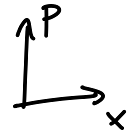
# Semiclassical states of LQG




Recall Harmonic oscillator $\hat{a}|z\rangle = z|z\rangle$

coherent state label $z = x+ip$

complex coordinate of phase space $\mathbb{R}^2 \simeq \mathbb{C}$

LQG coherent state : on a single edge

$$L^2(SU(2)) \ni \psi^t_g(h) = \sum_{\substack{j=0 \\ j\in \mathbb{Z}/2}}^{\infty} (2j+1)\, e^{-\frac{t}{2}j(j+1)}\, \chi_j(g h^{-1}) \qquad t = \frac{\ell_p^2}{a^2}$$

$h \in SU(2)$

SU(2) character of j-rep.

$t$: dimensionless semiclassicality parameter

$\vec{\tau} = -i\vec{\sigma}$

$$g = e^{-ip^j\tau^j/2}\, e^{\theta^j\tau^j/2} \in SL(2,\mathbb{C}) \qquad \chi_j = \mathrm{tr}_j D^j$$

$\hbar \to 0 \;\; \| \;\; t \to 0$

$SL(2,\mathbb{C})$ is a complexification of $SU(2)$

$$SL(2,\mathbb{C}) \simeq T^*SU(2) \simeq \mathbb{R}^3 \times S^3$$

$(p^j,\ h = e^{\vec{\theta}\cdot\vec{\tau}/2})$

phase space of LQG on an edge

Complex coordinate of $T^*SU(2)$

$$p^j = \frac{P^j}{a^2}$$

dimensionless flux.

a is a length unit.

On $\gamma$ $$\psi^t_{\vec{g}}(\vec{h}) = \prod_{e\in E(\gamma)} \psi^t_{g(e)}(h(e)) \in \mathcal{H}^0_\gamma$$ not SU(2) gauge inv

$\vec{g}$ complex coordinate of $(T^*SU(2))^{|E(\gamma)|}$

SU(2) gauge inv coherent state

$$\Psi^t_{[g]} = \int_{SU(2)^{|V(\gamma)|}} \prod_v d\mu_H(u_v)\ \psi^t_{\vec{g}^{\vec{u}}}$$

gauge orbit at $g$

$$\vec{g}^{\vec{u}} = \{ h_{s(e)}^{-1}\, g(e)\, h_{t(e)} \}$$

properties of $\psi^t_{g(e)}$ (on single edge $e$)

(1) semiclassicality : $$\langle \psi^t_{g(e)} | \hat{h}(e) | \psi^t_{g(e)} \rangle = e^{\theta^j\tau^j/2} + O(t)$$

$$g(e) = e^{-ip^j(e)\tau^j/2}\, e^{\theta(e)\tau^j/2}$$

$$\langle \psi^t_{g(e)} | \hat{p}^j(e) | \psi^t_{g(e)} \rangle = \underset{\sim}{p^j(e)} + \underline{\underline{O(t)}}$$

(2) overlap function $\left| \langle \psi^t_g | \psi^t_{g'} \rangle \right|^2 \sim O(t^\infty)$ as $t \to 0$

$g = e^{-ip^j\tau^j/2}\, e^{\theta^j\tau^j/2}$ (exponentially suppressed)

$g' = e^{-ip'^j\tau^j/2}\, e^{\theta'^j\tau^j/2}$ when $g$ is not close to $g'$

$$\left| \langle \psi^t_g | \psi^t_{g'} \rangle \right|^2 \sim e^{-\frac{\Delta p^2 + \Delta\theta^2}{t}}$$

when $g$ is close to $g'$

$\Delta p = |\vec{p} - \vec{p}'| \sim O(\sqrt{t})$

$\Delta\theta = |\vec{\theta} - \vec{\theta}'| \sim O(\sqrt{t})$

(3) overcompleteness $\tilde{\psi}^t_g = \frac{\psi^t_g}{\|\psi^t_g\|}$

$$\int_{SL(2,\mathbb{C})} dg\, | \tilde{\psi}^t_g \rangle \langle \tilde{\psi}^t_g | = \mathbb{1}_{L^2(SU(2))}$$

$$dg = \frac{c}{t^3}\, d\mu_H(e^{\theta^j\tau^j/2})\, d^3p \qquad c = \frac{2}{\pi} + O(t^\infty)$$

Expansion of volume operator (Giesel–Thiemann 2007)

