(23K - 2005)

Calculus Assignment # 01

Q1) (i) $\lim_{x \to 1^+} f(x) = x+6 = 1+6 = 7$

(ii) $\lim_{x \to 1^-} f(x) = (1)^3 + 4 = 1+4 = 5$

(iii) $\lim_{x \to 1} f(x) = 7$

Q2)

(i) $\lim_{x \to -2} f(x) = -(-2)^2 \Rightarrow -4$

(ii) $\lim_{x \to 2} f(x) = -(+2)-1 \Rightarrow -2-1 \Rightarrow -3.$

(iii) $\lim_{x \to 4} f(x) = -4-1 \Rightarrow -5$

(iv) $f(-2) = -(-2)^2 \Rightarrow -4$

(v) $f(2) = -2-1 \Rightarrow -3$

(vi) $f(4) = -4-1 \Rightarrow -5$

Q3) Sol

(i) lim

Sol:-

(ii)

Sol:-

$= 7$

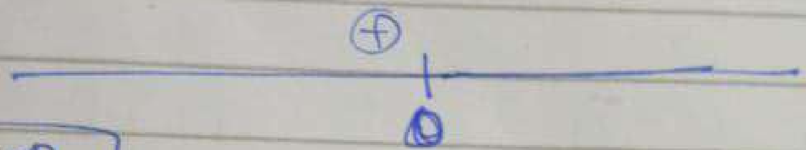
Date

Q(3) Sol

(i) $\lim_{x \to 0^-} \dfrac{3x+4}{x^2}$

Sol:-

Sign analysis :-



$\boxed{+\infty}$

(ii) $\lim_{x \to 3^+} \dfrac{x^2-9}{\sqrt{x-3}}$

Sol :-

Using L Hopital's rule.

$$\lim_{x \to 3^+} \frac{2x}{\frac{1}{2\sqrt{x-3}}(1)}$$

$$\Rightarrow \lim_{x \to 3^+} 2x \cdot 2\sqrt{x-3}$$

$$\Rightarrow 0.379$$

Date

(iii) $\lim\limits_{x \to -2} \dfrac{x^2 - 4}{x^2 - x - 6}$

Sol: Using L Hopital's rule.

$$\lim_{x \to -2} \frac{x^2 - 4}{x^2 - x - 6}$$

$$\lim_{x \to -2} \frac{\frac{d}{dx}(x^2 - 4)}{\frac{d}{dx}(x^2 - x - 6)}$$

$$\lim_{x \to -2} \frac{2x}{2x - 1}$$

$$\Rightarrow \frac{2(-2)}{2(-2) - 1}$$

$$\Rightarrow \frac{-4}{-4 - 1}$$

$$\Rightarrow \frac{-4}{-5}$$

$$\Rightarrow \frac{4}{5}$$

(iv) lim $t \to$

Sol:-

$\lim\limits_{t \to 1}$

$\lim\limits_{t \to 1}$

$\Rightarrow$ 3

$\Rightarrow$

$\Rightarrow$

$\lim\limits_{t \to 1^+}$

$\lim\limits_{t \to 1^-}$

(iv) $\lim_{t \to 1} \dfrac{t^3 + t^2 - 5t + 3}{t^3 - 3t + 2}$

<u>Sol</u> :-

$$\lim_{t \to 1} \frac{(d/dx)\, t^3 + t^2 - 5t + 3}{(d/dx)\, t^3 - 3t + 2}$$

$$\lim_{t \to 1} \frac{3t^2 + 2t - 5}{3t^2 - 3}$$

$$\Rightarrow \frac{3(1)^2 + 2(1)}{3(1)^2 - 3}$$

$$\Rightarrow \frac{3 + 2}{3 - 3}$$

$$\Rightarrow \frac{5}{0}$$


$\lim_{t \to 1^+} = \cancel{17.45}\ 1.322$

$\lim_{t \to 1^-} = 1.344 \quad$ so $\lim_{t \to 1} = 1.333$

Q(v) Sol :-

$$\lim_{x\to 0} \frac{d/dx\left(\sqrt{x+64}-8\right)}{d/dx\,(x)}$$

$$\Rightarrow \frac{\frac{1}{2\sqrt{x+64}}}{1}$$

$$\Rightarrow \frac{1}{2\sqrt{0+64}}$$

$$\Rightarrow \frac{1}{2\sqrt{64}}$$

$$\Rightarrow \frac{1}{2(8)}$$

$$\Rightarrow \frac{1}{16}$$ Ans.

