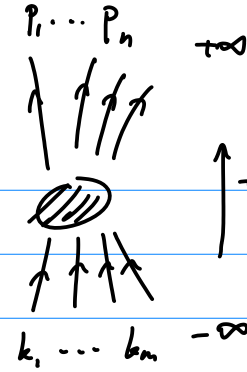
Scattering amplitude : LSZ reduction formular from $\langle 0 | O_1 \cdots O_n | 0 \rangle$



S-matrix : $S = {}_{out}\langle p_1 \cdots p_n | e^{iHT} | k_1 \cdots k_m \rangle_{in}$

free states　　free states

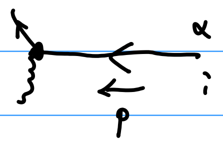
$S = 1 + iT$

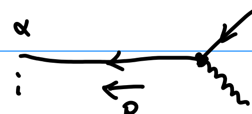
all interaction.

if $m = n$: $(2\pi)^4 \delta(p_1 - k_1) \cdots \delta(p_n - k_n)$

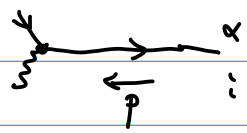
+ perm.

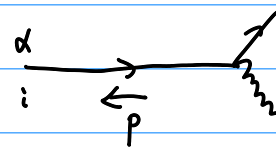
additional rules for T (external lines)

- 

$= u^\alpha_i(p)$ (quark state)


$= \bar{u}^\alpha_i(p)$

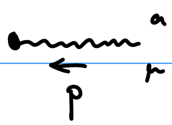


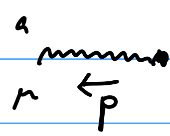
$= \bar{v}^\alpha_i(p)$ (anti-quark state)


$= v^\alpha_i(p)$

$(\gamma^\mu p_\mu - m) u(p) = 0$　　$\bar{u}(p) = u(p)^\dagger \gamma^0$

$(\gamma^\mu p_\mu + m) v(p) = 0$　　$\bar{v}(p) = v(p)^\dagger \gamma^0$

- 

$= \varepsilon^a_\mu(p)$


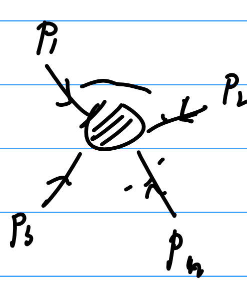
$= \varepsilon^a_\mu(p)^*$

$\partial^2 A^a_\mu = 0 \rightarrow p^2 \varepsilon^a_\mu(p) = 0$

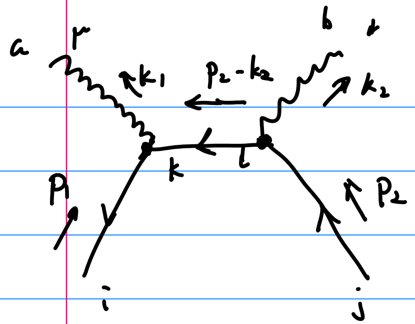
$\partial^\mu A^a_\mu = 0 \rightarrow p^\mu \varepsilon^a_\mu(p) = 0$

$A^a_\mu \sim A^a_\mu + D_\mu \chi^a \rightarrow \varepsilon^a_\mu(p) \sim \varepsilon^a_\mu + p_\mu \chi^a$

