$$\mathcal{Z} = \int DA\,D\psi\,D\bar\psi\,Dc\,D\bar c \; e^{i\int d^dx\,\mathcal{L}_{FP}}$$

$$\mathcal{L}_{FP} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \underline{\frac{1}{2\xi}(\partial^\mu A^a_\mu)(\partial^\nu A^a_\nu)} + \bar\psi^i (i\gamma^\mu D_\mu - m)^{ij}\psi^j$$

$$- \bar c^a (\partial^\mu D^{ac}_\mu) c^c$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D^{ij}_\mu \psi^j = \partial_\mu \psi^i + g A^a_\mu T^{a\,ij} \psi^j$$

$$D^{ac}_\mu c^c = \partial_\mu c^c + g f^{abc} A^b_\mu c^c$$

$g = 4$

$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} = \underline{-\frac{1}{2}\partial^\mu A^{\nu a}\partial_\mu A^a_\nu + \frac{1}{2}\partial^\mu A^{\nu a}\partial_\nu A^a_\mu}$$

$$- g f^{abc} A^{\mu a} A^{\nu b} \partial_\mu A^c_\nu \longleftarrow -g f^{abc}(-ik_\mu)\, g_{\nu\rho}\, A^{\mu a} A^{\nu b} A^{\rho c}$$

$$- \frac{1}{4} g^2 f^{abe} f^{cde} A^{\mu a} A^{\nu b} A^c_\mu A^d_\nu \longleftarrow$$

$$-\frac{1}{4} g^2 f^{abe} f^{cde}\, g_{\mu\rho}\, g_{\nu\sigma}\, A^{\mu a} A^{\nu b} A^{\rho c} A^{\sigma d}$$

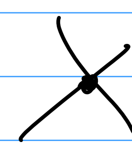
$$\mathcal{L}_{quark} = \bar\psi (i\not\partial - m)\psi$$

$$+ \underline{g A^a_\mu \bar\psi \gamma^\mu T^a \psi}$$

$\lambda\phi^4$

$\psi^{i=1\cdots N}_{\alpha=1\cdots 4}$

$a = 1 \cdots \dim(SU(N))$



$-i\lambda$

$$\mathcal{L}_{ghost} = -\bar c^a \partial^\mu \partial_\mu c^a - g\, \bar c^a \partial^\mu (f^{abc} A^b_\mu c^c)$$

$$\uparrow$$

$$+ g(\partial^\mu \bar c^a) f^{abc} A^b_\mu c^c \longrightarrow g(-ip^\mu)\bar c^a f^{abc} A^b_\mu c^c$$

$$\mathcal{L}^{(0)}_{gluon} = -\frac{1}{2}\partial^\mu A^{\nu a}\partial_\mu A^a_\nu + \frac{1}{2}\partial^\mu A^{\nu a}\partial_\nu A^a_\mu + \frac{1}{2\xi}(\partial^\mu A^a_\mu)(\partial^\nu A^a_\nu)$$

$$\delta\int \mathcal{L}^{(0)}_{gluon} = \int d^dx\, -\frac{1}{2}\Big(\partial^\mu \delta A^{\nu a}\partial_\mu A^a_\nu + \partial^\mu A^{\nu a}\partial_\mu \delta A^a_\nu$$

$$- \partial^\mu \delta A^{\nu a} \partial_\nu A^a_\mu - \partial^\mu A^{\nu a} \partial_\nu \delta A^a_\mu)$$

$$- \frac{1}{2\xi} (\partial^\mu \delta A^a_\mu \partial^\nu A^a_\nu + \partial^\mu A^a_\mu \partial^\nu \delta A^a_\nu)$$

$$= \int d^dx \; -\frac{1}{2} \Big[- \delta A^{\nu a} (\partial^\mu \partial_\mu A^a_\nu) - (\partial_\mu \partial^\mu A^{\nu a}) \delta A^a_\nu$$

$$+ \delta A^{\nu a} (\partial^\mu \partial_\nu A^a_\mu) + (\partial_\nu \partial^\mu A^{\nu a}) \delta A^a_\mu \Big]$$

$$- \frac{1}{2\xi} \Big[- \delta A^a_\mu (\partial^\mu \partial^\nu A^a_\nu) - (\partial^\nu \partial^\mu A^a_\mu) \delta A^a_\nu \Big]$$

EOM of free gluon :

$$\partial^\mu \partial_\mu A^a_\nu - \partial^\mu \partial_\nu A^a_\mu + \frac{1}{\xi} \partial_\nu \partial^\mu A_\mu = 0$$

$$\underline{\partial^\mu \partial_\mu A^a_\nu} - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial_\nu A^a_\mu = 0 \qquad \xi = 1$$

Feynman gauge.

gluon propagator :

$$g^\sigma_\nu \partial^\mu \partial_\mu A^a_\sigma - \left(1 - \frac{1}{\xi}\right) \partial_\nu \partial^\mu g_\mu{}^\sigma A^a_\sigma = 0$$

$$\left(g^{\nu\sigma} \partial^2 - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu g_\mu{}^\sigma \right) A^a_\sigma = 0$$