(vi) Sol :-

$$\lim_{y\to 0} \frac{5y^3+8y^2}{3y^4-16y^2}$$

$$\lim_{y\to 0} \frac{d/d \ldots}{d/d \ldots}$$

$$\lim_{y\to 0} \frac{15y^2 + \ldots}{12y^3 - \ldots}$$

$$\lim_{y\to 0} \frac{30y + \ldots}{36y^2 \ldots}$$

$$\Rightarrow \frac{30(0) \ldots}{36(0 \ldots}$$

$$\Rightarrow \frac{16}{-32}$$

$$\Rightarrow -\frac{1}{2}$$

Q4)

(i) $\lim_{x\to -4^-}$

(ii) $\lim_{x\to -2^-}$

Date

$$\lim_{y\to 0} \frac{d/dx\,(5y^3+8y^2)}{d/dx\,(3y^4-16y^2)}$$

$$\lim_{y\to 0} \frac{15y^2+16y}{12y^3-32y}$$

$$\lim_{y\to 0} \frac{30y+16}{36y^2-32}$$

$$\Rightarrow \frac{30(0)+16}{36(0)-32}$$

$$\Rightarrow \frac{16}{-32}$$

$$\Rightarrow -\frac{1}{2}$$ Ans.

Q4)

(i) $\lim_{x\to -4^-} f(x) = \text{DNE}$

(ii) $\lim_{x\to -2^-} f(x) = 2$

No.

(iii) $\lim_{x \to -1} f(x) = DNE$

(iv) $\lim_{x \to 3} f(x) = DNE$

Q5) Sol :-

(i) $g(x) = \dfrac{\tan 3x}{(x+7)^4}$

$$= \frac{d}{dx}\left[\frac{(\tan 3x)}{(x+7)^4}\right]$$

$$= \frac{(x+7)^4 \frac{d}{dx}(\tan 3x) - \tan 3x \frac{d}{dx}(x+7)^4}{[(x+7)^4]^2}$$

$$= \frac{(x+7)^4 \cdot \sec^2 3x \cdot (3) - \tan 3x \cdot 4(x+7)^3 \cdot (1)}{(x+7)^8}$$

$$= \frac{3\sec^2 3x (x+7)^4 - 4\tan 3x (x+7)^4}{(x+7)^8}$$

$$= \frac{3\sec^2 3x - 4\tan 3x}{(x+7)^8}$$

(ii) $r = \tan\sqrt{\theta}\,\sec\left(\frac{1}{\theta}\right)$

Sol:-

$$\frac{dr}{d\theta} = \frac{d}{d\theta}\left[\tan\sqrt{\theta}\,\sec\left(\frac{1}{\theta}\right)\right]$$

$$\frac{dr}{d\theta} = \tan\sqrt{\theta}\,\frac{d}{d\theta}\left(\sec\theta^{-1}\right) + \sec{}^{1}/_{\theta}\,\frac{d}{d\theta}\tan\sqrt{\theta}$$

$$\frac{dr}{d\theta} = \tan\sqrt{\theta}\cdot \sec{}^{1}/_{\theta}\tan{}^{1}/_{\theta}\left(-\frac{1}{\theta^2}\right) + \sec\left(\frac{1}{\theta}\right)\sec^2\sqrt{\theta}\cdot\frac{1}{2\sqrt{\theta}}\cdot(1)$$

$$\frac{dr}{d\theta} = \frac{-\sec{}^{1}/_{\theta}\tan{}^{1}/_{\theta}\cdot\tan\sqrt{\theta}}{\theta^2} + \frac{1}{2\sqrt{\theta}}\cdot\sec(1/\theta)\cdot\sec^2\sqrt{\theta}$$

$$\cancel{\frac{dr}{d\theta} = +2\sqrt{\theta}\left(-\sec\theta\tan\theta\cdot\tan\sqrt{\theta}\right) + \theta^2}$$

$$\frac{dr}{d\theta} = \sec{}^{1}/_{\theta}\left[-\frac{\tan\sqrt{\theta}\cdot\tan{}^{1}/_{\theta}}{\theta^2} + \frac{\sec^2\sqrt{\theta}}{2\sqrt{\theta}}\right]$$

$$= \sec\frac{1}{\theta}\left[\frac{\theta^2\sec^2\sqrt{\theta} - 2\sqrt{\theta}\tan\sqrt{\theta}\tan{}^{1}/_{\theta}}{\theta^2(2\theta)}\right.$$

