14.4

Q1) $y^2+z^2=9$

$z=\sqrt{9-y^2}$

$z_x=0,\ z_y=\dfrac{-y}{\sqrt{9-y^2}}$

$z_y^2=\dfrac{y^2}{9-y^2}$

$$SA=\int_{-3}^{3}\int_{0}^{2}\sqrt{\frac{y^2}{9-y^2}+1}\ dx\,dy$$

$$=\int_{-3}^{3}\int_{0}^{2}\sqrt{\frac{9}{9-y^2}}\ dx\,dy$$

$$=\int_{-3}^{3}\int_{0}^{2}\frac{3}{\sqrt{9-y^2}}\ dx\,dy$$

$$=\int_{-3}^{3}\left[\frac{3x}{\sqrt{9-y^2}}\right]_0^2 dy$$

$$S.A=\int_{-3}^{3}\frac{6}{\sqrt{9-y^2}}\,dy$$

$$\cancel{2\int_{-3}^{3}\frac{3}{\sqrt{9-y^2}}\,dy}$$

$$=6\int_{-3}^{3}\frac{1}{3\sqrt{1-\left(\frac{y}{3}\right)^2}}\,dy$$

$$=6\int_{-3}^{3}\frac{1/3}{\sqrt{1-\left(\frac{y}{3}\right)^2}}\,dy$$

$$=6\left[\sin^{-1}\left(\frac{y}{3}\right)\right]_{-3}^{3}$$

$$=6\left(\frac{\pi}{2}-\left(\frac{\pi}{2}\right)\right)$$

$$\boxed{=6\pi}$$

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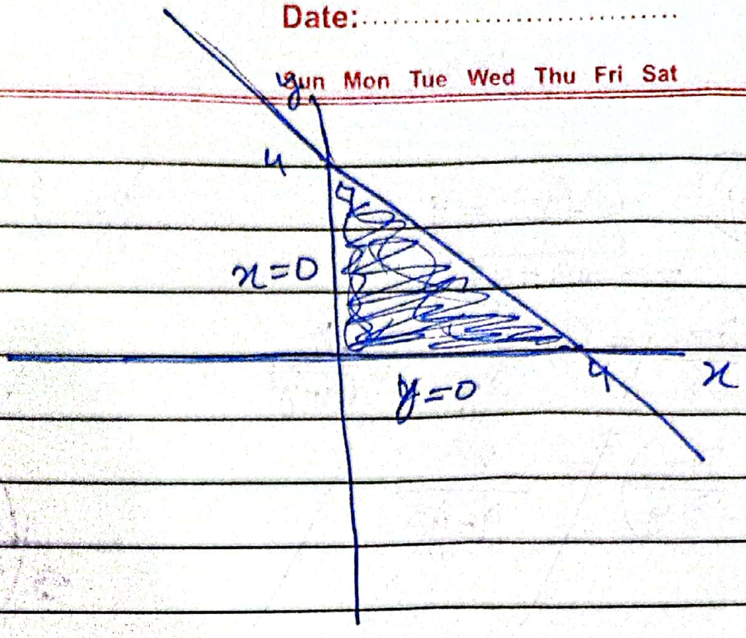


