ASSIGNMEN #2

23K-2023

(4.1)

(Q5) All real numbers in the form (x, y) where $x \geq 0$ with standard operations on R

① Let $u = (1, 2)$, $v = (2, -2)$, $w = (3, 1)$, $k = -2$, $m = -3$

① $u + v \in$

$(1, 2) + (2, -2) = (3, 0)$ proved

② $u + v = v + u$

$(3, 0) = (3, 0)$ proved

③ $u + (v + w) = (u + v) + w$

$(1, 2) + [(2, -2) + (3, 1)]$

$= (1, 2) + (5, -1) \Rightarrow (6, 1)$ ~~pr~~ $=$ L.H.S

R.H.S

$[(1, 2) + (2, -2)] + (3, 1)$

$= (3, 0) + (3, 1) = (6, 1)$

R.H.S = L.H.S proved

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(4) $u + x = u$

$(1, 2) + (0, 0) = (1, 2)$ proved

(5) $u + (-u) = 0$

$u = (1, 2)$, $-u = (-1, -2)$

but x should be $\neq 0$ so

Axiom 5 fails.

(6) $ku \in V$

$-2(1, 2) = (-2, -4)$

Axiom 6 also fails

(7) $k(u+v) = ku + kv$

L.H.S

$-2(3, 0) \Rightarrow (-6, 0)$

R.H.S

$-2(1, 2) + -2(2, -2)$

$= (-2, -4) + (-4, 4)$

$= (-6, 0)$

R.H.S = L.H.S proved

(8) $(k+m)u = \cancel{(k+m)u}$ $ku + mu$

$(-2-3)(1, 2) = (-5, -10) =$ L.H.S

$(-2, -4) + (-3, -6) = (-5, -10) =$ R.H.S proved

(9) $k(mu) = (km)u$

$-2(-3,-6) = (6)(1,2)$

$(6,12) = (6,12)$ proved

(10) $1u = u$

$1(1,2) = (1,2)$ proved

(Q9) Set of all 2x2 ~~matr invertible~~ matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$u = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, v = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}, w = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, k = -3$

$m = -2$

(1) $u + v \in V$

$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$ proved

(2) $u+v = v+u$

$\begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$ proved

(3) $u + (v+w) = (u+v)+w$

$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + \left[\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\right] = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

$=$

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$$\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 4\end{bmatrix} = \begin{bmatrix}2 & 0\\ 0 & 6\end{bmatrix}$$

$$= \begin{bmatrix}2 & 0\\ 0 & 6\end{bmatrix} = \begin{bmatrix}2 & 0\\ 0 & 6\end{bmatrix}$$ proved

(4) $u + x = u$

$$x = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\cancel{u+} \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} + \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} = \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$$ proved

(5) $u + (-u) = x$; $x = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$$-u = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$ proved

(6) $ku \in V$

$$-3\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}-3 & 0\\ 0 & -6\end{bmatrix}$$ proved $\in V$

(7) $k(u+v) = ku + kv$

$$-3\begin{bmatrix}-1 & 0\\ 0 & 5\end{bmatrix} = -3\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} + -3\begin{bmatrix}-2 & 0\\ 0 & 3\end{bmatrix}$$

$$= \begin{bmatrix}3 & 0\\ 0 & -15\end{bmatrix} = \begin{bmatrix}-3 & 0\\ 0 & -6\end{bmatrix} + \begin{bmatrix}6 & 0\\ 0 & -9\end{bmatrix}$$

$$\begin{bmatrix}3 & 0\\ 0 & -15\end{bmatrix} = \begin{bmatrix}3 & 0\\ 0 & -15\end{bmatrix}$$ proved

(8) $(k+m)u = ku + mu$

$$(-3-2)\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} = -3\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} + -2\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$$

$$= -5\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}-3 & 0\\ 0 & -6\end{bmatrix} + \begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix}$$

$$= \begin{bmatrix}-5 & 0\\ 0 & -10\end{bmatrix} = \begin{bmatrix}-5 & 0\\ 0 & -10\end{bmatrix}$$