- overall $(2\pi)^4 \delta^{(4)}(\Sigma p)$ of momentum conservation



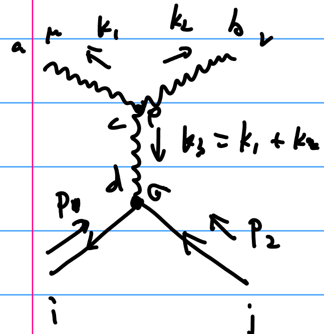
example: $q\bar{q} \to 2$ gluons @ order g^2



$$(2\pi)^4 \delta^{(4)}(k_1+k_2-p_1-p_2) \qquad (ig)^2$$

$$\bar{v}^i(p_1)\gamma^\mu (T^a)^i_k \frac{i\,\delta_{kl}}{\not{p}_2 - \not{k}_2 - m} \gamma^\nu (T^b)^l_j u^j(p_2)$$

$$\varepsilon^a_\mu(k_1)^* \; \varepsilon^b_\nu(k_2)^*$$



$$(2\pi)^4 \delta^{(4)}(k_1+k_2-p_1-p_2) \qquad ig^2$$

$$\frac{-i\,\delta^{cd} g^{\rho\sigma}}{k_3^2} \bar{v}^i(p_1)\, \gamma_\sigma (T^d)^i_j u^j(p_2)$$

$$V^{\mu\nu\rho}_{abc}(k_1, k_2, k_3)\; \varepsilon^a_\mu(k_1)^* \, \varepsilon^b_\nu(k_2)^* \Big|_{k_3 = k_2 + k_1}$$

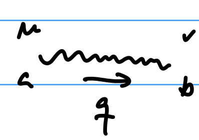
$$V^{\mu\nu\rho}_{abc} = -f^{abc}\left[g^{\mu\nu}(k_1-k_2)^\rho + g^{\nu\rho}(k_2-k_3)^\mu + g^{\mu\rho}(k_3-k_1)\right]$$

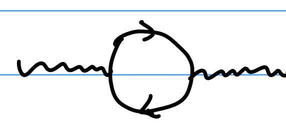
HW amplitude of 

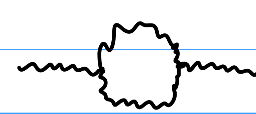
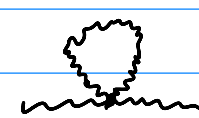
radiative corrections (1-loop divergence)

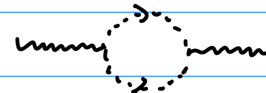
gluon 2-pt function

$$O(g^0)$$

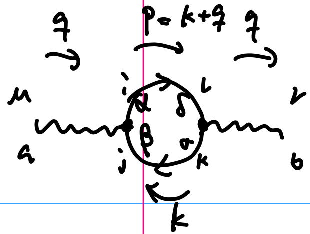
$\langle 0| T A^a_\mu(x) A^b_\nu(y) |0\rangle \xrightarrow{FT}$ 

$+$ 

$+$  $+$ 

$+$ 

$$O(g^2)$$



$$(-1)\int\frac{d^4k}{(2\pi)^4}(ig)^2\,(\gamma^\nu)_{\sigma\delta}(T^b)_{kl}\delta_{li}\left(\frac{i}{\not p-m}\right)_{\delta\alpha}(\gamma^\mu)_{\alpha\beta}(T^a)_{ij}\left(\frac{i}{\not k-m}\right)_{\beta\sigma}\delta_{jk}\Bigg|_{p=k+q}$$

↑ fermion loop

$$=-(ig)^2\,\underbrace{tr\left(T^aT^b\right)}_{C(N)\,\delta^{ab}}\int\frac{d^4k}{(2\pi)^4}\,tr\left(\gamma^\mu\frac{i}{\not k-m}\gamma^\nu\frac{i}{\not p-m}\right)_{p=k+q}$$

$$=-(ig)\,C(N)\,\delta^{ab}\int\frac{d^4k}{(2\pi)^4}\,tr\left(\gamma^\mu\frac{i(\not k+m)}{k^2-m^2}\gamma^\nu\frac{i(\not p+m)}{(k+q)^2-m^2}\right)$$

trace identities : $tr(\gamma^\mu)=0$ $\quad tr(\gamma^\mu\gamma^\nu)=4g^{\mu\nu}$ $\quad \{\gamma^\mu,\gamma^\nu\}=-2g^{\mu\nu}$

