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Q1) ⓐ $\int_1^2 \int_4^6 \frac{x}{y^2}\,dx\,dy$

$\int_1^2 \left.\frac{x^2}{2y^2}\right]_4^6 dy$

$\int_1^2 \frac{10}{y^2}\,dy$

$= \left.-\frac{10}{y}\right]_1^2$

$\boxed{= -5+10 = 5}$

ⓑ $\iint x^2+y^2\,dx\,dy$

$= \int \frac{x^3}{3} + xy^2\,dy$

$= \frac{x^3y}{3} + \frac{xy^3}{3}$

$\boxed{\text{ans} = \frac{x^3y+xy^3}{3}}$

ⓒ $\int_0^1 \int_1^2 \frac{xe^x}{y}\,dy\,dx$

$= \int_0^1 \left. xe^x \ln y\right]_1^2 dx$

$\int_0^1 xe^x \ln 2\,dx$

$= \ln 2 \int_0^1 xe^x\,dx$

d	I
$+\ x$	e^x
$-\ 1$	e^x
$+\ 0$	e^x

$= \ln 2 \left[xe^x - e^x\right]_0^1$

$= \ln 2(0-(-1))$

$\boxed{= \ln 2}$

$\boxed{\text{ans}) \ln 2}$

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Q2 $P(L,k) = 70L^{0.6}k^{0.4}$

$5000 \leq L \leq 6000$

$20 \leq k \leq 30$

$A = (6000-5000)(30-20)$

$A = 10000$

$$Out = \frac{1}{A}\int_{20}^{30}\int_{5000}^{6000} 70L^{0.6}k^{0.4}\,dL\,dk$$

$$= \frac{1}{10000}\int_{20}^{30}\left[\frac{70L^{1.6}k^{0.4}}{1.6}\right]_{5000}^{6000} dk$$

$$\cancel{= \frac{1}{10000}\int_{20}^{30}}$$

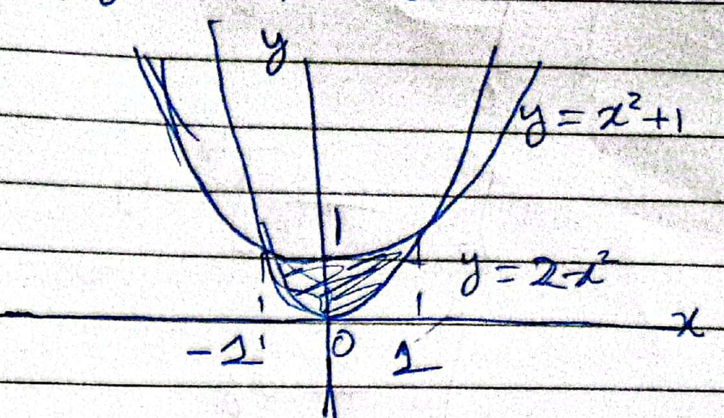
$$= 1227.925057\int_{20}^{30} k^{0.4}\,dk$$

$$= 1227.925\left[\frac{k^{1.4}}{1.4}\right]_{20}^{30}$$

$$= 1227.925(36.181)$$

$$\boxed{Out = 44426.95558}$$

Q3 (a) $\iint_D x+2y \, dA$ $\quad D \{ y=2x^2, \; y=1+x^2$



$$\int_{-1}^{1}\int_{2x^2}^{x^2+1} x+2y \, dy\, dx$$

$$\int_{-1}^{1} xy+y^2 \Big]_{2x^2}^{x^2+1} dx$$

$$\int_{-1}^{1} x(x^2+1)+(x^2+1)^2 - x(2x^2) - (2x^2)^2 dx$$

$x^2+1 = 2x^2$

$x^2=1$

$x = \pm 1$

$$\int_{-1}^{1} x^3+x + x^4+2x^2+1 - 2x^3 - 4x^4 \, dx$$

$$\int_{-1}^{1} x^4-4x^4+x^3-2x^3+2x^2+x+1 \, dx$$

$$\int_{-1}^{1} -3x^4 - x^3 + 2x^2 + x+1 \, dx$$

$$\frac{-3x^5}{5} - \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + x \Big]_{-1}^{1}$$

$$\frac{79}{60} - \left(-\frac{49}{60}\right)$$

$$\boxed{= \frac{32}{15}}$$

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(b) $\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx \, dy$

$$\int_0^4 \left. \frac{x^2y^2}{2} \right]_0^{\sqrt{y}} dy$$

$$\int_0^4 \frac{y^3}{2} \, dy$$

$$\left. \frac{y^4}{8} \right]_0^4$$

$$\boxed{=32}$$

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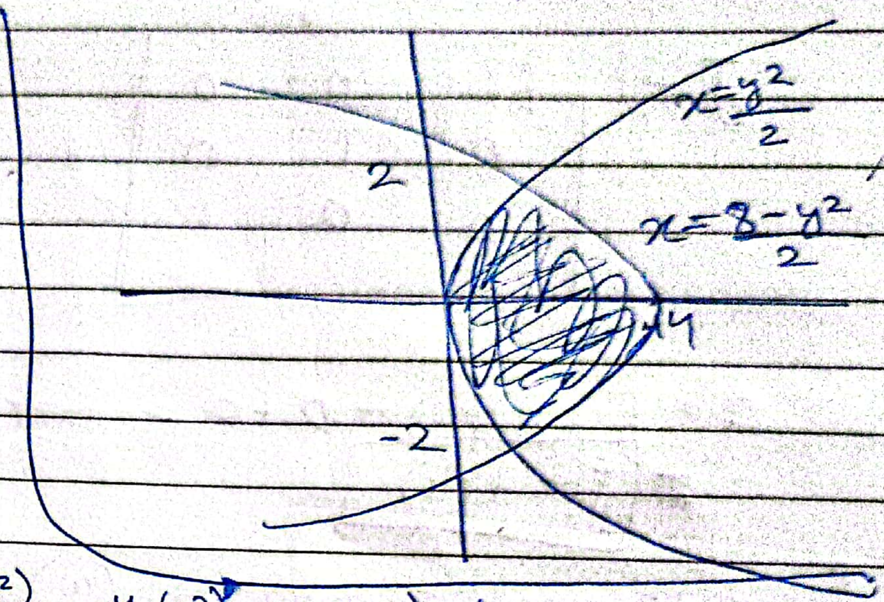
Q3C

