Tuesday, April 23, 2024 10:56 AM

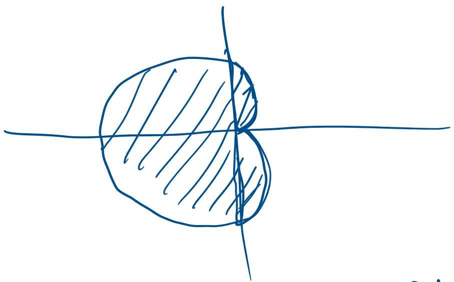
Double integral in Polar Coordinates

Ex #14.3

Q7 $r = 1 - \cos\theta$ $\le \theta \le$

$\theta = 2\pi$ $\theta = 0$

$r = 1 - 1 = 0$ $r = 0$



$0 \le r \le 1 - \cos\theta$

$0 \le \theta \le 2\pi$

using formula for area of R

$$\text{area of } R = \iint_R r\,dr\,d\theta = \int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{1-\cos\theta} d\theta$$

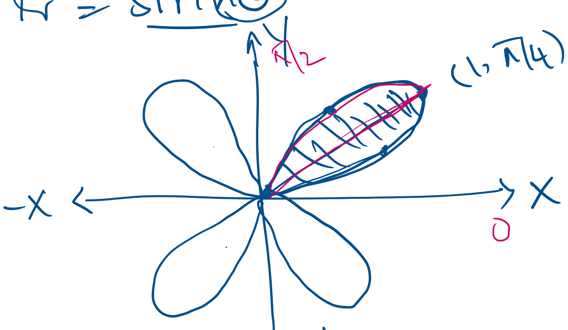
$$= \frac{1}{2}\int_0^{2\pi} \{(1-\cos\theta)^2 - 0\}\,d\theta$$

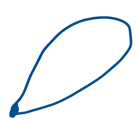
$$= \frac{1}{2}\int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\,d\theta$$

= Complete it

Q8 $r = \sin 2\theta$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
r	0	$\sqrt{3}/2$	1	$\sqrt{3}/2$	0





$2\pi/6 = \pi/3$

$\sin(2\pi/4) = 1$

$\sin(2\pi/3) = \frac{\sqrt{3}}{2}$

$$\text{area of } R = 4\int_0^{\pi/2}\int_0^{\sin 2\theta} r\,dr\,d\theta = 4\int_0^{\pi/2}\int_0^{\sin 2\theta} r\,dr\,d\theta$$

$$\text{area of } R = 4\int_0\int_0 r\,dr\,d\theta = 4\int_0\int_0 r\,dr\,d\theta$$

$$\text{area of } R = \cancel{4}^{2}\int_0^{\pi/2}\left[\frac{r^2}{\cancel{2}}\right]_0^{\sin 2\theta} d\theta = 2\int_0^{\pi/4}(\sin^2 \underset{t}{2\theta} - 0)\,d\theta$$

$$\text{area of } R = \cancel{2}\int_0^{\pi/2}\left(\frac{1-\cos 4\theta}{\cancel{2}}\right) d\theta$$

$$= \left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$$

$$= \left(\frac{\pi}{2} - 0\right) - \frac{1}{4}\left[\sin\left(\cancel{4}^{2}\times\frac{\pi}{\cancel{2}}\right) - \sin(0)\right]$$

$$\boxed{\text{area of } R = \pi/2 - \frac{1}{4}(0-0) = \pi/2}$$ Ans.

R.W

$$\sin^2 t = \frac{1-\cos 2t}{2}$$

$$\sin^2 2\theta = \frac{1-\cos 4\theta}{2}$$

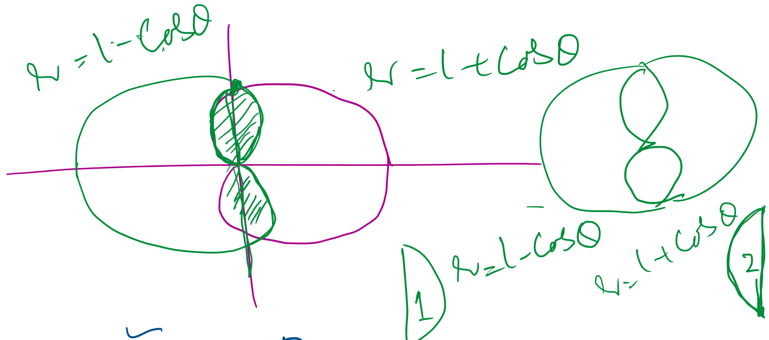
Find the area of Region enclosed by the intersection of two cardioid, $r = 1+\cos\theta$ & $r = 1-\cos\theta$

