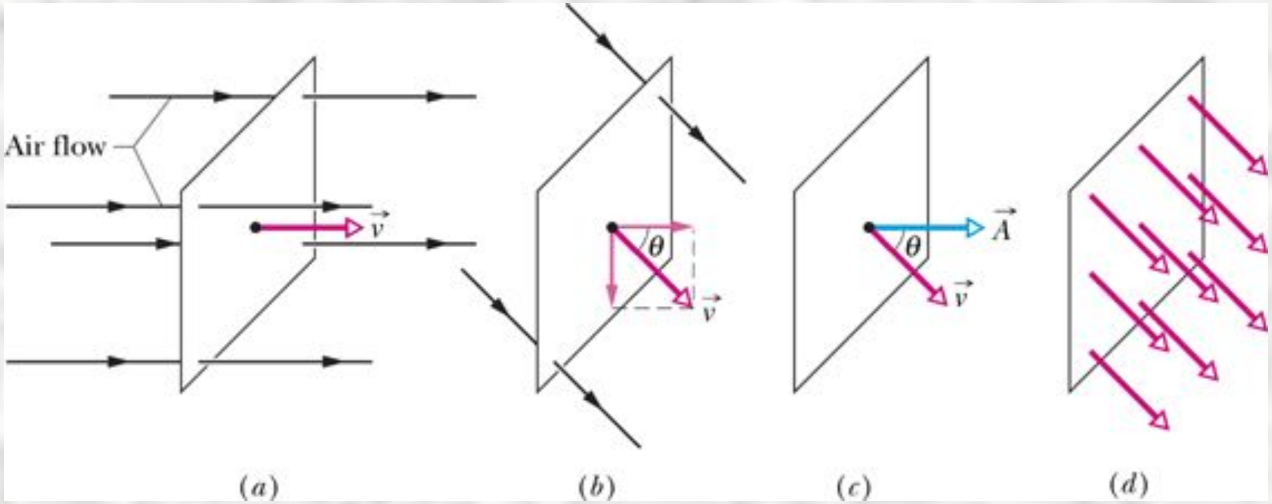
Flux



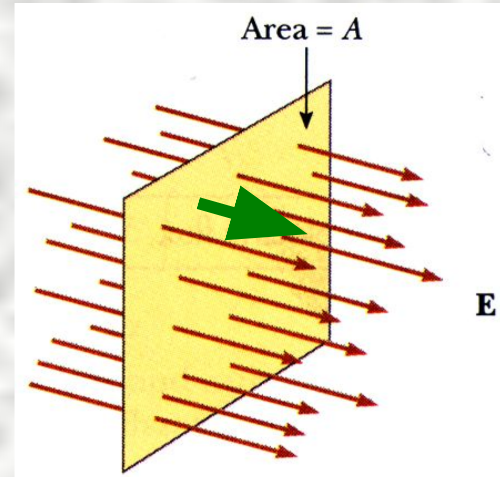
The rate of volume flow through the loop is :

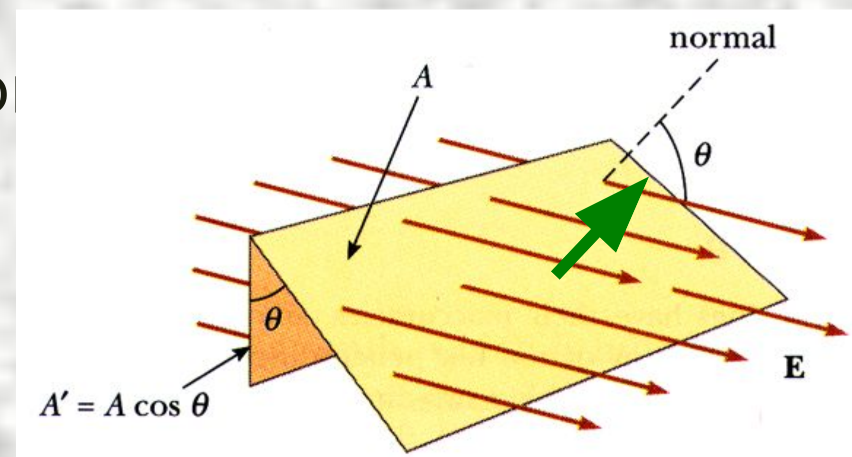
$$\Phi = (\nu \cos\theta) A .$$

$$\Phi = \nu A \cos\theta = \overrightarrow{\nu} \cdot \overrightarrow{A} ,$$

Definition of Electric Flux

- The amount of field, material or other physical entity passing through a surface.
- Surface area can be represented as vector defined normal to the surface it is describing
- Defined by the equatio