$$\iint 4-y^2 \, dA$$

R $\{ y^2 = 2x, \; y^2 = 8-2x$



$$\int_{-2}^{2}\int_{y^2/2}^{\frac{8-y^2}{2}} 4-y^2 \, dx\, dy$$

$$\int_{-2}^{2} 4x - y^2 x \Big]_{y^2/2}^{\frac{8-y^2}{2}} dy$$

$$\int_{-2}^{2} \frac{4(8-y^2)}{2} - \frac{y^2(8-y^2)}{2} - \frac{4(y^2)}{2} + \frac{y^2(y^2)}{2} \, dy$$

$$\int_{-2}^{2} \frac{32-4y^2-8y^2+y^4-4y^2+y^4}{2} \, dy$$

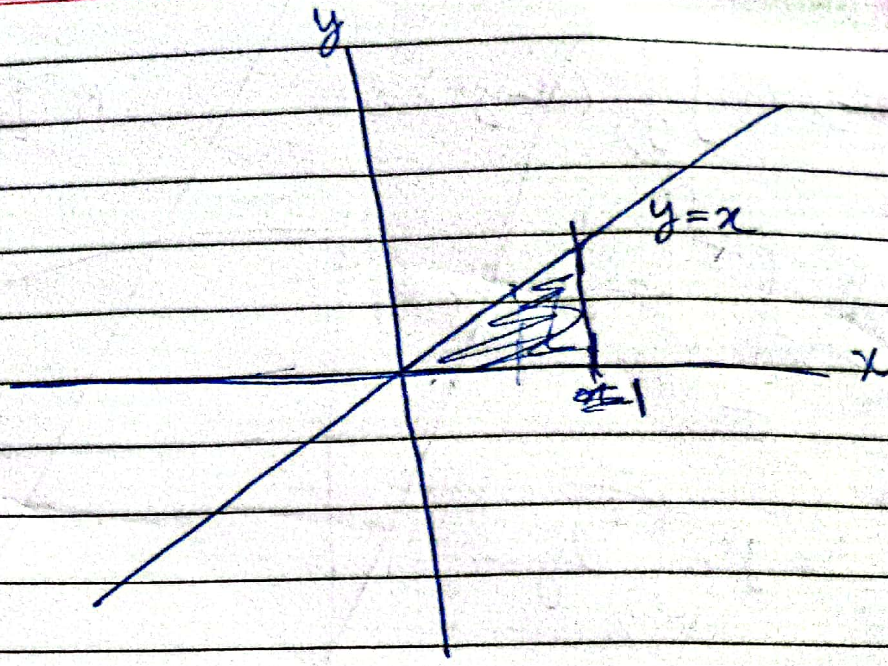
$$\int_{-2}^{2} \frac{32-16y^2+2y^4}{2} \, dy$$

$$\int_{-2}^{2} y^4-8y^2+16 \, dy$$

$$= \frac{y^5}{5} - \frac{8y^3}{3} + 16y \Big]_{-2}^{2}$$

$$\boxed{= \frac{256}{15} + \frac{256}{15} = \frac{512}{15} = 34.133}$$

Q4 $f(x,y) = 3 - x - y$



$$V = \int_0^1 \int_0^x 3 - x - y \, dy \, dx$$

$$\int_0^1 3y - xy - \frac{y^2}{2} \Big]_0^x \, dx$$

$$\int_0^1 3x - x^2 - \frac{x^2}{2} \, dx$$

$$\int_0^1 3x - \frac{3}{2} x^2 \, dx$$

$$\frac{3x^2}{2} - \frac{x^3}{2} \Big]_0^1$$

$$\boxed{V = \frac{3}{2} - \frac{1}{2} = 1}$$

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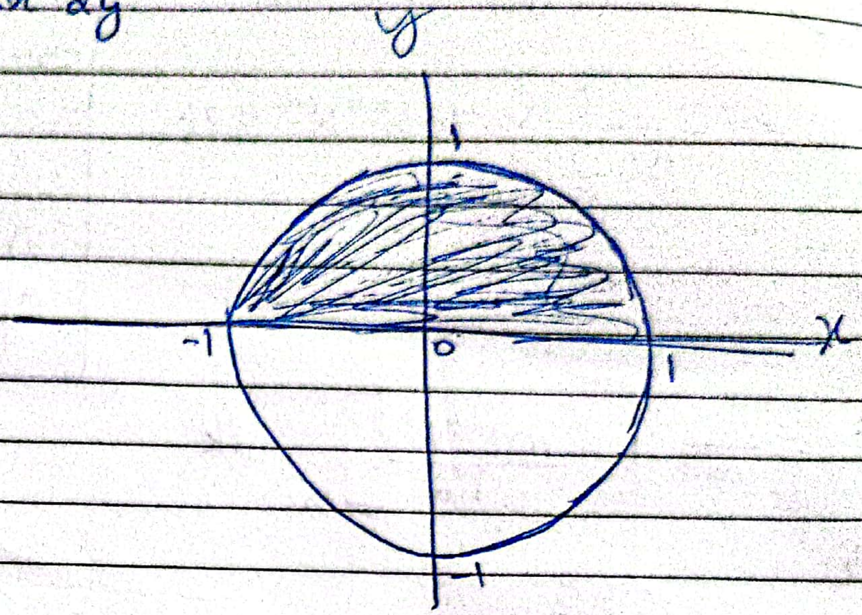
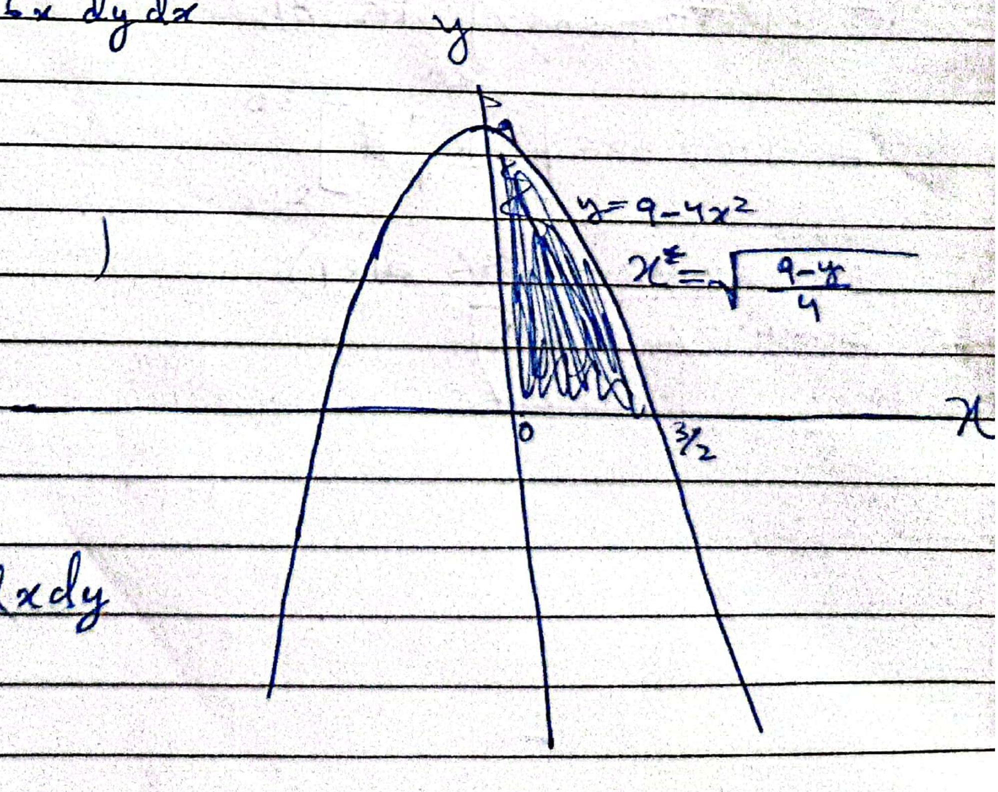
Q5 (a) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y \, dx \, dy$



