Data Structures Lab 9

Course: Data Structures (CL2001) **Semester:** Fall 2024
Instructor: Muhammad Nouman Hanif

Note:

- Lab manual cover following below:
  {Self-balancing Binary Trees, AVL Trees}
- Maintain discipline during the lab.
- Just raise your hand if you have any problem.
- Completing all tasks of each lab is compulsory.
- Get your lab checked at the end of the session.
- Don't just blatantly copy the same code and make changes to it accordingly

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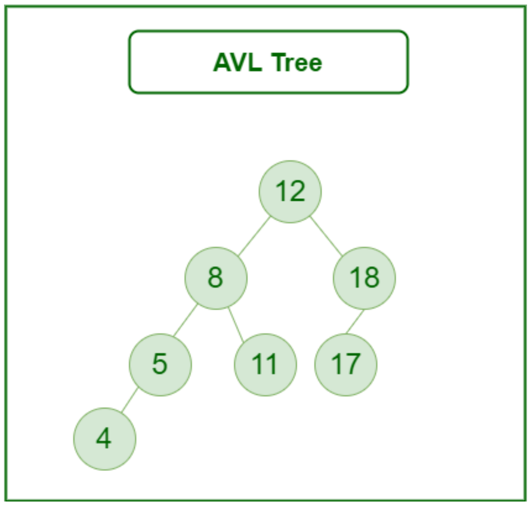
AVL Tree Data Structure

An **AVL tree** defined as a self-balancing **Binary Search Tree** (BST) where the difference between heights of left and right subtrees for any node cannot be more than one. The difference between the heights of the left subtree and the right subtree for any node is known as the **balance factor** of the node. The AVL tree is named after its inventors, Georgy **A**delson-**V**elsky and Evgenii **L**andis, who published it in their 1962 paper "An algorithm for the organization of information".

Example of AVL Trees:



AVL tree

The above tree is AVL because the differences between the heights of left and right subtrees for every node are less than or equal to 1. Once the difference exceeds one, the tree automatically executes the balancing algorithm until the difference becomes one again.

BALANCE FACTOR = HEIGHT(LEFT SUBTREE) – HEIGHT(RIGHT SUBTREE)

Operations on an AVL Tree:

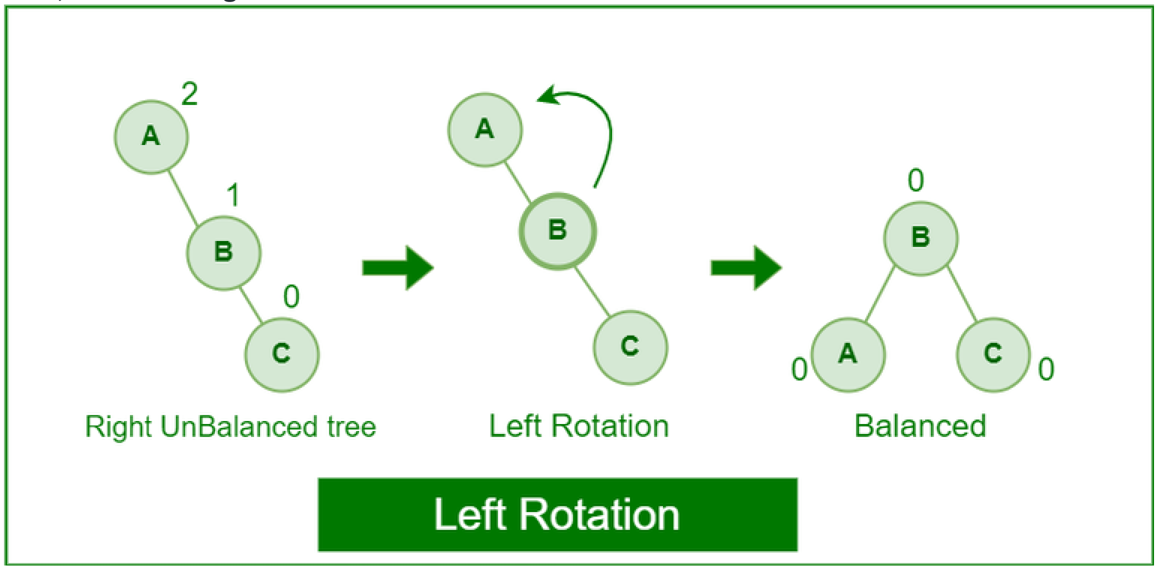
- Insertion
- Deletion
- Searching [It is similar to performing a search in BST]

Rotating the subtrees in an AVL Tree while inserting:

The AVL Tree may rotate in one of the following four ways to keep itself balanced:

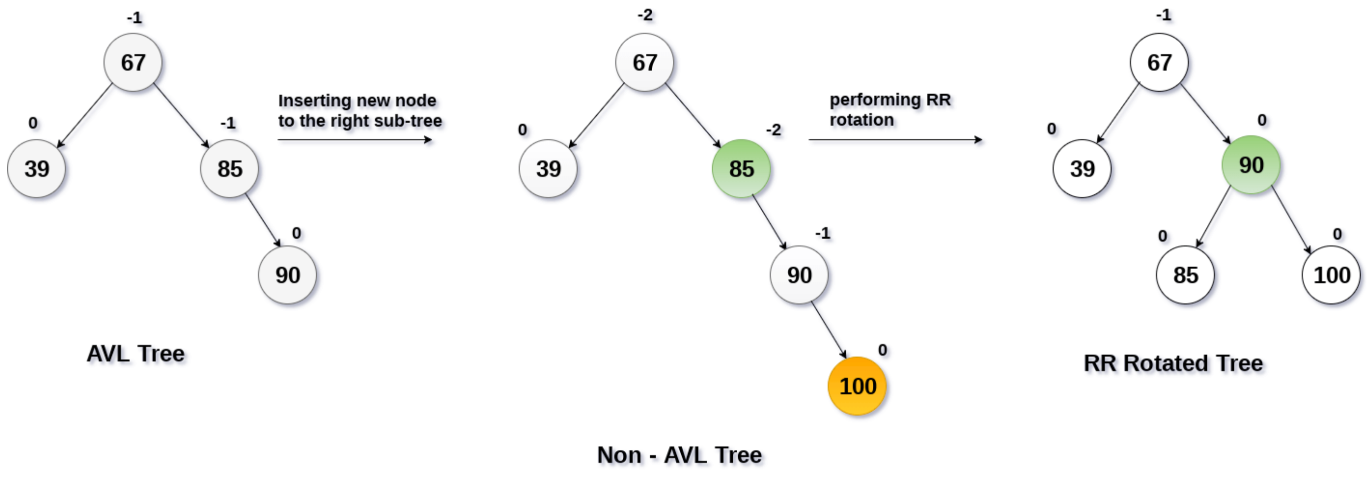
Left Rotation:

When a node is added into the right subtree of the right subtree, if the tree gets out of balance, we do a single left rotation.

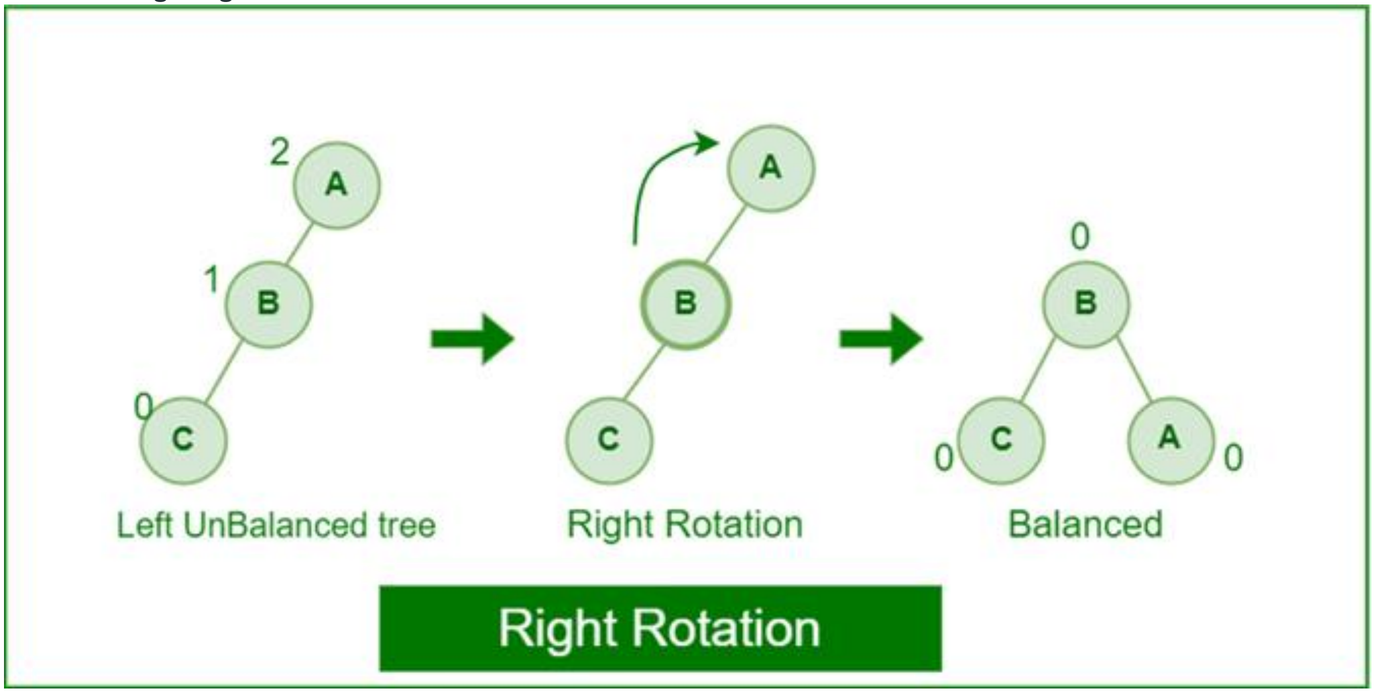


Left-Rotation in AVL tree

Example:

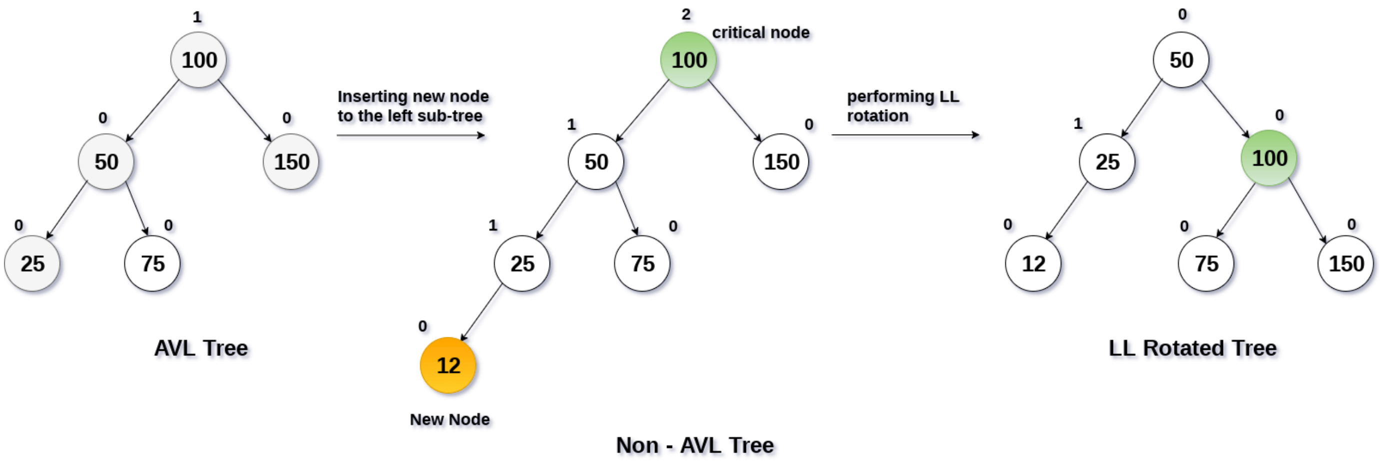


Right Rotation:
If a node is added to the left subtree of the left subtree, the AVL tree may get out of balance, we do a single right rotation.

