Hamiltonian constraint.

$$C(x) = \underbrace{\frac{1}{k} F^j_{ab} \frac{\varepsilon_{jkl} E^a_k E^b_l}{\sqrt{\det q}}}_{C_0(x)} - \underbrace{\frac{1}{k}(\beta^2+1)\, \varepsilon_{jmn} K^m_a K^n_b \overbrace{\frac{\varepsilon_{jkl} E^a_k E^b_l}{\sqrt{\det q}}}^{h\{h^{-1}\, V\}}}_{C_L(x)}$$

Euclidean term $\qquad$ Lorentz term $\qquad K^i_a = \frac{A^i_a - \Gamma^i_a(E)}{\beta}$

$$\hat{C}_{0,v} := -\frac{1}{i l_p^2 k \beta} \sum_{\substack{s_1 s_2 s_3 = \pm 1 \\ I_1 I_2 I_3 = 1,2,3}} s_1 s_2 s_3\, \varepsilon^{I_1 I_2 I_3}\, \mathrm{tr}\left(\hat{h}(\alpha_{I_1 s_1 I_2 s_2}(v))\, \hat{h}(e_{v I_3 s_3}) \left[\hat{h}(e_{v I_3 s_3})^{-1}, V_v \right] \right)$$

(2) Lorentz term

$$\underline{\underline{C_L(x)}} = \frac{2}{k}(\beta^2+1)\, \mathrm{tr}\left([K_a, K_b] \frac{[E^a, E^b]}{\sqrt{\det q}} \right) \qquad K_a = K^i_a \frac{\tau_i}{2}$$

we are quantizing $C_L(x)\, \mathrm{sgn}(\det e)$

$$\mathrm{sgn}(\det e) \frac{[E^a, E^b]}{\sqrt{\det q}} = -\frac{4}{k\beta}\, \varepsilon^{abc} \{A_c(x), V(R)\}$$

$x \in R$

Lemma (1) $K^j_a(x) = \frac{\delta \bar{K}}{\delta E^a_j(x)} = \frac{-2}{k\beta} \{A^j_a(x), \bar{K}\} \qquad \bar{K} = \int_\Sigma d^3x\, K^i_a E^a_i$

(2) $\bar{K} = -\frac{1}{\beta^2} \left\{ \int_\Sigma d^3x\, \mathrm{sgn}(\det e)\, C_0,\ V(\Sigma) \right\}$

Proof (1) $\frac{\delta K^j_b(x)}{\delta E^a_j(y)} = 0$ because $\underline{\{E^a_i(x), K^j_b(y)\} = \frac{k}{2} \delta^j_i \delta^a_b \delta^{(3)}(x,y)}$

(2) skip.

$$\mathrm{sgn}(\det e)\, C_L = \frac{2}{k}(\beta^2+1)\, \mathrm{tr}\left([K_a K_b] \{A_c\ V(R)\} \right) \varepsilon^{abc} \left(-\frac{4}{k\beta}\right)$$

$$= -\frac{8}{k^2\beta}(\beta^2+1)\, \mathrm{tr}\left((K_a K_b - K_b K_a) \{A_c\ V(R)\} \right) \varepsilon^{abc}$$

$$= -\frac{16}{k^2\beta}(\beta^2+1)\, tr\left(K_a K_b \{A_c\ V(R)\}\right) \varepsilon^{abc}$$

$$= -\frac{64}{k^4\beta^3}(\beta^2+1)\, tr\left(\{A_a\ \bar{K}\}\underbrace{\{A_b, \bar{K}\}}\underbrace{\{A_c\ V(R)\}}\right) \varepsilon^{abc}$$

discretization :

$$h(e_I)\{h(e_I)^{-1}\ V_v\} = -\int dx^I \{A_I(\vec{x})\ V_v\} + O(\mu^2)$$

$$h(e_I)\{h(e_I)^{-1}, \bar{K}\} = -\int dx^I \{A_I(\vec{x})\ \bar{K}\} + O(\mu^2)$$

$$\underline{\bar{K} = -\frac{1}{\beta^2}\left\{\sum_v C_{0,v}\ ,\ \sum_v V_v\right\}}$$

$$\Rightarrow C_{L,v} := \frac{8}{k^4\beta^3}(\beta^2+1)\sum_{\substack{s_1 s_2 s_3\\ =\pm 1}}\ \sum_{I_1 I_2 I_3} s_1 s_2 s_3\ \varepsilon^{I_1 I_2 I_3}$$

$$tr\left(h(e_{v,I_1 s_1})\{h(e_{vI_1s_1})^{-1}, \bar{K}\}\, h(e_{vI_2s_2})\{h(e_{vI_2s_2})^{-1}, \bar{K}\}\, h(e_{vI_3s_3})\{h(e_{vI_3s_3})^{-1}, V_v\}\right)$$

quantization , $\{\ ,\ \} \rightarrow \frac{1}{i\hbar}[\ ,\ ]$

$$\hat{C}_{L,v} := \frac{8}{(i\hbar)^3 k^4\beta^3}(\beta^2+1)\sum_{\substack{s_1 s_2 s_3\\ =\pm 1}}\ \sum_{I_1 I_2 I_3} s_1 s_2 s_3\ \varepsilon^{I_1 I_2 I_3}$$

$$tr\left(\hat{h}(e_{vI_1s_1})[\hat{h}(e_{vI_1s_1})^{-1}, \hat{\bar{K}}]\, \hat{h}(e_{vI_2s_2})[\hat{h}(e_{vI_2s_2}), \hat{\bar{K}}]\, \hat{h}(e_{vI_3s_3})[h(e_{vI_3s_3})^{-1}, V_v]\right)$$

$$\hat{\bar{K}} = -\frac{1}{i\hbar\beta^2}\left[\sum_v \hat{C}_{0,v}\ ,\ \sum_v \hat{V}_v\right]$$