$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

Flux of Electric Field

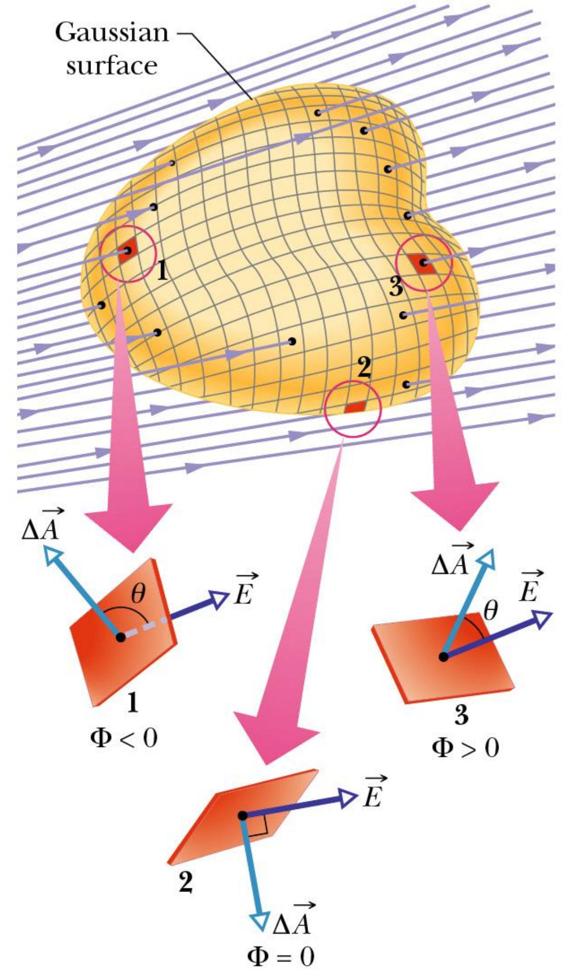
- Like the flow of water, or light energy, we can think of the electric field as flowing through a surface (although in this case nothing is actually moving).
- We represent the flux of electric field as Φ (greek letter phi), so the flux of the electric field through an element of area ΔA is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{A} = E\,\Delta A\cos\theta$$

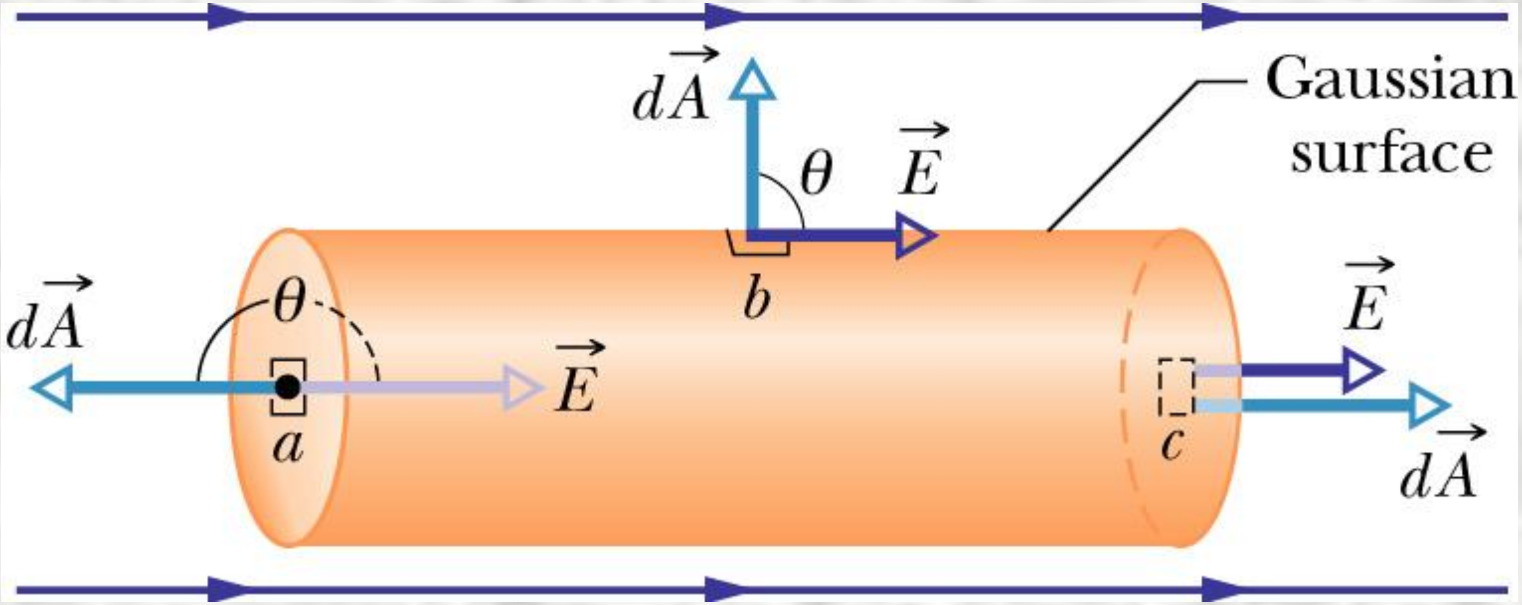
- When $\theta < 90^\circ$, the flux is positive (out of the surface), and when $\theta > 90^\circ$, the flux is negative.

$$d\Phi = \vec{E}\cdot d\vec{A} = E\,dA\cos\theta$$

- When we have a complicated surface, we can divide it up into tiny elemental areas:



