L-Hopital's Rule

Indeterminate Forms:

1. $\frac{0}{0}$
2. $\frac{\infty}{\infty}$
3. $\infty - \infty \,/ -\infty + \infty$
4. $0 * \infty / \infty * 0$
5. $\infty^0 /\ 0^0 / 1^\infty$

EXERCISE SET 6.5

Find the limits.

Question:

14. $\lim_{x \to +\infty} \frac{e^{3x}}{x^2}$

Answer:

$$\lim_{x \to +\infty} \frac{3e^{3x}}{2x} = \lim_{x \to +\infty} \frac{9e^{3x}}{2} = +\infty.$$

Question:

15. $\lim_{x \to 0+} \frac{\cot x}{\ln x}$

Answer:

$$\lim_{x \to 0+} \frac{-\csc^2 x}{1/x} = \lim_{x \to 0+} \frac{-x}{\sin^2 x} = \lim_{x \to 0+} \frac{-1}{2 \sin x \cos x} = -\infty.$$

Question:

20. $\lim_{x \to \pi^-} (x - \pi) \tan \frac{1}{2}x$

Answer:

$$\lim_{x\to\pi} (x-\pi)\tan(x/2) = \lim_{x\to\pi} \frac{x-\pi}{\cot(x/2)} = \lim_{x\to\pi} \frac{1}{-(1/2)\csc^2(x/2)} = -2.$$

Question:

22. $\lim_{x \to 0+} \tan x \ln x$

Answer:

$$\lim_{x\to0+} \tan x \ln x = \lim_{x\to0+} \frac{\ln x}{\cot x} = \lim_{x\to0+} \frac{1/x}{-\csc^2 x} = \lim_{x\to0+} \frac{-\sin^2 x}{x} = \lim_{x\to0+} \frac{-2\sin x\cos x}{1} = 0.$$

Question:

30. $\lim_{x \to +\infty} [\cos(2/x)]^{x^2}$

Answer:

$$y = [\cos(2/x)]^{x^2},\ \lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{\ln\cos(2/x)}{1/x^2} = \lim_{x\to+\infty} \frac{(-2/x^2)(-\tan(2/x))}{-2/x^3} = \lim_{x\to+\infty} \frac{-\tan(2/x)}{1/x} =$$

$$\lim_{x\to+\infty} \frac{(2/x^2)\sec^2(2/x)}{-1/x^2} = -2,\ \lim_{x\to+\infty} y = e^{-2}.$$

Question:

38. $\lim_{x \to 0+} (e^{2x} - 1)^x$

Answer:

$$y = (e^{2x}-1)^x, \ln y = x\ln(e^{2x}-1),\ \lim_{x\to0+} \ln y = \lim_{x\to0+} \frac{\ln(e^{2x}-1)}{1/x} = \lim_{x\to0+} \frac{2e^{2x}}{e^{2x}-1}(-x^2) =$$

$$= \lim_{x\to0+} \frac{x}{e^{2x}-1} \lim_{x\to0+} (-2xe^{2x}) = \lim_{x\to0+} \frac{1}{2e^{2x}} \lim_{x\to0+} (-2xe^{2x}) = \frac{1}{2}\cdot 0 = 0,\ \lim_{x\to0+} y = e^0 = 1.$$

Question:

34. $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

Answer:

$$\lim_{x\to+\infty} \ln\frac{x}{1+x} = \lim_{x\to+\infty} \ln\frac{1}{1/x+1} = \ln(1) = 0.$$