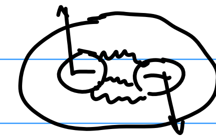
Superconductivity

$\vec{E}=0 \quad \vec{j}\neq 0 \quad , \quad \vec{E}=R\vec{j}$ Ohm's law $\Rightarrow R=0$

electron $\leftrightarrow$ lattice (phonon) $\leftrightarrow$ electron   low temperture strong coupling $\rightarrow$ Confinement of electrons.



Cooper pair $\phi(x)$

Landau-Ginzberg free energy of mean field theory (2nd order phase transition)

$$-F = \mathcal{L}\Big|_{\phi=\phi(\vec{x}),\ \vec{A}=\vec{A}(\vec{x}),\ A^0=0}$$

$$= \frac{1}{2}(\vec{\nabla}\times\vec{A})^2 + \left|(\vec{\nabla}-ie\vec{A})\phi\right|^2 + \mu|\phi|^2 + \lambda|\phi|^4$$

$\mu = \alpha(T-T_c)$, T_c : critical tempretme. $\alpha>0$

gauge symm. $\phi \to e^{i\Lambda(x)}\phi$, $\vec{A}\to\vec{A}+\frac{1}{e}\vec{\nabla}\Lambda$

order parameter : ϕ

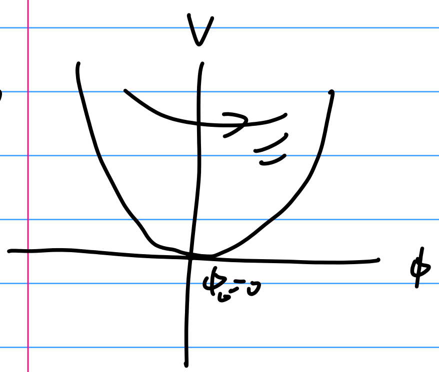
ground state : minimize free energy $\rightarrow$ Ginzberg-Landau Eqn.

- $\mu\phi + 2\lambda|\phi|^2\phi - (\vec{\nabla}-ie\vec{A})^2\phi = 0$
- $\vec{\nabla}\times\vec{B} = \vec{j}$, $\vec{j} = -ie(\phi^*\vec{\nabla}\phi - \phi\vec{\nabla}\phi^*) - 2e^2|\phi|^2\vec{A}$

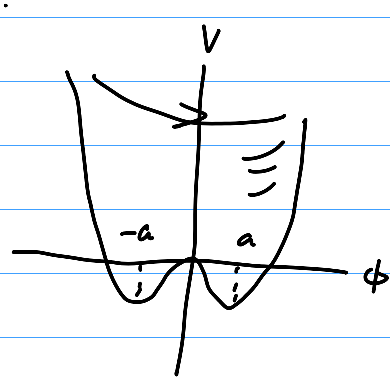
$(\vec{B}=\vec{\nabla}\times\vec{A})$

ground state : $\vec{A}=0$, $\phi=\phi_0$ const. $\rightarrow \mu\phi_0 + 2\lambda|\phi_0|^2\phi_0 = 0$

$\mu>0$



$\mu<0$



$\phi_0 = \sqrt{\frac{-\mu}{2\lambda}}$ or $\phi_0 = 0$

$\equiv a = \sqrt{\frac{\alpha}{2\lambda}}(T_c-T)^{\frac{1}{2}}$

symmetric phase $\phi_0 = 0$ ~~Symm~~ phase $\phi_0 = a \neq 0$

$$\phi(x) = \left(a + \frac{\phi_1(x)}{\sqrt{2}} \right) e^{i\theta(x)}$$

removed by gauge transf

$\rightarrow$ eaten by $\vec{A}$ and give mass to $\vec{A}$

$$\vec{\nabla} \times \vec{B} = \vec{j} \qquad \vec{j} = -ie\left(\phi^* \vec{\nabla} \phi - \phi \vec{\nabla} \phi^* \right) - 2e^2 |\phi|^2 \vec{A}$$

$$\left(\begin{array}{c} \phi_0 = a \ , \ \vec{A} = 0 \\ \text{ground state} \end{array} \right)$$

$$= -2e^2 a^2 \vec{A} \cancelto{0}{- ie\left[\left(a + \frac{\phi_1}{\sqrt{2}}\right) \vec{\nabla} \left(a + \frac{\phi_1}{\sqrt{2}}\right) - \left(a + \frac{\phi_1}{\sqrt{2}}\right) \vec{\nabla} \left(a + \frac{\phi_1}{\sqrt{2}}\right) \right]} + O(\phi_1 A)$$

$$\simeq -2e^2 a^2 \vec{A} = -\frac{e^2 \alpha}{\lambda} (T_c - T) \vec{A} \qquad \text{"London Eq"}$$