Find the electric flux through a cylindrical surface in a uniform electric field **E**



$$\Phi = \oint \vec{E} \bullet d\vec{A} \qquad = \oint E \cos\theta\, dA$$

$$d\vec{A} = \hat{n}\, dA$$

a. $\Phi = \int E \cos 180\, dA = -\int E dA = -E\pi R^2$

b. $\Phi = \int E \cos 90\, dA = 0$

c. $\Phi = \int E \cos 180\, dA = \int E dA = E\pi R^2$

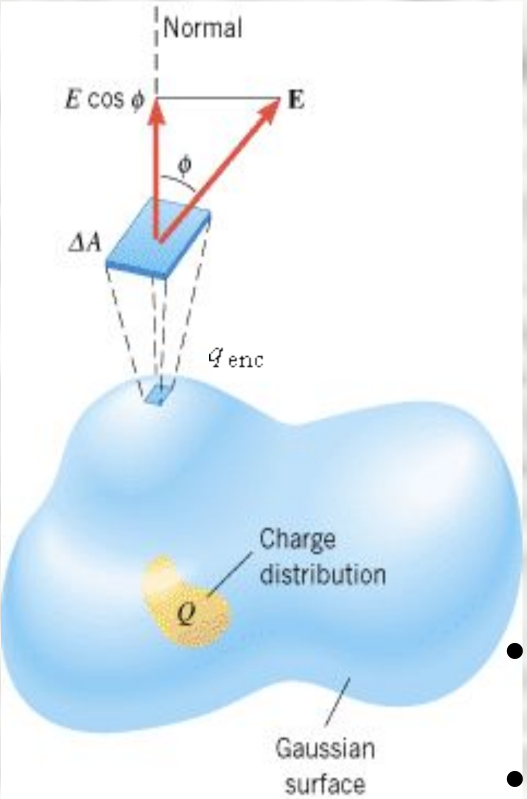
What is the flux if the cylinder were vertical ?

Suppose it were any shape?

Flux from a. + b. + c. =

Gauss' Law

For charge distribution Q:



The electric flux through a Gaussian surface times by ε_0 (the permittivity of free space) is equal to the net charge *Q* enclosed :

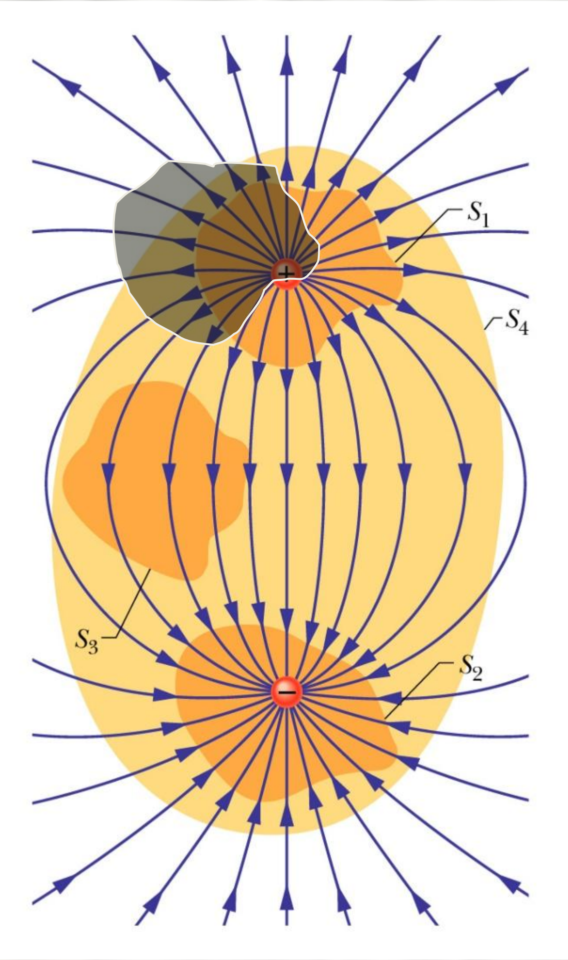
$$\varepsilon_0 \Phi = q_{\text{enc}} \qquad \text{(Gauss' law)},$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad \text{(Gauss ' law)}.$$

- The net charge q_{enc} is the algebraic sum of all the *enclosed* charges.
- Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc}.

Example of Gauss' Law

- Consider a dipole with equal positive and negative charges.
- Imagine four surfaces S_1, S_2, S_3, S_4, as shown.
- S_1 encloses the positive charge. Note that the field is everywhere outward, so the flux is positive.
- S_2 encloses the negative charge. Note that the field is everywhere inward, so the flux through the surface is negative.
- S_3 encloses no charge. The flux through the surface is negative at the upper part, and positive at the lower part, but these cancel, and there is no net flux through the surface.
- S_4 encloses both charges. Again there is no net charge enclosed, so there is equal flux going out and coming in—no net flux through the surface.