(9) $k(mu) = (km)\,u$

$$-3\left[(-2)\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\right] = (6)\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$$

$$= -3\begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix} = \begin{bmatrix}6 & 0\\ 0 & 12\end{bmatrix}$$

$$= \begin{bmatrix}6 & 0\\ 0 & 12\end{bmatrix} = \begin{bmatrix}6 & 0\\ 0 & 12\end{bmatrix}$$ proved

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(10) $1u = u$

$1\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$ proved

(Q11) Set of all pairs of $\mathbb{R}$ of the form ~~(1, y)~~ $(1, x)$ with the operations

$(1, y) + (1 + y') = (1, y + y')$, $u(1, y) = (1, y$

(1) $u + v \in V$

$(1, y) + (1, y') = (1, y + y')$

(2) $u + v = v + u$

$(1, y + y') = (1, y + y')$

(3) $u + (v + w) = (u + v) + w$

$(1, y) + [(1, y') + (1, y'')]$

$(1, y) + (1, y' + y'') = (1, y + y' + y'') = L.H.$

(4) $u + \alpha = u$

$(1, y) + (1, 0) = (1, y) = u$ proved

(5) $u + (-u) = 0$

$-u = (1, -y) \Rightarrow (1, y) + (1, -y) = (1, 0)$
proved

(6) $ku \in V$

$k(1,y) = (1, ky)$

(7) $k(u+v) = ku + kv$

$k(1, y+y') = (1, ky+ky')$ L.H.S

R.H.S

$\cancel{k} \; k(1,y) + k(1,y')$

$= (1, ky) + (1, ky')$

$= (1, ky + ky')$

R.H.S = L.H.S

(8) $(k+m)u = ku + mu$

$(k+m)(1,y) = (1, (k+m)y) = (1, ky+my)$ ∴ L.H.S

(9) $k(mu) = (km)u$

$k[m(1,y)] \Rightarrow k(1, my) \Rightarrow (1, kmy)$ ∴ L.H.S

R.H.S

$km(1,y) = (1, kmy)$ proved

(10) $1u = u$

$1(1,y) = (1,y)$ proved

(4.2)

(Q4)a) Set of all $n \times n$ matrices A such that $A^T = -A$
we will check the skew symmetric matrices.

$A^T = -A$, $B^T = -B$, $(A+B)^T = A^T + B^T = -(A+B)$
It is closed under addition.

$(cA)^T = c(A^T) = c(-A) = -(cA)$
hence $(cA)^T = -(cA)$ proved

(b) $Ax = 0$

$$A+B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Both A and B are invertible but their addition is not.

SCALOR MULTIPLICATION

if $x = 0$ then $xA = 0$; and zero matrix is not.

(c) $n \times n$ matrices such that $AB = BA$, fixed B.

$A_1 B = B A_1$ and $A_2 B = B A_2$

$\therefore A_1 + A_2$

$= (A_1 + A_2) B = B(A_1 + A_2)$

$= A_1 B + A_2 B = BA_1 + BA_2$

It is closed under addition.

$(cA)B = B(cA)$

$= (cA)B = c(AB) = c(BA) = B(cA)$.

It is closed under scalor multiplication.

(d) set of all $n \times n$ invertible matrices.

Matric is not invertible if scalor $= 0$

so this is not closed under scalor multiplication.

(Q5) a) All polynomials $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ for which $a_0 = 0$.

~~SAt~~ ADDITION

$p(x) + q(x) = (0 + a_1 x + a_2 x^2 + a_3 x^3) +$
$(0 + b_1 x + b_2 x^2 + b_3 x^3)$

$= (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3$

$p(x) + q(x) = w(x)$ proved

MULTIPLICATION

$c \cdot p(x) = c(a_1 x + a_2 x^2 + a_3 x^3) = (ca_1)x +$
$(ca_2)x^2 + (ca_3)x^3$

It is a subspace of P_3

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(b) All polynomials $a_0+a_1x+a_2x^2+a_3x^3$ for which $a_0+a_1+a_2+a_3=0$.

$p(x)+q(x) = (a_0+b_0)+(a_1+b_1)x+(a_2+b_2)x^2$
$+(a_3+b_3)x^3$.

According to the condition this satisfies $a_0+a_1+a_2+a_3=0$. Closed under addition.

$c \cdot p(x) = c(a_0+a_1x+a_2x^2+a_3x^3)$
$=(ca_0)+(ca_1)x^2+(a_3c)x^3$

Now checking sum of cofficients $=0$ proved

(4.3)

(Q6) Express as linear combination.