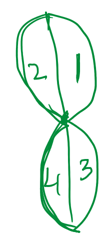
$1 - \cos\theta = 1 + \cos\theta$

$2\cos\theta = 0$

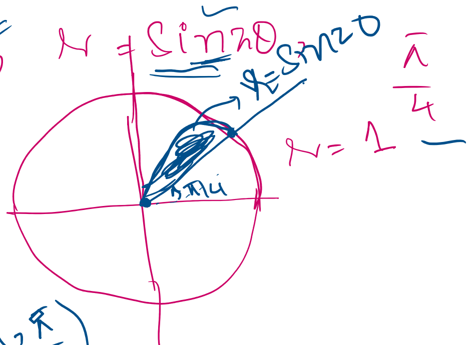
$\theta = \cos^{-1}(0)$

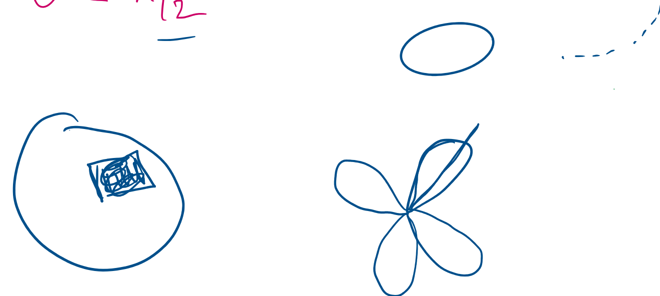
$\frac{\pi}{2}, \frac{3\pi}{2}$

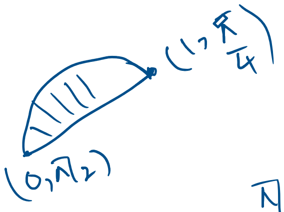




Q9 $r = 1$, $r = \sin 2\theta$, $\frac{\pi}{4} \le \theta \le \pi/2$







$$\sin 2\theta \le r \le 1, \quad \frac{\pi}{4} \le \theta \le \pi/2$$

$$\int^{\pi/2}\int^{1}$$

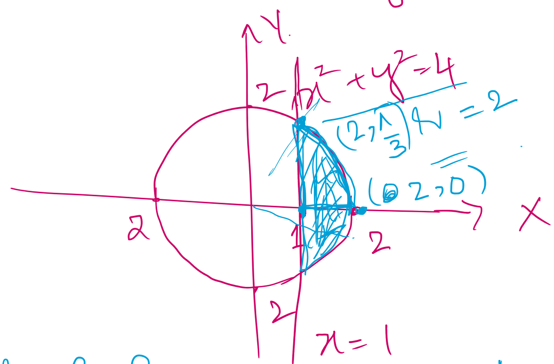
$(0, \pi/2)$ $\qquad\qquad\qquad\qquad \frac{\pi}{4} - \theta - \pi/2$

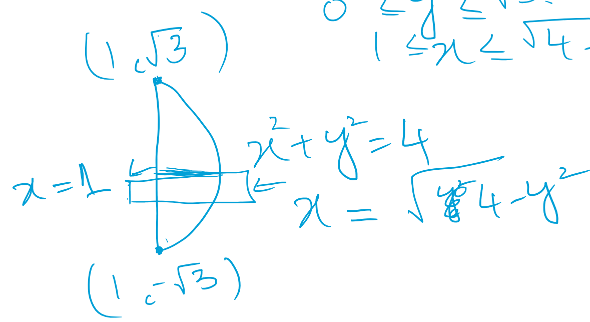
$$\text{area of } R = \int_{\pi/4}^{\pi/2} \int_{\sin 2\theta}^{1} r \, dr \, d\theta = \pi/16 .$$

Compute it.

Q10 inside the circle $x^2+y^2=4$ and right to the line $x=1$

Sol





$0 \le y \le \sqrt{3}$
$1 \le x \le \sqrt{4-y^2}$

equate

$r=2$ & $r = \sec\theta$

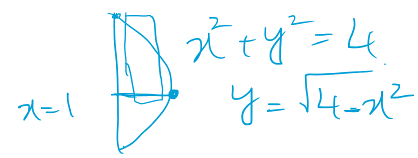
$2 = \sec\theta$

$2 = \frac{1}{\cos\theta}$

$\Rightarrow \cos\theta = 1/2 \Rightarrow \boxed{\theta = \pi/3}$

$r\cos\theta = 1$

$r = \sec\theta$



$0 \le y \le \sqrt{4-x^2}$
$1 \le x \le 2$

$\sec\theta \le r \le 2 \qquad 0 \le \theta \le \pi/3$

Method 1

$$\text{area of } R = 2\int_0^{\pi/3} \int_{\sec\theta}^{2} r \, dr \, d\theta = \frac{4\pi}{3} - \sqrt{3}$$

Method 2

$$\text{area of } R = 2\int_1^2 \int_0^{\sqrt{4-x^2}} dy \, dx = 2\int_0^{\sqrt{3}} \int_1^{\sqrt{4-y^2}} dx \, dy = \frac{4\pi}{3} - \sqrt{3}$$

SOME STANDARD CURVES IN POLA COORDINATE

left
Right
up
down

1) Cardioid

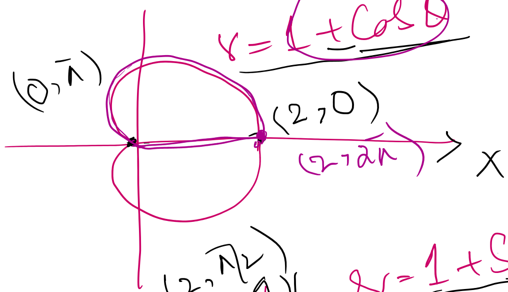
$\theta = 2\pi$ $\qquad r = 1 - \cos\theta$