Electric lines of flux and
Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E \cos\theta dA = \oint EdA$ **E ll n** **Cos 0 = 1**

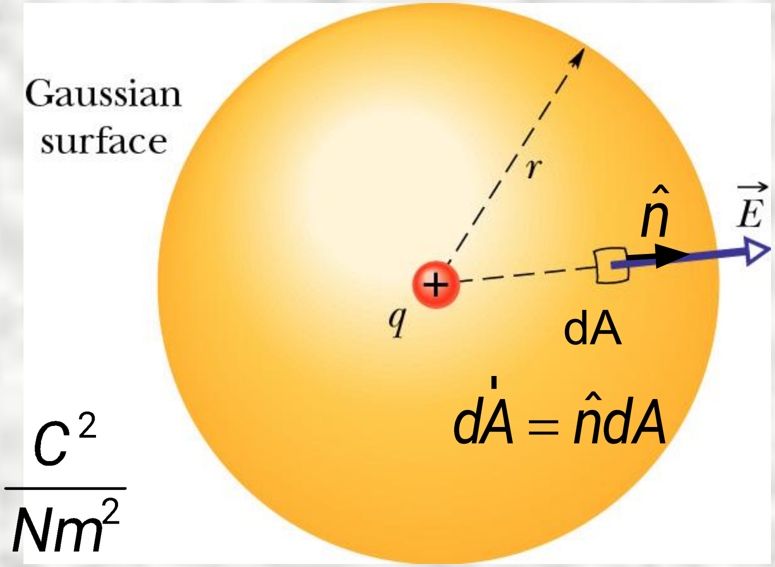
For a Point charge **E=kq/r²**

$$\Phi = \oint EdA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2 (4\pi r^2)$$

$$\Phi = 4\pi kq$$

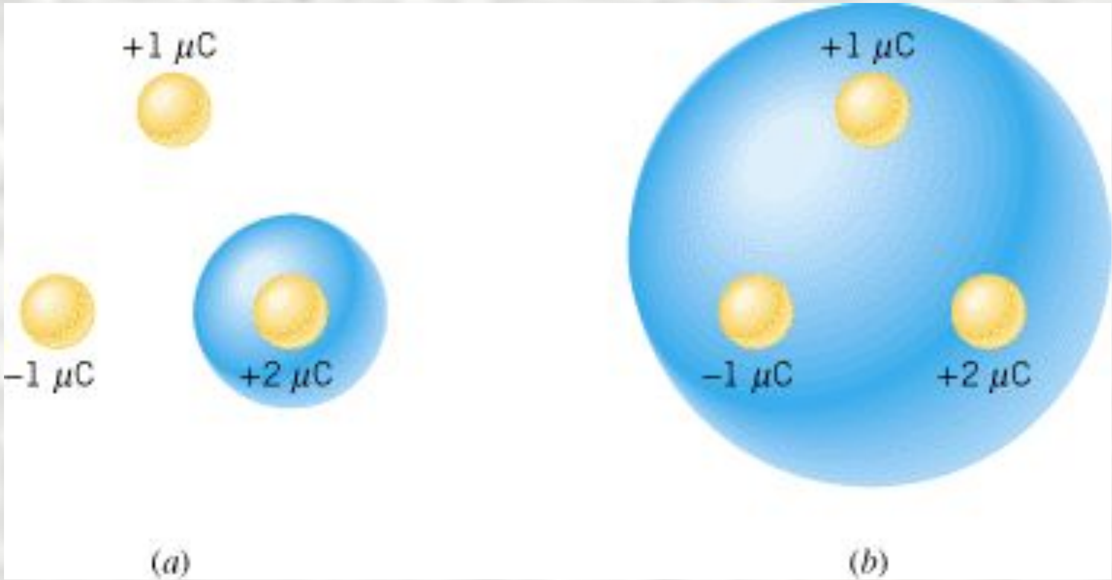
$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12} \frac{C^2}{Nm^2}$$



$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$ Gauss' Law

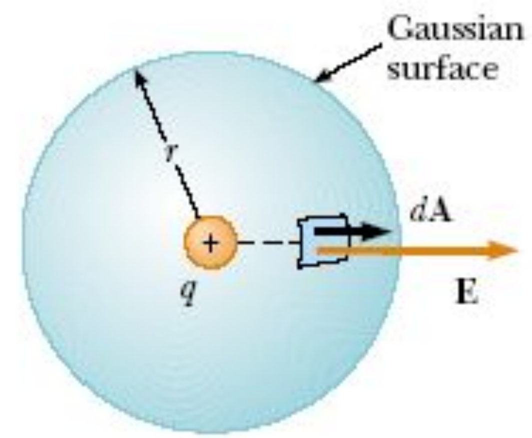
Check Your Understanding

The drawing shows an arrangement of three charges. In parts (*a*) and (*b*) different Gaussian surfaces are shown. Through which surface, if either, does the greater electric flux pass?



The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.