ans $= \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} 3y \, dy \, dx$

(b) $\int_0^{3/2} \int_0^{9-4x^2} 16x \, dy \, dx$



ans $= \int_0^9 \int_0^{\frac{\sqrt{9-y}}{2}} 16x \, dx \, dy$

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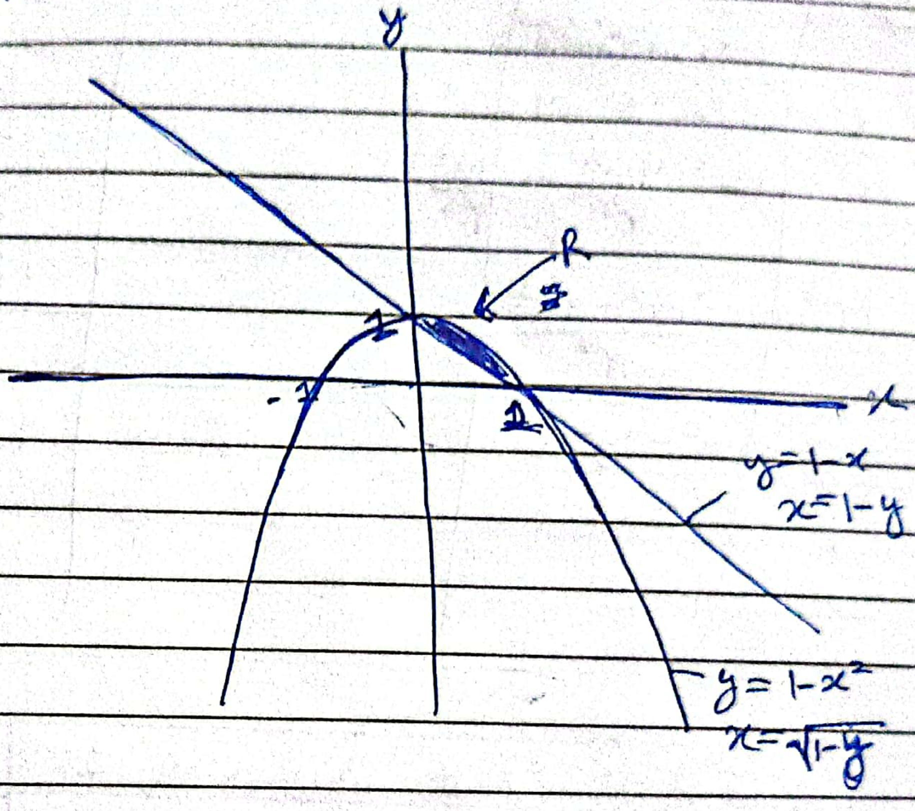
(c) $\int_0^1 \int_{1-x}^{1-x^2} dy\, dx$



$$\int_0^1 \int_{1-y}^{\sqrt{1-y}} dx\, dy$$

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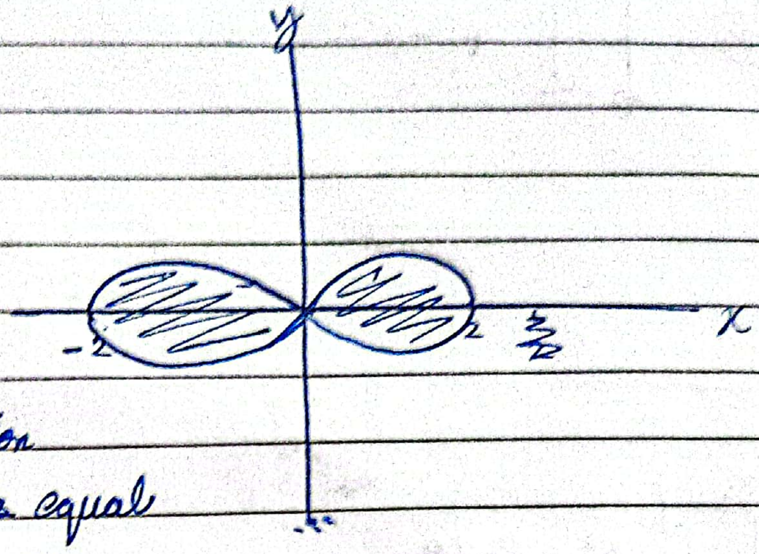
Q6) $r^2 = 4\cos 2\theta$



$\theta = 0, r = \pm 2$

$\theta = \pm \cancel{30}\ \pi/6, r = \pm\sqrt{2}$

$\theta = \pm \pi/4, r = 0$

sin Cos(θ) is even function
the value of r for $-\pi/4$ to 0 is equal to the value of r for $0-\pi/4$.

Area of ~~one~~ right half :-

$$A_r = \int_{-\pi/4}^{\pi/4}\int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta$$

if we consider the height to be 1, the volume and area will have same magnitude.

