15.2 (7-14, 19-22)

Date:..............................

Sun Mon Tue Wed Thu Fri Sat

Q7) $F(x,y) = 8i + 8j$ $\underset{t=0}{P(-4,4)}$, $\underset{t=1}{Q(-4,5)}$

$x(t) = -4$

$y(t) = t+4$

$r = x(t)i + y(t)j$

$= -4i + (t+4)j$

$r' = (j)\,dt$

$$\int_0^1 F\cdot dr$$

$$= \int_0^1 (8i+8j)\cdot(0i+1j)\,dt$$

$$= \int_0^1 8\,dt$$

$$\boxed{= 8}$$

Q8) $F(x,y) = 2i + 5j$ $P(1,-3)$, $Q(4,-3)$

$r(t) = ti - 3j$ $x(t) = t$ $1 \le t \le 4$

$r'(t) = (i)\,dt$ $y(t) = -3$

$$\int_1^4 (2i+5j)\cdot(1i+0j)\,dt$$

$$= \int_1^4 2\,dt$$

$$\boxed{= 8-2 = 6}$$

Date:..............................

Sun Mon Tue Wed Thu Fri Sat

Q9 $F(x,y) = 2x\,j$ $\quad P(-2,4), Q(-2,11)$

$r(t) = -2i + tj$ $\quad x(t) = -2$ $\quad 4 \leq t \leq 11$

$r'(t) = j\ dt$ $\quad y(t) = t$

$$\int_4^{11} (2x\,j)\cdot(1j)\,dt$$

$$= \int_4^{11} 2x\,dt$$

$$= \int_4^{11} -4\,dt$$

$$= -44 + 16$$

$$\boxed{= -28}$$

Q10 $F(x,y) = -8xi + 3yj$ $\quad P(-1,0), Q(6,0)$

$r(t) = ti$ $\quad x(t) = t$ $\quad -1 \leq t \leq 6$

$r'(t) = i\ dt$ $\quad y(t) = 0$

$$\int_{-1}^{6} (-8xi + 3yj)\cdot(i)\,dt$$

$$= \int_{-1}^{6} -8t\,dt$$

$$\boxed{= -4t^2\Big]_{-1}^{6} = 140}$$

Scanned with CamScanner

Date:

Sun Mon Tue Wed Thu Fri Sat

Q11 $x = 2t$, $y = t^2$ $0 \le t \le 1$

$r(t) = 2ti + t^2 j$

$r'(t) = (2i + 2tj)\, dt$

$|r'(t)| = \sqrt{4 + 4t^2}\, dt$

$ds = 2\sqrt{1+t^2}\, dt$

(a) $\int_0^1 (x - \sqrt{y})\, ds$

$\int_0^1 2t - t\, ds$

$\int_0^1 t \cdot 2\sqrt{1+t^2}\, dt$

$\int_0^1 2t\sqrt{1+t^2}\, dt$ $u = 1+t^2$

$du = 2t\,dt$

$\int_1^2 \sqrt{u}\, du$

$\boxed{\frac{2}{3}u^{3/2}\Big]_1^2 = \frac{2}{3}(\sqrt{8} - 1)}$

(b) $dx = 2dt$

$\int_0^1 t \cdot 2dt = \int_0^1 2t\, dt$

$\boxed{= t^2\Big]_0^1 = 1}$

(c) $dy = 2t\, dt$

$\int_0^1 t \cdot 2t\, dt$

$= \int_0^1 2t^2\, dt$

$= \frac{2}{3}t^3\Big]_0^1$

$\boxed{= \frac{2}{3}}$

Scanned with CamScanner

Date: ....