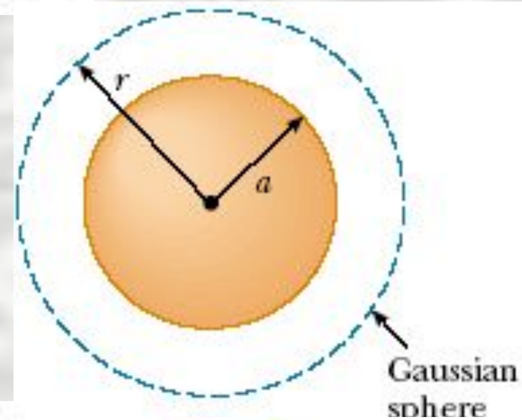
$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\, dA = \frac{q}{\epsilon_0}$$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2} = k_e \frac{q}{r^2}$$

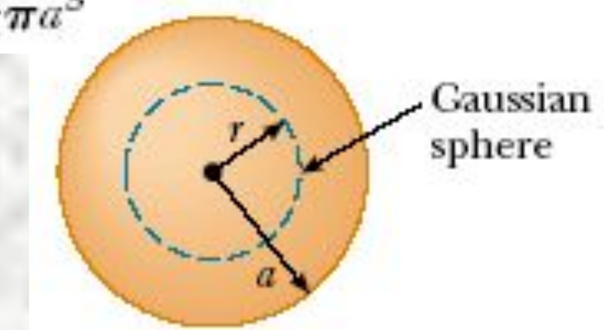
A Spherically Symmetric Charge Distribution

$$E = k_e \frac{Q}{r^2} \qquad \text{(for } r > a\text{)}$$



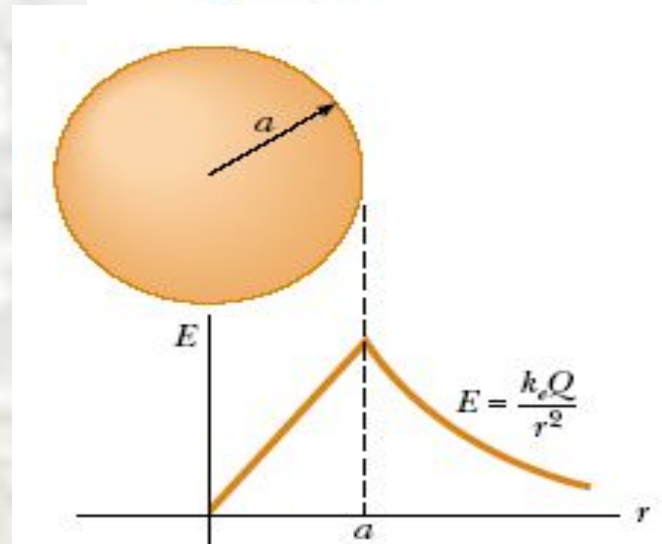
$$q_{\text{in}} = \rho V' = \rho\left(\tfrac{4}{3}\pi r^3\right)$$

$$\rho = Q / \tfrac{4}{3}\pi a^3$$



$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r$$



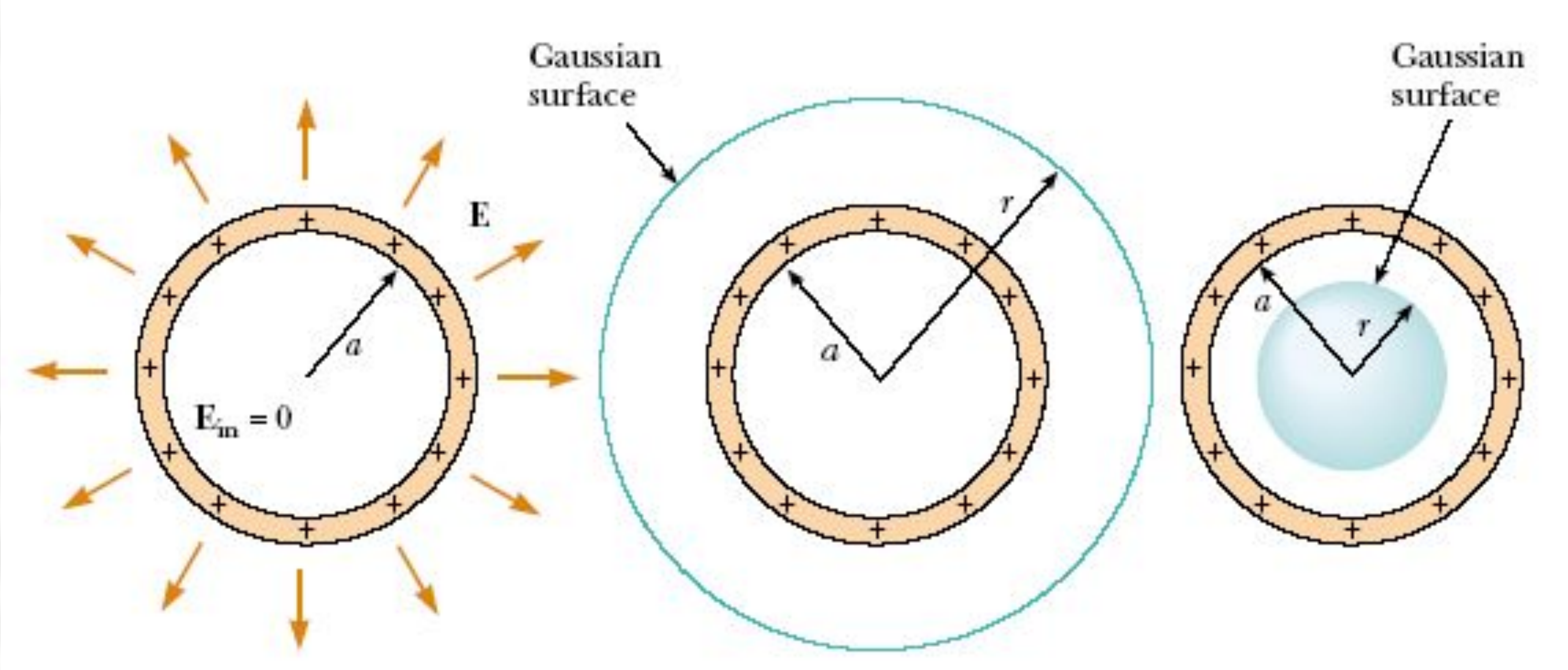
$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \frac{k_e Q}{a^3} r \qquad \text{(for } r < a\text{)}$$