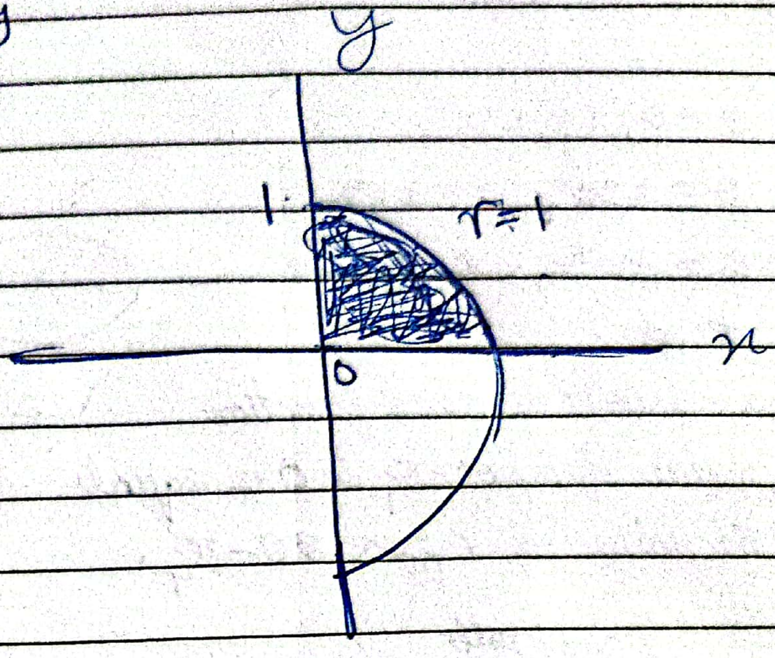
$$A_r = \int_{-\pi/4}^{\pi/4} \frac{\cancel{4}^{2}\cos 2\theta}{\cancel{2}_{1}}\,d\theta$$

$$A_r = \sin 2\theta \Big]_{-\pi/4}^{\pi/4}$$

$$A_r = 1-(-1) = 2$$

$$\boxed{\text{Total Area} = 2A_r = 2(2) = 4}$$

Q7 (a) $\int_0^1 \int_0^{\sqrt{1-y^2}} x^2 + y^2 \, dx \, dy$



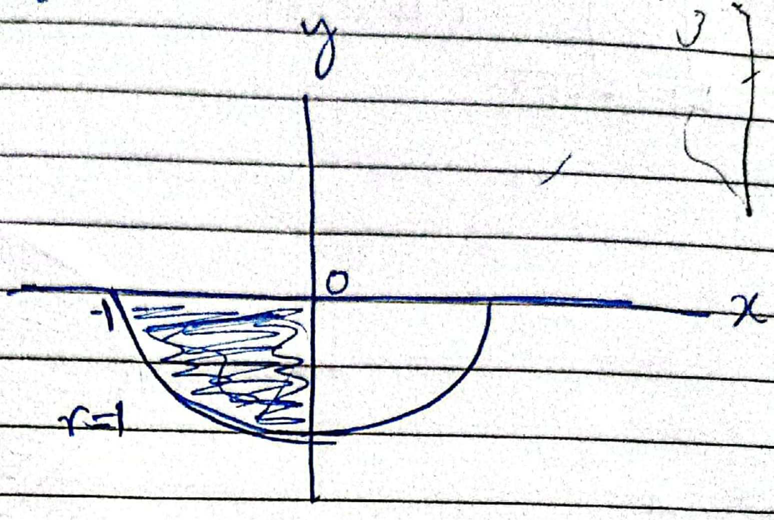
$$\int_0^{\pi/2} \int_0^1 r^2 \cdot r \, dr \, d\theta$$

$$\int_0^{\pi/2} \left[\frac{r^4}{4} \right]_0^1 d\theta$$

$$\int_0^{\pi/2} \frac{1}{4} \, d\theta$$

$$\boxed{\text{ans} = \pi/8}$$

(b) $\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\, dy\, dx$



$\int_{-\pi}^{-3\pi/2}\int_{0}^{1} \frac{2}{1+r}\, r\,dr\,d\theta$

$u = 1+r$ $du = dr$	$r = u-1$

$\int_{-\pi}^{-\pi/2} 2\int_{1}^{2} \frac{u-1}{u}\, du\, d\theta$ $= 2\theta\Big[1-\ln 2\Big]_{-\pi}^{-\pi/2}$

$= 2\Big[-\frac{\pi}{2}[1-\ln 2]+\pi[1-\ln 2]\Big]$

$\int_{-\pi}^{-\pi/2} 2\int_{1}^{2} 1-\frac{1}{u}\, du\, d\theta$

$(1-\ln 2)(\pi-\frac{\pi}{2})2$

$2\int_{-\pi}^{-\pi/2} u-\ln u\Big]_{1}^{2}\, d\theta$

$\boxed{ans = 2\left(\frac{\pi}{2}-\frac{\pi}{2}\ln 2\right)}$

$=2\int_{-\pi}^{-\pi/2} 1-\ln 2\, d\theta$

$\boxed{ans = 0.964}$

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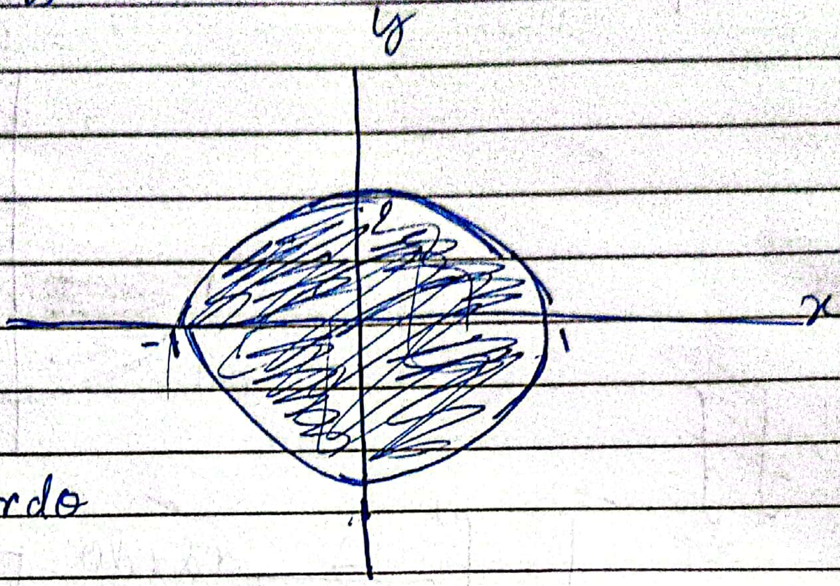
(c) $$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y)^2}\,dy\,dx$$