(a) $-9-7x-15x^2$

~~$-9 = 2k_1+k_2+4k_3$~~ $-9 = 2k_1+k_2+3k_3$

~~$-7 =$~~ $-7 = k_1-k_2+2k_3$

$-15 = 4k_1+3k_2+5k_3$

$$= \begin{bmatrix} 2 & 1 & 3 & -9 \\ 1 & -1 & 2 & -7 \\ 4 & 3 & 5 & -15 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} = \begin{bmatrix} 1 & -1 & 2 & -7 \\ 2 & 1 & 3 & -9 \\ 4 & 3 & 5 & -15 \end{bmatrix}$$

$$\begin{matrix} -2R_1+R_2 \\ -4R_1+R_2 \end{matrix} = \begin{bmatrix} 1 & -1 & 2 & -7 \\ 0 & 3 & -1 & 5 \\ 0 & 7 & -3 & 13 \end{bmatrix} = R_2 \times \frac{1}{3} = \begin{bmatrix} 1 & -1 & 2 & -7 \\ 0 & 1 & -1/3 & 5/3 \\ 0 & 7 & -3 & 13 \end{bmatrix}$$

$-7R_2+R_3 = \begin{bmatrix} 1 & -1 & 2 & -7 \\ 0 & 1 & -1/3 & 5/3 \\ 0 & 0 & -2/3 & 4/3 \end{bmatrix}$

$R_3 \times \frac{-3}{2} = \begin{bmatrix} 1 & -1 & 2 & -7 \\ 0 & 1 & -1/3 & 5/3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

① $p_1 - p_2 + 2p_3 = -7 \Rightarrow p = -2$

② $p_2 - \frac{1}{3}p_3 = \frac{5}{3} \Rightarrow p_2 = 1$

③ $p_3 = -2$

unique solution

$-9-7x-15x^2 = -2p_1 + p_2 - 2p_3$

(b) $6+11x+6x^2$

$6 = 2k_1 + k_2 + 3k_3$

$11 = k_1 - k_2 + 2k_3$

$6 = 4k_1 + 3k_2 + 5k_3$

$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 11 \\ 2 & 1 & 3 & 6 \\ 4 & 3 & 5 & 6 \end{bmatrix}$

$-2R_1+R_2 =$
$-4R_1+R_3$ $\begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 7 & -3 & -38 \end{bmatrix} \Rightarrow R_2 \times \frac{1}{3} = \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 7 & -3 & -38 \end{bmatrix}$

$-7R_2+R_3 = \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 0 & -2/3 & -2/3 \end{bmatrix} \Rightarrow R_3 \times \frac{-3}{2} = \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

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① $p_1 - p_2 + 2p_3 = \cancel{8}\ 11 \Rightarrow p_1 = 4$

$p_2 - \frac{1}{3}p_3 = -16/3 \Rightarrow p_2 = -5$

$p_3 = 1$

unique solution

$6 + 11x + 6x^2 = 4p_1 - 5p_2 + p_3$

(c) as the vector is 0

$0 = 0p_1 + 0p_2 + 0 \cdot p_3$

(d) $7 + 8x + 9x^2$

$$7 = 2k_1 + k_2 + 3k_3$$
$$8 = k_1 - k_2 + 2k_3$$
$$9 = 4k_1 + 3k_2 + 5k_3$$
$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 8 \\ 2 & 1 & 3 & \cancel{8}\,7 \\ 4 & 3 & 5 & 9 \end{bmatrix}$$

$$\begin{matrix} -2R_1 + R_2 \\ -4R_1 + R_3 \end{matrix} = \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 3 & -1 & -9 \\ 0 & 7 & -3 & -23 \end{bmatrix} \Rightarrow R_2 \times \frac{1}{3} = \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 1 & -1/3 & -3 \\ 0 & 7 & -3 & -23 \end{bmatrix}$$

$$-7R_2 + R_3 = \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 1 & -1/3 & -3 \\ 0 & 0 & -\frac{2}{3} & \cancel{-\frac{7}{8}}\ -2 \end{bmatrix} \Rightarrow R_3 \times -\frac{3}{2} = \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 1 & -1/3 & -2 \\ 0 & 0 & 1 & 3\ \cancel{\frac{15}{7}} \end{bmatrix}$$