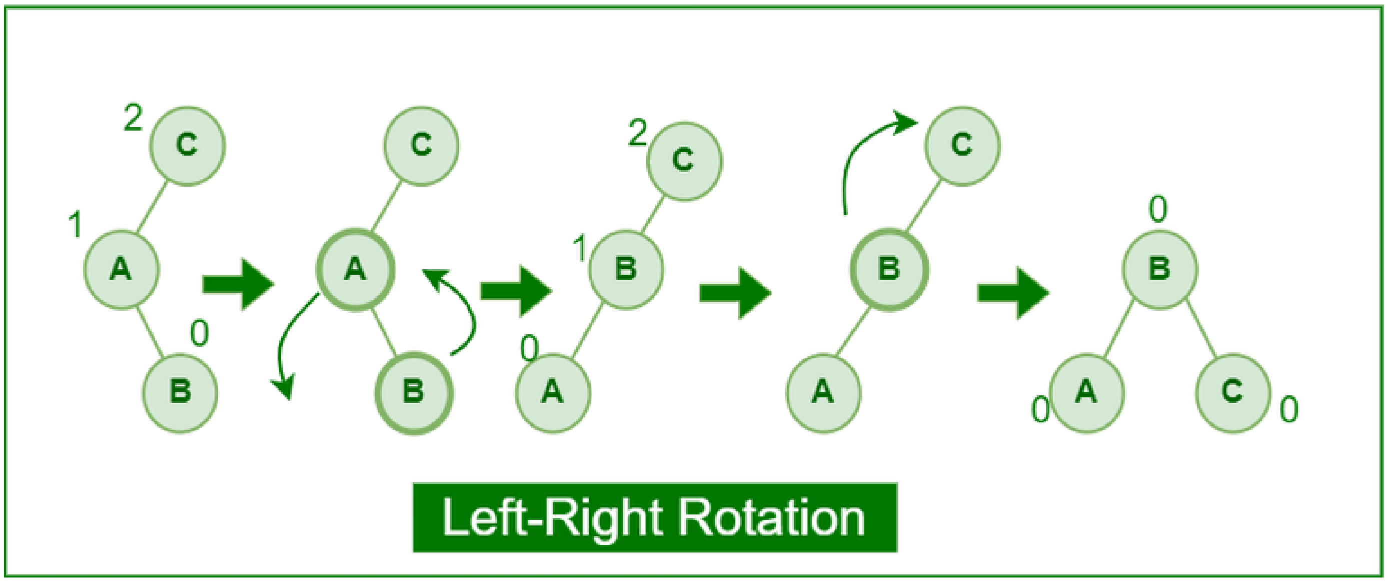


Right Rotation in AVL tree

Example:

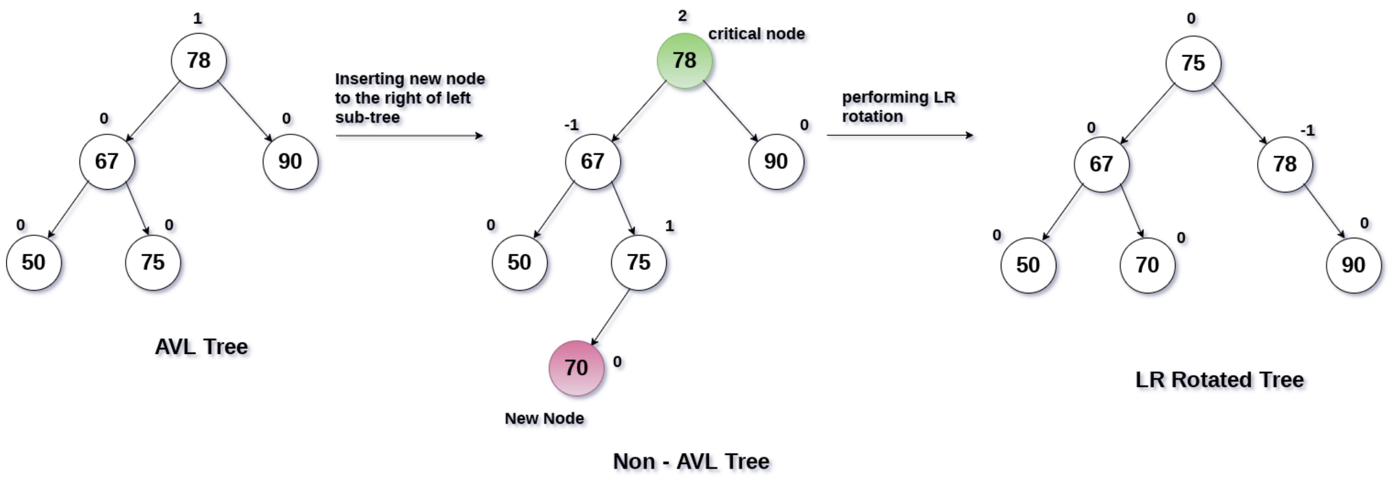


Left-Right Rotation:
A left-right rotation is a combination in which first left rotation takes place after that right rotation executes.