Sun Mon Tue Wed Thu Fri Sat

Q12) $x = t, y = 3t^2, z = 6t^3 \quad 0 \le t \le 1$

$dx = dt, dy = 6t\,dt, dz = 18t^2\,dt$

$r = ti + 3t^2 j + 6t^3 k$

$r' = (i + 6tj + 18t^2 k)dt$

$|r'| = \sqrt{324t^4 + 36t^2 + 1}\;dt$

$= \sqrt{(18t^2+1)^2}$

$ds = (18t^2+1)\,dt$

$324t^4 + 36t^2 + 1$

$(324t^4 + 18t^2) + (18t^2 + 1)$

$18t^2(18t^2+1) + 1(18t^2+1)$

$= (18t^2+1)^2$

(a) $\int_0^1 t(3t^2)(6t^3)^2(18t^2+1)\,dt$

$= \int_0^1 108t^9(18t^2+1)\,dt$

$= 108\int_0^1 18t^{11} + t^9\,dt$

$= 108\left[\frac{3}{2}t^{12} + \frac{t^{10}}{10}\right]_0^1$

$= 108\left(\frac{3}{2} + \frac{1}{10}\right)$

$\boxed{= \frac{864}{5}}$

(b) $\int_0^1 108t^9\,dt$

$= 108\left[\frac{t^{10}}{10}\right]_0^1$

$\boxed{= \frac{108}{10} = \frac{54}{5}}$

(c) $\int_0^1 108t^9 \times 6t\,dt$

$= 648\int_0^1 t^{10}\,dt$

$= 648\left[\frac{t^{11}}{11}\right]_0^1$

$\boxed{= \frac{648}{11}}$

(d) $\int_0^1 108t^9 \cdot 18t^2\,dt$

$\cdot\,1944\int_0^1 t^{11}\,dt$

$= 1944\left[\frac{t^{12}}{12}\right]_0^1$

$\boxed{= 162}$

~~Q13 a) C: x=t, y=t, 0 ≤ t ≤ 1~~

Q13) $\int_C (3x+2y)\,dx + (2x-y)\,dy$

a) $(0,0) \to (1,1)$

$x=t, y=t \quad 0 \leq t \leq 1$

$dx = dt, dy = dt$

$$\int_0^1 (3t+2t+2t-t)\,dt$$

$$= \int_0^1 6t\,dt$$

$$\boxed{= 3t^2\Big]_0^1 = 3}$$

b) $y = x^2 \qquad (0,0) \to (1,1)$

$x=t, y=t^2 \qquad 0 \leq t \leq 1$

$dx = dt, dy = 2t\,dt$

$$\int_0^1 (3t+2t^2)\,dt + (2t-t^2)\,2t\,dt$$

$$\int_0^1 (3t+2t^2+4t^2-2t^3)\,dt$$

$$= \int_0^1 3t+6t^2-2t^3 \quad dt$$

$$= \frac{3}{2}t^2 + 2t^3 - \frac{1}{2}t^4 \Big]_0^1$$

$$\boxed{= 3}$$

Date:..............................

Sun Mon Tue Wed Thu Fri Sat

(c) $x = t \,, \quad y = \sin\left(\frac{\pi t}{2}\right) \qquad 0 \le t \le 1$

$dx = dt \,, \quad dy = \frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right) dt$

$$\int_0^1 \left(3t + 2\sin\left(\frac{\pi t}{2}\right)\right) dt + \left(2t - \sin\left(\frac{\pi t}{2}\right)\right)\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right) dt$$

$$\int_0^1 \left[3t + 2\sin\left(\frac{\pi t}{2}\right) + \left[2t - \sin\left(\frac{\pi t}{2}\right)\right]\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)\right] dt$$

$$\int_0^1 3t + 2\sin\left(\frac{\pi t}{2}\right) + \pi t\cos\left(\frac{\pi}{2}t\right) - \frac{\pi}{2}\sin\left(\frac{\pi t}{2}\right)\cos\left(\frac{\pi t}{2}\right) \, dt$$

$$\int_0^1 3t + 2\sin\left(\frac{\pi t}{2}\right) + \pi t\cos\left(\frac{\pi}{2}t\right) dt + \int_0^1 \cancel{\frac{\pi}{2}}\cos\left(\frac{\pi t}{2}\right)\left(-\frac{\pi}{2}\sin\left(\frac{\pi t}{2}\right)\right) dt$$

$u = \cos\left(\frac{\pi t}{2}\right)$

$du = -\frac{\pi}{2}\sin\left(\frac{\pi t}{2}\right) dt$

$$\left.\frac{3t^2}{2} - 2\cos\left(\frac{\pi t}{2}\right)\cdot\frac{2}{\pi}\right]_0^1 + \int_0^1 \pi t\cos\left(\frac{\pi t}{2}\right) dt + \int_1^0 u \, du$$

	d	I
+	πt	$\cos\left(\frac{\pi t}{2}\right)$
−	π	$\frac{2\sin\left(\frac{\pi t}{2}\right)}{\pi}$
+	0	$-\frac{4\cos\left(\frac{\pi t}{2}\right)}{\pi^2}$

$$= \frac{3}{2} + \frac{4}{\pi} + \left[2t\sin\left(\frac{\pi t}{2}\right) + \frac{4\cos\left(\frac{\pi t}{2}\right)}{\pi}\right]_0^1 - \left.\frac{u^2}{2}\right]_0^1$$

$$= \frac{3}{2} + \cancel{\frac{4}{\pi}} + 2 - \cancel{\frac{4}{\pi}} - \frac{1}{2}$$

$$\boxed{= 3}$$

Scanned with CamScanner

Date:.............................