The Electric Field Due to a Thin Spherical Shell

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

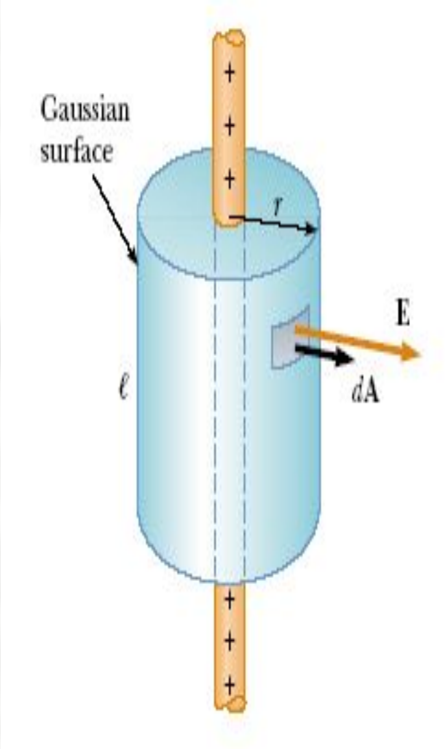
$$E = k_e \frac{Q}{r^2} \qquad (\text{for } r > a)$$

$E = 0$ in the region $r < a$.



A Cylindrically Symmetric Charge Distribution

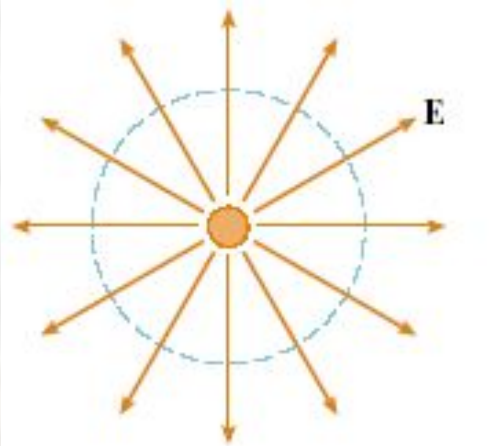
$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$



