23K-2005

Calculus Assignment #02

Q1) $f(x) = 4x - x^2$; $[0,4]$

Sol:

As we know that,

$$\text{Area} = \lim_{x \to \infty} \sum_{k=1}^{n} f(x_k^*) \Delta x \to (A)$$

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$$

$$x_k^* = a + k\Delta x$$

$$= 0 + k\left(\frac{4}{n}\right)$$

$$x_k^* = \frac{4k}{n}$$

$$f(x_k^*)\Delta x = \left[4\left[\frac{4k}{n}\right] - \left[\frac{4k}{n}\right]^2\right]\left(\frac{4}{n}\right)$$

$$= \left[\frac{16k}{n} - \frac{16k^2}{n^2}\right]\left(\frac{4}{n}\right)$$

$$f(x_k^*)\Delta x = \frac{64k}{n^2} - \frac{64k^2}{n^3}$$

Date

Apply sumission.

$$\sum_{K=1}^{n} f(x_K^*)\,\Delta x = \sum_{K=1}^{n}\left[\frac{64K}{n^2} - \frac{64K^2}{n^3}\right]$$

$$= \frac{64}{n^2}\sum_{K=1}^{n}(K) - \frac{64}{n^3}\sum_{K=1}^{n}(K^2)$$

$$= \frac{64}{n^2}\left[\frac{n(n+1)}{2}\right] - \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{32}{n}(n+1) - \frac{32}{3n^2}(n+1)(2n+1)$$

Limit :-

$$= \lim_{n\to\infty} \frac{32}{n}(n+1) - \lim_{n\to\infty} \frac{32}{3n^2}(n+1)(2n+1)$$

$$= \lim_{n\to\infty} 32\left(1+\frac{1}{n}\right) - \lim_{n\to\infty} \frac{32}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$= 32\left(1+\frac{1}{\infty}\right) - \frac{32}{3}\left(1+\frac{1}{\infty}\right)\left(2+\frac{1}{\infty}\right)$$

$$= 32(1+0) - \frac{32}{3}(1+0)(2+0)$$

$$= 32 - \frac{64}{3}$$

$$= \boxed{32/3} \quad 3 \rightarrow \text{Ans}$$

Date

Q2) $\int 8x^4 \cos(2x)\, dx$

u		v
$8x^4$	$+$	$\cos 2x$
$32x^3$	$-$	$\sin 2x/2$
$96x^2$	$+$	$\cos 2x/4$
$192x$	$-$	$\sin 2x/8$
192	$+$	$\cos 2x/16$
0		$\sin 2x/32$

$$\Rightarrow 8x^4\left(\frac{\sin 2x}{2}\right) + \frac{32x^3(\cos 2x)}{4} - \frac{96x^2 \sin 2x}{8}$$
$$- \frac{192}{16}x(\cos 2x) + \frac{192}{32}\sin 2x$$

$$\Rightarrow 4x^4 \sin 2x + 8x^3 \cos 2x - 12x^2 \sin 2x - 12x\cos 2x + 6\sin 2x$$

$$\Rightarrow \sin 2x(4x^4 - 12x^2 + 6) + \cos 2x(8x^3 - 12x)$$
$$\Rightarrow 2\sin 2x(2x^4 - 6x^2 + 3) + 4x\cos 2x(2x^2 - 3)$$

Date

Q4) (a) : $\int_{-\infty}^{2} \frac{dx}{x^2+4}$

Sol :-

This is improper integral.

$$\int_{-\infty}^{2} \frac{1}{x^2+4}\,dx = \lim_{a\to\infty} \int_{a}^{2} \frac{1}{x^2+4}\,dx$$

~~Let~~

$$\because \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$= \lim_{a\to\infty} \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_{a}^{2} + C$$

$$= \frac{1}{2}\lim_{a\to\infty} \left|\tan^{-1}\frac{x}{2}\right|_{a}^{2} + C$$

$$= \frac{1}{2}\lim_{a\to\infty} \left[\tan^{-1}(1) - \tan^{-1}\frac{a}{2}\right] + C$$

$$= \frac{1}{2}\left[\tan^{-1}(1) - \tan^{-1}\left(\frac{-\infty}{2}\right)\right] + C$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{2}\left(\frac{3\pi}{4}\right)$$

$$= \frac{3\pi}{8} \quad \text{Ans.}$$

(a) 4(b) $\displaystyle\int_0^1 \frac{1}{2x-1}\,dx$

This is an improper integral. because the function is discontinuous at 1/2.