Q2) $2x+2y+z=8$

$z=8-2x-2y$

$\underline{z=0}$:-

$2y=8-2x$

$y=4-x$

$z_x = -2$, $z_x^2=4$

$z_y = -2$, $z_y^2 = 4$

$$S.A = \int_0^4 \int_0^{4-x} \sqrt{4+4+1}\, dy\, dx$$

$$= \int_0^4 \int_0^{4-x} \cancel{4}\,3\, dy\, dx$$

$$= \int_0^4 \cancel{3}\,12-3x\, dx$$

$$= \Big[12x - \frac{3x^2}{2} \Big]_0^4$$

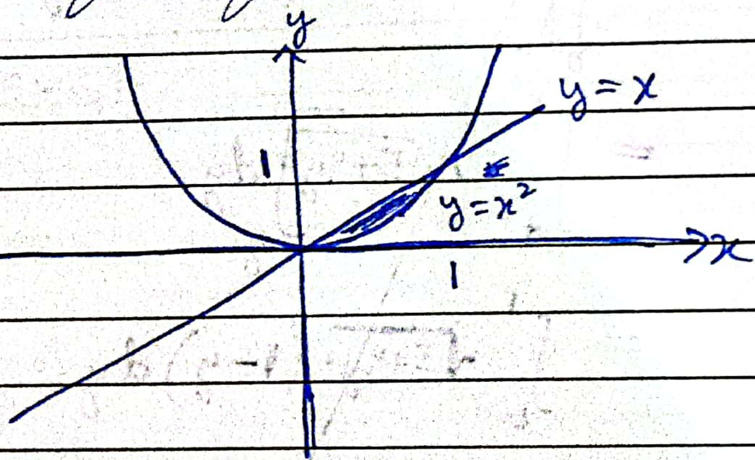
$$\boxed{= 48 - 24 = 24}$$

Q3) $z^2 = 4x^2 + 4y^2$

$z = \sqrt{4x^2+4y^2}$

$z = 2\sqrt{x^2+y^2}$

$z_x = \dfrac{2x}{\sqrt{x^2+y^2}}, \; z_x^2 = \dfrac{4x^2}{x^2+y^2}$



$z_y = \dfrac{2y}{\sqrt{x^2+y^2}}, \; z_y^2 = \dfrac{4y^2}{x^2+y^2}$

$$S.A = \int_0^1 \int_{x^2}^{x} \sqrt{\frac{4(x^2+y^2)}{x^2+y^2}+1}\; dy\, dx$$

$$= \int_0^1 \int_{x^2}^{x} \sqrt{5}\; dy\, dx$$

$$= \int_0^1 y\sqrt{5}\Big]_{x^2}^{x} dx$$

$$= \int_0^1 x\sqrt{5} - x^2\sqrt{5}\; dx$$

$$= \frac{x^2\sqrt{5}}{2} - \frac{x^3\sqrt{5}}{3}\Big]_0^1$$

$$= \frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{3}$$

$$\boxed{= \frac{\sqrt{5}}{6}}$$

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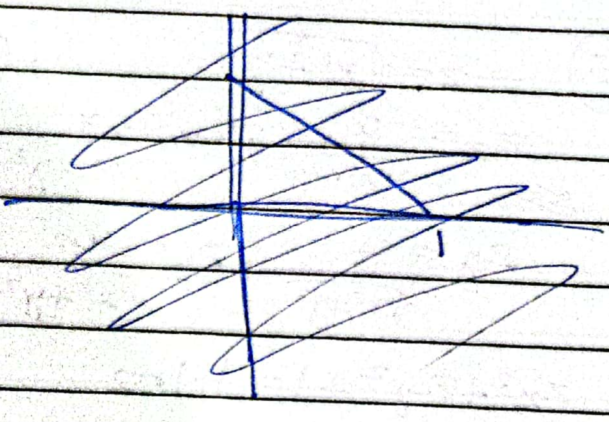
Q4

$z = 2x + y^2$

$z_x = 2, \quad z_x^2 = 4$

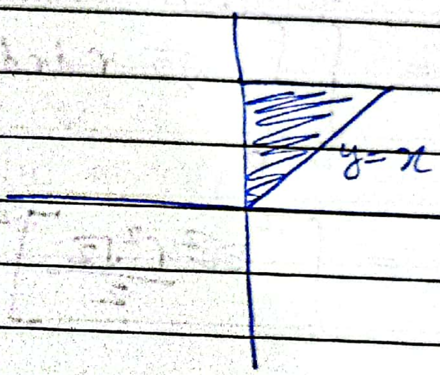
$z_y = 2y, \quad z_y^2 = 4y^2$

$z_x^2 + z_y^2 + 1 = 5 + 4y^2$



$$S.A = \int_0^1 \int_0^y \sqrt{5+4y^2}\, dx\, dy$$

$$= \int_0^1 y\sqrt{5+4y^2}\, dy \qquad u = 5+4y^2, \quad \frac{du}{8} = y\, dy$$



$$= \frac{1}{8}\int_5^9 \sqrt{u}\, du$$

$$= \frac{1}{8}\left(\frac{2}{3} u^{3/2}\right)\Big]_5^9$$

$$= \frac{1}{12}\left[u^{3/2}\right]_5^9$$

$$\boxed{S.A = \frac{27 - 5\sqrt{5}}{12}}$$

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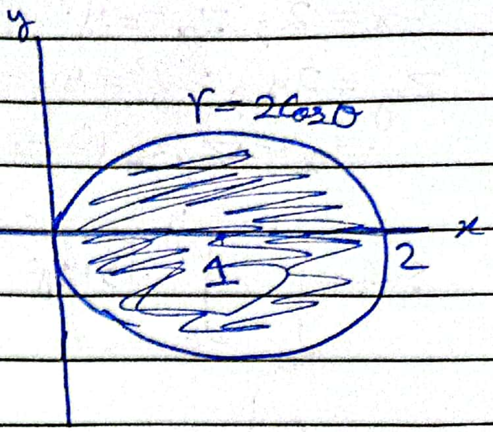
(5) $z=\sqrt{x^2+y^2}$, $R=\{x^2+y^2=2x\}$