① $p_1 - p_2 + 2p_3 = 8 \quad = 0$

② $p_2 - \frac{1}{3}p_3 = -3 \Rightarrow p_2 = -2$

③ $p_3 = \cancel{\frac{15}{7}}\ 3$

$7 + 8x + 9x^2 = 0p_1 - 2p_2 + 3p_3$

(Q8) $v_1 = (2,1,0,3)$, $v_2 = (3,-1,5,2)$,
$v_3 = (-1,0,2,1)$, span $\{v_1, v_2, v_3\}$

(a) $(2,3,-7,3)$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 3 & -1 & 2 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{matrix} -2R_1+R_2 = \\ -3R_1+R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 5 & -1 & -4 \\ 0 & 5 & 2 & -7 \\ 0 & 5 & 1 & -6 \end{bmatrix} \Rightarrow R_2 \times \frac{1}{5} = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 5 & 2 & -7 \\ 0 & 5 & 1 & -6 \end{bmatrix}$$

$$\begin{matrix} -5R_2+R_3 = \\ -5R_2+R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 2 & -2 \end{bmatrix} \Rightarrow \begin{matrix} R_3 \times \frac{1}{3} \\ -R_3+R_4 \\ \cancel{R_4 \times \frac{1}{2}} \end{matrix} = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In order to exist system must have unique

(1) $p_1 - p_2 = 3 \Rightarrow p_1 = 2$

(2) $p_2 - 1/5\, p_3 = -4/5 \Rightarrow p_2 = -1$

(3) $p_3 = -1$

The system is consistent so it spans

(b) (0,0,0,0)

The system spans as

$0(2,1,0,3) + 0(3,-1,5,2) + 0(-1,0,2,1) = (0,0,0,0)$

(c) (1,1,1,1) $\Rightarrow$

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & 3 & -1 & 1 \\ 0 & 5 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$$

$-2R_1+R_2$, $-3R_1+R_4$

$$= \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 5 & -1 & -1 \\ 0 & 5 & 2 & 1 \\ 0 & 5 & 1 & -2 \end{bmatrix} \Rightarrow R_2 \times \frac{1}{5}, -5R_2+R_3, -5R_2+R_4 = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & -1/5 & -1/5 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 2 & -1 \end{bmatrix}$$

$$R_3 \times \frac{1}{3}, -2R_3+R_4 = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & -1/5 & -1/5 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & -7/3 \end{bmatrix}$$

system is inconsistent so it doesnt span

(d) (-4, 6, -13, 4)

$$= \begin{bmatrix} 1 & -1 & 0 & 6 \\ 2 & 3 & -1 & -4 \\ 0 & 5 & 2 & -13 \\ 3 & 2 & 1 & 4 \end{bmatrix} \Rightarrow -2R_1+R_2, -3R_1+R_4 = \begin{bmatrix} 1 & -1 & 0 & 6 \\ 0 & 5 & -1 & -16 \\ 0 & 5 & 2 & -13 \\ 0 & 5 & 1 & -14 \end{bmatrix}$$

-24
-14

$$R_2 \times \frac{1}{5} = \begin{bmatrix} 1 & -1 & 0 & 6 \\ 0 & 1 & -1/5 & -16/5 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 2 & 2 \end{bmatrix} \begin{matrix} \\ \\ -5R_2+R_3 \\ -5R_2+R_4 \end{matrix} \quad R_3 \times \frac{1}{3} = \begin{bmatrix} 1 & -1 & 0 & 6 \\ 0 & 1 & -1/5 & -16/5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ -2R_3+R_4 \end{matrix}$$

System is consistent so it spans.

(Q18) $u_1 = (0,1,1)$, $u_2 = (2,-1,1)$, $u_3 = (1,1,-2)$

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

$$T_A(u_1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$T_A(u_2) = (1, -2)$, $T_A(u_3) = (2, 3)$

$$(b_1, b_2) = k_1(1,0) + k_2(1,-2) + k_3(2,3)$$

$\Rightarrow$ $k_1 + k_2 + 2k_3 = b_1$

$0k_1 - 2k_2 + 3k_3 = b_2$

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & -2 & 3 \end{pmatrix} \Rightarrow R_2 \times -\frac{1}{2} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -3/2 \end{bmatrix}$$

The rank is 2, this means the vector spans $\mathbb{R}^2$.