1) Cardioid

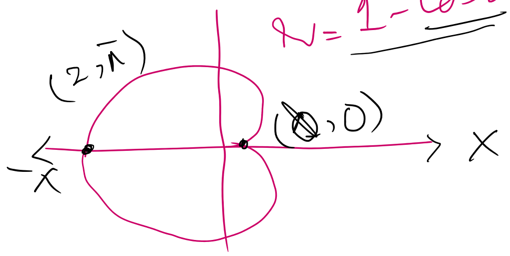
$\theta = 2\pi$

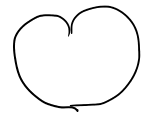
up
down

$r = 1 + \cos\theta$

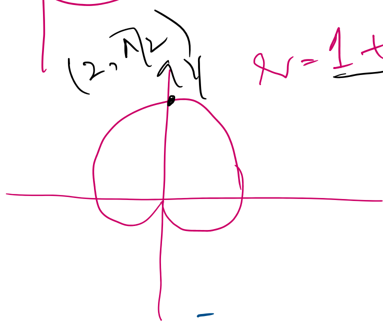


$r = 1 - \cos\theta$

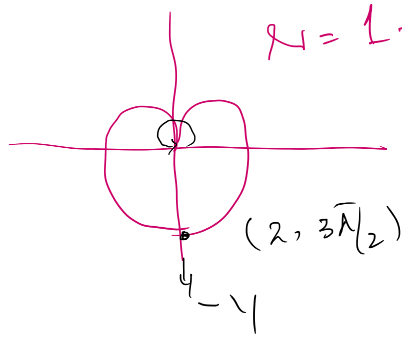




$r = 1 + \sin\theta$

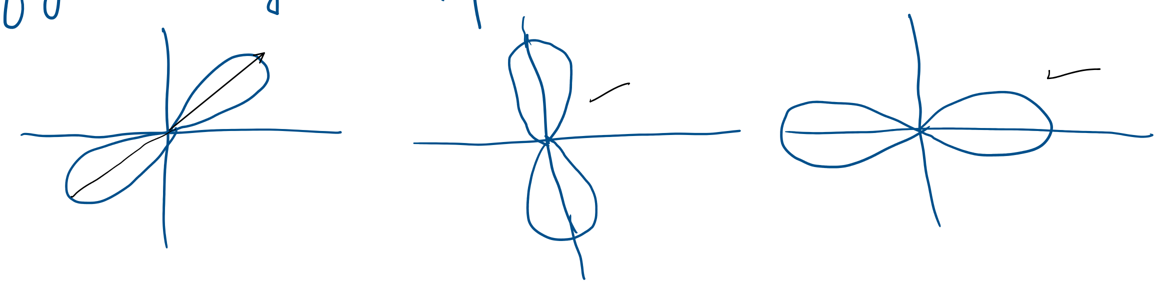


$r = 1 - \sin\theta$



2) Lemniscate

A figure eight shape is called Lemniscate.

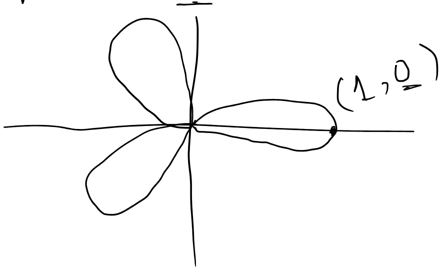


3) Rose curves

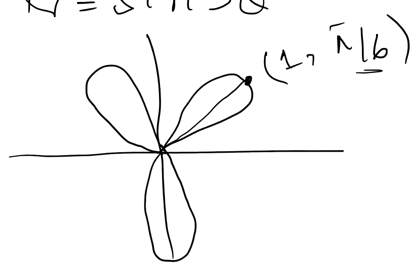
$\theta = \pi/6$

3- Petal Rose

$r = \cos 3\theta$



$r = \sin 3\theta$



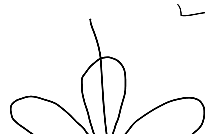
② 4-Petal Rose

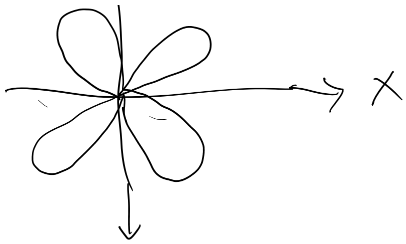
$r = \sin 2\theta$

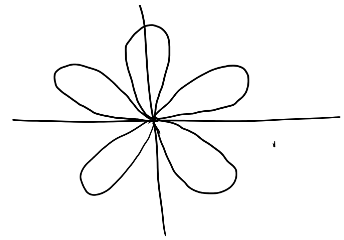
e.g $r = \cos 2\theta$

③ 5-Petal rose

$r = \sin 5\theta$

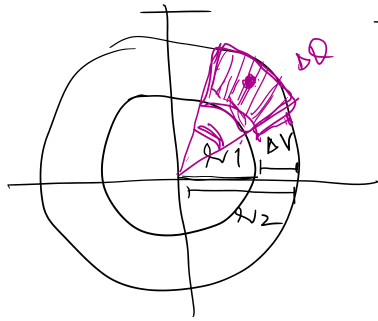


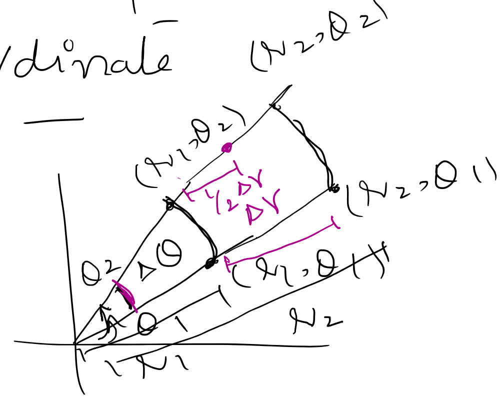




Double integrals in Polar coordinate

I. Area of a Sector $= \frac{1}{2} r^2 \theta$



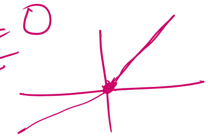


$dA = dx\,dy$

If $f(r,\theta)$ is continuous and non-negative on region R, then

$$V = \iint_R f(r,\theta)\,dA = \iint_R f(r,\theta)\, r\,dr\,d\theta$$

$r = 0$



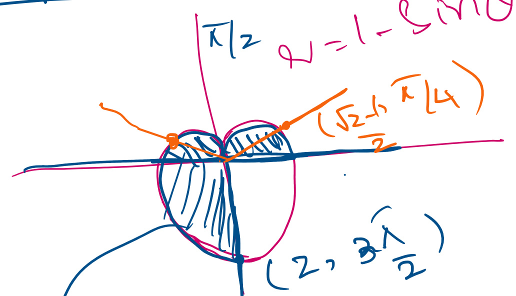
Q1-Q6 Ex # 14.3 Q1 – Q10

Evaluate the iterated integrals.