Formally $\hat{V}_v = (\hat{Q}_v^2)^{1/4} = \left( \hat{Q}_v^2 - \langle Q_v \rangle^2 + \langle Q_v \rangle^2 \right)^{1/4}$

$$\overset{\shortparallel}{\sqrt{|\hat{Q}|}} = \langle Q_v \rangle^{1/2} \Big( 1 + \underbrace{\frac{\hat{Q}_v^2 - \langle Q_v \rangle^2}{\langle Q_v \rangle^2}}_{\to\ \hat{X}} \Big)^{1/4}$$

$$\langle Q_v \rangle = \langle \psi^t_{\vec{g}} | \hat{Q}_v | \psi^t_{\vec{g}} \rangle = Q_v[\vec{g}] + O(t) > 0$$

$\langle \hat{X} \rangle$ is quantum fluctuation of $\hat{Q}_v$ $\sim O(t)$

$$\hat{V}_v = \langle Q_v\rangle^{1/2}\left[1+\frac{1}{4}\frac{\hat{Q}^2-\langle Q_v\rangle^2}{\langle Q_v\rangle^2} - \frac{3}{32}\left(\frac{\hat{Q}_v^2-\langle Q_v\rangle^2}{\langle Q_v\rangle^2}\right)^2 + \dots\right]$$

truncate this series ; $\hat{V}_v$ is formally written as a polynomial operator of $\hat{p}^j{}_{ref}$

this series may not be convergent , at most an asymptotic expansion (semiclassical expansion)

Indeed the expansion makes sense in computing expectation values of

e.g. $\langle \psi^t_{\vec{g}}|\hat{V}_v|\psi^t_{\vec{g}}\rangle$ , $\langle \psi^t_{\vec{g}}|\hat{C}_v|\psi^t_{\vec{g}}\rangle$

$\langle \psi^t_{\vec{g}}|\hat{C}_{jv}|\psi^t_{\vec{g}}\rangle$ , $\langle \psi^t_{\vec{g}}|\hat{M}|\psi^t_{\vec{g}}\rangle$

$\langle \psi^t_{\vec{g}}|\hat{H}|\psi^t_{\vec{g}}\rangle$ .

if this $\vec{g}$ is the same as $\vec{g}$ in $\langle \hat{Q}_v\rangle$, you can replace $\hat{V}_v$ by the Giesel-Thiemann expansion , and truncate to finite order, higher orders terms give higher order in $t = \frac{\ell_p^2}{a^2}$

$$\Rightarrow \quad \langle \psi^t_{\vec{g}}|\hat{V}_v|\psi^t_{\vec{g}}\rangle = V_v[\vec{g}] + O(t)$$

$$\langle \psi^t_{\vec{g}}|\hat{C}_v|\psi^t_{\vec{g}}\rangle = \underline{\underline{C_v[\vec{g}]}} + O(t)$$

$$\langle \psi^t_{\vec{g}}|\hat{C}_{jv}|\psi^t_{\vec{g}}\rangle = C_{jv}[\vec{g}] + O(t)$$

$$\langle \psi^t_{\vec{g}}|\hat{M}|\psi^t_{\vec{g}}\rangle = M[\vec{g}] + O(t)$$

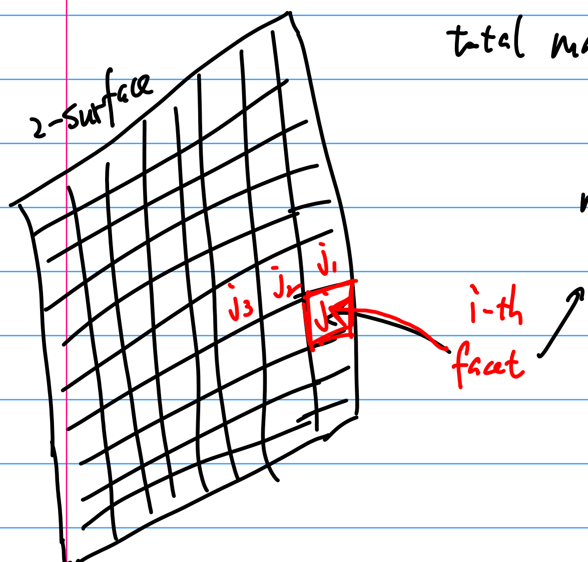
$$\langle \psi^t_{\vec{g}}|\hat{H}|\psi^t_{\vec{g}}\rangle = H[\vec{g}] + O(t)$$

expectation values on $\gamma$ discrete classical quantities on $\gamma$

semiclassical limits of these operators are all correct.