(b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix}$

$T_A(u_1) = (1,-2)$, $T_A(u_2) = (-1,-2)$, $T_A(u_3) = (1,8)$

$B = \begin{bmatrix} 1 & -1 & 1 \\ -2 & -2 & 8 \end{bmatrix}$

$2R_1 + R_2 = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -4 & 10 \end{bmatrix} \Rightarrow R_2 \times \frac{-1}{4} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -5/2 \end{bmatrix}$

rank is 2 is vector spans

(4.4)

(Q6) Determine values for k for which it is L.I in M_{22}

$\begin{bmatrix} 1 & 0 \\ 1 & k \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ k & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

$c_1 \begin{bmatrix} 1 & 0 \\ 1 & k \end{bmatrix} + c_2 \begin{bmatrix} -1 & 0 \\ k & 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & k & 1 \\ k & 1 & 3 \end{bmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 1 & k & 1 \\ k & 1 & 3 \end{vmatrix}$

$1 \begin{vmatrix} k & 1 \\ 1 & 3 \end{vmatrix} - 1 \begin{vmatrix} -1 & 2 \\ 1 & 3 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ k & 1 \end{vmatrix}$

$= (3k-1) - 1(-3-2) + k(1-2k)$

$= 3k - 1 + 5 + k - 2k^2 \Rightarrow -2k^2 \neq 0$

$-2k^2+2k+4 \neq 0$

$k=2 \quad , k=-1$

matrices are L.I when $k \neq -1, k \neq 2$

(Q11) $v_1 = (\lambda, -1/2, -1/2), v_2 = (-1/2, \lambda, -1/2)$

$v_3 = (-1/2, -1/2, \lambda)$

$$= \begin{vmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{vmatrix}$$

(11.N)

$$\lambda \begin{vmatrix} \lambda & -1/2 \\ -1/2 & \lambda \end{vmatrix} + \frac{1}{2} \begin{vmatrix} -1/2 & -1/2 \\ -1/2 & \lambda \end{vmatrix} - \frac{1}{2} \begin{vmatrix} -1/2 & -1/2 \\ \lambda & -1/2 \end{vmatrix}$$

$= \lambda^3 - \frac{3}{4}\lambda - \frac{1}{4} = 0$; It is L.I when $\lambda = 1 , \lambda = -\frac{1}{2}$

(Q13) a) $u_1 = (1,2) , u_2 = (-1,1)$

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$$

$$u_1 = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$u_2 = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$|A| = -2+8$

$= 6$ it is L.I and span

(b) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

$u_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix}$

$= 0$ It is 1.I.

(4.5)

(Q15)

$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $A_2 \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $A_3 \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$; $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$1 = k_1$

$0 = k_1 + k_2 \Rightarrow k_2 = -1$

$1 = k_1 + k_2 + k_3 \Rightarrow k_3 = 1$

$0 = k_1 + k_2 + k_3 + k_4 \Rightarrow k_4 = -1$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

coordinate vector is $(A)_s = (1, -1, 1, -1)$

linear express = $A = A_1 - A_2 + A_3 - A_4$

The vector also spans; $|A| = 1$

(Q18) Show that $S = \{p_1, p_2, p_3\}$ is a basis of P_2. Then express p as L.C in S then find coordinate vector of $(p)_S$.

$p_1 = 1 + x + x^2$, $p_2 = x + x^2$, $p_3 = x^2$

$p = 7 - x + 2x^2$

$7 = k_1$

$-1 = k_1 + k_2 \Rightarrow k_2 = -8$

$2 = k_1 + k_2 + k_3 \Rightarrow k_3 = 3$

$p_1 = 1 + 2x + x^2$, $p_2 = 2 + 9x$, $p_3 = 3 + 3x + 4x^2$

$p = 2 + 17x - 3x^2$

$2 = k_1 + 2k_2 + 3k_3$

$17 = 2k_1 + 9k_2 + 3k_3$

$-3 = k_1 + 0k_2 + 4k_3$

checking for span and L.I

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 9 \\ 1 & 0 \end{matrix}$$

$(36 + 6) - (16 + 27) \Rightarrow 42 - 43 = -1 \neq 0$

so it span

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$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 9 & 3 & 17 \\ 1 & 0 & 4 & -3 \end{bmatrix} \Rightarrow \begin{array}{l} -2R_1+R_2 = \\ -R_1+R_3 \end{array} \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 5 & -3 & 13 \\ 0 & -2 & 1 & -5 \end{bmatrix}$$