$r = 1 - \sin\theta$

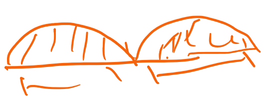
Q5 $\int_0^{\pi}\int_0^{1-\sin\theta} r^2 \cos\theta\,dr\,d\theta$

$1 - 1/\sqrt{2}$



$0 \le r \le 1 - \sin\theta$

$0 \le \theta \le \pi$



$\frac{\pi}{2} \le \theta \le 0$

$\frac{\pi}{4} \le \theta \le 3\pi/4$

$\frac{3\pi}{2} \le \theta \le 0$

$$\int_0^{\pi}\int_0^{1-\sin\theta} r^2 \cos\theta\,dr\,d\theta = \int_0^{\pi} \left[\cos\theta\ \frac{r^3}{3}\right]_0^{1-\sin\theta} d\theta$$

$$= \frac{1}{3}\int_0^{\pi} \cos\theta \{(1-\sin\theta)^3 - 0\} \, d\theta$$

$$= \frac{1}{3}\int_0^{\pi} \cos\theta \, (\underbrace{1-\sin\theta}_{u})^3 \, \underbrace{d\theta}_{(-du)}$$

R.W

$u = 1-\sin\theta$

$du = -\cos\theta \, d\theta$

$\boxed{-du = \cos\theta \, d\theta}$

$\int \cos\theta (1-\sin\theta)^3 d\theta$

$= -\int u^3 \, du$

$= -\frac{u^4}{4}$

$= -\frac{(1-\sin\theta)^4}{4}$

$$= -\frac{1}{3}\left[\frac{(1-\sin\theta)^4}{4}\right]_0^{\pi}$$

$$= -\frac{1}{12}\left[\{1-\sin(\pi)\}^4 - \{1-\sin(0)\}^4\right]$$

$$= -\frac{1}{12}\left[(1-0)^4 - (1-0)^4\right]$$

$$= -\frac{1}{12}(1-1) = 0 \quad \text{Ans}$$

$$V = \iint \underbrace{f(x,y)}_{1} \, dx\,dy = \iint f(x,y) \, dA$$

FINDING AREA USING POLAR DOUBLE INTEGRAL

The area of a region R in the xy plane can be expressed as

$$\text{area of } R = \iint_R 1 \, dA = \iint_R dx\,dy$$

So

area of R in the polar coordinate is given as

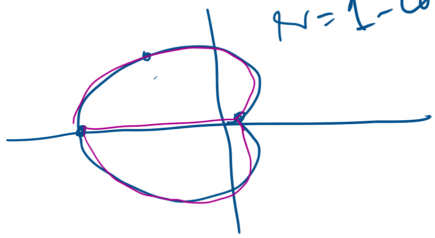
$$\text{area of } R = \iint_R dA = \iint_R r \, dr \, d\theta.$$

....

$$\iint_R dA = \iint_R r\,dr\,d\theta$$

EX #14.3

Q7 The region enclosed by the cardioid $r = 1-\cos\theta$



$0 \le r \le 1-\cos\theta$

$0 \le \theta \le 2\pi$

$$\text{area of } R = \iint_R r\,dr\,d\theta = \int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta.$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{1-\cos\theta} d\theta$$

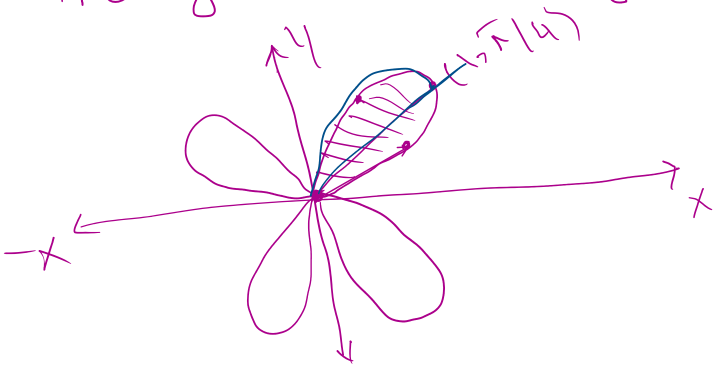
$$= \frac{1}{2}\int_0^{2\pi} \left\{(1-\cos\theta)^2 - 0\right\} d\theta$$

$$\text{area of } R = \frac{1}{2}\int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\,d\theta$$

Complete it.

Q8 The region enclosed by $r = \sin 2\theta$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
r	0	$\sqrt{3}/2$	1	$\sqrt{3}/2$	0



$2\theta = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

$2\theta = \frac{2\pi}{3}$

$\sin 2\theta = \sin(2\pi/3) = \frac{\sqrt{3}}{2}$

$2\theta = 2(\pi/2) = \pi$

$\sin \pi = 0$

$$\text{area of } R = \iint_R r\,dr\,d\theta$$

$$= 4\int^{\pi/4}\int^{\sin 2\theta} r\,dr\,d\theta$$

$$= 4\int_0^{\pi/2}\int_0 r\,dr\,d\theta$$

$$= \frac{4}{2}\int_0^{\pi/2}\left[\frac{r^2}{2}\right]_0^{\sin 2\theta} d\theta = 2\int_0^{\pi/2}(\sin^2 2\theta - 0)\,d\theta$$