Semiclassical limit of LQG dynamics (Reduced phase space formulation) reproduces classical Einstein gravity : see 2005.00988

## Quantum statistics of a LQG surface

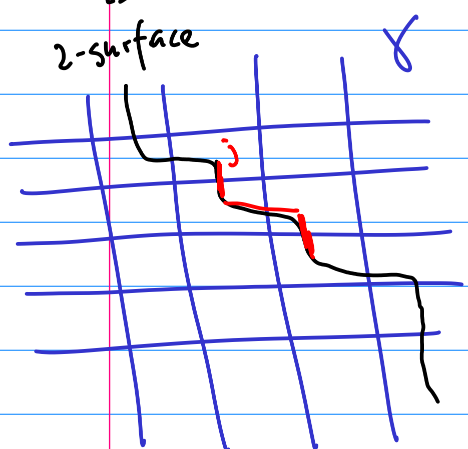



total macroscopic area $A$

microscopic area $8\pi G\hbar\beta\sqrt{j_i(j_i+1)}$

$$A = \sum_i 8\pi G\hbar\beta\sqrt{j_i(j_i+1)}$$

\# of facets $\to \infty$




| | 2-surface in LQG | Gas in a box |
|---|---|---|
| macrostate | $A$ (here we don't fix total number of facets) | $E$ total energy $N$ total # of particles |
| microstates | $\{j_i\}_i$ | $\{\varepsilon_i\}_i$ particle energy |
| | $A = \sum_i 8\pi G\hbar\beta\sqrt{j_i(j_i+1)}$ | $E = \sum_i \varepsilon_i$ |

ensemble of identical facets carrying quantum number $j$

distribution of $j$ $\{n_j\}$ where $n_j$ is the number of facets carrying the same $j>0$

Given a distribution $\{n_j\}$, total number of microstate

$$d[\{n_j\}] = \Big(\sum_{j>0} n_j\Big)! \prod_{j>0} \frac{(2j+1)!}{n_j!}$$

what's the most probably distribution? i.e. maximum of $d[\{n_j\}]$ by variating $\{n_j\}$

Constraint: $C[\{n_j\}] = A - \sum_j n_j \left(8\pi G\hbar\beta \sqrt{j(j+1)}\right) = 0$

$$\delta \ln d[\{n_j\}] - \underset{\substack{\uparrow \\ \text{Lagrangian multiplier.}}}{T^{-1}} \frac{\delta C[\{n_j\}]}{8\pi G\hbar\beta} = 0 \quad + \text{ stirling's approx.}$$

solution $\{\bar{n}_j\}$ Boltzmann distribution $\dfrac{\bar{n}_j}{\sum_j \bar{n}_j} = (2j+1)\, e^{-T^{-1}\sqrt{j(j+1)}}$

$$\sum_{\substack{j=1 \\ j\in\mathbb{Z}/2}}^{\infty} (2j+1)\, e^{-T^{-1}\sqrt{j(j+1)}} = \frac{\sum_j \bar{n}_j}{\sum_j \bar{n}_j} = 1$$

$$\Rightarrow \frac{T_0^{-1}}{2\pi} = 0.274\ldots$$

$$\ln d[\{\bar{n}_j\}] = \ln \underbrace{\Big(\sum_j \bar{n}_j\Big)}_{N}! - \sum_j \ln \bar{n}_j! + \sum_j \bar{n}_j \ln(2j+1)$$

$$= \cancel{N \ln N} - \cancel{N} - \underbrace{\sum_j \left(\bar{n}_j \ln \bar{n}_j - \cancel{\bar{n}_j}\right)} + \cancel{\sum_j \bar{n}_j \ln(2j+1)}$$

$$- \sum_j \underbrace{N(2j+1)\, e^{-T^{-1}\sqrt{j(j+1)}}}_{\bar{n}_j} \left(\ln N + \ln(2j+1) - T^{-1}\sqrt{j(j+1)}\right)$$

$$\hookrightarrow -\underbrace{\cancel{\sum_j (2j+1) e^{-T^{-1}\sqrt{j(j+1)}}}}_{1} N \ln N$$

$$- \sum_j \bar{n}_j \cancel{\ln (2j+1)} + \sum_j \bar{n}_j T^{-1} \sqrt{j(j+1)}$$

$$= \frac{T_0^{-1} A}{8\pi G\hbar \beta} \qquad A = \sum_j \bar{n}_j \sqrt{j(j+1)} \, (8\pi G \hbar \beta)$$

Entropy of the LQG surface: $S = \ln d[\{n_j\}] = \dfrac{T_0^{-1}}{8\pi G\hbar\beta} A$

area law

$$S = \frac{T_0^{-1}/2\pi}{\beta} \cdot \underbrace{\frac{A}{4G\hbar}}_{\text{"BH entropy"}} = \frac{A}{4G\hbar} \quad \text{if } \beta = \beta_0 = 0{,}274\ldots$$

$$T_0^{-1}/2\pi = 0{,}274\ldots$$

This computation can be applied to LQG black holes, see 1703.09149 by A. Perez.

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see 1610.02134.