$\hat{C}_v = \hat{C}_{0,v} + \hat{C}_{L,v}$ actually quantize $sgn(\det e)\ C(x)$

$$\hat{C}_v \circlearrowleft \mathcal{H}_r$$

$\uparrow$

SU(2) gauge inv.

Diffeomorphism constraint $\quad C_a = \frac{2}{\kappa\beta} F^k_{ab} E^b_k$ cannot be quantized

$$C_j \equiv 2 \underset{\substack{\uparrow \\ \text{triad}}}{e^a_j} C_a = \frac{4}{\kappa\beta} F^k_{ab} \underbrace{\frac{E^a_j E^b_k}{\sqrt{\det q}}}$$

$$\underline{\underline{\mathrm{tr}\left(\tau_j F_{ab} [E^a, E^b]\right)}} = \mathrm{tr}\left(\tau_j F^i_{ab} \frac{\tau_i}{2} E^a_k E^b_l \underbrace{\left[\frac{\tau_k}{2} \frac{\tau_l}{2}\right]}_{\varepsilon_{klm}\frac{\tau_m}{2}}\right)$$

$$= \frac{1}{4} F^i_{ab} E^a_k E^b_l \, \varepsilon_{klm} \underbrace{\mathrm{tr}\left(\tau_j \tau_i \tau_m\right)} \qquad \vec{\tau} = -i\vec{\sigma}$$

$$= i \, \mathrm{tr}(\sigma_j \sigma_i \sigma_m)$$

$$= i \cdot 2i \, \varepsilon_{jim}$$

$$= -\frac{1}{2} F^i_{ab} E^a_k E^b_l \underbrace{\varepsilon_{klm} \varepsilon_{jim}}_{2\delta^j_{[k}\delta^i_{l]}}$$

$$= - F^i_{ab} E^a_j E^b_i$$

$$C_j(x) = -\frac{4}{\kappa\beta} \mathrm{tr}\left(\tau_j F_{ab} \frac{[E^a, E^b]}{\sqrt{\det q}} \right)$$

can be quantized similar to C_0

- $$\mathrm{sgn}(\det e) \frac{[E^a \; E^b]}{\sqrt{\det q}} = -\frac{4}{\kappa\beta} \varepsilon^{abc} \{ A_c \; V(R) \}$$

K. Giesel & T. Thiemann AQG I 2006

$$\mathrm{sgn}(\det e)\, C_j = \frac{16}{\kappa^2\beta^2} \varepsilon^{abc} \mathrm{tr}\left(\tau_j F_{ab} \{A_c, V(R)\}\right)$$

- $$h(\alpha_{IJ}) = 1 + \int_{S_{IJ}} dx^I dx^J F_{IJ} + O(\mu^3)$$

$\uparrow$ (no sum)

$\partial S_{IJ} = \alpha_{IJ}$

- $$h(e_I) \{ h(e_I)^{-1}, V_v \} = -\int_{e_I} dx^I \{ A_I(\vec{x}), V_v \} + O(\mu^2)$$

average over 8 cubes at v

$$C_{j,v} = -\frac{2}{K^2\beta^2} \sum_{\substack{s_1 s_2 s_3 \\ =\pm 1}} \sum_{I_1 I_2 I_3 = 1}^{3} s_1 s_2 s_3 \, \varepsilon^{I_1 I_2 I_3}$$

$$\mathrm{tr}\left(\tau_j \, h(\alpha_{I_1 s_1 I_2 s_2}(v)) \, h(e_{v, I_3 s_3}) \left\{ h(e_{v I_3 s_3})^{-1}, V_v \right\} \right)$$

$$\hat{C}_{j,v} = -\frac{2}{i\hbar K^2 \beta} \sum_{\substack{s_1, s_2, s_3 \\ =\pm 1}} \sum_{I_1, I_2, I_3 = 1}^{3} s_1 s_2 s_3 \, \varepsilon^{I_1 I_2 I_3}$$

$$\mathrm{tr}\left(\tau_j \, \hat{h}(\alpha_{I_1 s_1 I_2 s_2}(v)) \, \hat{h}(e_{v I_3 s_3}) \left[\hat{h}(e_{v I_3 s_3})^{-1}, \hat{V}_v \right] \right)$$

$\hat{C}_{j,v}$ is gauge covariant $\quad \hat{C}_{j,v} \to \hat{C}_{k,v} \Lambda^k{}_j \quad \Lambda \in SO(3)$

$$\sum_{j=1}^{3} \hat{C}_j \hat{C}_j$$