Sun Mon Tue Wed Thu Fri Sat

(d) $x = y^3 \quad (0,0) \to (1,1)$

$y = t, \; x = t^3 \qquad 0 \leq t \leq 1$

$dy = dt, \; dx = 3t^2 dt$

$$\int_0^1 (3t^3 + 2t)\, 3t^2 dt + (2t^3 - t)\, dt$$

$$\int_0^1 (9t^5 + 6t^3 + 2t^3 - t)\, dt$$

$$\int_0^1 (9t^5 + 8t^3 - t)\, dt$$

$$= \frac{3}{2}t^6 + 2t^4 - \frac{t^2}{2} \Big]_0^1$$

$$\boxed{= \frac{3}{2} + 2 - \frac{1}{2} = 3}$$

Date:............................

Sun Mon Tue Wed Thu Fri Sat

14) $\int_c y\,dx + z\,dy - x\,dz$

(a) $(0,0,0) \rightarrow (1,1,1)$ $\quad x=t, y=t, z=t$

$dx=dt, dy=dt, dz=dt$

$0 \le t \le 1$

$$\int_0^1 t\,dt + t\,dt - t\,dt$$

$$= \int_0^1 (t+t-t)\,dt$$

$$= \int_0^1 t\,dt$$

$$\boxed{= \frac{t^2}{2}\Big]_0^1 = \frac{1}{2}}$$

(b) $x=t, y=t^2, z=t^3$ $\quad (0,0,0) \rightarrow (1,1,1)$

$dx=dt, dy=2t\,dt, dz=3t^2\,dt$

$$\int_0^1 t^2\,dt + t^3 \cdot 2t\,dt - t \cdot 3t^2\,dt$$

$$\int_0^1 t^2\,dt + 2t^4\,dt - 3t^3\,dt$$

$$\int_0^1 (t^2 + 2t^4 - 3t^3)\,dt$$

$$\frac{t^3}{3} + \frac{2t^5}{5} - \frac{3t^4}{4}\Big]_0^1$$

$$\boxed{= \frac{1}{3} + \frac{2}{5} - \frac{3}{4} = -\frac{1}{60}}$$

COPY

Scanned with CamScanner

Date:..............................

Sun Mon Tue Wed Thu Fri Sat

(c) $x = \cos \pi t$, $y = \sin \pi t$, $z = t$ $(1,0,0) \to (-1,0,1)$

$dx = -\pi \sin \pi t \, dt$, $dy = \pi \cos \pi t \, dt$, $dz = dt$

$$\int_0^1 \sin \pi t(-\pi \sin \pi t \, dt) + t(\pi \cos \pi t) dt - \cos \pi t \, dt$$

$$\int_0^1 \left(-\pi \sin^2 \pi t \, \cancel{dt} + \pi t \cos \pi t \, \cancel{dt} - \cos \pi t\right) dt$$

$$\int_0^1 -\pi \left(\frac{1-\cos(2\pi t)}{2}\right) dt + \int_0^1 \pi t \cos \pi t \, dt - \int_0^1 \cos \pi t \, dt$$

	d	I
+	πt	$\cos \pi t$
−	π	$\frac{\sin \pi t}{\pi}$
+	0	$-\frac{\cos \pi t}{\pi^2}$

$$= -\frac{\pi}{2} \int_0^1 1 - \cos(2\pi t) \, dt + \left[t \sin \pi t + \frac{\cos \pi t}{\pi}\right]_0^1 - \left[\frac{\sin \pi t}{\pi}\right]_0^1$$

$$= -\frac{\pi}{2}\left[t - \frac{\sin(2\pi t)}{2\pi}\right]_0^1 - \frac{1}{\pi} - \frac{1}{\pi} - 0$$

$$\boxed{= -\frac{\pi}{2} - \frac{2}{\pi}}$$

Date: ....