$$R_2 \times \frac{1}{5} = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -3/5 & 13/5 \\ 0 & -2 & 1 & -5 \end{bmatrix}$$

$$2R_2 + R_3 = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -3/5 & 13/5 \\ 0 & 0 & -1/5 & 1/5 \end{bmatrix}$$

$$\frac{-6+5}{5}$$

$$\frac{26-25}{5}$$

$$R_3 \times -\frac{1}{5} = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -3/5 & 13/5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

① $p_1 + 2p_2 + 3p_3 = 2 \Rightarrow p_1 = 1$

② $p_2 - 3/5\, p_3 = 13/5 \Rightarrow p_2 = 2$

③ $p_3 = -1$

$P = p_1 + 2p_2 - p_3$

$(P)_S = (1, 2, -1)$ Ans

(4.6) 18, 19c

(b) $x+y+z=0$

$3x+2y-2z=0$

$4x+3y-z=0$

$6x+5y+z=0$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 3 & 2 & -2 & 0 \\ 4 & 3 & -1 & 0 \\ 6 & 5 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{l} -3R_1+R_2 \\ -4R_1+R_3 \\ -6R_1+R_4 \end{array} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{bmatrix} \Rightarrow -R_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{bmatrix}$$

$$\begin{array}{l} R_2+R_3 \\ R_2+R_4 \end{array} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

~~RA~~ Rank(A) $=2$

$x_1+x_2+x_3=0 \Rightarrow x_1 = 4x_3 \Rightarrow x_1 = 4s$

$x_2+5x_3=0 \Rightarrow x_2 = -5x_3 \Rightarrow x_2 = -5s$

$; x_3 = s$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} 4 \\ -5 \\ 1 \end{bmatrix} \qquad \text{Dim} = 1$$

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(Q18) Find the basis for the subspace that is spanned

~~$v_1 = (1,0,0)$, $v_2 = (1,0,1)$, $v_3 = (2,0,1)$~~

~~$v_4 = (0,0,1)$~~

$v_1 = (1,1,1,1)$, $v_2 = (2,2,2,0)$

$v_3 = (0,0,0,3)$, $v_4 = (3,3,3,4)$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & 0 & 3 & 0 \\ 1 & 0 & 3 & 4 & 0 \end{array}\right]$$

$-R_1 + R_2 =$ $-R_1 + R_3$ $-R_1 + R_4$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 3 & 1 & 0 \end{bmatrix}$$

$R_4 \leftrightarrow R_2 =$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & -2 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \times -\frac{1}{2} =$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 1 & -3/2 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$w + 2x + 3z = 0$

$x - 3/2y - \frac{1}{2}z = 0$

~~$w = -2r - 3t$~~

~~$x = \frac{3s}{2} + \frac{1t}{2}$~~

$$w = -2\left(\frac{-3}{2}r - \frac{1}{2}t\right) - 3t$$

$$x = \frac{-3}{2}r - \frac{1}{2}t$$

$$w = \cancel{6}\ 3r \cancel{+t} - 4t \quad ; y = r, z = t$$

$$x = \frac{-3}{2}r - \frac{1}{2}t$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3r - 4t \\ -3/2r - 1/2\,t \\ r \\ t \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ t \end{bmatrix} = r\begin{bmatrix} 3 \\ -3/2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -4 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}$$

Basis are; $\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right)$

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(Q19)c) $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right]$

$R_1+R_2 =$
R_1+R_3 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \Rightarrow -R_2+R_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$x = 0$

$y = 0$

$z = 0$

Since the solution is zero vector the dimensions are 0.