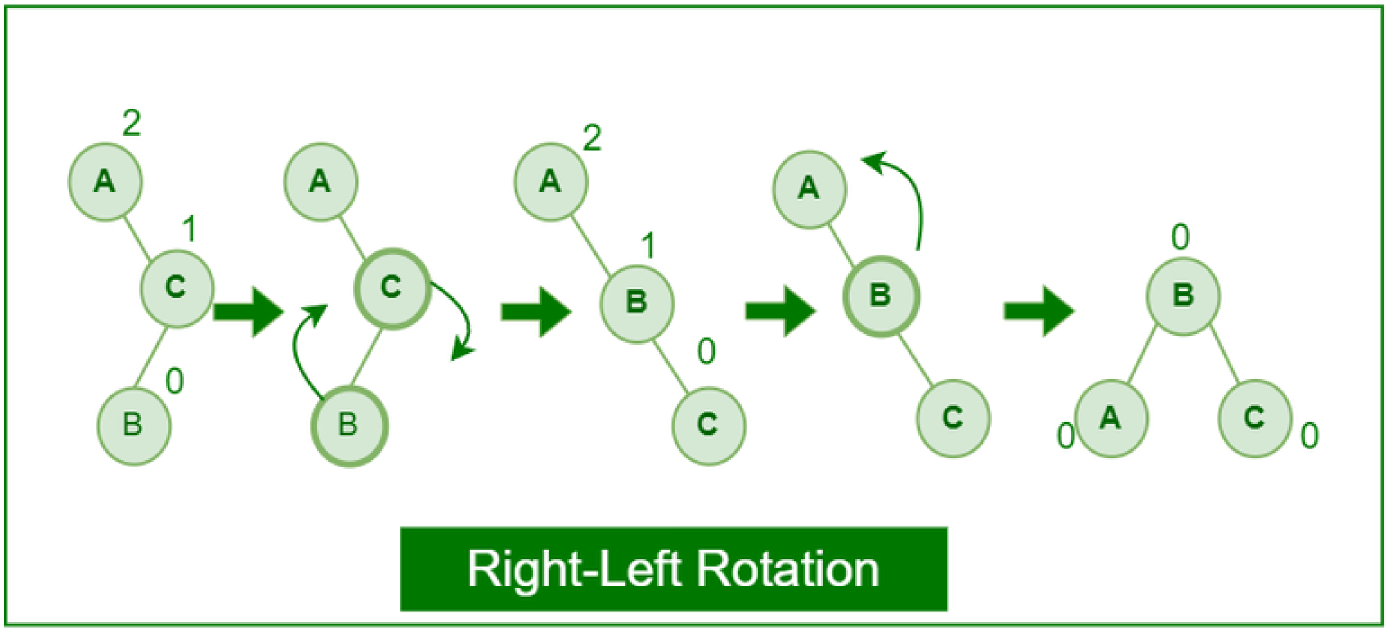


Left-Right Rotation in AVL tree

Example:

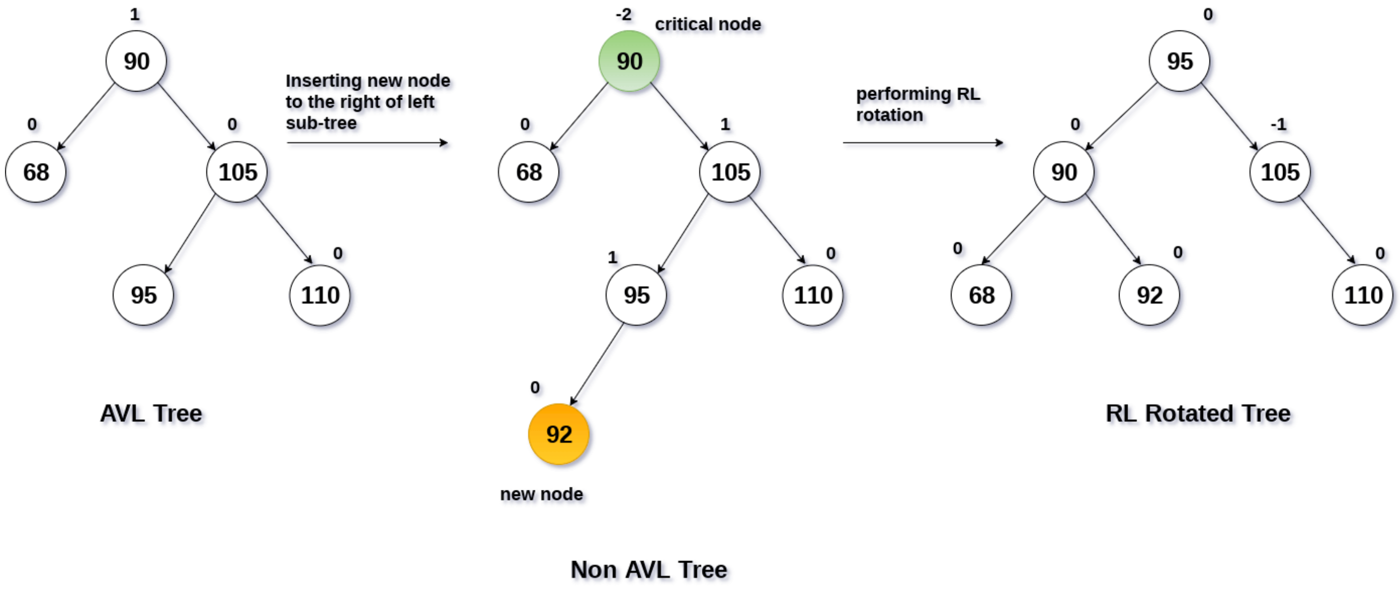


Right-Left Rotation:
A right-left rotation is a combination in which first right rotation takes place after that left rotation executes.