$$\int_0^2 \frac{dx}{2x-1} = \int_0^{1/2^-} \frac{dx}{2x-1} + \int_{1/2^+}^{1} \frac{dx}{2x-1}$$

Consider,

$$\int_0^{1/2^-} \frac{dx}{2x-1} = \lim_{b\to\frac{1}{2}^-} \int_0^b \frac{1}{2x-1}\,dx$$

$$= \lim_{b\to\frac{1}{2}^-} \frac{1}{2} \Big| \ln(2x-1) \Big|_0^b$$

$$= \lim_{b\to\frac{1}{2}^-} \frac{1}{2}\left[\ln(2b-1) - \ln(-1)\right]$$

$$= \lim_{b \to \frac{1}{2}^{+}} \frac{1}{2} \left[\ln(2b-1) \right]$$

$$\boxed{\because \ln(0) = 0}$$

Applying limit.

$$= -\infty \; ; \text{(Divergent)}$$

Not doing it for $\lim_{x \to \frac{1}{2}^{+}}$ because if one limit is divergent The other must also be divergent.

Date

$= \frac{1}{3} x e^{x^3} - \int e^{x^3} dx$

$= \frac{1}{3} x e^{x^3} - \frac{e^{x^3}}{3.x^2} dx$

Q5) (i) $\int_0^3 \frac{x}{\sqrt{3+2x}} dx$

Let $u = \sqrt{3+2x}$ $\Rightarrow$ $\boxed{x = \frac{u^2-3}{2}}$

$\frac{du}{dx} = \frac{1}{\cancel{2}\sqrt{3+2x}} \cdot \cancel{2}$

$du = \frac{dx}{\sqrt{3+2x}}$

$\Rightarrow \int_0^3 \left(\frac{u^2-3}{2}\right) du$

$\Rightarrow \frac{1}{2} \int_0^3 (u^2-3) du$

~~$u = \sqrt{3+2x}$~~

Date

$$\Rightarrow \frac{1}{2}\left[\int_0^3 u^2\,du - 3\int_0^3 du\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{u^3}{3} - 3u\right]_0^3$$

$$\because u = \sqrt{3+2x}$$

$$\Rightarrow \frac{1}{2}\left[\frac{(\sqrt{3+2x})^3}{3} - 3\sqrt{3+2x}\right]_0^3$$

$$\Rightarrow \frac{1}{2}\left[\left[\frac{(\sqrt{3+2(3)})^3}{3} - 3\sqrt{3+6}\right] - \left[\frac{(3+0)^{3/2}}{3} - 3\sqrt{3}\right]\right]$$

$$\Rightarrow \frac{1}{2}\left[\left\{\frac{(9)^{3/2}}{3} - 3(3)\right\} - \left\{\frac{(3)^{3/2}}{3} - (3)\sqrt{(3)}\right\}\right]$$

$$\Rightarrow \frac{1}{2}\left[\left\{\frac{(3^2)^{3/2}}{3} - 9\right\} - \left\{\frac{(3^{3/2})}{3} - 3\sqrt{3}\right\}\right]$$

$$\Rightarrow \frac{1}{2}\left[\left\{\frac{27}{3} - 9\right\} - \left\{\frac{3^{3/2}}{3} - 3\sqrt{3}\right\}\right]$$

$$\Rightarrow \frac{1}{2}\left[\cancel{9} - \cancel{9} - \frac{3\sqrt{3}}{3} + 3\sqrt{3}\right]$$

Date

$$\Rightarrow \frac{1}{\cancel{2}}\left[\cancel{2}\sqrt{3}\right]$$

$\Rightarrow \sqrt{3}$ units

Ans.