$z_x = \frac{x}{\sqrt{x^2+y^2}}$, $z_x^2 = \frac{x^2}{x^2+y^2}$

$z_y = \frac{y}{\sqrt{x^2+y^2}}$, $z_y^2 = \frac{y^2}{x^2+y^2}$



$z_x^2+z_y^2+1 = 1+1 = 2$

$$S = \int_0^{\pi}\int_0^{2\cos\theta} \sqrt{2}\, r\,dr\,d\theta$$

$$= \int_0^{\pi} \left[\frac{r^2\sqrt{2}}{2}\right]_0^{2\cos\theta} d\theta$$

$$= \int_0^{\pi} 2\sqrt{2}\cos^2\theta \, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi} \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \sqrt{2}\int_0^{\pi} 1+\cos 2\theta \, d\theta$$

$$= \sqrt{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$\boxed{= \sqrt{2}\,\pi}$$

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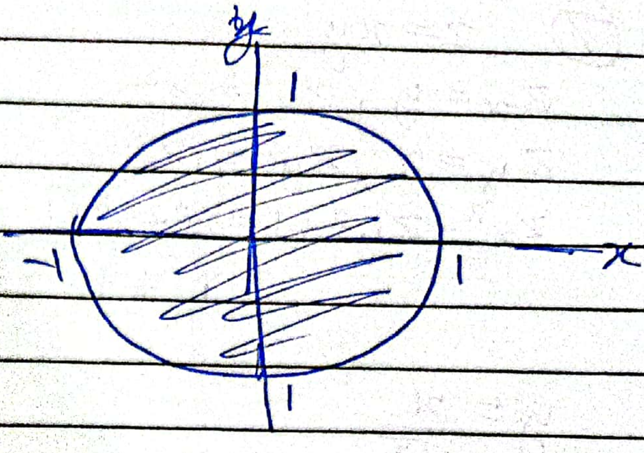
Q6)

