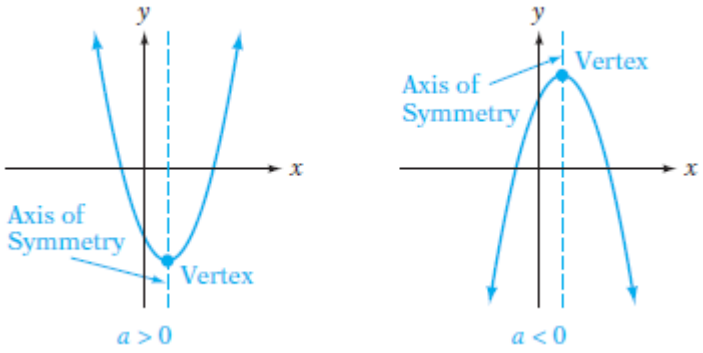
Module 15: More on Functions

Objective 15.1A – Graph a quadratic function

A **quadratic function** is a function that can be expressed by the equation $f(x) = ax^2 + bx + c,\ a \neq 0$. The graph of a quadratic function is called a **parabola**. The graph opens up when $a > 0$ and opens down when $a < 0$.



The **vertex** of a parabola is the point with the least *y*-coordinate when $a > 0$ and the point with the greatest *y*-coordinate when $a < 0$. The line that passes through the vertex and is parallel to the y-axis is called the **axis of symmetry**.

Vertex and Axis of Symmetry of a Parabola

Let $f(x) = ax^2 + bx + c,\ a \neq 0,$ be the equation of a parabola. Then

- the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$; and
- the equation of the axis of symmetry is $x = -\frac{b}{2a}$.

Example:

The quadratic function $f(x) = -2x^2 + 4x + 1$ has a graph that is a parabola. Find the coordinates of the vertex and the equation of the axis of symmetry for the parabola. Then sketch its graph. State the domain and range of the function.

Solution:

For this function, $a = -2,\ b = 4,$ and $c = 1$.

The x-coordinate of the vertex is $x=-\frac{b}{2a}=-\frac{4}{2(-2)}=1$. The y-coordinate is found by computing $f(1)=-2\cdot 1^2+4\cdot 1+1=3$. The vertex thus has coordinates $(1,3)$.

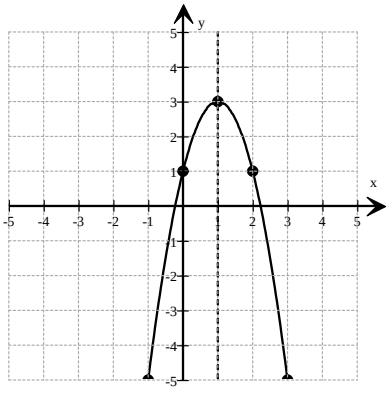
The axis of symmetry has equation $x=-\frac{b}{2a}=1$ from our work finding the vertex.

The parabola has a graph that opens downward since $a=-2<0$.

Find some ordered-pair solutions and record these in a table. Notice that the symmetry can be used to find points that correspond to one another on either side of $x=1$.

x	$f(x)$
-1	-5
0	1
2	1
3	-5

The graph is shown here. The axis of symmetry has been plotted as a dashed line.



Because $f(x)=2x^2+4x+1$ is a real number for all values of x, the domain of f is $\{x \mid x \in \text{real numbers}\}$. The vertex of the