Q5) (ii) $\displaystyle\int \frac{2^y}{2^y+5}\,dy$

Let $u = 2^y + 5$

$$\frac{du}{dy} = 2^y \ln 2$$

$$\frac{du}{\ln 2} = 2^y\,dy$$

Now,

$$\int \frac{1}{u} \cdot \frac{du}{\ln 2}$$

$$\Rightarrow \frac{1}{\ln 2}\int \frac{1}{u}\,du$$

$$\Rightarrow \frac{\ln u}{\ln 2} + C$$

Date

$$\Rightarrow \frac{\ln(2^y+5)}{\ln(2)} + C$$

Ans.

Q5 (iii) $\displaystyle\int \frac{1}{\sqrt{1+x^2}\cdot \sinh^{-1}x}\,dx$

Let $u = \sin^{-1}x$

$$\frac{du}{dx} = \frac{1}{\sqrt{1+x^2}}$$

$$du = \frac{dx}{\sqrt{1+x^2}}$$

Now,

$$\int \frac{1}{u}\,du$$

$\Rightarrow \ln u + C$

$\Rightarrow \ln(\sinh^{-1}x) + C$

Ans.

Date

Q5 (iv) : $\int \frac{\sec^2\theta}{\tan^3\theta - \tan^2\theta} \, d\theta$

Let $u = \tan\theta$
$du = \sec^2\theta \, d\theta$

Now,

$$\int \frac{1}{u^3 - u^2} \, du$$

$$\int \frac{1}{u^2(u-1)} \, du$$

Let $\frac{1}{u^2(u-1)} = \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u-1} \rightarrow A$

$1 = A(u)(u-1) + B(u-1) + C(u^2) \rightarrow 1$

For A, putting $u = 2$.

$1 = A(2)(1) + B(1) + C(4)$
$2A = 1 - B - 4C \longrightarrow 2$.

Date

For B, putting $u = 0$.

$$1 = A(0) + B(-1) + C(0)$$

$$1 = -B$$

$$\boxed{B = -1}$$

For C, putting $u = 1$.

$$1 = A(0) + B(0) + C$$

$$\boxed{C = 1}$$

Putting B & C in eq 2.

$$2A = 1 - B - 4C$$

$$2A = 1 + 1 - 4$$

$$\cancel{2}A = -\cancel{2}$$

$$\boxed{A = -1}$$

Now, putting the values of A, B and C in eq (A),

$$\frac{1}{u^2(u-1)} = \frac{-1}{u} - \frac{1}{u^2} + \frac{1}{u-1}$$

$$\Rightarrow \int \frac{1}{u^2(u-1)} = -\int \frac{1}{u} - \int \frac{1}{u^2} + \int \frac{1}{u-1}$$

$\int \frac{1}{u^2} \Rightarrow \int u^{-2} \Rightarrow \frac{u^{-1}}{-1} + c \Rightarrow -\frac{1}{u} + c$

$$\cancel{-\frac{\ln(u)}{1} - \frac{\ln(u^{-1})}{1}}$$

$$= -\frac{\ln(u)}{1} - \left(-\frac{1}{u}\right) + \frac{\ln(u-1)}{1} + c$$

$$= -\ln u + \frac{1}{u} + \ln(u-1) + c$$

$$= \ln(u-1) - \ln(u) + \frac{1}{u} + c$$

$$= \frac{\ln(\tan\theta - 1)}{\ln(\tan\theta)} + \frac{1}{\tan\theta} + c$$

Ans.

Q5 (vi) $\int x^5 e^{x^3} dx$

Sol: $\int x^3 . e^{x^3} . x^2 dx$

Let $u = x^3$

$\frac{du}{3} = x^2 dx$

$\Rightarrow \frac{1}{3} \int u . e^u du$

Date

By table method:

u	v
u	e^u
1	e^u
0	e^u

$\oplus$ $\ominus$

$$\Rightarrow \frac{1}{3}\left(u \cdot e^u - e^u + C\right)$$

$$\Rightarrow \frac{1}{3}\left(x^3 \cdot e^{x^3} - e^{x^3}\right) + C$$

Ans.

