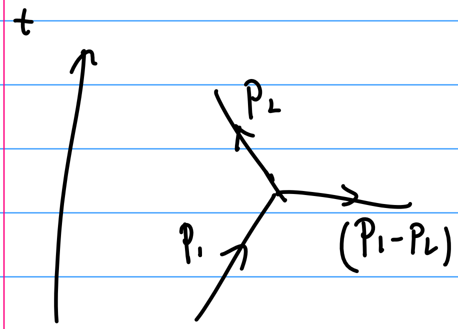
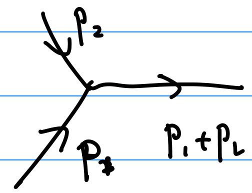
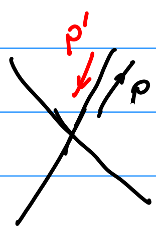
$$\hbar = 1 = c \qquad [E] = [m] = [\lambda]^{-1} \qquad E = \hbar \frac{1}{t}, \quad E = mc^2$$



$$(p_1 - p_2)^2 = p_1^2 + p_2^2 - 2 p_1 \cdot p_2 < 0$$



$$(p_1 + p_2)^2 = p_1^2 + p_2^2 + 2 p_1 \cdot p_2 < 0$$



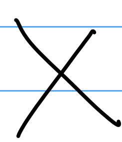
$P = -P'$

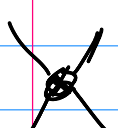
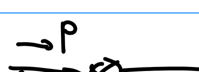
$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 - \frac{1}{2} m_0^2 \phi_0^2 - \frac{\lambda_0 \mu^{4-d}}{4!} \phi_0^4$$

$$= \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda \mu^{4-d}}{4!} \phi^4 \quad \leftarrow \mathcal{L}_{ren}$$

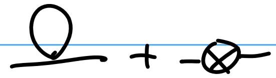
$$+ \frac{1}{2} \delta_Z (\partial_\mu \phi)^2 - \frac{1}{2} \delta_m \phi^2 - \frac{\delta_\lambda \mu^{4-d}}{4!} \phi^4 \quad \leftarrow \delta \mathcal{L}$$

At this moment, all m^2, λ, δ_Z, δ_m, δ_λ are free parameters

 $= \dfrac{i}{p^2 - m^2 + i\varepsilon}$  $= -i\lambda \mu^{4-d}$

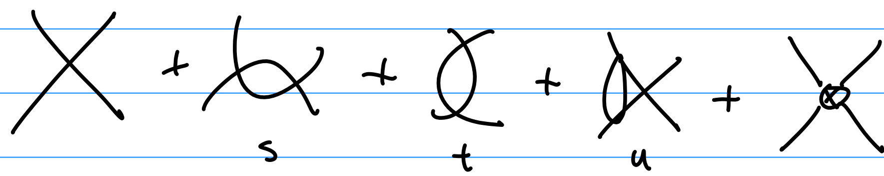
 $= -i\delta_\lambda \mu^{4-d}$  $i(p^2 \delta_Z - \delta_m)$

2 pt func $=$ 

 $\equiv -i\Sigma^2(p^2)$

$$= \frac{i}{p^2 - m^2 - \Sigma^2(p^2) + i\varepsilon}$$

$$-i\Sigma^2(p^2) = \frac{i\lambda m^2}{16\pi^2\varepsilon} + \frac{i\lambda m^2}{32\pi^2}\left[1 - \gamma + \ln\left(\frac{4\pi\mu^2}{-m^2}\right)\right] + i\left(p^2\delta_2 - \delta_m\right) + O(\varepsilon)$$

4pt function $=$ 

$$\Gamma^{(4)}(p_1 \cdots p_4) = -i\lambda\mu^\varepsilon + \frac{3i\lambda^2\mu^\varepsilon}{16\pi^2\varepsilon} - \frac{i\lambda^2\mu^\varepsilon}{32\pi^2}\left[3\gamma + F(s,m,\mu) + F(t,m,\mu) + F(u,m,\mu)\right] - i\delta_\lambda\mu^\varepsilon + O(\varepsilon)$$

determine $\underline{\delta_m, \delta_2, \delta_\lambda}$ in terms of λ, m. $\to$ renormalization condition

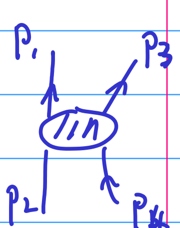
- (1) 2pt function $= \dfrac{i}{p^2 - m^2 + i\varepsilon}$ $(*)$ i.e $\Sigma^2(p^2) = 0$

we need to set an energy scale, M

massive $(m^2 \neq 0)$, $(*)$ is satisfied at $p^2 = m^2$

$p_1 = p_2 = (m, 0, 0, 0)$ massless $(m^2 = 0)$, $(*)$ is satisfied at $p^2 = -M^2$

