

```
1 ---
2 title: "PSTAT 10 Homework 3"
3 author: "Sou Hamura"
4 date: "2024-07-16"
5 output:
6   pdf_document: default
7   html_document: default
8 ---
9
10 ```{r setup, include=FALSE}
11 knitr::opts_chunk$set(echo = TRUE)
12 ```
13
14 ## Problem 1: Rolling dice
15
16 ```{r}
17
18 trials <- 10^4
19 success <- 0
20 for (i in seq_len(trials)){
21   throw <- sample(1:6, size=30, replace=TRUE)
22   count <- table(throw)
23   if (all(count >= 3)){
24     success <- success + 1
25   }
26 }
27
28 success / trials
29
30 ```
```

[1] 0.492

```
31 ## Problem 2: Binomial Distribution
```

```
32
```

```
33 1. About 71% of the Earth is covered with water. I toss a globe into the air 12
    times, each time catching it with one hand and noting if the tip of my index
    finger is over water. What is the probability my finger landed on water 8
    times?
```

```
34
```

```
35 {r}
```

```
36
```

```
37 dbinom(8, 12, 0.71)
```

```
38
```

```
39
```

```
[1] 0.226081
```

```
40
```

```
41 2. About 8% of men are color blind. A researcher needs three colorblind men for
    an experiment and begins checking potential subjects. What is the probability
    that she finds three or more colorblind men in the first nine she examines?
```

```
42
```

```
43 {r}
```

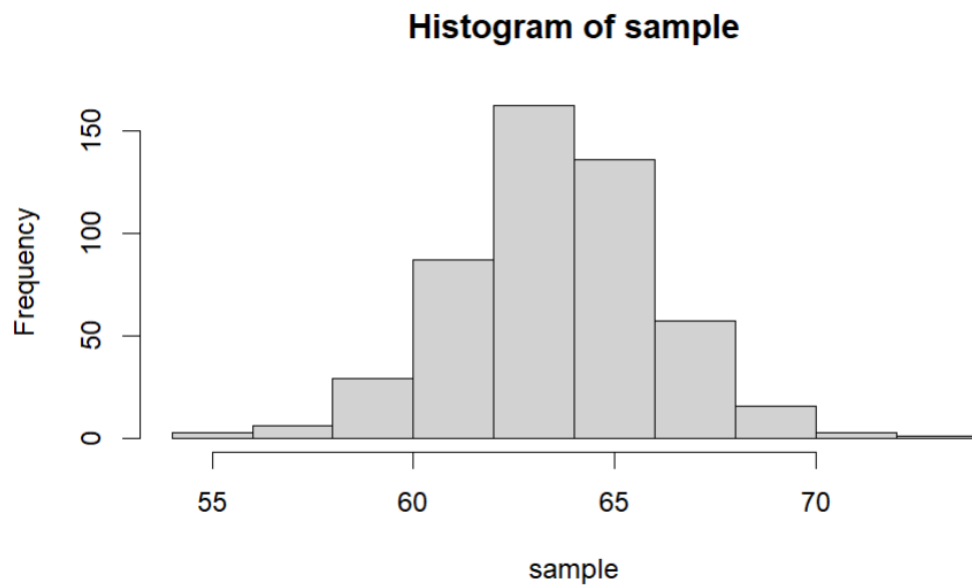
```
44
```

```
45 1 - pbinom(2, 9, 0.08)
```

```
46
```

```
47
```

```
[1] 0.02979319
```



```
48
49 ## Problem 3: Normal Distribution
50
```

```
51 1. Compute  $P(X < 60 \text{ OR } X > 65)$ .
```

```
52
53 ```{r}
54
55 mean <- 63.6
56 sd <- 2.5
57
58 pnorm(60, mean, sd) + (1 - pnorm(65, mean, sd))
59
60 ```
```

```
[1] 0.3626734
```

```
61 2. What percentage of women in this population must duck when walking through a
62 door that is 72 inches high?
```

```
63 ```{r}
64 1 - pnorm(72, mean, sd)
65
66 ```
```

```
[1] 0.0003897124
```

```
67 3. Generate a sample of 500 observations of  $X$ . Here, we know that the sample is
68 normally distributed. Nevertheless, provide a visual check to see if the sample
69 is normally distributed.
```

```
70
71 ```{r}
72
73 sample <- rnorm(500, mean, sd)
74
75 hist(sample)
```

```
76
77 ## Problem 4: Geometric Distribution
```

```
78
79 ```{r}
```

```
80
81 prob <- 1/8
```

```
82
83 ```
```

```
84
85 1. What is the support of X? Is X a discrete or continuous r.v.?
```

```
86
87 # X is supported by the set of all positive integers since there can be zero
88 # or more failure before the first success.
```

```
89
90 # X is a discrete random variable because its values are distinct and
91 # countable.
```

```
92
93 2. Using the base R functions, determine  $P(X = 4)$ 
```

```
94
95 ```{r}
```

```
96
97 dgeom(4, prob)
```

```
98
99 ```
```

```
[1] 0.07327271
```

```
100
101 3. Determine  $P(1 < X \leq 4)$ 
```

```
102
103 ```{r}
```

```
104
105 pgeom(4, prob) - pgeom(1, prob)
```

```
106
107 ```
```

```
[1] 0.2527161
```

```
108
109 4. It can be shown that  $E[X] = (1 - p)/p$ . Set a seed of 123 and then generate
110 1000 observations of X and estimate EX. In terms of absolute distance, how
111 close was the approximation to the theoretical value?
```

```
112
113 ```{r}
```

```
114
115 theo_v <- (1 - prob)/prob
```

```
116
117 set.seed(123)
```

```
118
119 sample_x <- rgeom(1000, prob)
```

```
120
121 estimate <- mean(sample_x <= 3)
```

```
122
123 abs(estimate - theo_v)
```

```
124
125 ```
```

```
[1] 6.572
```

```

120 ## Problem 5: Lining up
121
122 1. Ten people, labeled "A" through "J", are lined up randomly so that each
123    possible arrangement is equally likely.
124
125 {r}
126
127 trials <- 1000
128 success <- vector(length=trials)
129
130 for (i in 1:trials){
131   letters <- sample(LETTERS[1:10])
132   where_a <- which(letters == "A")
133   where_b <- which(letters == "B")
134   success[i] <- (abs(where_a - where_b) == 1)
135 }
136
137 cum_success <- cumsum(success)
138 probability <- cum_success / seq_len(trials)
139
140
141
142 2. It can be shown that the theoretical probability is  $1/5 = 0.20$ . Create the
143    following plot of your approximation as the number of replications increases.
144
145 {r}
146 plot(probability,
147       type = "l",
148       xlim = c(0, trials),
149       ylim = c(0.2, 1),
150       main = "Simulating Lining Up",
151       xlab = "Replication",
152       ylab = "Probability")
153
154 abline(h = 1/5, col = "red")
155
156

```