Q5 (vii) : $\displaystyle\int \frac{1}{\sqrt{x^2-4x}}\,dx$

Sol :-

$$\int \frac{1}{\sqrt{(x)^2 - 2(x)(2) + (2)^2 - (2)^2}}\,dx$$

$$\int \frac{1}{\sqrt{(x-2)^2 - 4}}\,dx$$

Let $u = x - 2$

$$\frac{du}{dx} = 1$$

Date

$du = dx$

Now,

$$\int \frac{1}{\sqrt{u^2-(2)^2}}\,du$$

$$\because \int \frac{1}{\sqrt{u^2-a^2}}\,dx = \ln\left(u+\sqrt{u^2-a^2}\right)+C$$

$$\Rightarrow \ln\left(u+\sqrt{u^2-a^2}\right)+C$$

$$\Rightarrow \ln\left[(x-2)+\sqrt{(x-2)^2-(2^2)}\right]+C$$

$$\Rightarrow \ln\left[(x-2)+\sqrt{(x-2)^2-4}\right]+C$$

Ans.

(viii): $$\int \frac{x}{\sqrt{x^2+4x+5}}\,dx$$

Sol:- $$\int \frac{x}{\sqrt{(x)^2+2(x)(2)+(2)^2-(2)^2+5}}\,dx$$

$$\Rightarrow \int \frac{x}{\sqrt{(x+2)^2+1}}\,dx$$

Date

Let $u = x+2 \Rightarrow \boxed{x = u-2}$

$$\frac{du}{dx} = 1$$

$$du = dx$$

Now,

$$\int \frac{u-2}{\sqrt{u^2+1}}\, du$$

$$\Rightarrow \int \frac{u}{\sqrt{u^2+1}}\, du - 2\int \frac{1}{\sqrt{u^2+1}}\, du \rightarrow (1)$$

Consider,

$$\int \frac{u}{\sqrt{u^2+1}}\, du$$

Let $u^2+1 = t$

$$2u\,du = dt$$

$$u\,du = \frac{dt}{2}$$

Now,

$$\int \frac{1}{\sqrt{t}} \cdot \frac{dt}{2}$$

$$\int \frac{1}{\sqrt{u^2+1}} = \int (u^2+1)^{-1/2} \Rightarrow \frac{\sqrt{u^2+1}}{1/2} \Rightarrow 2\sqrt{u^2+1}$$

Date

$$\Rightarrow \frac{1}{2} \int t^{-1/2}\, dt$$

$$\Rightarrow \frac{1}{\cancel{2}} \cdot (\cancel{2}\sqrt{t}) + c$$

$$\Rightarrow \sqrt{t} + c$$

$$\Rightarrow \sqrt{u^2+1} + c$$

Now, eq ① becomes,

$$\Rightarrow \int \frac{4}{\sqrt{u^2+1}}\, du - 2 \int \frac{1}{\sqrt{u^2+1}}\, du$$

$$\Rightarrow \left[\sqrt{u^2+1} + c\right] - 2 \left(2\sqrt{u^2+1}\right)$$

$$\Rightarrow \sqrt{u^2+1} - 4\sqrt{u^2+1} + c$$

$$\Rightarrow -3\sqrt{u^2+1} + c$$

$$\Rightarrow -3\sqrt{(x+2)^2+1} + C$$

Ans.

Date

Q5 (ix): $\int \ln(2x+3)\,dx$

Let $2x+3 = t$

$2\,dx = dt$

$dx = \frac{dt}{2}$

Now,

$\frac{1}{2}\int \ln t \, dt$

Let $u = \ln t$, $v = 1$

$du = \frac{1}{t}$, $\int v = t$

$\because \int u \cdot v = u\int v - \int u'\int v$

$= \frac{1}{2}\left[\ln t\,(t) - \int\left(\frac{1}{t}\right)(t)\right]$

$= \frac{1}{2}\left[t\ln t\,() - t\right] + C$

$= \frac{1}{2}\left[(2x+3)\ln(2x+3) - (2x+3)\right] + C$

$= \frac{(2x+3)}{2}\left[\ln(2x+3) - 1\right] + C$

Ans.