$$A = 2\pi r \ell$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

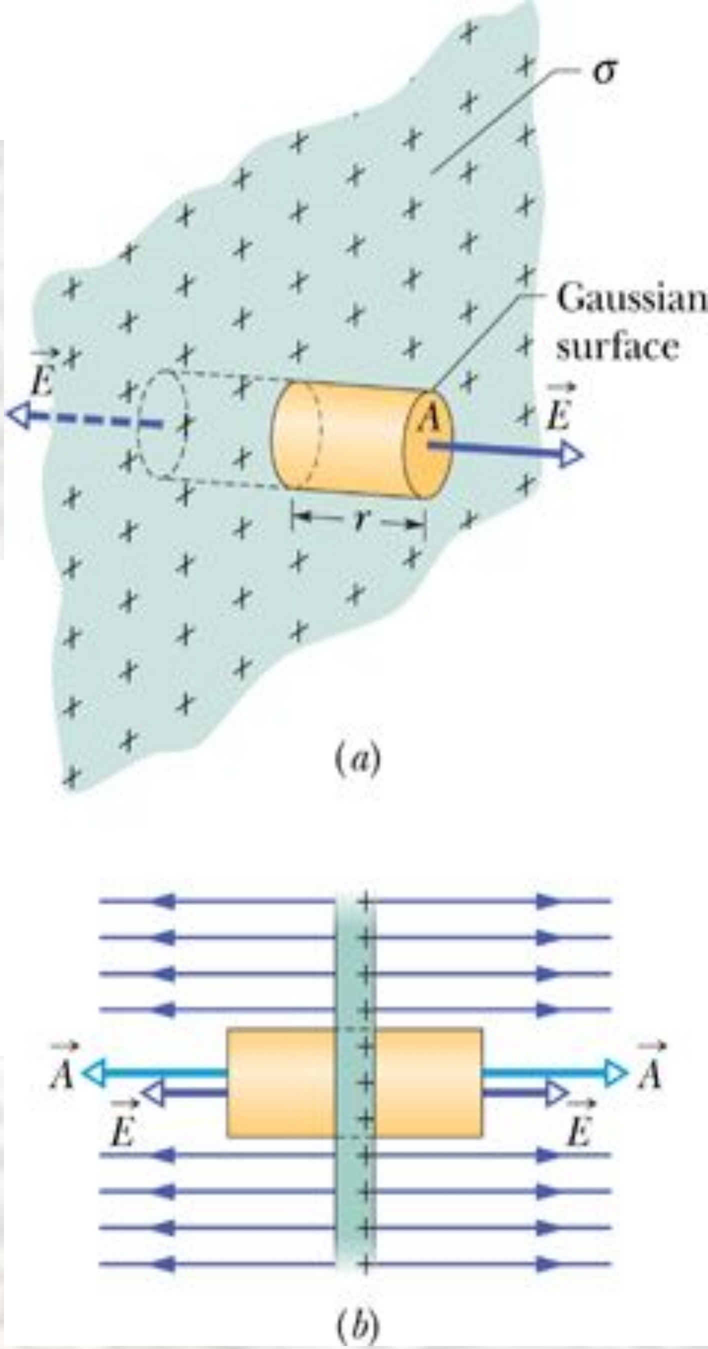
$$E = \frac{\lambda}{2\pi \epsilon_0 r} = \boxed{2k_e \frac{\lambda}{r}}$$



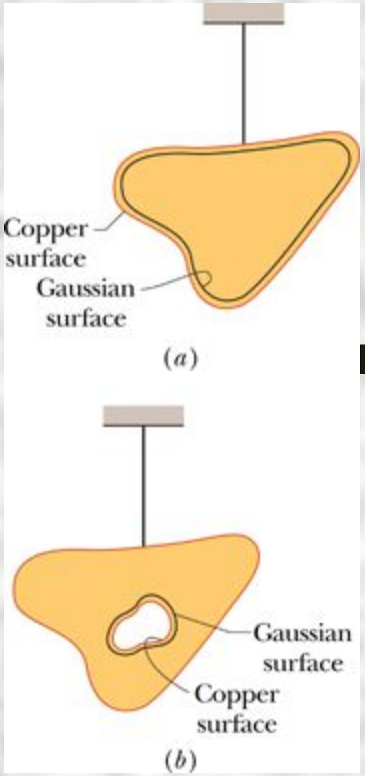
A Nonconducting Plane of Charge

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

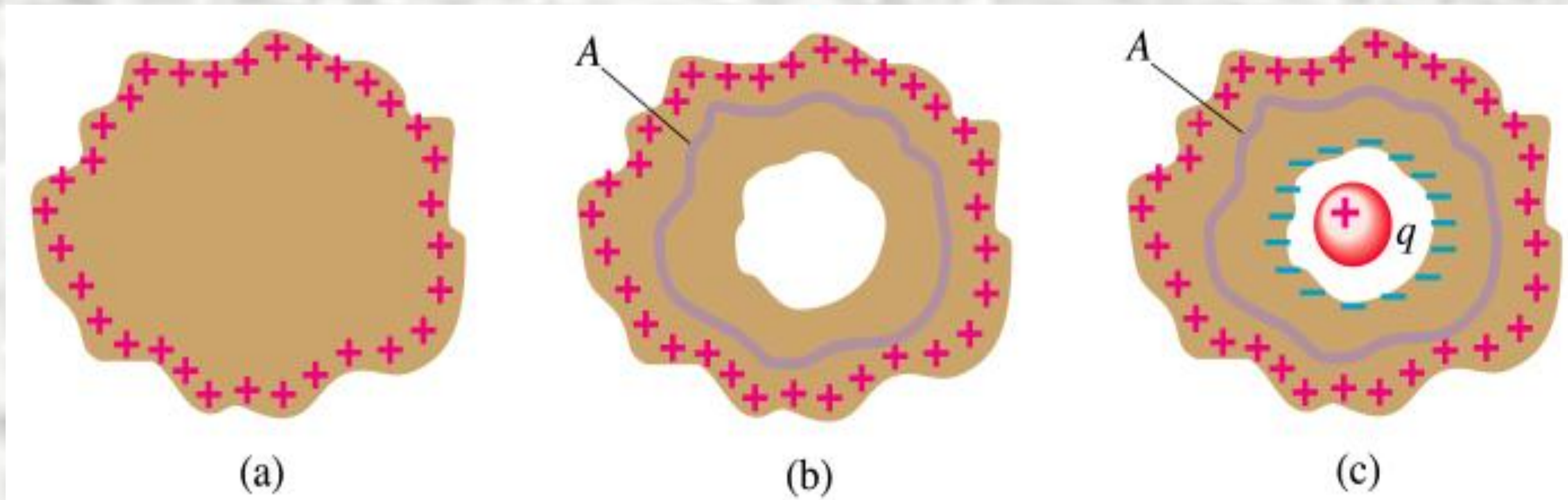
$$E = \boxed{\frac{\sigma}{2\epsilon_0}}$$