$$= 2\int_0^{\pi/2}\sin^2 2\theta\,d\theta$$

RoW

$\sin^2 t = \frac{1-\cos 2t}{2}$

$\sin^2 2\theta = \frac{1-\cos 4\theta}{2}$

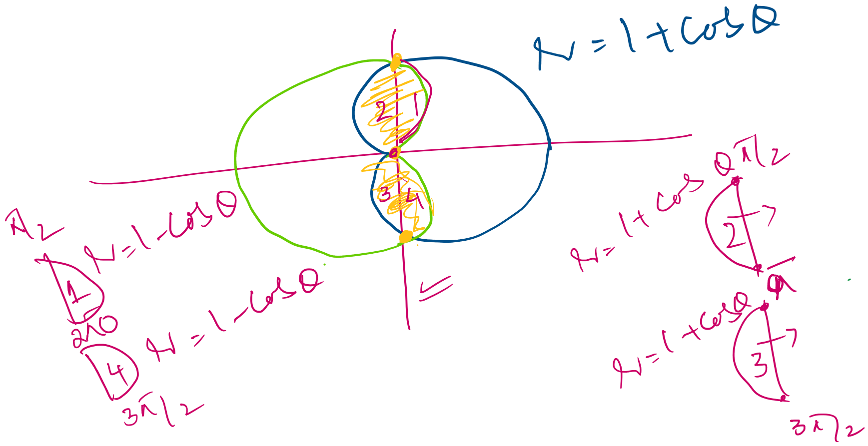
$$= \cancel{2}\int_0^{\pi/2}\left\{\frac{1-\cos 4\theta}{\cancel{2}}\right\}d\theta$$

$$= \int_0^{\pi/2}(1-\cos 4\theta)\,d\theta = \left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$$

$$= \left(\frac{\pi}{2}-0\right) - \frac{1}{4}\left[\sin\left(\cancel{4}^{2}\times\frac{\pi}{\cancel{2}}\right) - \sin(0)\right]$$

area of R $= \pi/2 - 0 = \pi/2$ Ans ✓

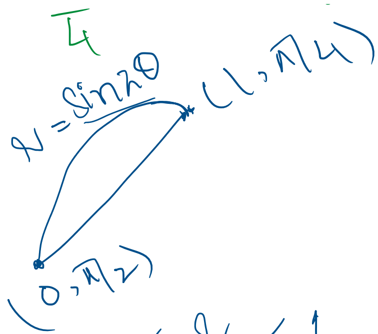
Find the area of the region enclosed by the intersection of two cardioid $r = 1+\cos\theta$ and $r = 1-\cos\theta$

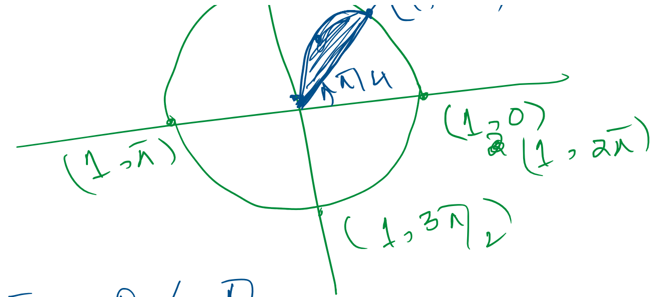


Q9 The region enclosed by $r = 1$ and $r = \sin 2\theta$

$\frac{\pi}{4} \le \theta \le \pi/2$

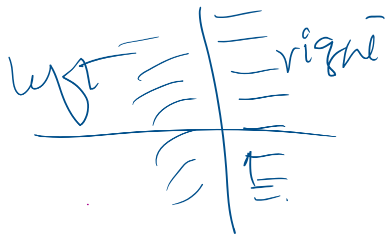
$\sin 2\theta \quad (1, \pi/4)$



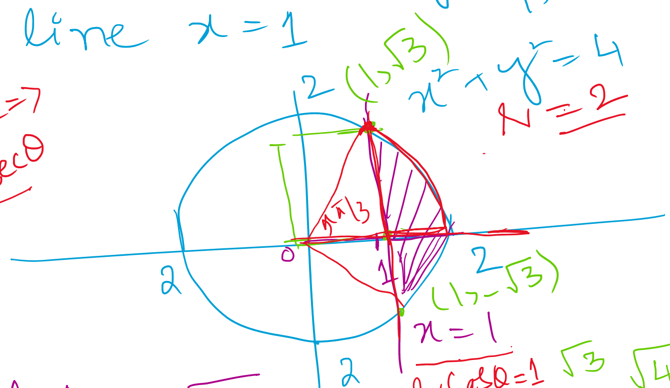


$\sin 2\theta \le r \le 1$ $\qquad$ $\frac{\pi}{4} \le \theta \le \pi/2$

$$\text{area of } R = \int_{\pi/4}^{\pi/2}\int_{\sin 2\theta}^{1} r \, dr \, d\theta = \pi/16 \quad \text{Ans.}$$

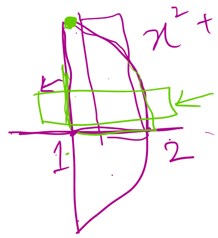


Q10 The region enclosed by the interior of circle $(x^2+y^2=4)$ and to the right of line $x = 1$



$r\cos\theta = 1 \Rightarrow r = \sec\theta$

Method 1 $\quad x = 1$



$y = \sqrt{4-x^2}$

$x = \sqrt{4-y^2}$

Method 1

$$\text{area of } R = 2\int_{1}^{2}\int_{0}^{\sqrt{4-x^2}} dy \, dx = 2\int_{0}^{\sqrt{3}}\int_{1}^{\sqrt{4-y^2}} dx \, dy = \frac{4\pi}{3} - \sqrt{3}$$

$1 \le x \le 2$

$0 \le y \le \sqrt{4-x^2}$

type II region.