$$\int_{0}^{2\pi}\int_{0}^{1} \frac{2}{(1+r^2)^2}\, r\,dr\,d\theta$$

$$\int_{0}^{2\pi}\int_{0}^{1} \frac{2r}{(1+r^2)^2}\, \cancel{r}\,dr\,d\theta$$

$u = 1+r^2$.
$du = 2r\,dr$

$$\int_{0}^{2\pi}\int_{1}^{2} \frac{1}{u^2}\,du\,d\theta$$

$$\int_{0}^{2\pi} -\frac{1}{u}\Big]_{1}^{2}\,d\theta$$

$$\int_{0}^{2\pi} 1-\frac{1}{2}\,d\theta$$

$$\int_{0}^{2\pi} \frac{1}{2}\,d\theta$$

~~ans~~ $= \frac{1}{2}\theta\Big]_{0}^{2\pi}$

$\boxed{ans = \pi}$

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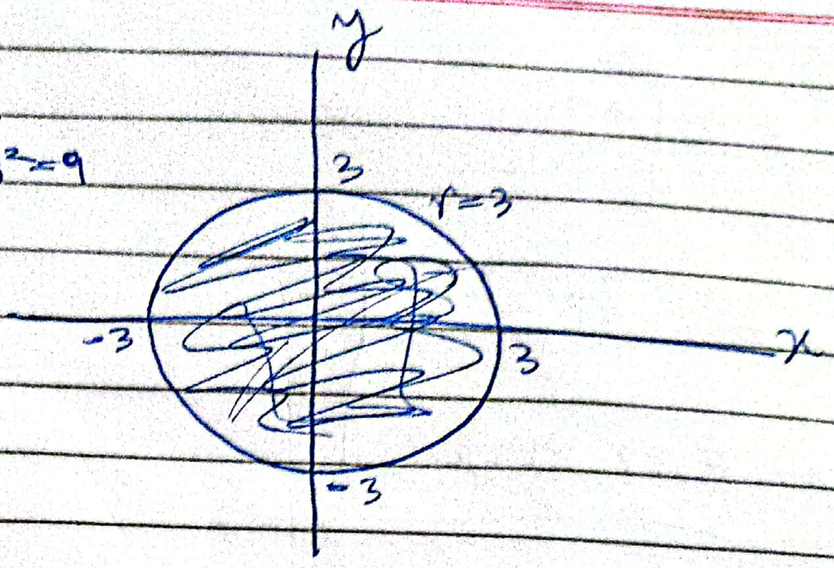
Q8) (a) $z = x^2 + y^2$, $z = 9$

, $x^2 + y^2 = 9$



$\frac{dz}{dx} = 2x$

$\frac{dz}{dy} = 2y$

$$S = \iint \sqrt{(2x)^2 + (2y)^2 + 1} \; dx\,dy$$

$$= \iint \sqrt{4x^2 + 4y^2 + 1} \; dx\,dy$$

$$= \int_0^{2\pi}\int_0^3 \sqrt{4r^2 + 1} \; r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^3 r\sqrt{4r^2+1} \; dr\,d\theta$$

$u = 4r^2 + 1$

$\frac{du}{8} = r\,dr$

$$= \int_0^{2\pi} \frac{1}{8}\int_1^{37} \sqrt{u} \; du\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{8}\left[\frac{2}{3}u^{3/2}\right]_1^{37}$$

$$\int_0^{2\pi} \left[\frac{1}{12}(4r^2+1)^{3/2}\right]_0^3 d\theta$$

$$\int_0^{2\pi} \frac{37^{3/2}}{12} - \frac{1}{12} \; d\theta$$

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$$\int_0^{2\pi} \frac{37^{3/2}-1}{12} \, d\theta$$

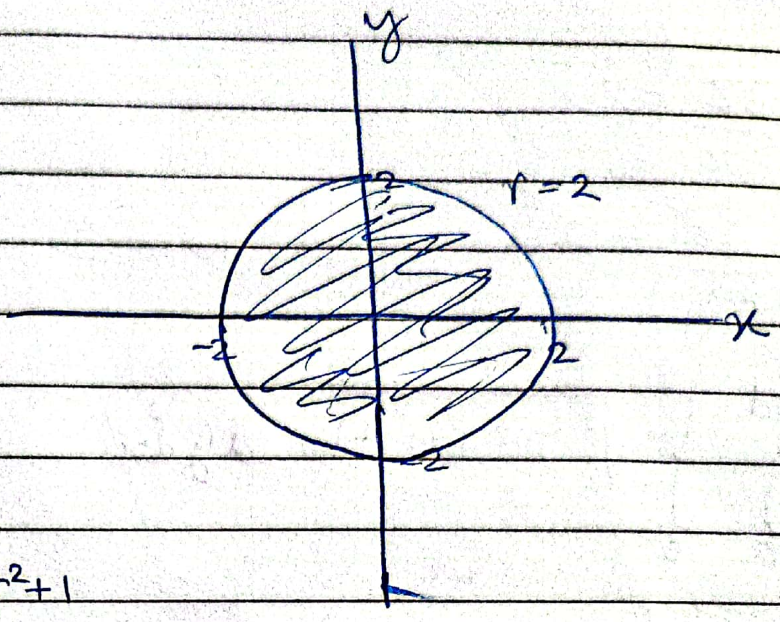
$$\boxed{\begin{gathered} \text{ans} = \frac{37^{3/2}-1}{6}\pi \\ \text{ans} = 37.344\pi \end{gathered}}$$

(b) $x^2+y^2+z=4$, above xy plane $(z=0)$

$z=4-x^2-y^2$

$\frac{\partial z}{\partial x} = -2x$

$\frac{\partial z}{\partial y} = -2y$



$$\int_0^{2\pi}\int_0^2 \sqrt{4r^2+1}\; r\,dr\,d\theta$$

$u = 4r^2+1$

$\frac{du}{8} = r\,dr$

$$\int_0^{2\pi} \frac{1}{8}\int_1^{17} \sqrt{u}\;du\,d\theta$$

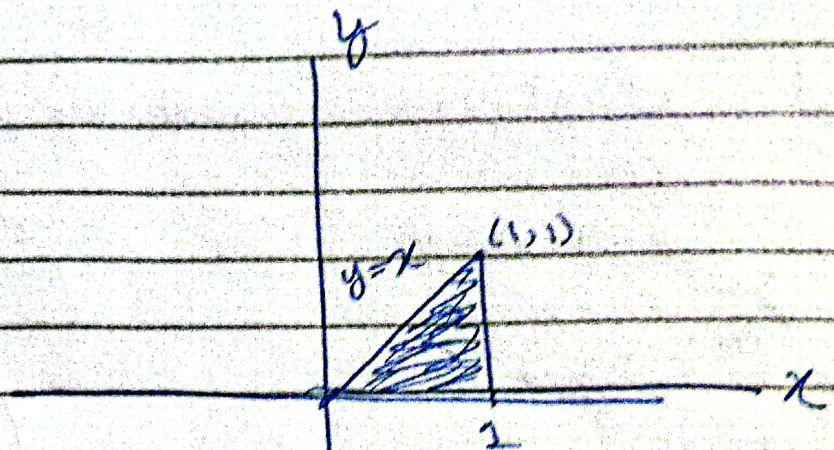
$$\int_0^{2\pi} \frac{1}{12}u^{3/2}\Big]_1^{17} d\theta$$