Sun Mon Tue Wed Thu Fri Sat

Q19) $\int_C \frac{1}{1+x} ds \quad C: r(t) = t i + \frac{2}{3} t^{3/2} j \quad 0 \le t \le 3$

$r'(t) = (i + t^{1/2} j) dt$

$ds = |r'(t)| = \sqrt{1+t} \quad dt$

$\int_0^3 \frac{1}{1+t} \sqrt{1+t}\, dt$

$\int_0^3 \frac{1}{\sqrt{1+t}} dt \qquad u = \sqrt{1+t}$

$du = \frac{1}{2\sqrt{1+t}} dt$

$2du = \frac{1}{\sqrt{1+t}} dt$

$\int_1^2 2\, du$

$\boxed{2u\Big]_1^2 = 2}$

Q20) $\int_C \frac{x}{1+y^2} ds \quad c: x = 1+2t, y = t \quad 0 \le t \le 1$

$r(t) = (1+2t) i + t j$

$r'(t) = (2i + j)\, dt$

$ds = |r'(t)| = \sqrt{4+1} \quad dt = \sqrt{5}\, dt$

$\int_0^1 \frac{1+2t}{1+t^2} \sqrt{5}\, dt$

$\sqrt{5}\left[\int_0^1 \frac{1}{1+t^2} dt + \int_0^1 \frac{2t}{1+t^2} dt\right.$

$\sqrt{5}\left[\tan^{-1}(t)\Big]_0^1 + \ln(1+t^2)\Big]_0^1\right.$

$\boxed{= \sqrt{5}\left(\pi/4 + \ln 2\right)}$

Date:..............................

Sun Mon Tue Wed Thu Fri Sat

21) $\int_C 3x^2yz\,ds \quad C: x=t, y=t^2, z=\frac{2}{3}t^3 \quad 0\leq t\leq 1$

$r(t) = ti + t^2j + \frac{2}{3}t^3k$

$r'(t) = (i + 2tj + 2t^2k)dt$

$|r'(t)| = \sqrt{1+4t^2+4t^4}\,dt$

$= \sqrt{(2t^2+1)^2}\,dt$

$ds = (2t^2+1)\,dt$

$4t^4+4t^2+1$

$4t^4+2t^2+2t^2+1$

$2t^2(2t^2+1)+1(2t^2+1)$

$(2t^2+1)(2t^2+1)$

$(2t^2+1)^2$

$\int_0^1 3t^2(t^2)(\frac{2}{3}t^3)\cdot(2t^2+1)\,dt$

$\int_0^1 2t^7(2t^2+1)\,dt$

$= \int_0^1 4t^9 + 2t^7\,dt$

$= \frac{2}{5}t^{10} + \frac{1}{4}t^8\Big]_0^1$

$\boxed{= \frac{2}{5} + \frac{1}{4} = \frac{13}{20}}$

Scanned with CamScanner

Date:..............................

Sun Mon Tue Wed Thu Fri Sat

22) $\int_C \frac{e^{-z}}{x^2+y^2}\, ds$ $\quad C: r(t) = 2\cos t\, i + 2\sin t\, j + t\, k$

$r'(t) = (-2\sin t\, i + 2\cos t\, j + k)\, dt \quad 0 \le t \le 2\pi$

$|r'(t)| = \sqrt{4\sin^2 t + 4\cos^2 t + 1}\; dt$

$= \sqrt{4+1}\, dt$

$ds = \sqrt{5}\, dt$

$$\int_0^{2\pi} \frac{e^{-t}}{4\cos^2 t + 4\sin^2 t} \sqrt{5}\, dt$$

$$\frac{\sqrt{5}}{4} \int_0^{2\pi} e^{-t}\, dt$$

$$\frac{\sqrt{5}}{4} \left[-e^{-t} \right]_0^{2\pi}$$

$$= \frac{\sqrt{5}}{4} \left(-e^{-2\pi} - (-e^0) \right)$$

$$\boxed{= \frac{\sqrt{5}}{4} \left(1 - \frac{1}{e^{2\pi}} \right)}$$

15.4 (3–10)

$y^2 - 3xy \Big]_1^2$

$3 - 3x$

$3x - \frac{3x^2}{2} \Big]_{-2}^{4}$

Date:..........................