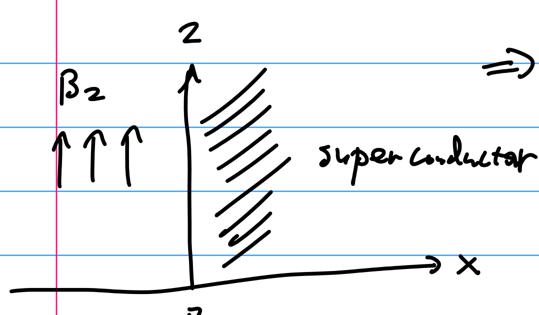
correspond to the mass term of $\vec{A}$

U(1) gauge symm is spontaneously broken

$$\vec{E} = \frac{\partial \vec{A}}{\partial t} = 0 \quad \text{but we have} \quad \vec{j} = -\frac{e^2 \alpha}{\lambda} (T_c - T) \vec{A} \neq 0$$

$$\vec{E} = R \vec{j} \quad \Rightarrow \quad R = 0 \qquad \text{superconductivity}$$

$$\vec{\nabla} \times \vec{B} = \vec{j} = -2e^2 a^2 \vec{A} \quad \Rightarrow \quad \underbrace{\vec{\nabla} \times \vec{\nabla} \times \vec{B}}_{\vec{\nabla} \cdot \underbrace{(\vec{\nabla} \cdot \vec{B})}_{\vec{\nabla} \cdot \vec{B} = 0} - \nabla^2 \vec{B}} = -2 e^2 a^2 \underbrace{\vec{\nabla} \times \vec{A}}_{\vec{B}}$$

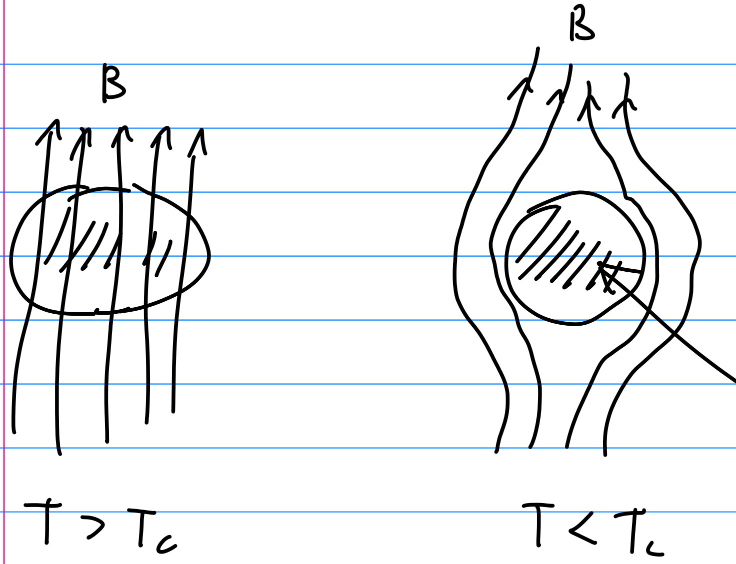


$$\Rightarrow \quad \nabla^2 \vec{B} = 2 e^2 a^2 \vec{B}$$

$$\vec{B} = B^x(x) \hat{x} + B^y(x) \hat{y} + B^z(x) \hat{z}$$

$$\partial_x^2 B^i = 2a^2e^2 B^i \rightarrow B^i(x) = Ce^{-\sqrt{2}a^2e^2x}$$

$$\frac{1}{\sqrt{2}a^2e^2} = \text{Loudon penetration depth}$$



Meissner effect

<u>Lattice gauge theory</u> (QCD)

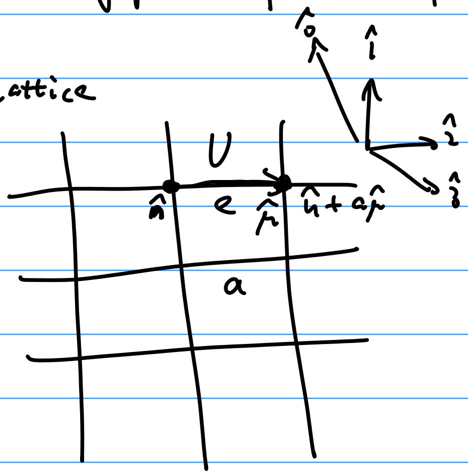
- Wilson action
- fermion coupling (species doubling)

$$S_{QCD} = \frac{1}{4}F^2 + \bar{\psi}\cancel{D}\psi$$

Motivation: Asymptotic freedom of QCD.