$$\int_0^{2\pi} \frac{17^{3/2}}{12} - \frac{1}{12}\,d\theta$$

$$\int_0^{2\pi} \frac{17^{3/2}-1}{12}\,d\theta$$

$$\boxed{\begin{array}{l} \text{ans} = \dfrac{17^{3/2}-1}{6}\pi \\ \text{ans} = 36.177\pi \end{array}}$$

5 ⓒ $z = x^2 + 2y$



$$\frac{\partial z}{\partial x} = 2x$$

$$\frac{\partial z}{\partial y} = 2$$

$$\int_0^1 \int_0^x \sqrt{4x^2 + 4 + 1}\, dy\, dx$$

$$\int_0^1 \int_0^x \sqrt{4x^2+5}\, dy\, dx$$

$$\int_0^1 y\sqrt{4x^2+5}\Big]_0^x dx$$

$u = 4x^2 + 5$

$\frac{du}{8} = x\, dx$

$$\int_0^1 x\sqrt{4x^2+5}\, dx$$

$$\frac{1}{8}\int_5^9 \sqrt{u}\, du$$

$$\frac{1}{8}\left[\frac{2}{3}u^{3/2}\right]_5^9$$

$$= \frac{1}{12} u^{3/2}\Big]_5^9$$

$$= \frac{27}{12} - \frac{5^{3/2}}{12}$$

$$\boxed{\text{ans} = \frac{27 - 5^{3/2}}{12} = 1.318}$$

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Q9) a) $\cancel{\int_0^{\sqrt{2}}\int_0^{\sqrt{2}}\int_0}$

$$\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{x^2+y^2}^{2} x\,dz\,dy\,dx$$

$$\int_{x^2+y^2}^{2} x\,dz$$

$$= xz\Big]_{x^2+y^2}^{2}$$

$$= 2x - x^3 - xy^2$$

$$\int_0^{\sqrt{2-x^2}} 2x - x^3 - xy^2\,dy$$

$$= 2xy - x^3y - \frac{xy^3}{3}\Big]_0^{\sqrt{2-x^2}}$$

$$= 2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{x(2-x^2)^{3/2}}{3}$$

$$\int_0^{\sqrt{2}} 2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{x(2-x^2)^{3/2}}{3}\,dx$$

$u = 2-x^2$

$\frac{du}{-2} = x\,dx$

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$$\int_0^{\sqrt{2}} 2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{x(2-x^2)^{3/2}}{3}\,dx$$

$u = 2-x^2$

$\frac{du}{-2} = x\,dx$

$$\int_0^{\sqrt{2}} \left[2\sqrt{2-x^2} - x^2\sqrt{2-x^2} - \frac{(2-x^2)^{3/2}}{3}\right] x\,dx$$

$$-\frac{1}{2}\int_2^0 2\sqrt{2-x^2} - x^2\sqrt{2-x^2} - \frac{(2-x^2)^{3/2}}{3}\,du$$

$$\frac{1}{2}\int_0^2 2\sqrt{2-x^2} - x^2\sqrt{2-x^2} - \frac{(2-x^2)^{3/2}}{3}\,du$$

$$\frac{1}{2}\int_0^2 2\sqrt{u} - x^2\sqrt{u} - \frac{u^{3/2}}{3}\,du$$

$$\frac{1}{2}\int_0^2 2\sqrt{u} - (2-u)\sqrt{u} - \frac{u^{3/2}}{3}\,du$$

$$\frac{1}{2}\int_0^2 2\sqrt{u} - 2\sqrt{u} + u^{3/2} - \frac{u^{3/2}}{3}\,du$$

$$\frac{1}{2}\int_0^2 \frac{2}{3}u^{3/2}\,du$$

$$\frac{1}{3}\int_0^2 u^{3/2}\,du$$

$$\frac{1}{3}\left[\frac{2}{5}u^{5/2}\right]_0^2$$

$$= \frac{2}{15}u^{5/2}\Big]_0^2$$

$$\boxed{\text{ans} = \frac{2(2)^{5/2}}{15} = 0.754}$$

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Q9) (b) $\int_0^3 \int_0^2 \int_0^1 (xyz)^2 \, dx \, dy \, dz$

$$\int_0^1 x^2 y^2 z^2 \, dx$$

$$= y^2 z^2 \int_0^1 x^2 \, dx$$

$$= y^2 z^2 \left[\frac{x^3}{3} \right]_0^1 = \frac{y^2 z^2}{3}$$

$$\int_0^2 \frac{y^2 z^2}{3} \, dy$$

$$= \frac{z^2}{3} \int_0^2 y^2 \, dy$$

$$= \frac{z^2}{3} \left[\frac{y^3}{3} \right]_0^2 = \frac{8z^2}{9}$$

$$\int_0^3 \frac{8z^2}{9} \, dz$$

$$= \frac{8}{9} \int_0^3 z^2 \, dz$$

$$\boxed{\text{ans} = \frac{8}{9} \left[\frac{z^3}{3} \right]_0^3 = 8}$$

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(c) $\int_0^{\pi/4} \int_0^{\ln(\sec(t))} \int_{-\infty}^{2s} e^r \, dr\, ds\, dt$

$$\int_{-\infty}^{2s} e^r \, dr$$

$$= e^r \Big]_{-\infty}^{2s}$$

$$= e^{2s} - e^{-\infty}$$

$$= e^{2s} - 0 = e^{2s}$$

$$\int_0^{\ln(\sec(t))} e^{2s} \, ds$$

$$= \frac{e^{2s}}{2} \Big]_0^{\ln(\sec(t))}$$

$$= \frac{e^{2\ln(\sec(t))} - 1}{2}$$

$$e^{2\ln(\sec t)} = \left(e^{\ln(\sec t)}\right)^2 = (\sec t)^2 = \sec^2 t$$

$$\int_0^{\pi/4} \frac{e^{2\ln[\sec(t)]} - 1}{2} \, dt$$

$$= \frac{1}{2} \int_0^{\pi/4} \sec^2 t - 1 \, dt$$

$$= \frac{1}{2} \tan t - \frac{1t}{2} \Big]_0^{\pi/4}$$

~~ans = $\frac{1}{2} - \frac{\pi}{8}$~~

~~ans = $2 - \frac{\pi}{4} = -0.285$~~

$$\boxed{ans = \frac{1}{2} - \frac{\pi}{8}}$$

$$\boxed{ans = 0.107}$$

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(d) $\iiint yz^2 \sin(xyz)\, dx\, dy\, dz$