$1 \le x \le \sqrt{4-y^2}$

$0 \le y \le \sqrt{3}$

Method 2: Via Polar Coordinates

$r = 2$ and $r = \sec\theta$

on equating

$2 = \sec\theta \Rightarrow 2 = \frac{1}{\cos\theta} \Rightarrow \cos\theta = 1/2 \Rightarrow \boxed{\theta = \pi/3}$

$0 \le \theta \le \pi/3$

$$\text{area of } R = 2\int_{0}^{\pi/3}\int_{\sec\theta}^{2} r \, dr \, d\theta = \frac{4\pi}{3} - \sqrt{3}$$

area of $R = \int_{-\pi}^{\pi}\int_{\sec\theta} r\,dr\,d\theta = \frac{4\pi}{3} - \sqrt{3}$

Date:- 01/05/2024 MVC - BCS 2B & 2F

TRIPLE INTEGRATION:-

EX # 14.5 Q1 – Q8

Q2 $\displaystyle\int_{1/3}^{1/2}\int_{0}^{\pi}\int_{0}^{1} z\,x\sin(xy)\,dz\,dy\,dx$

$= \displaystyle\int_{1/3}^{1/2}\int_{0}^{\pi}\left[x\sin(xy)\frac{z^2}{2}\right]_0^1 dy\,dx$

$= \displaystyle\frac{1}{2}\int_{1/3}^{1/2}\int_{0}^{\pi} x\sin(xy)\,(1-0)\,dy\,dx$

$= \displaystyle-\frac{1}{2}\int_{1/3}^{1/2}\left[\cos(xy)\right]_0^{\pi} dx$

$= \displaystyle-\frac{1}{2}\int_{1/3}^{1/2}\{\cos(\pi x)-\cos(0)\}\,dx$

$= \displaystyle-\frac{1}{2}\int_{1/3}^{1/2}(\cos\pi x - 1)\,dx$

$= \displaystyle-\frac{1}{2}\left[\frac{\sin\pi x}{\pi} - x\right]_{1/3}^{1/2} = -\frac{1}{2}\left\{\left(\frac{\sin(\pi/2)}{\pi} - \frac{\sin(\pi/3)}{\pi}\right) - \left(\frac{1}{2}-\frac{1}{3}\right)\right\}$

$= \displaystyle-\frac{1}{2}\left[\left(\frac{1}{\pi}-\frac{\sqrt{3}}{2\pi}\right)-\frac{1}{6}\right] = \frac{\sqrt{3}}{4\pi} - \frac{1}{2\pi} + \frac{1}{12}$ Ans

R.W

Let $u = xy$

$du = x\,dy$

$\int x\sin(xy)\,dy$

$\int \sin(u)\,du$

$-\cos u$

$-\cos(xy)$

Q8 $= \displaystyle\int_{1}^{2}\int_{z}^{2}\int_{0}^{\sqrt{3}y} \frac{y}{x^2+y^2}\,dx\,dy\,dz$

R.W.

$\displaystyle\int\frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

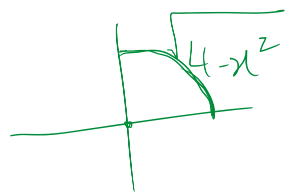
Sol $\displaystyle\int^{2}\int^{2}\left[y/x\ \tan^{-1}\left(\frac{x}{\ }\right)\right]^{\sqrt{3}y} dy\,dz$

Sol $$\int_1^2 \int_z^2 \left[\cancel{y} \times \frac{1}{\cancel{y}} \tan^{-1}\left(\frac{x}{y}\right) \right]_0^{\sqrt{3}y} dy\, dz$$

$\tan^{-1}(\sqrt{3}) = \pi/3$

$$= \int_1^2 \int_z^2 \left[\tan^{-1}\left(\frac{\sqrt{3}\cancel{y}}{\cancel{y}}\right) - \tan^{-1}(0) \right] dy\, dz$$

$$= \int_1^2 \int_z^2 (\pi/3 - 0)\, dy\, dz$$



$$= \int_1^2 \left[\frac{\pi}{3} y \right]_z^2 dz$$

$$= \frac{\pi}{3} \int_1^2 (2 - z)\, dz$$

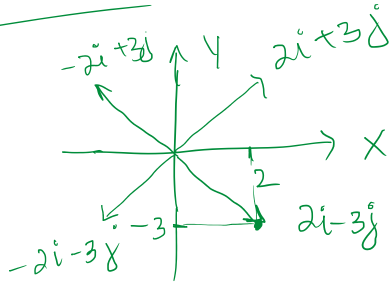
$$= \frac{\pi}{3} \left[2z - \frac{z^2}{2} \right]_1^2 = \frac{\pi}{3} \left\{ (4 - 2) - \frac{1}{2}(4 - 1) \right\}$$

$$= \frac{\pi}{3} \left(2 - \frac{3}{2} \right) = \pi/6 \text{ Ans}$$

CH #15
Vector Field

$V = 2i - 3j$

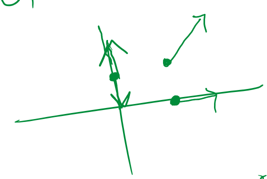
$(2, 3)$



$F(x, y) = xi + yj$

$F(1, 1) = i + j$ $\quad F(0, 1) = j$

$F(1, 0) = i$



$F(x, y) = xi - yj$

$F(0, 1) = -j$

Q22 EX #15.1 Q17 – Q28

$$F = \ln x\, i + e^{xyz} j + \tan^{-1}(z/x) K$$

$$F = \underbrace{\ln x}_{f}\, i + \underbrace{e^{xyz}}_{g}\, j + \underbrace{\tan^{-1}(z/x)}_{h}\, K$$

For div F

$$\text{div}\, f = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial g}{\partial y} = xz\, e^{xyz}, \quad \frac{\partial h}{\partial z} = \frac{1}{1+\left(\frac{z}{x}\right)^2} \times \frac{1}{x} = \frac{x^2}{x^2+z^2} \times 1/x$$