(2) 4pt function $\to$ $\Gamma^{(4)}(p_1 \cdots p_4) = -i\lambda\mu^\varepsilon$ $(**)$ $s + t + u = 4m^2$



$s = (p_1 + p_2)^2$
$= p_1^2 + p_2^2$
$2p_1 \cdot p_2 = 4m^2$
$t = (p_1 + p_3)^2 = 0$

massive $(m^2 \neq 0)$, $(**)$ is satisfied at $s = 4m^2$, $\underline{\underline{t = u = 0}}$

massless $(m^2 = 0)$, $(**)$ is satisfied at $s = t = u = -M^2$

$$\text{2pt function} = \frac{i}{(1 + i\delta_2)p^2 - m^2\left(1 - \frac{i\lambda}{16\pi^2\varepsilon} + \frac{i\lambda}{32\pi^2}\left[1 - \gamma + \ln\left(\frac{4\pi\mu^2}{-m^2}\right)\right]\right) - \delta_m}$$

Apply (1) : $\delta_2 = 0$ $\quad \delta_m = m^2\left(-\frac{i\lambda}{16\pi^2\varepsilon} + \frac{i\lambda}{32\pi^2}\left[1-\gamma+\ln\left(\frac{4\pi\mu^2}{-m^2}\right)\right]\right)$

only for 1-loop renormalization, there are non trivial corrections from 2-loop & higher

4pt function :

$$F(t,m,\mu) = \int_0^1 dz \ln\left[\frac{-t\,z(1-z)-m^2}{4\pi\mu^2}\right]$$

$$\Gamma^{(4)}(p_1 \cdots p_4) = -i\lambda\mu^{\varepsilon} + \frac{3i\lambda^2\mu^{\varepsilon}}{16\pi^2\varepsilon} - \frac{i\lambda^2\mu^{\varepsilon}}{32\pi^2}\left[3\gamma + F(s,m,\mu) + F(t,m,\mu) + F(u,m,\mu)\right]$$
$$-i\delta_\lambda\mu^{\varepsilon} + O(\varepsilon)$$

Apply (2) : for $m^2 \neq 0$ $\Rightarrow$ $\delta_\lambda = \frac{3\lambda^2}{16\pi^2\varepsilon} - \frac{\lambda^2}{32\pi^2}\left[3\gamma + F(4m^2, m, \mu) + 2F(0, m, \mu)\right]$

$m^2 = 0$, $s = t = u = -M^2$ $\quad \delta_\lambda = \frac{3\lambda^2}{16\pi^2\varepsilon} - \frac{\lambda^2}{32\pi^2}\left[3\gamma + 3F(-M^2, m, \mu)\right] \equiv \delta_\lambda(M)$

All 1-loop div. are cancelled. higher loops: only need to correct values of $\delta_m, \delta_\lambda, \delta_Z$ to cancel higher loop divergence

$\Rightarrow$ $\lambda\phi^4$ theory is finite order by order (in λ) by fixing 3 local counter terms (with 3 renormalization const. $\delta_m, \delta_\lambda, \delta_Z$)

$\lambda\phi^4$ theory is renormalizable.

<u>Def</u>: A QFT is renormalization if you need only finite # of local counter terms to render it UV finite order by order

otherwise it's called non-renormalizable

4 fund. interactions $\begin{cases} \left.\begin{array}{l}\text{strong} \\ \text{weak} \\ \text{EM}\end{array}\right\} \text{standard model renormalization} \\ \text{gravity} \leftarrow \text{non-renormalizable}\end{cases}$

(non local)

Renormalization group (RG) for babies

QFT dep. on M , $\underbrace{\lambda = \lambda(M)}_{\text{running coupling}}$, $m^2 = m^2(M)$, $\delta_\lambda = \delta_\lambda(M)$

$\delta_Z = \delta_Z(M)$, $\delta_m = \delta_m(M)$

$\mathcal{L}_{Ren}$ dep. on M

How QFT change w.r.t. M , $\beta(M) := M \frac{\partial}{\partial M} \lambda(M)$

QFT_{M_1}

$\Downarrow$ RG flow

QFT_{M_2}

$$\lambda(M) = \lambda_0 Z^2 - \delta_\lambda(M)$$

$$= \lambda_0 (1 + \cancel{\delta_Z(M)})^2 - \delta_\lambda(M)$$

1-loop $\delta_Z = 0$

$$\beta(M) = M \frac{\partial}{\partial M} \lambda(M) = - M \frac{\partial}{\partial M} \delta_\lambda(M)$$