Sun Mon Tue Wed Thu Fri Sat

Q3) $\oint_C \underbrace{3xy}_{P}\, dx + \underbrace{2xy}_{Q}\, dy \qquad -2 \le x \le 4$

$\qquad 1 \le y \le 2$

$$= \iint \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] dA$$

$$= \int_1^2 \int_{-2}^{4} 2y - 3x \, dx\, dy$$

$$= \int_1^2 2xy - \frac{3x^2}{2} \Big]_{-2}^{4} dy$$

$$= \int_1^2 8y - 24 - (-4y - 6)\, dy$$

$$= \int_1^2 12y - 18 \, dy$$

$$= 6y^2 - 18y \Big]_1^2$$

$$= 0$$

Scanned with CamScanner

Date:.................................

Sun Mon Tue Wed Thu Fri Sat

Q1) $\oint_C (x^2-y^2)\,dx + x\,dy$ $\quad C: x^2+y^2=9$

~~$x=\cos t,\ y=\sin t$~~

$0 \le r \le 3$

$0 \le \theta \le 2\pi$

$x = 3\cos\theta$

$y = 3\sin\theta$

$$\int_0^{2\pi}\int_0^{3} \left(1-(-2y)\right) r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{3} (1+6\sin\theta)\, r\,dr\,d\theta$$

$$\frac{1}{2}\int_0^{2\pi} 9(1+6\sin\theta)\,d\theta$$

$$\frac{9}{2}\int_0^{2\pi} 1+6\sin\theta\,d\theta$$

$$\frac{9}{2}\left[\theta - 6\cos\theta\right]_0^{2\pi}$$

$$\frac{9}{2}\left[(2\pi-6)-(-6)\right]$$

$$\boxed{= \frac{9}{2}(2\pi) = 9\pi}$$

COPY

Scanned with CamScanner

Sun Mon Tue Wed Thu Fri Sat

(5) $\oint_C x\cos y\,dx - y\sin x\,dy$ $\quad 0 \le x \le \pi/2$, $\quad 0 \le y \le \pi/2$

$$= \int_0^{\pi/2}\int_0^{\pi/2} \left(-y\cos x + x\sin y\right) dx\,dy$$

$$= \int_0^{\pi/2} \left[-y\sin x + \frac{x^2\sin y}{2}\right]_0^{\pi/2} dy$$

$$= \int_0^{\pi/2} -y + \frac{\pi^2}{8}\sin y \; \cancel{} \, dy$$

$$= \left[\frac{-y^2}{2} - \frac{\pi^2}{8}\cos y\right]_0^{\pi/2}$$

$$\boxed{= \frac{-\pi^2}{8} - \left(\frac{-\pi^2}{8}\right) = 0}$$

Q6) $\oint_C y\tan^2 x\,dx + \tan x\,dy$ $\quad x^2 + (y+1)^2 = 1$

$r = -2\sin\theta$

$0 \le r \le -2\sin\theta$

$0 \le \theta \le \pi$

$$\int_0^{\pi}\int_0^{-2\sin\theta} \sec^2 x - \tan^2 x \; dA$$

$$\int_0^{\pi}\int_0^{-2\sin\theta} \sec^2 x - \sec^2 x + 1 \; dA$$

$$\int_0^{\pi}\int_0^{-2\sin\theta} 1 \; dA$$

This ~~...~~ is area of the region so:-

ans $\boxed{= \pi r^2 = \pi(1)^2 = \pi}$

Scanned with CamScanner

Sun Mon Tue Wed Thu Fri Sat

Q7) $\oint_C (x^2-y)\,dx + x\,dy$

$x^2+y^2=4$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$= \int_0^{2\pi}\int_0^{2} (1-(-1))\,dA$