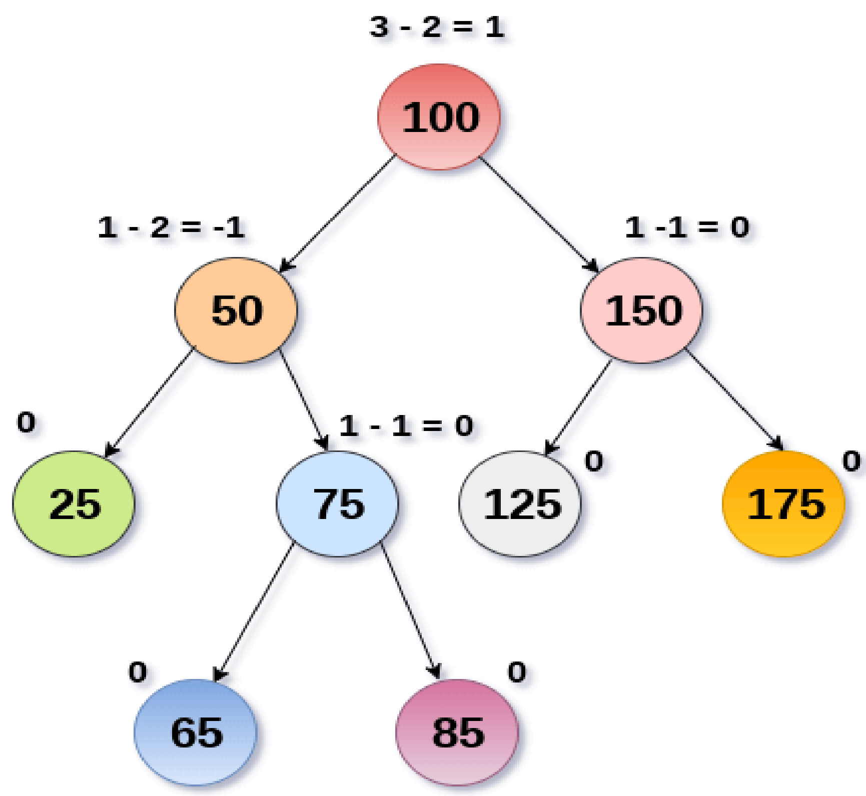


Right-Left Rotation in AVL tree

Example:



An **AVL tree** is given in the following figure. We can see that, balance factor associated with each node is in between -1 and +1. therefore, it is an example of AVL tree.



AVL Tree

Advantages of AVL Tree:

1. AVL trees can self-balance themselves.
2. It is surely not skewed.
3. Better searching time complexity compared to other trees like binary tree.
4. Height cannot exceed log(N), where, N is the total number of nodes in the tree.

Disadvantages of AVL Tree:

1. It is difficult to implement.
2. It has high constant factors for some of the operations.
3. Due to its rather strict balance, AVL trees provide complicated insertion and removal operations as more rotations are performed.
4. Take more processing for balancing.

Case for Insertion:

```
START
   if node == null then:
      return new node
   if key < node.key then:
      node.left = insert (node.left, key)
   else if (key > node.key) then:
      node.right = insert (node.right, key)
   else
      return node
   node.height = 1 + max (height (node.left), height (node.right))
   balance = getBalance (node)
   if balance > 1 and key < node.left.key then:
      rightRotate
   if balance < -1 and key > node.right.key then:
      leftRotate
   if balance > 1 and key > node.left.key then:
      node.left = leftRotate (node.left)
      rightRotate
   if balance < -1 and key < node.right.key then:
      node.right = rightRotate (node.right)
      leftRotate (node)
   return node
END
```

Steps to follow for insertion:

Let the newly inserted node be **w**

- Perform standard **BST** insert for **w**.
- Starting from **w**, travel up and find the first **unbalanced node**. Let **z** be the first unbalanced node, **y** be the **child** of **z** that comes on the path from **w** to **z** and **x** be the **grandchild** of **z** that comes on the path from **w** to **z**.
- Re-balance the tree by performing appropriate rotations on the subtree rooted with **z.** There can be 4 possible cases that need to be handled as **x, y** and **z** can be arranged in 4 ways.
- Following are the possible 4 arrangements:
  - y is the left child of z and x is the left child of y (Left Left Case)
  - y is the left child of z and x is the right child of y (Left Right Case)
  - y is the right child of z and x is the right child of y (Right Right Case)
  - y is the right child of z and x is the left child of y (Right Left Case)

Case for Deletion

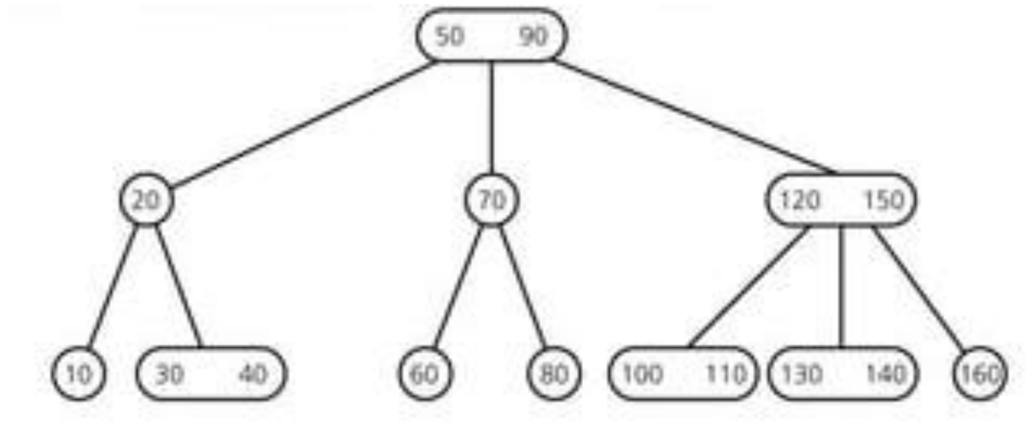
```
START
   if root == null: return root
   if key < root.key:
      root.left = delete Node
   else if key > root.key:
      root.right = delete Node
   else:
      if root.left == null or root.right == null then:
         Node temp = null
         if (temp == root.left)
            temp = root.right
         else
            temp = root.left
         if temp == null then:
            temp = root
            root = null
         else
            root = temp
         else:
            temp = minimum valued node
            root.key = temp.key
            root.right = delete Node
      if (root == null) then:
         return root
         root.height = max (height (root.left), height
(root.right)) + 1
         balance = getBalance
      if balance > 1 and getBalance (root.left) >= 0:
         rightRotate
      if balance > 1 and getBalance (root.left) < 0:
         root.left = leftRotate (root.left);
         rightRotate
      if balance < -1 and getBalance (root.right) <= 0:
         leftRotate
      if balance < -1 and getBalance (root.right) > 0:
         root.right = rightRotate (root.right);
         leftRotate
      return root
END
```