$$z=0$$

$$z = 1 - x^2 - y^2 \quad , \quad \cancel{z} \; x^2+y^2=1$$

$$z_x = \cancel{z} -2x \quad , \quad z_x^2 = 4x^2$$

$$z_y = -2y \quad , \quad z_y^2 = 4y^2$$



$$z_x^2+z_y^2+1 = 4x^2+4y^2+1$$

$$= 4r^2+1$$

$$S.A = \int_0^{2\pi}\int_0^{1} \sqrt{4r^2+1}\; r\,dr\,d\theta$$

$$u = 4r^2+1$$

$$\frac{du}{8} = r\,dr$$

$$= \frac{1}{8}\int_0^{2\pi}\int_1^{5} \sqrt{u}\; du\,d\theta$$

$$= \frac{1}{8}\int_0^{2\pi} \frac{2}{3}u^{3/2}\Big]_1^{5}\, d\theta$$

$$= \frac{1}{12}\int_0^{2\pi} 5\sqrt{5}-1\, d\theta$$

$$= \frac{5\sqrt{5}-1}{12}\,\theta\Big]_0^{2\pi}$$

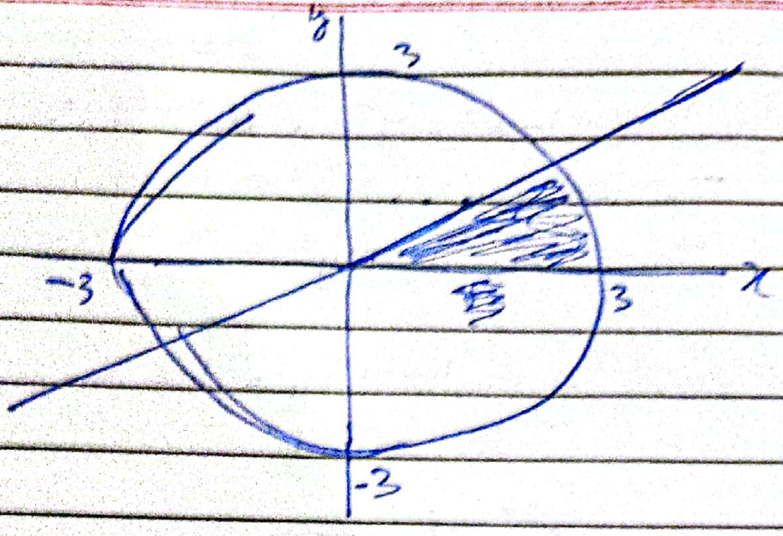
$$\boxed{= \frac{5\sqrt{5}-1}{6}\pi}$$

7) $z = xy$

$z_x = y \;,\; z_x^2 = y^2$

$z_y = x \;,\; z_y^2 = x^2$

$z_x^2 + z_y^2 + 1 = x^2 + y^2 + 1$
$= r^2 + 1$



$y = \frac{x}{\sqrt{3}}$

$r\sin\theta = \frac{r\cos\theta}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \pi/6$

$$S = \int_0^{\pi/6} \int_0^3 r\sqrt{r^2+1}\, dr\, d\theta$$

$u = r^2 + 1$

$\frac{du}{2} = r\,dr$

$$= \frac{1}{2}\int_0^{\pi/6} \int_1^{10} \sqrt{u}\, du\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/6} \frac{2}{3} u^{3/2} \Big]_1^{10} d\theta$$

$$= \frac{1}{3}\int_0^{\pi/6} 10\sqrt{10} - 1\, d\theta$$

$$= \frac{10\sqrt{10} - 1}{3} \theta \Big]_0^{\pi/6}$$

$$\boxed{= \frac{10\sqrt{10} - 1}{18} \pi}$$

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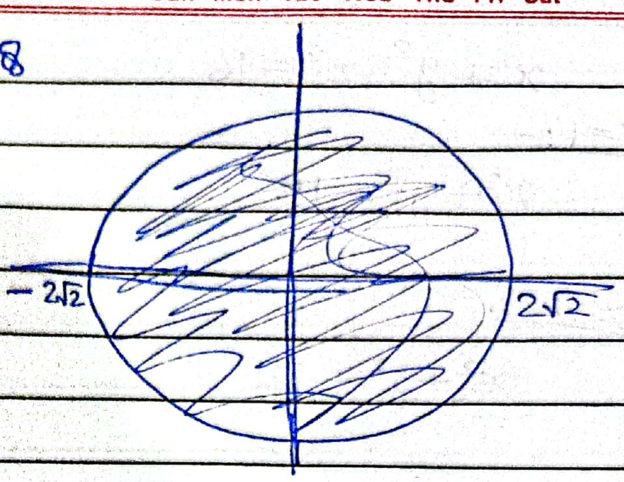
(8) $2z = x^2+y^2$ $\qquad x^2+y^2=8$

$z = \frac{x^2+y^2}{2}$



$z_x = x, \quad z_x^2 = x^2$

$z_y = y, \quad z_y^2 = y^2$

$z_x^2 + z_y^2 + 1 = x^2+y^2+1$

$= r^2+1$

$$S = \int_0^{2\pi}\int_0^{2\sqrt{2}} r\sqrt{r^2+1}\, dr\, d\theta \qquad u = r^2+1, \quad \frac{du}{2} = r\,dr$$

$$= \frac{1}{2}\int_0^{2\pi}\int_1^{9} \sqrt{u}\, du\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{2}{3}u^{3/2}\Big]_1^{8}$$

$$= \frac{1}{3}\int_0^{2\pi} 27 - 1\, d\theta$$

$$= \frac{26}{3}\theta\Big]_0^{2\pi}$$

$$\boxed{= \frac{52}{3}\pi}$$

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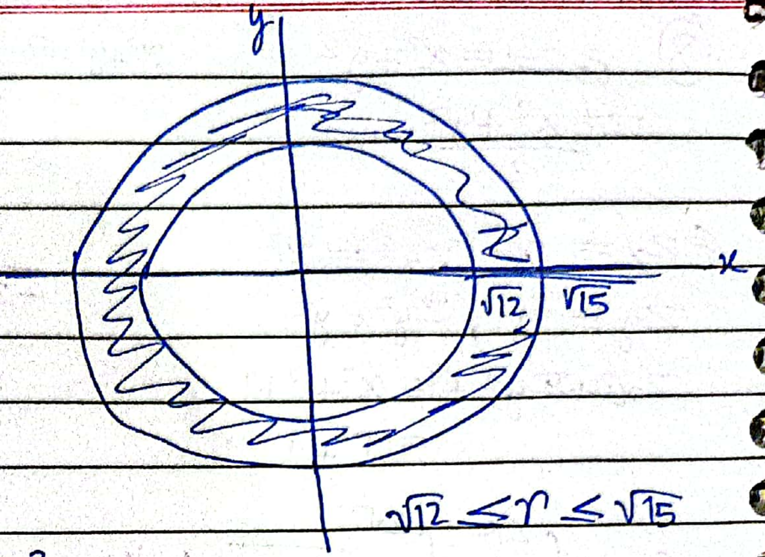
(9) $x^2+y^2+z^2=16$