Date

(x) $\int \sin^4 2x \, dx$

Let $u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

Now,

$$\frac{1}{2}\int \sin^4 u \, du$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{4}\sin^3 u \cos u + \frac{3}{4}\int \sin^2 u \, du\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{4}\sin^3 u \cos u + \frac{3}{4}\int \frac{1-\cos 2u}{2} du\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{4}\sin^3 u \cos u + \frac{3}{4}\left\{\int \frac{1}{2} du - \int \frac{\cos 2u \, du}{2}\right\}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{4}\sin^3 u \cos u + \frac{3}{4}\left\{\frac{1}{2}u - \frac{1}{4}\sin 2u\right\}\right] + C$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{4}\sin^3 u \cos u + \frac{3}{8}u - \frac{3}{16}\sin 2u\right] + C$$

Ans.

Date

(xii) : $\int_0^{\ln 2} \sqrt{e^x - 1}\, dx$

Sol:-

Let $u = \sqrt{e^x - 1} \Rightarrow \boxed{e^x = u^2 + 1}$

$$\frac{du}{dx} = \frac{1}{2\sqrt{e^x - 1}} \cdot e^x$$

$$\frac{du}{dx} = \frac{e^x}{2u}$$

$$dx = \frac{2u\,du}{e^x}$$

Now,

$$\int_0^{\ln 2} u \left(\frac{2u\,du}{u^2+1}\right)$$

$$\Rightarrow 2\int_0^{\ln 2} \frac{u^2}{u^2+1}\, du$$

$$\Rightarrow 2\int_0^{\ln 2} \frac{u^2+1-1}{u^2+1}\, du$$

$$\Rightarrow 2\int_0^{\ln 2} \left(\frac{\cancel{u^2+1}}{\cancel{u^2+1}} - \frac{1}{u^2+1}\right) du$$

$$\Rightarrow 2\int_0^{\ln 2}\left(1-\frac{1}{u^2+1}\right)du$$

$$\Rightarrow 2\left[\int_0^{\ln 2} du - \int_0^{\ln 2}\frac{1}{u^2+1}\,du\right]$$

$$\Rightarrow 2\left[u-\frac{1}{1}\tan^{-1}\frac{u}{1}\right]_0^{\ln 2}$$

$$\Rightarrow 2\left[u-\tan^{-1}u\right]_0^{\ln 2}$$

$$\Rightarrow 2\left[\sqrt{e^x-1}-\tan^{-1}\sqrt{e^x-1}\right]_0^{\ln 2}$$

$$\Rightarrow 2\left[\left\{\sqrt{e^{\ln 2}-1}-\tan^{-1}\sqrt{e^{\ln 2}-1}\right\}-\left\{\sqrt{e^0-1}-\tan^{-1}\sqrt{e^0-1}\right\}\right]$$

$$\Rightarrow 2\left[\left\{\sqrt{2-1}-\tan^{-1}\sqrt{2-1}\right\}-\left\{\sqrt{1-1}-\tan^{-1}\sqrt{1-1}\right\}\right]$$

$$\Rightarrow 2\left[\left\{1-\frac{\pi}{4}\right\}-\left\{0-0\right\}\right]$$

$$\Rightarrow \cancel{2}^{1}\left(\frac{4-\pi}{\cancel{4}_{2}}\right)$$

$$\Rightarrow \frac{4-\pi}{2}$$ Ans.

Date

Q11) $\int \frac{\sqrt{1+4x^2}}{x}\,dx$

Sol:-

$\Rightarrow \int \frac{\sqrt{4\left(\frac{1}{4}+x^2\right)}}{x}\,dx$

$\Rightarrow 2\int \frac{\sqrt{\left(\frac{1}{2}\right)^2+x^2}}{x}\,dx \rightarrow (1)$

Let $x = a \tan\theta$

$x = \frac{1}{2}\tan\theta \Rightarrow \tan\theta = 2x$

$\frac{dx}{d\theta} = \frac{1}{2}\sec^2\theta$

$dx = \frac{1}{2}\sec^2\theta\,d\theta$

Now,

$\cancel{2}\int \frac{\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\tan\theta\right)^2}}{1/2\tan\theta}\left(\frac{1}{\cancel{2}}\sec^2\theta\,d\theta\right)$