(4.8) 13, 26

(Q7) b)

For $Ax = b$,

$x_1 + x_2 + 2x_3 = 5$
$x_1 + x_3 = -2$
$2x_1 + x_2 + 3x_3 = 3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 5 \\ 1 & 0 & 1 & -2 \\ 2 & 1 & 3 & 3 \end{bmatrix}$$

$$\begin{array}{l} -R_1+R_2 \\ -2R_1+R_3 \end{array} = \begin{bmatrix} 1 & 1 & 2 & 5 \\ 0 & -1 & -1 & -7 \\ 0 & -1 & -1 & -7 \end{bmatrix} \Rightarrow \begin{array}{l} -R_2 \\ -R_2+R_3 \end{array} = \begin{bmatrix} 1 & 1 & 2 & 5 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + 2z = 5 \Rightarrow x = 5 - 7 + s - 2s$

$y + z = 7 \Rightarrow y = 7 - \cancel{z} \Rightarrow y = 7 - s$

$x = \cancel{-2+s} - 2 - s$

$y = 7 - s$

$z = s$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2-s \\ 7-s \\ s \end{bmatrix}$$

$$= s \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 7 \\ 0 \end{bmatrix}$$

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~~when~~ vector form $Ax=0$ is

$(x_1, x_2, x_3) = (-1, -1, 0)$

(Q13) a)

$$A = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$$

$$\begin{array}{l} 2R_1+R_2 \\ R_1+R_3 \\ 3R_1+R_4 \\ \\ \end{array} = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 2 & 6 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{l} -R_2+R_3 \\ -2R_2+R_4 \\ -R_3+R_4 \\ \\ \end{array} = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for row space:-

$r_1 = (1, -2, 5, 0, 3)$, $r_2 = (0, 1, 3, 0, 1)$, $r_3 = (0, 0, 0, 1, 0)$

Basis for column space:-

$c_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $c_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $c_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $A^T = \begin{bmatrix} 1 & -2 & -1 & -3 \\ -2 & 5 & 3 & 8 \\ 5 & -7 & -2 & -9 \\ 0 & 0 & 1 & 1 \\ 3 & -6 & -3 & -9 \end{bmatrix}$

reduced row echelon form is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$c_1' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; c_2' = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, c_3' = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Basis of column spaces of A^T are formed by:-

$$C_1 = \begin{bmatrix} 1 \\ -2 \\ 5 \\ 0 \\ 3 \end{bmatrix}; C_2 = \begin{bmatrix} -2 \\ 5 \\ 7 \\ 0 \\ -6 \end{bmatrix}, C_3 = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 1 \\ -3 \end{bmatrix}$$

Since columns of A^T are rows of A, row basis are

$r_1 = [1, -2\ 5\ 0\ 3]$, $r_2 = [-2\ 5\ -7\ 0\ 6]$

$r_3 = [-1\ 3\ -2\ 1\ -3]$.

(Q26)
$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & 1 & 4 & 0 \\ 1 & -7 & 5 & 0 \end{array}\right]$$

$$\begin{array}{l} -2R_1+R_2 = \\ -R_1+R_3 \end{array} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -2 & 0 \\ 0 & -5 & 2 & 0 \end{bmatrix}$$

$$R_2 \times \frac{1}{5} = \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & -2/5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$5R_2+R_3$

$$x_1 = \frac{-11}{5}t, \quad x_2 = \frac{2}{5}t, \quad x_3 = t$$

(b) multiplying $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 1 & -7 & 5 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{bmatrix} 2 \\ 7 \\ -1 \end{bmatrix}$ therefore $x_1 = x_2 = x_3 = 1$ is a solution of the non-homogenous system.

(c) $(x_1, x_2, x_3) = (1, 1, 1) + t(-11/5, 2/5)$

(d) $\begin{pmatrix} 1 & -2 & 3 & 2 \\ 2 & 1 & 4 & 7 \\ 1 & 7 & 5 & 1 \end{pmatrix}$

$-2R_1 + R_2 =$
$-R_1 + R_2$ $\begin{bmatrix} 1 & -2 & 3 & 2 \\ 0 & 5 & -2 & 3 \\ 0 & 5 & -2 & +3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 11/5 & 16/5 \\ 0 & 1 & -2/5 & 3/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = \frac{16}{5} - \frac{11}{5}s$, $x_2 = \frac{3}{5} + \frac{2}{5}s$, $x_3 = s$

$s = 1 + t$ then solution is same as part c.