$z=1$:-

$x^2+y^2=15$

$z=2$:-

$x^2+y^2=12$



$\sqrt{12}\le r\le\sqrt{15}$

$z^2=16-x^2-y^2$

$z_x=\frac{-x}{\sqrt{16-x^2-y^2}}$, $z_x^2=\frac{x^2}{16-x^2-y^2}$

$z_y=\frac{-y}{\sqrt{16-x^2-y^2}}$, $z_y^2=\frac{y^2}{16-x^2-y^2}$

$z_x^2+z_y^2+1=\frac{16}{16-x^2-y^2}=\frac{16}{16-r^2}$

$S=\int_0^{2\pi}\int_{\sqrt{12}}^{\sqrt{15}}\sqrt{\frac{16}{16-r^2}}\,r\,dr\,d\theta$

$u=16-r^2$

$-\frac{du}{2}=r\,dr$

$=\int_0^{2\pi}\int_{\sqrt{12}}^{\sqrt{15}}\frac{4r}{\sqrt{16-r^2}}\,dr\,d\theta$

$=\int_0^{2\pi}-\frac{1}{2}\int_4^1\frac{4}{\sqrt{u}}\,du\,d\theta$

$=\frac{2}{\cancel{4}}\int_0^{2\pi}\int_1^4\frac{1}{\sqrt{u}}\,du\,d\theta$

$=\frac{2}{\cancel{2}}\int_0^{2\pi}\left[2u^{1/2}\right]_1^4$

$=4\int_0^{2\pi}2-1\,d\theta$

$\boxed{=8\pi}$

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Q10 $x^2+y^2+z^2=8$

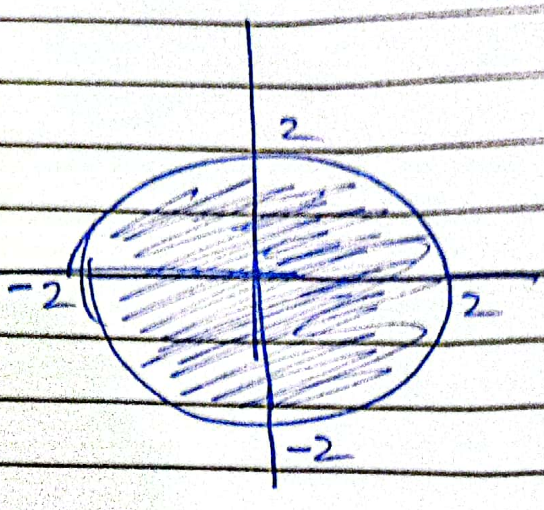
$z=\sqrt{x^2+y^2}$

$z^2=x^2+y^2$

$x^2+y^2+x^2+y^2=8$

$2(x^2+y^2)=8$

$x^2+y^2=4$



$\cancel{z_x=\sqrt{}}$

$z_x=\dfrac{-x}{\sqrt{8-x^2-y^2}}$, $z_x^2=\dfrac{x^2}{8-x^2-y^2}$

$z_y=\dfrac{-y}{\sqrt{8-x^2-y^2}}$, $z_y^2=\dfrac{y^2}{8-x^2-y^2}$

$z_x^2+z_y^2+1=\dfrac{8}{8-x^2-y^2}=\dfrac{8}{8-r^2}$

$S=\int_0^{2\pi}\int_0^{2}\dfrac{2\sqrt{2}\,r}{\sqrt{8-x^2}}\,dr\,d\theta$ $u=8-r^2$

$-\dfrac{du}{2}=r\,dr$

$=\int_0^{2\pi}-\dfrac{1}{2}\int_8^{4}\dfrac{2\sqrt{2}}{\sqrt{u}}\,du\,d\theta$

$=\sqrt{2}\int_0^{2\pi} 2\sqrt{u}\Big]_4^8$

$=2\sqrt{2}\int_0^{2\pi} 2\sqrt{2}-2\ d\theta$

$=\cancel{8}\,(8-4\sqrt{2})\theta\Big]_0^{2\pi}$

$\boxed{=\cancel{2}(16-8\sqrt{2})\pi}$