Now

$$\text{div}\, F = \frac{1}{x} + xz\, e^{xyz} + x/x^2+z^2 \quad \text{Ans}$$

For Curl F :-

$$\text{curl}\, f = \nabla \times F = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix} = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \ln x & e^{xyz} & \tan^{-1}\left(\frac{z}{x}\right) \end{vmatrix}$$

$$\text{curl}\, f = i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ e^{xyz} & \tan^{-1}\left(\frac{z}{x}\right) \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ \ln x & \tan^{-1}(z/x) \end{vmatrix} + K \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ \ln x & e^{xyz} \end{vmatrix}$$

$$\text{Rule } \frac{\partial}{\partial x}\left(\frac{z}{x}\right) = z \times (-x^{-2}) = -z/x^2$$

$$\text{Curl}\, f = i\left\{ \partial/\partial y \left(\tan^{-1} z/x\right) - \frac{\partial}{\partial z}\left(e^{xyz}\right)\right\} - j\left\{ \partial/\partial x \left(\tan^{-1}(z/x)\right) - \partial/\partial z (\ln x)\right\}$$
$$+ K\left\{ \partial/\partial x \left(e^{xyz}\right) - \partial/\partial y (\ln x)\right\}$$

$$\text{curl}\, f = i\left\{ 0 - xy\, e^{xyz}\right\} - j\left\{ \frac{\cancel{x^2}}{x^2+z^2} \times \left(-\frac{z}{\cancel{x^2}}\right) - 0\right\} + K\left\{ yz\, e^{xyz} - 0\right\}$$

$$\text{curl}\, f = -xy\, e^{xyz}\, i + z/x^2+z^2\, j + yz\, e^{xyz}\, K \quad \text{Ans}$$

Q24 $\quad \nabla \cdot (F \times G)$

Sol

For $F \times G$

$$F = \underbrace{yz}_{f_1}\, i + \underbrace{xz}_{f_2}\, j + \underbrace{xy}_{f_3}\, K$$

$$G = \underbrace{xy}_{g_2}\, j + \underbrace{xyz}_{g_3}\, K$$

$$F \times G = \begin{vmatrix} i & j & K \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$F \times G = \begin{vmatrix} i & j & k \\ f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{vmatrix}$$

$$F \times G = \begin{vmatrix} i & j & k \\ yz & xz & xy \\ 0 & xy & xyz \end{vmatrix} = i\,(x^2yz^2 - x^2y^2) - j\,(xy^2z^2 - 0) + k\,(xy^2z - 0)$$

$$F \times G = \underbrace{(x^2yz^2 - x^2y^2)}_{f}\,i - \underbrace{xy^2z^2}_{g}\,j + \underbrace{xy^2z}_{h}\,k$$

Now

$$\nabla \cdot (F \times G) = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

$$\nabla \cdot (F \times G) = \cancel{2xyz^2} - 2xy^2 + \cancel{2xyz^2} + xy^2 = -xy^2 \quad \text{Ans.}$$

$$\nabla \times (F \times G) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix}$$

Date: 01/05/2024 MVC - BCV 2A & 2B

TRIPLE INTEGRALS EX # 14.5

Q1 – Q8

$$\int_{1/3}^{1/2}\int_{0}^{\pi}\int_{0}^{1} z\,x\sin(xy)\,dz\,dy\,dx$$

Sol

$$\int_{1/3}^{1/2}\int_{0}^{\pi}\left[x\sin(xy)\,\frac{z^2}{2}\right]_0^1 dy\,dx$$

$$= \frac{1}{2}\int_{1/3}^{1/2}\int_{0}^{\pi} x\sin(xy)\,(1-0)\,dy\,dx$$

$$= \frac{1}{2}\int_{1/3}^{1/2}\int_{0}^{\pi} x\sin(xy)\,dy\,dx$$

R.W

Let $u = xy$

$du = x\,dy$

$\int x\sin(xy)\,dy$

$\int \sin(u)\,du$

$-\cos(u)$

$= -\cos(xy)$

$$= \frac{1}{2}\int_{1/3}^{1/2}\int_{0}^{\pi} x \sin(xy)\,dy\,dx$$

$$= -\frac{1}{2}\int_{1/3}^{1/2} \left[\cos(xy)\right]_0^{\pi} dx = -\frac{1}{2}\int_{1/3}^{1/2}\{\cos(\pi x) - \cos(0)\}\,dx$$

$$= -\frac{1}{2}\int_{1/3}^{1/2} (\cos \pi x - 1)\,dx$$

$$= -\frac{1}{2}\left[\frac{\sin \pi x}{\pi} - x\right]_{1/3}^{1/2}$$

$$= -\frac{1}{2}\left\{\left(\frac{\sin(\pi/2)}{\pi} - \frac{\sin(\pi/3)}{\pi}\right) - \left(\frac{1}{2} - \frac{1}{3}\right)\right\}$$

$$= -\frac{1}{2}\left\{\frac{1}{\pi} - \frac{\sqrt{3}}{2\pi} - \frac{1}{6}\right\} = \frac{\sqrt{3}}{4\pi} - \frac{1}{2\pi} + \frac{1}{12} \quad \text{Ans}$$

Q8 $$\int_1^2 \int_z^2 \int_0^{\sqrt{3}y} \frac{y}{x^2+y^2}\,dx\,dy\,dz$$

R.W.

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

here $y = a$

$$\int \frac{dx}{x^2+y^2} = \frac{1}{y}\tan^{-1}\left(\frac{x}{y}\right)$$

Sol $$\int_1^2\int_z^2 \left[\cancel{y} \times \frac{1}{\cancel{y}}\tan^{-1}\left(\frac{x}{y}\right)\right]_0^{\sqrt{3}y} dy\,dz$$

$$\Rightarrow \int_1^2\int_z^2 \left\{\tan^{-1}\left(\frac{\sqrt{3}\cancel{y}}{\cancel{y}}\right) - \tan^{-1}(0)\right\} dy\,dz$$

$$\Rightarrow \int_1^2\int_z^2 (\pi/3 - 0)\,dy\,dz$$

$$\Rightarrow \int^2 \left(\frac{\pi}{3} \quad y\right]_z^2 dz$$

$$= \int^{2} \left(\frac{\pi}{3} - 1\right)(2 - z)\,dz$$

$$= \left(\frac{\pi}{3}\right)\left[2z - \frac{z^2}{2}\right]_1^2 = \left(\frac{\pi}{3} - 1\right)\left[(4-2) - \frac{1}{2}(4-1)\right]$$

$$= \left(\frac{\pi}{3}\right)\left[2 - 3/2\right] = \left(\frac{\pi}{3}\right) 1/2 = \pi/6 \text{ Ans}$$

