$$Z = \underline{\int DAD\psi D\bar{\psi}\; e^{iS}} \qquad S = \int d^dx \left(-\frac{1}{4}\mathrm{tr}(F^2) + \bar{\psi}(\gamma^\mu D_\mu - m)\psi\right)$$

$SU(N)$

$$\frac{Z}{[\text{gauge volume}]} \qquad G(A) = \partial^\mu A_\mu^a - \omega^a(x)$$

$$G(A^\alpha) = \partial^\mu\left(A_\mu^a + \frac{1}{g}D_\mu \alpha^a\right) - \omega^a(x)$$

FP trick $\qquad 1 = \int \delta(G(A^\alpha))\, D\alpha\, \underbrace{\det\left(\frac{\delta G}{\delta \alpha}\right)(A^\alpha)}_{\text{FP determinant}}$

$$\int DAD\psi D\bar{\psi}\; e^{iS} = \int D\alpha\, DAD\psi D\bar{\psi}\; e^{iS[A,\psi,\bar{\psi}]}\; \delta(G(A^\alpha))\det\left(\frac{\delta G}{\delta \alpha}\right)(A^\alpha)$$

$$= \int D\alpha\; DA^\alpha D\psi^\alpha D\bar{\psi}^\alpha\; e^{iS[A^\alpha,\psi^\alpha,\bar{\psi}^\alpha]}\; \delta(G(A^\alpha))\det\left(\frac{\delta G}{\delta \alpha}\right)(A^\alpha)$$

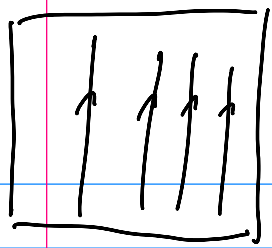
due to $\quad DAD\psi D\bar{\psi} = DA^\alpha D\psi^\alpha D\bar{\psi}^\alpha$

$\qquad S[A,\psi,\bar{\psi}] = S[A^\alpha,\psi^\alpha,\bar{\psi}^\alpha]$

change of variable $A^\alpha \to A$

$$Z = \underbrace{\int D\alpha}_{\substack{\text{Volume}\\\text{of gauge}\\\text{obit}}} \int DAD\psi D\bar{\psi}\; e^{iS[A,\psi,\bar{\psi}]}\; \delta(G(A))\det\left(\frac{\delta G}{\delta \alpha}\right)(A)$$

$$\frac{Z}{\text{gauge volume}} = \underbrace{\int DAD\psi D\bar{\psi}\; e^{iS[A,\psi,\bar{\psi}]}\; \delta(G(A))\det\left(\frac{\delta G}{\delta \alpha}\right)(A)}_{\text{indep. of gauge fixing in particular indep. of } \omega^a(x)}$$



$$\underbrace{\int D\omega^a \, e^{-i\int d^dx \frac{\omega^a\omega^a}{2\xi}}}_{\text{overall factor}} \left(\frac{Z}{\text{gauge volume}}\right) \equiv \mathcal{Z}$$

$$\int D\omega^a \, e^{-i\int d^dx \frac{\omega^a\omega^a}{2\xi}} \, \delta\left(\partial^\mu A^a_\mu - \omega^a\right) = e^{-i\int d^dx \frac{(\partial^\mu A^a_\mu)^2}{2\xi}}$$

$$\mathcal{Z} = \int DAD\psi D\bar\psi \, e^{iS[A,\psi,\bar\psi] - i\int d^dx \frac{(\partial^\mu A^a_\mu)^2}{2\xi}} \det\left(\frac{\delta G}{\delta\alpha}\right)(A)$$

FP determinant

$$\det M = \int dc_\alpha \, d\bar c_\beta \, \exp\left(-i\,\bar c_\alpha M_{\alpha\beta} c_\beta\right)$$

$M_{\alpha\beta}$

Grassmann variable

$$\det\left(\frac{\delta G}{\delta\alpha}\right) = \det\left(\frac{1}{g}\partial^\mu D_\mu\right) = \int Dc^a(x) D\bar c^a(x) \, e^{-i\int d^dx \, \bar c^a (\partial^\mu D^{ab}_\mu) c^b}$$

$c^a(x), \bar c^a(x)$: ghost field $\qquad D^{ab}_\mu c^b = \partial_\mu c^b + f^{abc} A^b_\mu c^c$

- scalars
- ferimion (Grassmann variable)
- adj. rep of $SU(N)$ (because D_μ in adj. rep)

} wrong spin-statistics

$$\mathcal{L}_{ghost} = -\bar c^a (\partial^\mu D^{ac}_\mu) c^c$$
$$= -\bar c^a \left[\partial^\mu \left(\partial_\mu c^a + g f^{abc} A^b_\mu c^c\right)\right]$$

$$\mathcal{Z} = \int DAD\psi D\bar\psi \, Dc^a D\bar c^a \, e^{i\int d^dx \, \mathcal{L}_{FP}}$$

$$\mathcal{L}_{FP} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{1}{2\xi} \underline{(\partial^\mu A^a_\mu)(\partial^\nu A^a_\nu)} + \bar\psi^i (i\gamma^\mu D^{ij}_\mu - m)\psi^j - \bar c^a (\partial^\mu D^{ac}_\mu) c^c$$

N×N matrix $A_\mu = A_\mu^a T^a \rightarrow U A_\mu U^{-1} + \frac{i}{g} U \partial_\mu U^{-1}$ $\qquad U = U^i{}_j \in SU(N)$ in fund. rep

$\psi \rightarrow U^i{}_j \psi^j \quad , \quad \bar\psi^i \rightarrow \bar\psi^j (U^{-1}{}_j{}^i)$

$c = c^a T^a \rightarrow U c U^{-1}$

$\bar c = \bar c^a T^a \rightarrow U \bar c U^{-1}$

HW check the gauge transf. of $\mathcal{L}_{FP}$.

BRST Lagrangian & BRST inv. (Becchi - Rouet - Stora - Tyutin)

$$Z = \int DA D\psi D\bar\psi Dc D\bar c \, \underset{\substack{\uparrow \\ \text{auxiliary} \\ \text{scalar field}}}{DB^a(x)} \, e^{i\int d^dx \mathcal{L}_{BRST}}$$

$$\mathcal{L}_{BRST} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar\psi(i\cancel{\partial} - m)\psi + \bar c^a(-\partial^\mu D^{ac}_\mu)c^c$$
$$- \frac{\xi}{2} B^a B^a + B^a \partial^\mu A^a_\mu$$

$\mathcal{L}_{BRST}$ has BRST symmetry (global $\underset{\text{fermionic}}{\underbrace{}}$ symmetry)

$$\delta_{BRST} A^a_\mu = \varepsilon D^{ac}_\mu c^c(x)$$

ε fermionic

$$\delta_{BRST} \psi^j = ig\, \varepsilon c^a (T^a)^j{}_k \psi^k$$

$\varepsilon c(x)$: gauge parameter

$$\delta_{BRST} c^a = -\frac{1}{2} g \varepsilon f^{abc} c^b c^c$$

$$\delta_{BRST} \bar c^a = \varepsilon B^a$$

$$\delta_{BRST} B^a = 0$$

HW check $\int d^dx \mathcal{L}_{BRST}$ BRST inv.

BRST is remnant of gauge symmetry (∞-dim) after gauge fixing (1-dim)

HW $\delta^2_{BRST}(A, \psi, c, \bar{c}, B) = 0$

$dd f = 0$