(iii) $y = \left[\frac{t^2}{t^3 - 4t}\right]^3$

Sol:-

$$\frac{d}{dx}(y) = \frac{d}{dx}\left[\frac{t^2}{t^3 - 4t}\right]^3$$

$$= 3\left[\frac{t^2}{t^3-4t}\right]^2 \cdot \frac{d}{dx}\left[\frac{t^2}{t^3-4t}\right]$$

$$= 3\left[\frac{t^2}{t^3-4t}\right]^2 \cdot \left[\frac{(t^3-4t)(2t)-(t^2)(3t^2-4)}{(t^3-4t)^2}\right]$$

$$= 3\left[\frac{t^2}{t^3-4t}\right]^2 \cdot \left[\frac{2t^4-8t^2-3t^4+4t^2}{(t^3-4t)^2}\right]$$

$$= \frac{3t^4\left[-t^4-4t^2\right]}{(t^3-4t)^4}$$

$$= \cancel{\frac{-3t^8-12t^6}{(t^3-4t)^4}}$$

$$\frac{dy}{dx} = \cancel{-3(t^8} \quad \frac{-3t^4(t^4+4t^2)}{(t^3-4t)^4}$$

Date

(iv) $q = \tan\left(\frac{\cos t}{t}\right)$

Sol

$$\frac{dq}{dt} = \frac{d}{dt}\tan\left[\frac{\cos t}{t}\right]$$

$$= \sec^2\left(\frac{\cos t}{t}\right)\cdot\frac{d}{dx}\left(\frac{\cos t}{t}\right)$$

$$= \sec^2\left(\frac{\cos t}{t}\right)\left[\frac{t(-\sin t) - \cos t(1)}{t^2}\right]$$

$$= \sec^2\left(\frac{\cos t}{t}\right)\left[\frac{-t\sin t - \cos t}{t^2}\right]$$

$$= -\sec^2\left(\frac{\cos t}{t}\right)\left[\frac{t\sin t + \cos t}{t^2}\right]$$

(v) $y = \left(t^{-3/4}\sin t\right)^{4/3}$

Sol:-

$$\frac{dy}{dt} = \frac{d}{dt}\left[t^{-3/4}\sin t\right]^{4/3}$$

$$= \frac{4}{3}\left(t^{-3/4}\sin t\right)^{1/3}\cdot\frac{d}{dt}\left[t^{-3/4}\sin t\right]$$

$$\frac{dy}{dt} = \frac{4}{3}\left(t^{-3/4}\sin t\right)^{1/3}\left[t^{-3/4}(\cos t) + \sin t\left(\frac{-3t}{4}\right)^{1/4}\right]$$

$$\frac{dy}{dt} = \frac{4}{3}\left(t^{-3/4}\sin t\right)^{1/3}\left[t^{-3/4}\cos t + \sin t\left(-\frac{3t}{4}\right)^{1/4}\right]$$

Ans.

(vi) $f(x) = [x^4 - \sec(4x^2 - 2)]^{-4}$

Sol:

$$f'(x) = \frac{d}{dx}[x^4 - \sec(4x^2-2)]^{-4}$$

$$= -4[x^4 - \sec(4x^2-2)]^{-5} \cdot [4x^3 - \sec(4x^2-2)\tan(4x^2-2)(8x)]$$

$$= -4[x^4 - \sec(4x^2-2)]^{-5} [4x^3 - \sec(4x^2-2)\tan(4x^2-2)(8x)]$$

$$= -4[x^4 - \sec(4x^2-2)]^{-5} \cdot [4x^3 - 8x\sec(4x^2-2)\tan(4x^2-2)]$$

Ans.