Vector Field CH #15

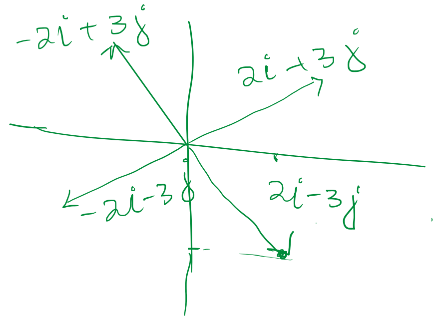
Vector $a = 2i - 3j$

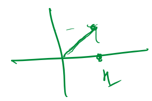
$(2,3) \to 1^{st}$

$(-2,3) \to 2^{nd}$

$(-2,-3) \to 3^{rd}$

$(2,-3) \to 4^{th}$

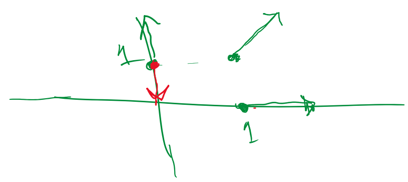




$F(x,y) = xi + yj$

$F(x,y) = xi - yj$

$F(0,1) = -j$



$F(1,0) = i$

$F(1,1) = i + j$

$F(0,1) = j$

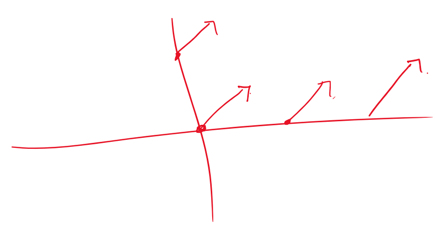
$F(x,y) = yi - xyj$

$F(x,y) = \underline{f(x,y)}i + g(x,y)j$

$f(x,y) = y^2 + y$

$g(x,y) = xy - x^2$

$F(x,y) = i + j$



Ex # 15.1 Q17 - Q28

Q22. $F(x,y,z) = \underbrace{\ln x}_{f(x,y)}\, i + \underbrace{e^{xyz}}_{g(x,y)}\, j + \underbrace{\tan^{-1}\left(\frac{z}{x}\right)}_{h(x,y)} k$

For div F

Using formula

$$\text{div } F = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

$$\text{div } F = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

$$\frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial g}{\partial y} = xz\, e^{xyz}, \quad \frac{\partial h}{\partial z} = \frac{1}{1+\left(\frac{z}{x}\right)^2} \times \frac{1}{x} = \frac{\cancel{x^2}}{z^2+x^2} \times \frac{1}{\cancel{x}}$$

Now

$$\text{div } F = \frac{1}{x} + xz\, e^{xyz} + \frac{x}{x^2+z^2} \quad \text{Ans}$$

For Curl F

using formula

$$\text{Curl } F = \nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \ln x & e^{xyz} & \tan^{-1}(z/x) \end{vmatrix}$$

$$x^{-1}z \qquad -x^{-2}z$$

$$\text{Curl } F = i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ e^{xyz} & \tan^{-1}\left(\frac{z}{x}\right) \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ \ln x & \tan^{-1}(z/x) \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ \ln x & e^{xyz} \end{vmatrix}$$

$$\text{Curl } F = i\left\{\partial/\partial y \left(\tan^{-1} z/x\right) - \partial/\partial z \left(e^{xyz}\right)\right\} - j\left\{\partial/\partial x \left(\tan^{-1} z/(x)\right) - \partial/\partial z (\ln x)\right\}$$
$$+ k\left\{\partial/\partial x \left(e^{xyz}\right) - \partial/\partial y (\ln x)\right\}$$

$$\text{Curl } F = i\left\{0 - xy\, e^{xyz}\right\} - j\left\{\frac{\cancel{x^2}}{x^2+z^2} \times \left(\frac{-z}{\cancel{x^2}}\right) - 0\right\} +$$
$$k\left\{yz\, e^{xyz} - 0\right\}$$

$$\text{curl } F = -xy\, e^{xyz}\, i + \frac{z}{x^2+z^2}\, j + xy\, e^{xyz}\, k$$

Ans

Q24 $\nabla \cdot (F \times G)$

$$F(x,y,z) = \underbrace{yz}_{f_1} i + \underbrace{xz}_{f_2} j + \underbrace{xy}_{f_3} k, \quad G(x,y,z) = \underbrace{xy}_{g_2} j + \underbrace{xyz}_{g_3} k$$

For $F \times G$

$$\begin{vmatrix} i & j & k \end{vmatrix} \quad \begin{vmatrix} i & j & k \end{vmatrix}$$

for $F \times G$

$$F \times G = \begin{vmatrix} i & j & k \\ f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{vmatrix} = \begin{vmatrix} i & j & k \\ yz & xz & xy \\ 0 & xy & xyz \end{vmatrix}$$

$$F \times G = i(x^2yz^2 - x^2y^2) - j(xy^2z^2 - 0) + k(xy^2z - 0)$$

$$F \times G = \underbrace{(x^2yz^2 - x^2y^2)}_{f} i - \underbrace{xy^2z^2}_{g} j + \underbrace{xy^2z}_{h} k$$

we have $\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$

$$\nabla \cdot (F \times G) = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

$$= \cancel{2xyz^2} - 2xy^2 + (\cancel{-2xyz^2}) + xy^2$$

$$\boxed{\nabla \cdot (F \times G)} = -xy^2 \quad \text{Ans}$$

$\nabla \cdot F = \operatorname{div} F$

$\operatorname{div}(F \times G)$

$y \le$

$y = 1$

$x^2 + y^2 = 1$