$\Rightarrow 2\int \frac{\sqrt{\frac{1}{4}+\frac{1}{4}\tan^2\theta}}{\tan\theta}\sec^2\theta\,d\theta$

$$\frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta} = \frac{1}{\sin\theta} = \frac{1}{\operatorname{cosec}\theta}$$

Date

$$\Rightarrow 2\int \frac{\sqrt{\frac{1}{4}(1+\tan^2\theta)}}{\tan\theta} \cdot \sec^2\theta \, d\theta$$

$$\Rightarrow \cancel{2}\int \frac{1}{\cancel{2}} \frac{\sqrt{1+\tan^2\theta}}{\tan\theta} \cdot \sec^2\theta \, d\theta$$

$$\Rightarrow \int \frac{\sqrt{\sec^2\theta} \cdot \sec^2\theta \, d\theta}{\tan\theta}$$

$$\Rightarrow \int \frac{\sec^3\theta \, d\theta}{\tan\theta}$$

$$\Rightarrow \int \frac{\sec\theta \cdot (\sec^2\theta) \, d\theta}{\tan\theta}$$

$$\Rightarrow \int \frac{\sec\theta \cdot (1+\tan^2\theta) \, d\theta}{\tan\theta}$$

$$\Rightarrow \int \frac{\sec\theta + \sec\theta \tan^2\theta \, d\theta}{\tan\theta}$$

$$\Rightarrow \int \frac{\sec\theta \, d\theta}{\tan\theta} + \int \sec\theta \tan\theta \, d\theta$$

$$\Rightarrow \int \operatorname{cosec}\theta \, d\theta + \int \sec\theta \tan\theta \, d\theta$$

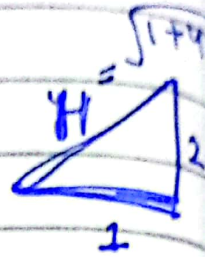
$H^2 = B^2 + P^2$

$H = \sqrt{1^2 + 4x^2}$

Date

$\Rightarrow \ln(\operatorname{cosec}\theta - \cot\theta) + \sec\theta + C$

? ? ?



$$\boxed{\tan\theta = \frac{P}{B} = \frac{2x}{1}}$$

$$\boxed{\cot\theta = \frac{B}{P} = \frac{1}{2x}}$$

$$\boxed{\operatorname{cosec}\theta = \frac{H}{P} = \frac{\sqrt{1+4x^2}}{2x}}$$

$$\boxed{\sec\theta = \frac{H}{B} = \sqrt{1+4x^2}}$$

$$\Rightarrow \ln\left[\frac{\sqrt{1+4x^2}}{2x} - \frac{1}{2x}\right] + \sqrt{1+4x^2} + C$$

Ans.

(v) : $\displaystyle\int \frac{x^3+8}{(x^2-1)(x-2)}\,dx$

Sol:-

$$\int \frac{x^3+8}{(x^2-1)(x-2)}\,dx$$

$$\Rightarrow \int \frac{x^3+8}{x^3-2x^2-x+2}\,dx$$

$$\begin{array}{r|l} & \quad 1 \\ \hline x^3-2x^2-x+2 & x^3+8 \\ & \pm x^3 \mp 2x^2 \mp x \pm 2 \\ \hline & 2x^2+x+6 \end{array}$$

$$\Rightarrow \int dx + \int \frac{2x^2+x+6}{(x-1)(x+1)(x-2)}\,dx \rightarrow (A)$$

Let

$$\frac{2x^2+x+6}{(x-1)(x+1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x-2)} \rightarrow (1)$$

$$\cancel{2x^2+x+6} = \frac{A}{(x+1)(x-2)} + \frac{B}{(x-1)(x-2)} + \frac{C}{(x-1)(x+1)} \rightarrow (2$$

$$2x^2+x+6 = A(x+1)(x-2) + B(x-1)(x-2) + C(x+1)(x-1) \rightarrow ($$

For A, putting $x = 1$.

$$2+1+6 = A(2)(-1) + B(0) + C(0)$$

$$9 = -2A$$

$$\boxed{A = \frac{-9}{2}}$$

For B, putting $x = -1$.