$\int yz^2 \sin(xyz)\, dx$ $\qquad u = xyz$
$\qquad du = yz\, dx$

$= \int z \sin u \, du$

$= -z\cos u$

$= -z\cos(xyz)$

$\int -z\cos(xyz)\, dy$ $\qquad u = xyz$
$\qquad du = xz\, dy$

$= -\frac{1}{x}\int \cos u \, du$ $\qquad \frac{du}{x} = z\, dy$

$= -\frac{1}{x}\sin(xyz)$

$\int -\frac{1}{x}\sin(xyz)\, dz$ $\qquad u = xyz$
$\qquad du = xy\, dz$

$= -\frac{1}{x}\cdot\frac{1}{xy}\int \sin u \, du$ $\qquad \frac{du}{xy} = dz$

$\boxed{\text{ans} = \frac{\cos(xyz)}{x^2 y}}$

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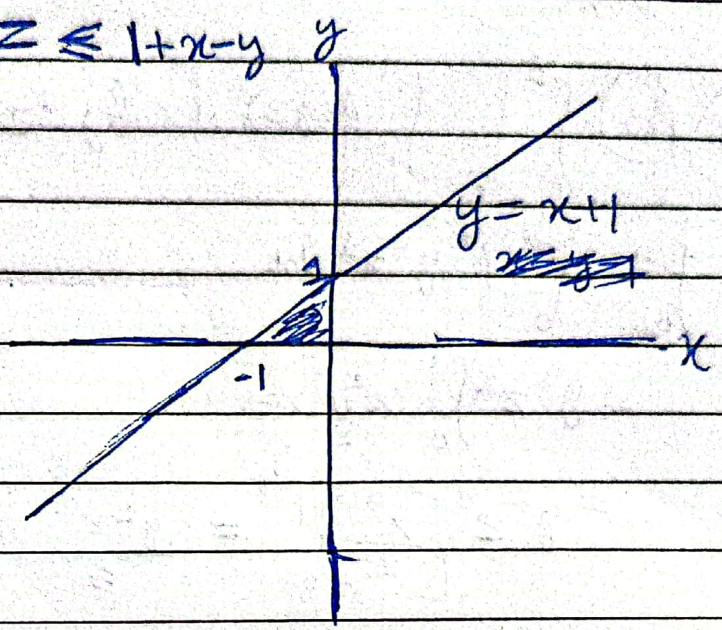
Q10 (a) $y=0$, $z=0$, $x=0$, $y-x+z=1$

$z=1+x-y$

$0 \leq z \leq 1+x-y$



$$V=\int_{-1}^{0}\int_{0}^{x+1}\int_{0}^{1+x-y} dz\, dy\, dx$$

$$=\int_{-1}^{0}\int_{0}^{x+1} z\Big]_{0}^{1+x-y} dy dx$$

$$=\int_{-1}^{0}\int_{0}^{x+1} 1+x-y\, dy dx$$

$$=\int_{-1}^{0} y+xy-\frac{y^2}{2}\Big]_{0}^{x+1}$$

$$=\int_{-1}^{0} x+1+x(x+1)-\frac{(x+1)^2}{2}\, dx$$

$$\int_{-1}^{0} x+1 + x(x+1) - \frac{(x+1)^2}{2}\, dx$$

$$= \int_{-1}^{0} x+1 + x^2 + x - \frac{x^2 + 2x + 1}{2}\, dx$$

$$= \int_{-1}^{0} x^2 + 2x + 1 - \frac{x^2 + 2x + 1}{2}\, dx$$

$$= \int_{-1}^{0} (x+1)^2 \left(1 - \frac{1}{2}\right) dx$$

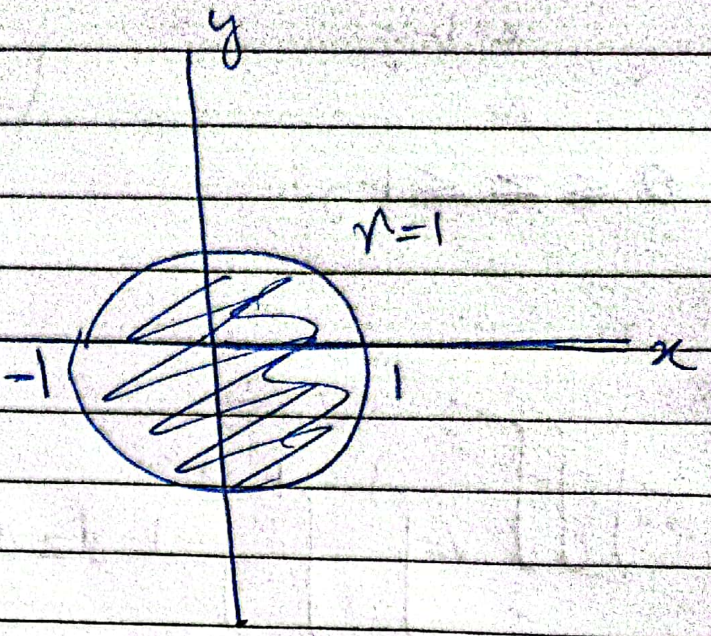
$$= \frac{1}{2}\int_{-1}^{0} x^2 + 2x + 1\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + x^2 + x\right]_{-1}^{0}$$

$$= 0 - \frac{1}{2}\left(-\frac{1}{3} + 1 - 1\right)$$

$$\boxed{V = \frac{1}{6}}$$

Q10 (b) xy Plane $(z=0)$, $x^2+y^2=1$, $x+y+z=3$



$0 \le z \le 3-x-y$

$0 \le z \le 3-r(\cos\theta+\sin\theta)$

$$V=\int_0^{2\pi}\int_0^1\int_0^{3-x-y} dz\, r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 r(3-r\cos\theta-r\sin\theta)\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 3r-r^2\cos\theta-r^2\sin\theta\,dr\,d\theta$$