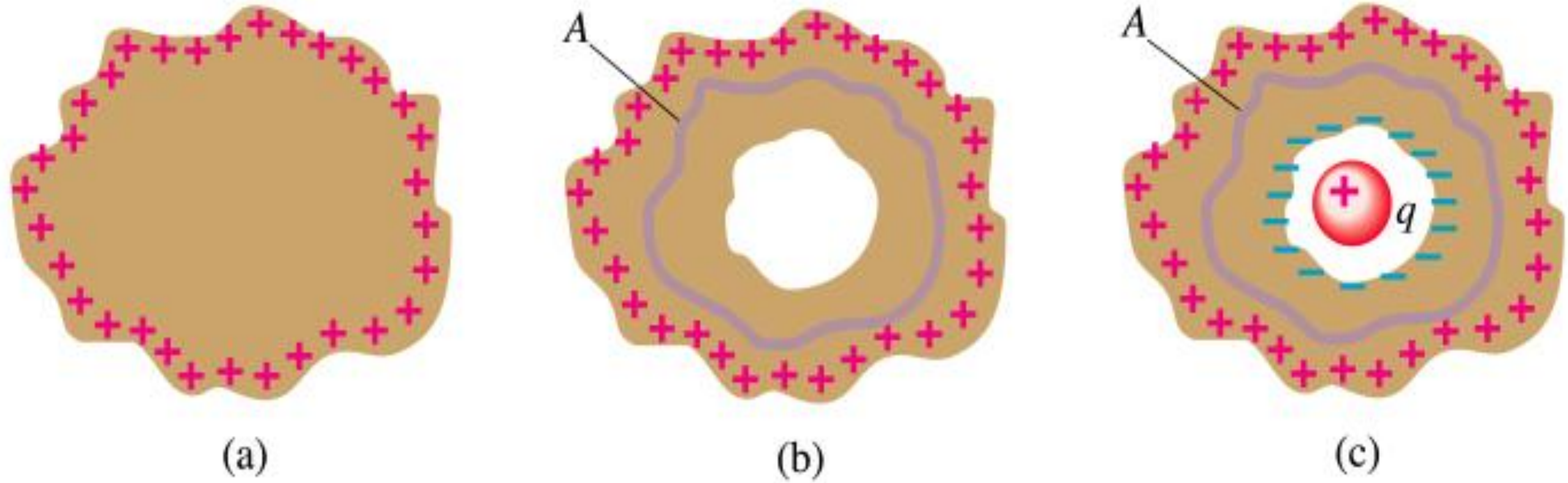
A Charged Isolated Conductor



- If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.
- For an Isolated Conductor with a Cavity, There is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor



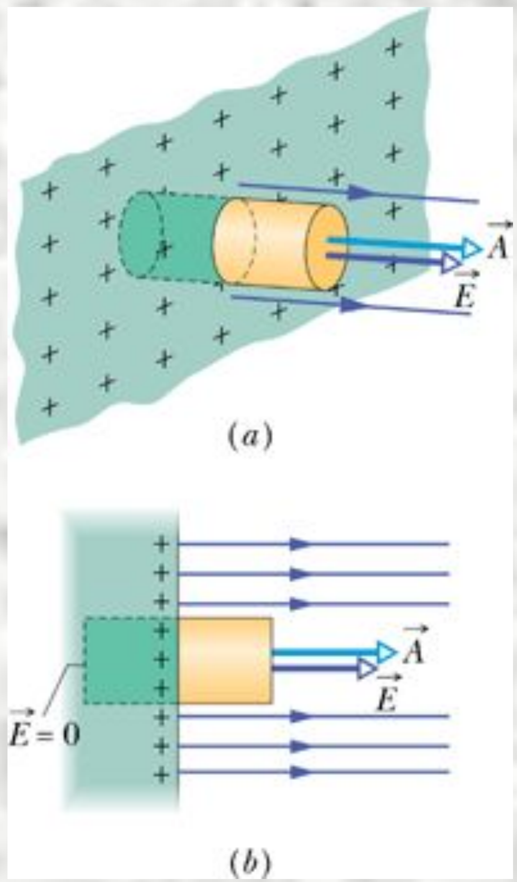
Find electric charge q on surface of hole in the charged conductor.



The solution of this problem lies in the fact that the electric field inside a conductor is zero and if we place our Gaussian surface inside the conductor (where the field is zero), the charge enclosed must be zero (+ q – q) = 0.

Find electric charge q on surface of hole in the charged conductor.

The External Electric Field of a Conductor



If σ is the charge per unit area,

according to Gauss' law,

$$\varepsilon_0 EA = \sigma A,$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface)}.$$