \langle\phi\rangle = 0 \\ \mu < 0 & \phi = \sqrt{\frac{-\mu}{2\lambda}} \equiv a \end{cases}$$

up to phase



$a,\ \phi_1(x) \in \mathbb{R}$

$$\phi(x) = \left(a + \frac{\phi_1(x)}{\sqrt{2}}\right) e^{i\theta(x)}$$

$\uparrow$ would be Goldstone mode

But now by gauge inv. $\quad \phi(x) \sim \phi(x) e^{i\theta(x)}$

$\Rightarrow$ set $\theta(x) = 0$ by gauge transf $\to$ no Goldstone mode

where the Goldstone DOF goes? $\quad A_\mu$ receive mass at the new vacuum.

$$\phi(x) = a + \frac{\phi_1(x)}{\sqrt{2}} \qquad A_\mu(x) = \underset{\uparrow}{0} + A_\mu(x)$$

$$D_\mu\phi(x) = (\partial_\mu + igA_\mu)\left(a + \frac{\phi_1(x)}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\partial_\mu\phi_1 + \frac{i}{\sqrt{2}} g A_\mu \phi_1 + i g a A_\mu$$

$$D_\mu\phi(x)(D^\mu\phi(x))^* = \underline{\frac{1}{2}\partial_\mu\phi_1\partial_\mu\phi_1 + g^2a^2A_\mu A^\mu} + \frac{2a}{\sqrt{2}} g A_\mu A^\mu \phi_1 + \frac{1}{2}g^2 A_\mu A^\mu \phi_1 + \cdots$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{g^2 a^2 A_\mu A^\mu}_{\text{mass of vector field.}} + \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \underline{\underline{2\lambda a^2 \phi_1^2}} + \text{couplings}$$

$$V(\phi) = \mu\left(a+\frac{\phi_1}{\sqrt{2}}\right)^2 + \lambda\left(a+\frac{\phi_1}{\sqrt{2}}\right)^4$$

$$= \underbrace{\mu a^2 + \lambda a^4}_{\downarrow} + \underbrace{\left(\sqrt{2}a\mu + 2\sqrt{2}\lambda a^3\right)}_{0}\phi_1 + \underbrace{\left(\frac{\mu}{2}+3\lambda a^2\right)}_{2\lambda a^2}\phi_1^2 + O(\phi_1^3)$$

$a = \sqrt{\frac{-\mu}{2\lambda}}$

$\mu < 0$

$a^3 = -\frac{\mu}{2\lambda}a$

$$\mu\left(\frac{-\mu}{2\lambda}\right) + \lambda\frac{\mu^2}{4\lambda^2} = -\frac{\mu^2}{4\lambda}$$

- $g^2 a^2 A_\mu A^\mu$ break $U(1)$ gauge symm. , $\langle\phi\rangle = a$ symm. breaking phase

A_μ obtain mass. $m^2 = 4g^2a^2 = 4g^2\langle\phi\rangle^2$

$\delta_A \mathcal{L} = 0$ $\quad \partial_\mu F^{\mu\nu} + m^2 A^\nu = 0$

act by ∂_ν $\Rightarrow$ $\underline{\underline{\partial_\nu A^\nu = 0}}$

$\underline{\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + m^2 A^\nu = 0} \Rightarrow \partial_\mu\partial^\mu A^\nu + m^2 A^\nu = 0$

4 KG eqns.

$$A_\mu = \int \frac{d^4k}{(2\pi)^4\sqrt{2E}} \sum_{\lambda=0}^{3}\left[\varepsilon_\mu^{(\lambda)}(k)\, a_k^{(\lambda)} e^{-ik\cdot x} + \varepsilon_\mu^{(\lambda)}(k)^* a_k^{(\lambda)*} e^{ik\cdot x}\right]_{k^0=E}$$

$$E = \sqrt{\vec{k}\cdot\vec{k}+m^2}$$

$\varepsilon_\mu^{(\lambda)}(k)$ $\quad \lambda = 0 .. 3$ $\quad$ 4 DOFs.

$\partial_\mu A^\mu = 0 \Rightarrow k^\mu \varepsilon_\mu^{(\lambda)}(k) = 0 \rightarrow 3$ DOFs

$U(1)$ gauge theory 2 DOF , massive vector field $\rightarrow$ 3 DOFs

symm phase : A_μ ; 2 DOFs $\quad$ complex ϕ : 2 DOFs per pt.

~~symm~~ phase : A_μ : 3 DOFs $\quad$ real ϕ_1 : 1 DOFs

$$2+2 = 3+1$$

- the goldstone mode from complex higgs field is eaten by A_μ, which becomes massive

Couple to fermions : $\mathcal{L}_{tot} = \underline{\underline{\mathcal{L}[A,\phi]}} + \mathcal{L}_f[\psi, A, \phi]$

$$\mathcal{L}_f = i\bar{\psi}\not{D}\psi + y\phi\bar{\psi}\psi$$

Yukawa interaction

standard model

$G = SU(2)\times U(1)$

$\cup$

$H = U(1)$

~~Symm~~ $\phi = a + \frac{\phi_1}{\sqrt{2}}$

$$y\phi\bar{\psi}\psi = y\left(a+\frac{\phi_1}{\sqrt{2}}\right)\bar{\psi}\psi = \underline{ya\bar{\psi}\psi} + \text{coupling}$$

$m_f = ya = y\langle\phi\rangle$

fermion receive mass.

<u>Superconductivity (Bardeen-Cooper-Schrieffer (BCS) theory)</u>

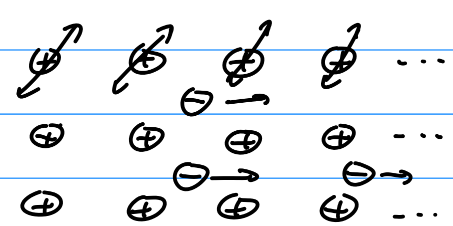
phenomenological paradism of superconductivity. (low temprature)

(Landau paradism)

Ohm's law $\vec{E} = R\vec{j}$ $\quad R = 0$ superconductor.

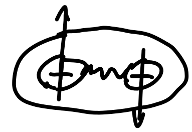
$\vec{E} = 0 \quad \vec{j} \neq 0 \quad \vec{B} = 0$

metal



lattice $\rightarrow$ phonon

electron $\xleftrightarrow{\text{coupling}}$ phonon $\xleftrightarrow{\text{coupling}}$ electron very low temprature → effective attractive force between 2 electrons



cooper pair.

can be effectively described by a complex scalar ϕ.

effective field of cooper pair

Landau-Ginzburg free energy of mean field theory (classical)

of 2nd order phase transition