Q6) **Proof :-**

$$y = x^3 + 3x + 1.$$

$$y' = \frac{d}{dx}(x^3 + 3x + 1)$$

$$\boxed{y' = 3x^2 + 3}$$

$$y'' = \frac{d}{dx}(3x^2 + 3)$$

$$\boxed{y'' = 6x}$$

$$y''' = \frac{d}{dx}(6x)$$

$$\boxed{y''' = 6.}$$

Now,

$$y''' + xy'' - 2y' = 0$$

$$6 + x(6x) - 2(3x^2 + 3) = 0$$

$\cancel{6} + \cancel{6x^2} - \cancel{6x^2} - \cancel{6} = 0$

$0 = 0$

Hence proved!

**Q7)** (i) $5y^2 = x^2y + \frac{2}{xy^2}$

**Sol :-**

$$\frac{d}{dx}(5y^2) = \frac{d}{dx}\left(x^2y + \frac{2}{xy^2}\right)$$

$$10y\frac{dy}{dx} = \left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right] + \frac{d}{dx}2x^{-1}y^{-2}$$

$$10y\frac{dy}{dx} = \left[x^2\frac{dy}{dx} + y\,2x\right] + 2\left[x^{-1}\frac{d}{dx}y^{-2} + y^{-2}\frac{d}{dx}x^{-1}\right]$$

$$10y\frac{dy}{dx} = \left[x^2\frac{dy}{dx} + 2xy\right] + 2\left[\frac{1}{x}(-2y^{-3})\frac{dy}{dx} + \frac{1}{y^2}\left(\frac{-1}{x^2}\right)\right]$$

$$10y\frac{dy}{dx} = \left[x^2\frac{dy}{dx} + 2xy\right] + 2\left[-\frac{2}{xy^3}\frac{dy}{dx} - \frac{1}{x^2y^2}\right]$$

Date

$$10y\frac{dy}{dx} = x^2\frac{dy}{dx} + 2xy - \frac{4}{xy^3}\frac{dy}{dx} - \frac{2}{x^2y^2}$$

$$\cancel{x^2\frac{dy}{dx} + 2xy}$$

$$10y\frac{dy}{dx} + \frac{4}{xy^3}\frac{dy}{dx} - x^2\frac{dy}{dx} = 2xy - \frac{2}{x^2y^2}$$

$$\frac{dy}{dx}\left(10y + \frac{4}{xy^3} - x^2\right) = \frac{2x^3y^3 - 2}{x^2y^2}$$

$$\frac{dy}{dx}\left(\frac{10xy^4 + 4 - x^3y^3}{\cancel{x}\cancel{y^3}}\right) = \frac{2x^3y^3 - 2}{x^{\cancel{2}}y^{\cancel{2}}}$$

$$\frac{dy}{dx}\left(\frac{10xy^4 + 4 - x^3y^3}{y}\right) = \frac{2x^3y^3 - 2}{x}$$

$$\frac{dy}{dx} = \frac{y(2x^3y^3 - 2)}{x(10xy^4 - x^3y^3 + 4)}$$

Ans.

(ii) $x^{3/2} + y^{3/2} = 2$

Sol :-

$$\frac{d}{dx}\left(x^{3/2} + y^{3/2}\right) = \frac{d}{dx}(2)$$

$$\frac{3}{2}(x)^{1/2} + \frac{3}{2}(y)^{1/2}\frac{dy}{dx} = 0$$

$$\frac{\cancel{3}}{\cancel{2}} y^{1/2} \frac{dy}{dx} = -\frac{\cancel{3}}{\cancel{2}}\sqrt{x}$$

$$\frac{dy}{dx} = \frac{-\sqrt{x}}{\sqrt{y}}$$

(iii) $x^5 + 3x^2y^3 + 3x^3y^2 + y^5 = 8$

Sol :-

$$\frac{d}{dx}\left(x^5 + 3x^2y^3 + 3x^3y^2 + y^5\right) = \frac{d}{dx}(8)$$

$$5x^4 + 3\left[x^2 3y^2\frac{dy}{dx} + y^3.6x\right] + 3\left[x^3 2y\frac{dy}{dx} + y^2.9x^2\right] + 5y^4\frac{dy}{dx} = 0$$

$$5x^4 + 9x^2y^2\frac{dy}{dx} + 18xy^3 + 6x^3y\frac{dy}{dx} + 27x^2y^2 + 5y^4\frac{dy}{dx} = 0$$