Steps to follow for Deletion:

1. Perform the normal BST deletion.
2. The current node must be one of the ancestors of the deleted node. Update the height of the current node.
3. Get the balance factor (left subtree height – right subtree height) of the current node.
4. If balance factor is greater than 1, then the current node is unbalanced and we are either in Left Left case or Left Right case. To check whether it is Left Left case or Left Right case, get the balance factor of left subtree. If balance factor of the left subtree is greater than or equal to 0, then it is Left Left case, else Left Right case.
5. If balance factor is less than -1, then the current node is unbalanced and we are either in Right Right case or Right Left case. To check whether it is Right Right case or Right Left case, get the balance factor of right subtree. If the balance factor of the right subtree is smaller than or equal to 0, then it is Right Right case, else Right Left case.

2-3 Trees Data Structure

A 2-3 Tree is a type of self-balancing search tree used in computer science to maintain data in a sorted order, enabling efficient search, insertion, and deletion operations. Each node in a 2-3 Tree can have either one key (forming a 2-node) or two keys (forming a 3-node), with each node having either two or three child nodes, respectively. This means that nodes can store multiple keys and adjust their structure dynamically to maintain balance, distributing keys in a way that ensures all leaf nodes are at the same depth.



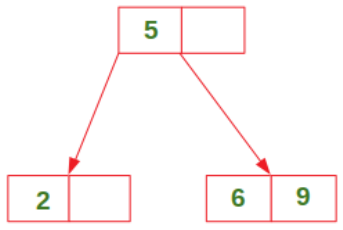
Operations on an 2-3 Tree:

- Insertion
- Deletion
- Searching

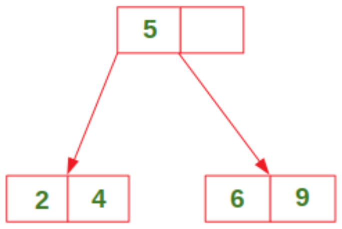
Insertion:

- **Case 1:** Insert in a node with only one data element

Insert 4 in the following 2-3 Tree:



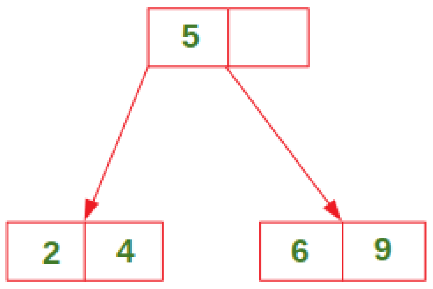
Initial



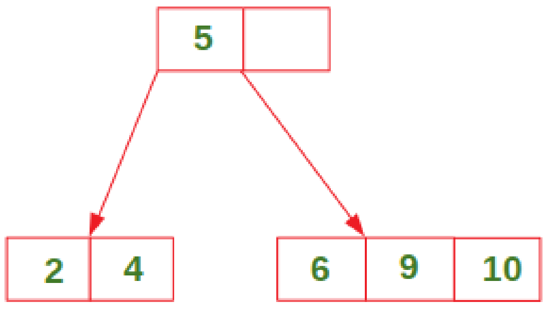
After Insertion

- **Case 2:** Insert in a node with two data elements whose parent contains only one data element.

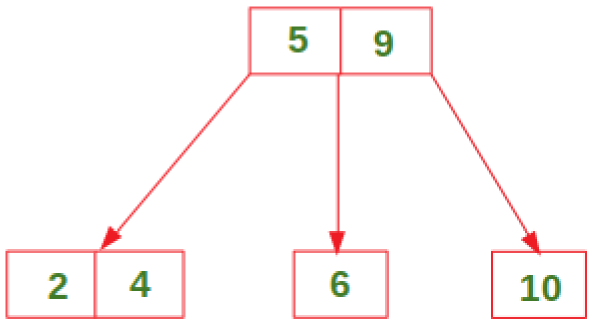
Insert 10 in the following 2-3 Tree:



Initial



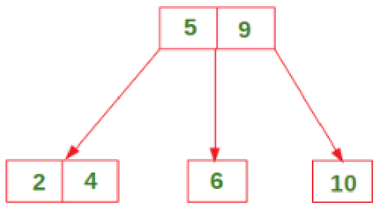
Temporary Node with 3 data elements



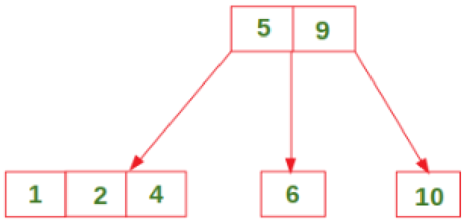
Move the middle element to parent and split the current Node

- **Case 3:** Insert in a node with two data elements whose parent also contains two data elements.

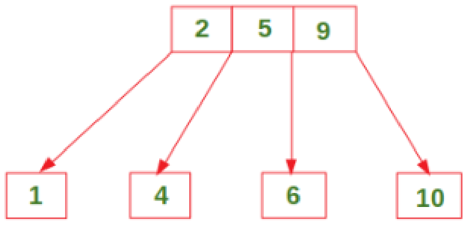
Insert 1 in the following 2-3 Tree:



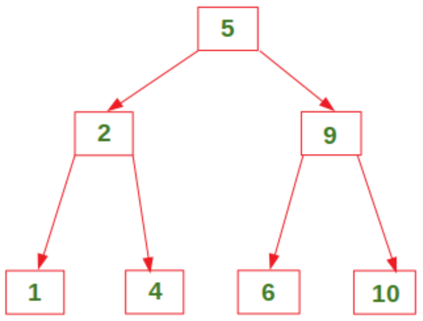
Initial



Temporary Node with 3 data elements



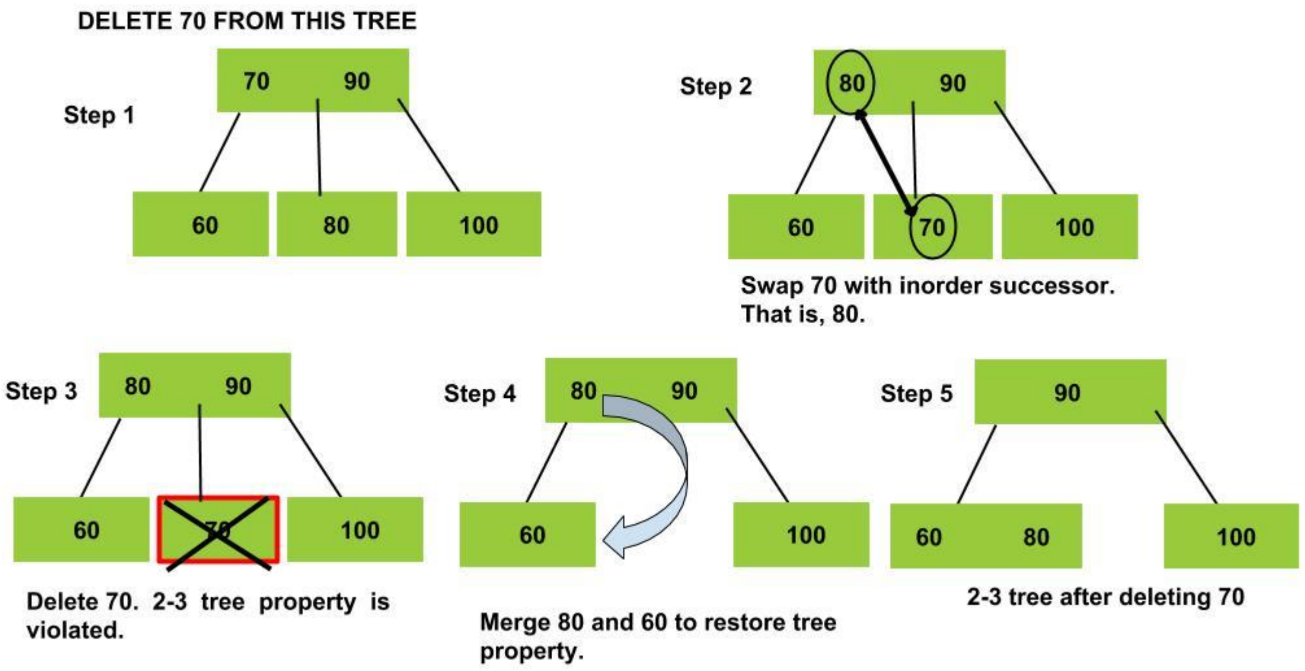
Move the middle element to the parent
and split the current Node



Move the middle element to the parent
and split the current Node

Deletion:

- To delete a value, it is replaced by its in-order successor and then removed.
- If a node is left with less than one data value then two nodes must be merged together.
- If a node becomes empty after deleting a value, it is then merged with another node.


DELETE 70 FROM THIS TREE
Step 1
70
90
60
80
100
Step 2
80
90
60
70
100
Swap 70 with inorder successor. That is, 80.
Step 3
80
90
60
70
100
Delete 70. 2-3 tree property is violated.
Step 4
80
90
60
100
Merge 80 and 60 to restore tree property.
Step 5
90
60
80
100
2-3 tree after deleting 70

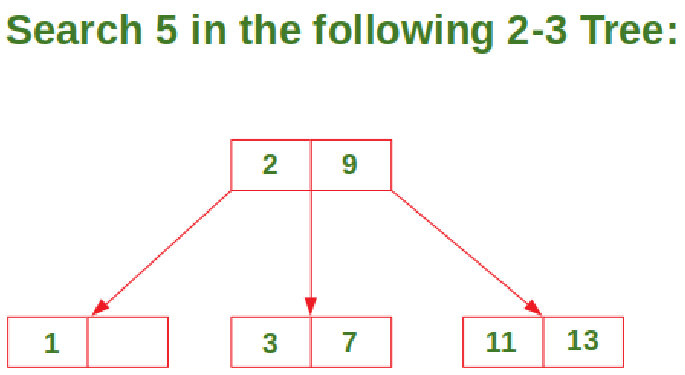
Searching:

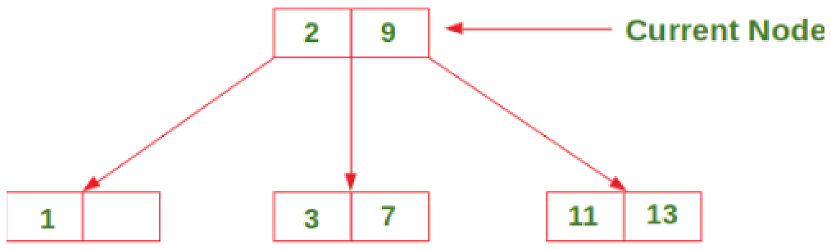
Base cases:

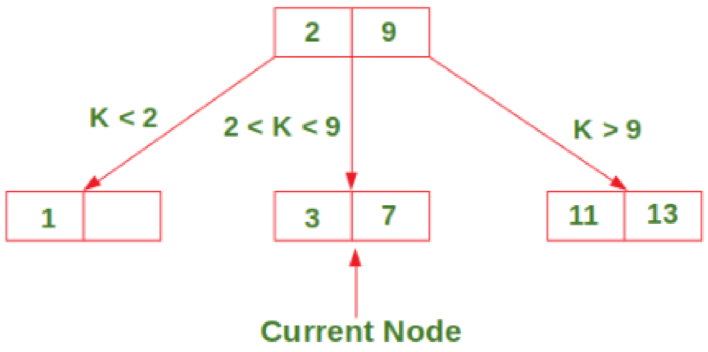
1. If **T** is empty, return False (key cannot be found in the tree).
2. If current node contains data value which is equal to **K**, return True.
3. If we reach the leaf-node and it doesn't contain the required key value **K**, return False.