$$\int DADFD\bar{\psi}\, e^{-S_{QCD}}$$

4 d. Lattice



$$U_\mu(\hat{n}) = P e^{ig\int_e A_\mu \hat{e}^\mu dt} \in SU(N)$$

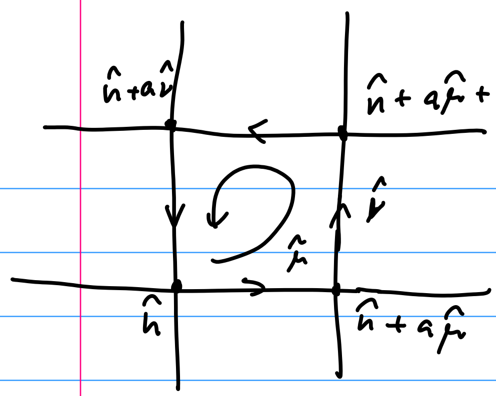
T^a Hermitian $\quad tr(T^aT^b) = \frac{1}{2}\delta_{ab}$

$$A_\mu = A_\mu^a T^a, \quad T^a \in su(N)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu A_\nu]$$

- $U_{-\mu}(\hat{n} + a\hat{\mu}) = U_\mu(\hat{n})^+ = U_\mu(\hat{n})^{-1}$
- gauge transformation : $U_\mu(\hat{n}) \rightarrow \underline{g(\hat{n})\, U_\mu(\hat{n})\, g(\hat{n}+a\hat{\mu})^+}$

$$A_\mu \rightarrow g A_\mu g^{-1} + g\partial_\mu g^{-1}$$



$$U_\mu(\hat{n})\, U_\nu(\hat{n}+a\hat{\mu})\, U_{-\mu}(\hat{n}+a\hat{\mu}+a\hat{\nu})\, U_{-\nu}(\hat{n}+a\hat{\nu})$$
$$= U_L(\hat{n})$$

gauge transf $\quad U_L(\hat{n}) \to g(\hat{n})\, U_L(\hat{n})\, g(\hat{n})^{-1}$

$P_{\mu\nu}$ plaquette in (μ,ν) plane $\quad F_{\mu\nu} \to g(x)\, F_{\mu\nu}(x)\, g(x)^{-1}$

$$\mathrm{tr}\left(U_L(\hat{n}) \cdots U_L(\hat{n}) \right) \quad \text{gauge inv.}$$

$$\mathrm{tr}\left(U_L(\hat{n}) \right) \quad \text{Wilson loop.}$$

Wilson action

$$S = \frac{1}{2} \sum_{P_{\mu\nu}} \frac{1}{g^2}\, \mathrm{tr}\left(U_L(\hat{n}) + U_L(\hat{n})^\dagger - 2 \right)$$

$P_{\mu\nu} \Leftrightarrow \hat{n}$ base pt. of $P_{\mu\nu}$

Nonabelian stocks Thm

for $P_{\mu\nu}$. $\quad U_L(\hat{n}) = \exp\left[i a^2 g\, U \left(\partial_\mu A_\nu - \partial_\nu A_\mu + ig [A_\mu, A_\nu] \right) U^\dagger \right]$

$$= \exp\left(i a^2 g\, U F_{\mu\nu}(\hat{n}) U^{-1} \right) \quad \text{as } a \to 0$$

$a \to 0$ classically : Naive continuum limit

$$U_L(\hat{n}) = 1 + i a^2 g\, U F_{\mu\nu} U^{-1} - \underline{\frac{a^4 g^2}{2} U F_{\mu\nu}^2 U^{-1}} + O(a^6)$$

$$S \simeq \frac{1}{2} \sum_{P_{\mu\nu}} \frac{1}{g^2} \mathrm{tr}\left[-\frac{a^4 g^2}{2} U F_{\mu\nu}^2 U^{-1} \times 2 + \cdots \right]$$

$$= -\frac{1}{2} \sum_{P_{\mu\nu}} a^4 \, \mathrm{tr}\left(F_{\mu\nu}^2 \right) \qquad a \to 0$$

$$\longrightarrow -\frac{1}{2} \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F_{\mu\nu} \right)$$

Euclidean pathintegral of QCD.

$$\int \prod_{e} dU_\mu(\hat{n}) \, \exp(S) \xrightarrow{a \to 0} \int DA_\mu \, e^{-\frac{1}{2}\int d^4x \, \mathrm{Tr}(F^2)}$$

intuitively

$dU_\mu(\hat{n})$ Haar measure on $SU(N)$

$\int dU_\mu(\hat{n}) = 1$ and left & right inv.