$+ 9x^2y^2\frac{d}{d}$

$\frac{dy}{dx}\left[9x^2y\right.$

$\frac{dy}{dx} =$

$\boxed{\frac{dy}{dx} =}$

Q8) (i)

$\frac{dy}{dx} = 3$

$\frac{dy}{dt} =$

$$+9x^2y^2\frac{dy}{dx} + 6x^3y\frac{dy}{dx} + 5y^4\frac{dy}{dx} = -5x^4 - 18xy^3 - 27x^2y^2$$

$$\frac{dy}{dx}\left[9x^2y^2 + 6x^3y + 5y^4\right] = -\left(5x^4 + 18xy^3 + 27x^2y^2\right)$$

$$\frac{dy}{dx} = \frac{-\left(5x^4 + 18xy^3 + 27x^2y^2\right)}{9x^2y^2 + 6x^3y + 5y^4}$$

$$\boxed{\frac{dy}{dx} = \frac{-x\left(5x^3 + 18y^3 + 27xy^2\right)}{y\left(9x^2y + 6x^3 + 5y^3\right)}}$$

Ans.

Q8) (i) $y = x^3$ & $x = \frac{1}{\sqrt{t^2+5}}$

$$\frac{dy}{dx} = 3x^2$$

$$\frac{dx}{dt} = \frac{d(t^2+5)^{-1/2}}{dt}$$

$$\frac{dx}{dt} = \frac{-1}{\cancel{2}\sqrt{(t^2+5)^3}}(\cancel{2}t)$$

$$\frac{dx}{dt} = \frac{-t}{(t^2+5)^{3/2}}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Date

$$\frac{dy}{dt} = 3x^2 \times \left(\frac{-t}{(t^2+5)^{3/2}}\right)$$

$$= \frac{-3tx^2}{(t^2+5)^{3/2}}$$

$$= \frac{-3(t)(t)^2}{(t^2+5)^{3/2}}$$

$$\boxed{\frac{dy}{dt} = \frac{-3t^3}{(t^2+5)^{3/2}}}$$

(ii) $y = x^2 + 3x + 2$ & $x = \frac{t-1}{t+1}$

Sol:-

$$\frac{dy}{dx} = 2x + 3$$

$$\frac{dx}{dt} = \frac{(t+1)(1) - (t-1)(1)}{(t+1)^2}$$

$$= \frac{\cancel{t} + 1 - \cancel{t} + 1}{(t+1)^2}$$

$$= \frac{2}{(t+1)^2}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

No.

Date

$$\frac{dy}{dt} = (2x+3)\left[\frac{2}{(t+1)^2}\right]$$

$$= \frac{2(2x+3)}{(t+1)^2}$$

$$\frac{dy}{dt} = \frac{2(2t+3)}{(t+1)^2}$$

Ans.

Date

(iii) $f(u) = \dfrac{5}{\left(u + \frac{1}{\sqrt{u}}\right)^4}$

<u>Sol</u> :-

$f(x) = \dfrac{5}{\left(\frac{u\sqrt{u} + 1}{\sqrt{u}}\right)^4}$

$= \dfrac{5}{\frac{(u\sqrt{u} + 1)^4}{u^2}}$

$f(x) = \dfrac{5u^2}{(u\sqrt{u} + 1)^4}$

Now,

$$f'(x) = \frac{d}{dx}\left[\frac{5u^2}{(u^{3/2}+1)^4}\right]$$

$$= \frac{(u^{3/2}+1)^4 \cdot 10u - 5u^2 \cdot \left(\overset{2}{\cancel{4}}(u^{3/2}+1)^3\right)\frac{3\sqrt{u}}{\cancel{2}}}{(u^{3/2}+1)^8}$$

$$= \frac{10u(u^{3/2}+1)^4 - 30u^{3/2}(u^{3/2}+1)^3}{(u^{3/2}+1)^8}$$

Ans.