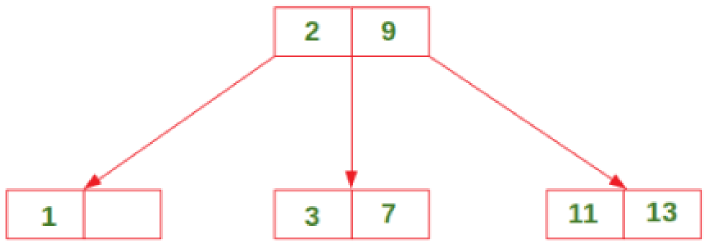
Recursive Calls:

1. If **K** < currentNode.leftVal, we explore the left subtree of the current node.
2. Else if currentNode.leftVal < **K** < currentNode.rightVal, we explore the middle subtree of the current node.
3. Else if **K** > currentNode.rightVal, we explore the right subtree of the current node.









5 Not Found. Return False

Pseudocodes

- **Insertion:**

```
FUNCTION insert(tree, key):
    IF tree is empty:
        CREATE a new root with key
        RETURN

    currentNode = tree.root

    # Step 1: Traverse to find the correct insertion point
    WHILE currentNode is not a leaf:
        IF currentNode is a 2-node:
            IF key < currentNode.key1:
                currentNode = currentNode.leftChild
            ELSE:
                currentNode = currentNode.rightChild
        ELSE IF currentNode is a 3-node:
            IF key < currentNode.key1:
                currentNode = currentNode.leftChild
            ELSE IF key > currentNode.key2:
                currentNode = currentNode.rightChild
            ELSE:
                currentNode = currentNode.middleChild

    # Step 2: Insert key in the leaf node
    INSERT key into currentNode in sorted order

    # Step 3: Handle splitting if the node becomes overfull
    WHILE currentNode has 3 keys:
        middleKey = currentNode.key2
        leftChild = Node(currentNode.key1)
        rightChild = Node(currentNode.key3)

        IF currentNode is the root:
            # Create a new root if splitting the root
            tree.root = Node(middleKey)
            tree.root.leftChild = leftChild
            tree.root.rightChild = rightChild
            RETURN

        parentNode = currentNode.parent
        INSERT middleKey into parentNode in sorted order
        REPLACE currentNode in parentNode’s children with leftChild and
rightChild
        currentNode = parentNode  # Move up to handle further splits if
needed
```

- **<u>Deletion:</u>**

```
FUNCTION delete(tree, key):
    # Step 1: Locate the node containing the key
    currentNode = tree.root
    WHILE currentNode is not NULL:
        IF key is found in currentNode:
            BREAK
        ELSE IF currentNode is a 2-node:
            IF key < currentNode.KEY1:
                currentNode = currentNode.LEFT
            ELSE:
                currentNode = currentNode.RIGHT
        ELSE IF currentNode is a 3-node:
            IF key < currentNode.KEY1:
                currentNode = currentNode.LEFT
            ELSE IF key > currentNode.KEY2:
                currentNode = currentNode.RIGHT
            ELSE:
                currentNode = currentNode.MIDDLE

    # Step 2: Delete the key
    IF key is in a leaf node:
        REMOVE key from the leaf node
    ELSE:
        # Key is in an internal node, find and replace with predecessor or
successor
        IF currentNode has LEFT child:
            predecessor = findMax(currentNode.LEFT)
            REPLACE key in currentNode with predecessor
            currentNode = predecessor  # Move to predecessor node to delete
key
        ELSE:
            successor = findMin(currentNode.RIGHT)
            REPLACE key in currentNode with successor
            currentNode = successor  # Move to successor node to delete key

    # Step 3: Handle underfull nodes after deletion
    WHILE currentNode is underfull:
        sibling = findSiblingWithExtraKey(currentNode)
        IF sibling is not NULL:
            # Borrow from sibling if possible
            borrowFromSibling(currentNode, sibling)
        ELSE:
            # Merge with sibling if borrowing is not possible
            parentNode = currentNode.PARENT
            mergeWithSibling(currentNode, parentNode)
            IF parentNode becomes underfull:
                currentNode = parentNode
            ELSE:
                BREAK

    # Step 4: Adjust root if it becomes underfull and empty
    IF tree.root has no keys:
        tree.root = tree.root.LEFT or tree.root.RIGHT  # Make child the new
root if empty
```

- **Searching:**

```
FUNCTION search(tree, key):
    # Step 1: Start at the root
    currentNode = tree.root

    # Step 2: Traverse the tree until key is found or leaf is reached
    WHILE currentNode is not NULL:
        IF currentNode is a 2-node:
            IF key == currentNode.KEY1:
                RETURN currentNode  # Key found
            ELSE IF key < currentNode.KEY1:
                currentNode = currentNode.LEFT  # Move left
            ELSE:
                currentNode = currentNode.RIGHT  # Move right

        ELSE IF currentNode is a 3-node:
            IF key == currentNode.KEY1 OR key == currentNode.KEY2:
                RETURN currentNode  # Key found
            ELSE IF key < currentNode.KEY1:
                currentNode = currentNode.LEFT  # Move left
            ELSE IF key > currentNode.KEY2:
                currentNode = currentNode.RIGHT  # Move right
            ELSE:
                currentNode = currentNode.MIDDLE  # Move middle

    # Step 3: If we reached a leaf without finding the key
    RETURN NOT_FOUND  # Key not found
```

Tasks

1. Implement the following insertions in the AVL tree (1,2,3,4,5,6,7)

2. Delete value 3 from the tree and balance it.

3. Do a pre-order, in order and post-order traversal of the tree before deletion and after deletion.

4. Search for any value in the tree if it is present print, it with its index (key) value otherwise inserts it into the tree and balances it with the appropriate rotations.

5. Find the kth smallest and largest value in the AVL tree and print its key also print both the left side and right-side height of the tree starting from root.

6. Implement a 2-3 Tree and perform all the operations you learned above like:
   Inserting, Deleting, Searching, and Display.