$$\underbrace{tr(\gamma^{\mu_1}\cdots\gamma^{\mu_n})}_{\text{odd number}}=0$$

$\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$

$$tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma)=4\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}+g^{\mu\sigma}g^{\nu\rho}\right)$$

$$tr(\gamma^5)=tr(\gamma^5\gamma^\mu\gamma^\nu)=0$$

$$tr(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma)=-4i\,\varepsilon^{\mu\nu\rho\sigma}$$

$$tr(\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_n})=tr(\gamma^{\mu_n}\cdots\gamma^{\mu_2}\gamma^{\mu_1})$$

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$$=(ig)^2\,n_f\,C(N)\,\delta^{ab}\;4\int\frac{d^4k}{(2\pi)^4}\,\frac{k^\mu(k+q)^\nu+k^\nu(k+q)^\mu-g^{\mu\nu}\left(k\cdot(k+q)-m^2\right)}{(k^2-m^2)\left((k+q)^2-m^2\right)}$$

↑ number of fermion species

$$k\to\infty\quad\int d^4k\,\frac{k^2}{k^4}\;\to\;\Lambda^2\text{-divergent}$$

more careful : → log- divergent

$m^2\Lambda^2$

$$\frac{1}{AB}=\int_0^1 dx\,\frac{1}{\left(xA+(1-x)B\right)^2}$$

x: Feynman parameter

general
$$\frac{1}{A_1^{m_1}\cdots A_n^{m_n}}=\int_0^1 dx_1\cdots dx_n\,\delta\left(\Sigma x_i-1\right)\frac{\prod_i x_i^{m_i-1}}{\left(\Sigma x_iA_i\right)^{\Sigma m_i}}\,\frac{\Gamma(m_1+\cdots+m_n)}{\Gamma(m_1)\cdots\Gamma(m_n)}$$

$$\frac{1}{(k^2-m^2)((k+q)^2-m^2)} = \int_0^1 dx \frac{1}{(k^2+2xk\cdot q+xq^2-m^2)^2} = \int_0^1 dx \frac{1}{(\ell^2-\Delta)^2}$$

$$\ell = k + xq \qquad k^2+2xk\cdot q+xq^2-m^2 = (k+xq)^2 - x^2q^2 + xq^2 - m^2$$

$$= \ell^2 + \underbrace{x(1-x)q^2 - m^2}_{-\Delta}$$

$$d^4k \to d^4\ell$$

numerator $= 2\ell^\mu\ell^\nu - g^{\mu\nu}\ell^2 - 2x(1-x)q^\mu q^\nu + g^{\mu\nu}(m^2 + x(1-x)q^2)$

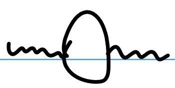
+ linear to ℓ

wick rotation $\ell^0 = i\ell_E \qquad \ell^2 = g_{\mu\nu}\ell^\mu\ell^\nu = \ell^{0\,2} - (\ell^i)^2 = -\ell_E^2 - (\ell^i)^2$

$$= -\ell_E^2$$

$$d^4k = d^4\ell = i\,d^4\ell_E$$

$$\int d^4\ell_E \frac{\ell_E^\mu}{(\cdots)} = 0 \qquad \int d^d\ell \frac{\ell^\mu\ell^\nu}{(\cdots)} = \frac{1}{d} g^{\mu\nu} \int d^d\ell \frac{\ell^2}{(\cdots)}$$

 $= -4g^2\, i\, n_f\, C(N)\, \delta^{ab} \int_0^1 dx \int \frac{d^d\ell_E}{(2\pi)^d} \frac{-\frac{2}{d}g^{\mu\nu}\ell_E^2 + g^{\mu\nu}\ell_E^2 - 2x(1-x)q^\mu q^\nu + g^{\mu\nu}(m^2+x(1-x)q^2)}{(\ell_E^2+\Delta)^2}$