$\langle 0 | T A^a_\mu(x) A^b_\nu(y) | 0 \rangle$

$= D^{ab}_{\mu\nu}(x-y) + O(g)$

$$\left(g^{\nu\sigma} \partial^2 - \left(1 - \frac{1}{\xi}\right) \partial^\nu \partial^\mu g^\sigma{}_\mu \right) i D^{ab}_{\sigma\rho}(x-y) = \delta^\nu_\rho \delta^{ab} \delta^{(d)}(x-y)$$

$$\rightarrow \left(-g^{\nu\sigma} k^2 + \left(1 - \frac{1}{\xi}\right) k^\sigma k^\nu \right) i D^{ab}_{\sigma\rho}(k) = \delta^\nu_\rho \delta^{ab}$$

Feynman propagator of gluon

$$D^{ab}_{\sigma\rho}(k) = \frac{-i \delta^{ab}}{k^2 + i\varepsilon} \left(g^{\rho\sigma} - (1 - \xi) \frac{k^\rho k^\sigma}{k^2} \right)$$

$$D^{ab}_{\mu\nu}(x-y) = \int \frac{d^dk}{(2\pi)^d} D^{ab}_{\mu\nu}(k) e^{-ik(x-y)}$$

ghost propagator :

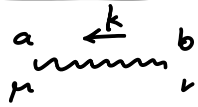
$$\langle 0 | T c^a(x) \bar{c}^b(y) | 0 \rangle = \int \frac{d^dk}{(2\pi)^d} e^{-ik(x-y)} \left[\frac{i}{k^2 + i\varepsilon} \delta^{ab} \right] + O(g)$$

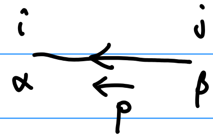
quark propagator :

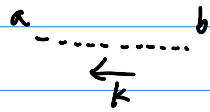
$$\langle 0 | T \psi^i_\alpha(x) \bar{\psi}^j_\beta(y) | 0 \rangle$$

$$= \int \frac{d^dk}{(2\pi)^d} e^{-ik(x-y)} \left[\frac{i}{\not{k} - m + i\varepsilon} \right]_{\alpha\beta} \delta^{ij} + O(g)$$

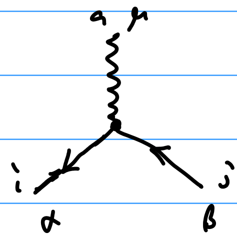
Feynman rules of correlation functions

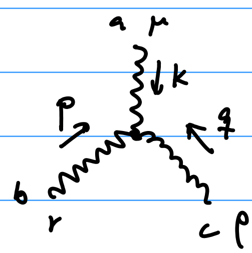
- gluon propagator  $= \dfrac{-i\delta^{ab}}{k^2+i\varepsilon}\left(g^{\mu\nu}-(1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right), \quad (\xi=1)$

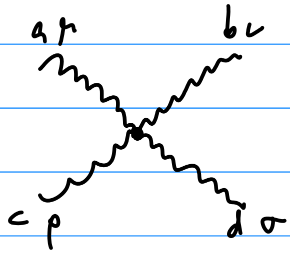
- quark propagator  $= \left(\dfrac{i}{\not{p}-m+i\varepsilon}\right)_{\alpha\beta}\delta^{ij}$

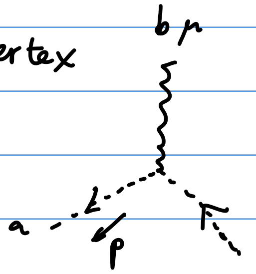
- ghost propagator  $= \dfrac{i}{k^2+i\varepsilon}\,\delta^{ab}$

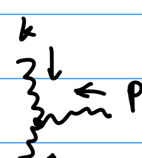
orientation of quark line = order of index

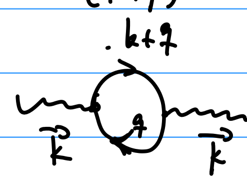
- gluon-quark vertex  $= ig(\gamma^\mu)_{\alpha\beta}(T^a)^i{}_j$

- 3-gluon vertex 
$$= g f^{abc}\left[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\right]$$

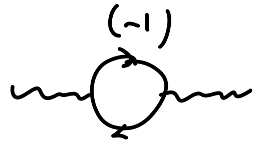
- 4-gluon vertex 
$$= -ig^2\left[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})\right]$$

- gluon-ghost vertex  $= -g f^{abc} p^\mu$

- momentum conservation at each vertex  $q=-(k+p)$

- each internal loop $\rightarrow \displaystyle\int \frac{d^dq}{(2\pi)^d}$ 

– each fermion loop : add overall (-1)



– divide symm. factor

$\langle 0 | T(\psi \cdots \psi A \cdots A \psi \cdots \psi) | 0 \rangle =$

in momentum rep.