$$\int_0^{2\pi}\left[\frac{3r^2}{2}-\frac{r^3\cos\theta}{3}-\frac{r^3\sin\theta}{3}\right]_0^1 d\theta$$

$$\int_0^{2\pi}\frac{3}{2}-\frac{\cos\theta}{3}-\frac{\sin\theta}{3}\,d\theta$$

$$\frac{1}{6}\int_0^{2\pi} 9-2\cos\theta-2\sin\theta\,d\theta$$

$$V=\frac{1}{6}\Big[9\theta-2\sin\theta+2\cos\theta\Big]_0^{2\pi}$$

$$V=\frac{1}{6}\Big[18\pi+2-(2)\Big]$$

$$\boxed{V=3\pi}$$

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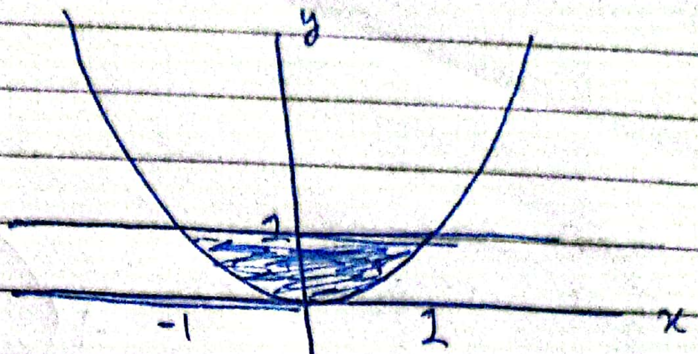
(c) $y+z=1$, $y=x^2$, xy plane $(z=0)$

$z=1-y$

$0 \le z \le 1-y$



$$V=\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} 1 \, dz\, dy\, dx$$

$$\int_{-1}^{1}\int_{x^2}^{1} z\Big]_0^{1-y} dy\, dx$$

$$=\int_{-1}^{1}\int_{x^2}^{1} 1-y \; dy\, dx$$

$$=\int_{-1}^{1} y-\frac{y^2}{2}\Big]_{x^2}^{1}$$

$$=\int_{-1}^{1} \frac{1}{2}-x^2+\frac{x^4}{2} \, dx$$

$$=\frac{1}{2}x-\frac{x^3}{3}+\frac{x^5}{10}\Big]_{-1}^{1}$$

$$=\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{10}\right)-\left(\frac{-1}{2}+\frac{1}{3}-\frac{1}{10}\right)$$

$$=\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{10}\right)+\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{10}\right)$$

$$=2\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{10}\right)$$

$$\boxed{V=\frac{8}{15}}$$

Q11) (a) $x = f(u), y = v, z = w$

$$J = \begin{vmatrix} f'(u) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= f'(u)(1-0) - 0 + 0 =$

$\boxed{J = f'(u)}$

(b) $x = f(u), y = g(v), z = h(w)$

$$J = \begin{vmatrix} f'(u) & 0 & 0 \\ 0 & g'(v) & 0 \\ 0 & 0 & h'(w) \end{vmatrix}$$

$J = f'(u)(g'(v)h'(w))$

$\boxed{J = f'(u)g'(v)h'(w)}$

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Q12) (a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $x=au$, $y=bv$, $z=cw$

$$J=\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc$$

$\frac{(au)^2}{a^2}+\frac{(bv)^2}{b^2}+\frac{(cw)^2}{c^2}=1$

$u^2+v^2+w^2=1$

sphere with radius 1

$$\iiint 1 \, dx\,dy\,dz$$

$$= abc \iiint 1 \, du\,dv\,dw$$

$\iiint 1 \, du\,dv\,dw$ = volume of sphere with $r=1$

$\iiint 1 \, du\,dv\,dw = \frac{4}{3}\pi(1)^2 = \frac{4}{3}\pi$

$$\boxed{V=\frac{4\pi}{3}(abc)}$$

Q12 (b) $R\ \{(1,0),(4,0),(0,4),(0,1)\}$ | $x = u - uv$

$y = uv$

$x+y = u$

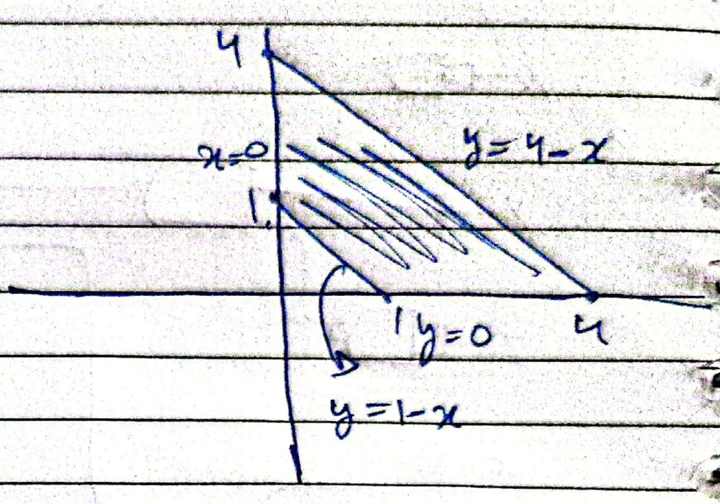
$$\iint \frac{1}{x+y}\, dA$$

$y = 4-x$

$y = 1-x$

$y = 0$

$x = 0$



$x+y=4$

$\boxed{u=4}$

$x+y=1$

$\boxed{u=1}$

$uv = 0$

$\boxed{u=0,\ v=0}$

$u - uv = 0$

$u(1-v) = 0$

$\boxed{u=0,\ v=1}$

$$J = \begin{vmatrix} 1-v & -u \\ v & u \end{vmatrix}$$

$J = u - uv - (-uv)$

$= u - uv + uv$

$\boxed{J = u}$

$1 \leq u \leq 4$

$0 \leq v \leq 1$

u cannot be between [0−1] as if it is, it will ~~contradict~~ result in x and y values that contradict the original region. ~~As~~ As if v is 0 and u is [0−1] the x y coordinates would be (a, 0) where a is less than 1.

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$$\int_0^1 \int_1^4 \frac{1}{u} \cdot u \, du \, dv$$

$$= \int_0^1 \int_1^4 1 \, du \, dv$$

$$= \int_0^1 u\Big]_1^4 dv$$

$$= \int_0^1 3 \, dv$$

$$= 3v\Big]_0^1$$

$$\boxed{ans = 3}$$