$= 2 \underbrace{\iint 1\,dA}_{\text{area of the region}}$

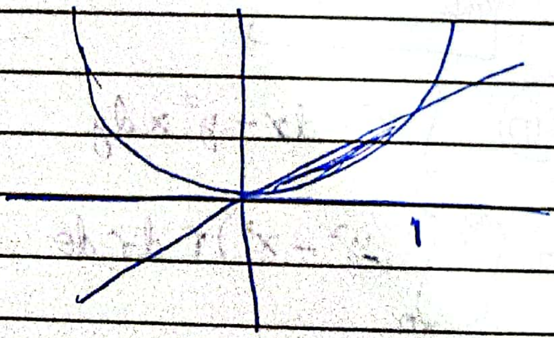
$A = \pi r^2 = \pi(2)^2 = 4\pi$

$= 2(4\pi)$

$\boxed{\text{ans} = 8\pi}$

Q8) $\int_C (e^x+y^2)\,dx + (e^y+x^2)\,dy$

$y=x^2, \; y=x$



$= \int_0^1 \int_{x^2}^{x} 2x-2y \; dy\,dx$

$= \int_0^1 \left. 2xy - y^2 \right]_{x^2}^{x}$

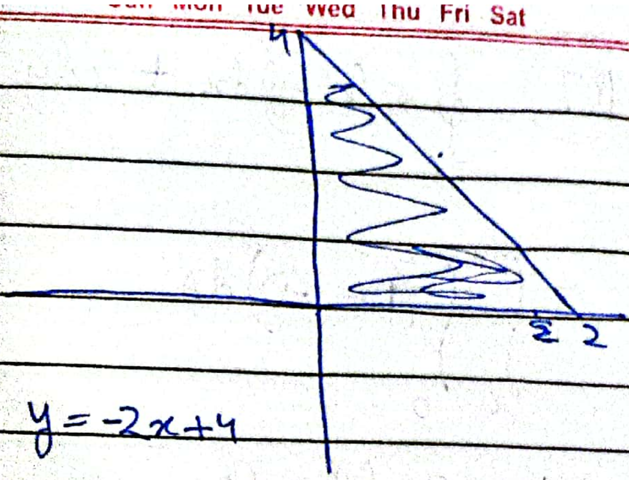
$= \int_0^1 2x^2 - x^2 - (2x^3 - x^4)\,dx$

$= \int_0^1 x^2 - 2x^3 + x^4 \; dx$

$= \left. \frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} \right]_0^1$

$\boxed{= \frac{1}{3} - \frac{1}{2} + \frac{1}{5} = \frac{1}{30}}$

(9) $\int_C \ln(1+y)\,dx - \frac{xy}{1+y}\,dy$



$= \int_0^2\int_0^{-2x+4} \frac{-y}{1+y} - \frac{1}{1+y}\,dy\,dx$

$y=-2x+4$

$= \int_0^2\int_0^{-2x+4} \frac{-\cancel{(1+y)}}{\cancel{1+y}}\,dy\,dx$

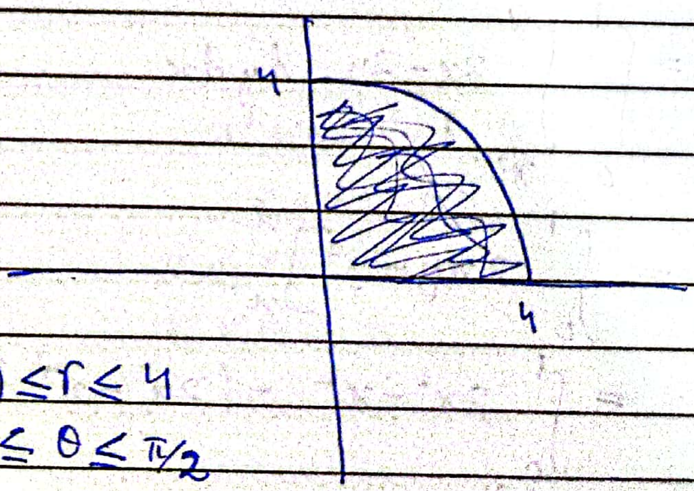
$= \boxed{\int_0^2\int dA}$

area of the region

$A = \frac{1}{2}bh = 4$

$\boxed{\text{ans} = -4}$

Q10) $\int_C x^2y\,dx - y^2x\,dy$



$= \int_0^{\pi/2}\int_0^4 (-y^2 - x^2)\,r\,dr\,d\theta$

$= \int_0^{\pi/2}\int_0^4 (-r^2)\,r\,dr\,d\theta$

$0 \le r \le 4$

$0 \le \theta \le \pi/2$

$= \int_0^{\pi/2} -\frac{r^4}{4}\Big]_0^4\,d\theta$

$= \int_0^{\pi/2} -64\,d\theta$

$\boxed{= -32\pi}$