$$2-1+6 = A(0) + B(-2)(-3) + C(0)$$
$$7 = 6B$$
$$\boxed{B = \frac{7}{6}}$$

For C, putting $x = 2$.

$$8+2+6 = A(0) + B(0) + C(3)(1)$$
$$16 = 3C$$
$$\boxed{C = \frac{16}{3}}$$

Putting in eq (1).

$$\cancel{2x^2+} (1) \Rightarrow \frac{-9}{2(x-1)} + \frac{7}{6(x+1)} + \frac{16}{3(x-2)}$$

$$(1) \Rightarrow \frac{-9}{2}\int \frac{1}{(x-1)}dx + \frac{7}{6}\int \frac{1}{(x+1)}dx + \frac{16}{3}\int \frac{dx}{x-2}$$

Putting in eq (A).

$$\Rightarrow \int dx + \int \frac{2x^2+6+x}{(x+1)(x-1)(x-2)}dx$$

Date

$\Rightarrow \cancel{\int dx + \frac{9}{2}}$

$$\Rightarrow x - \frac{9}{2}\ln(x-1) + \frac{7}{6}\ln(x+1) + \frac{16}{3}\ln(x-2) + C$$

Ans.

X ——————————— X

Question # 03 (b):

$$\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx$$

Let $u = \sqrt{x^2+1} \Rightarrow x = \sqrt{u^2-1}$

$$\frac{du}{dx} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}}$$

$$du = \frac{x\,dx}{\sqrt{x^2+1}}$$

Now,

$$\int_0^1 x^2 \cdot \frac{x\,dx}{\sqrt{x^2+1}}$$

$$\Rightarrow \int_0^1 (u^2-1)\,du$$

Date

$$\Rightarrow \int_0^1 u^2\,du - \int_0^1 du$$

$$\Rightarrow \left[\frac{u^3}{3} - u\right]_0^1$$

$$\Rightarrow \cancel{\left[\frac{(1)^3}{3} - 1\right] - \left[\frac{(0)}{3} - 0\right]}$$

$$\Rightarrow \cancel{\left[\frac{1}{3} - 1\right]}$$

$$\Rightarrow \left[\frac{(x^2+1)^{3/2}}{3} - \sqrt{x^2+1}\right]_0^1$$

$$\Rightarrow \left[\frac{(1+1)^{3/2}}{3} - \sqrt{1+1}\right] - \left[\frac{(0+1)^{3/2}}{3} - \sqrt{0+1}\right]$$

$$\Rightarrow \left[\frac{2\sqrt{2}}{3} - \sqrt{2}\right] - \left[\frac{1}{3} - \sqrt{1}\right]$$

$$\Rightarrow 0.195$$

Ans.

Date

Question # 03(a)

$$\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx$$

Sol:-

$$\int_0^1 \overset{u}{x^2} \cdot \overset{v}{\frac{x}{\sqrt{x^2+1}}}\,dx$$

$$\because \int u \cdot v = u\int v - \int u' \int v$$

$$\Rightarrow \left[x^2 \int \frac{x\,dx}{\sqrt{x^2+1}} - \int 2x \int \frac{x}{\sqrt{x^2+1}}\,dx\right]_0^1$$

Let $u = x^2 + 1$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x\,dx$$

Now,

$$\Rightarrow \left[x^2 \frac{1}{\cancel{2}} \int \frac{du}{\sqrt{u}} - \left(\cancel{2} \frac{x}{\cancel{2}} \cdot \int \frac{du}{\sqrt{u}}\right)\right]_0^1$$

Date

$$\Rightarrow \left[\frac{x^2}{\cancel{2}} \cdot \cancel{2}\sqrt{u} - \left(2x \cdot \sqrt{u}\right)\right]_0^1$$

$$\Rightarrow \left[x^2 \cdot \sqrt{x^2+1} - \frac{2x}{3}\left(\sqrt{x^2+1}\right)^3\right]_0^1$$

$$\Rightarrow \left[\sqrt{2} - \frac{2}{3}(2)^{3/2}\right] \cancel{\left[0 - 0\right]} + \frac{2}{3}$$

$$\Rightarrow -0.471 + \frac{2}{3}$$

$\Rightarrow$ 0.195 units

Ans.