Ex 4.1

Date:

Q5) axiom ~~4~~ 5 will fail, as ~~for~~ $-u$ vector only exists when $x=0$ and as such there is no other vector other than $(0,y)$ that holds axiom 5.

axiom 6 also fails as scalar multiplication is the standard ~~op~~ operation in R^2 so for any $k<0$ it will not hold ~~as~~ unless $x=0$.

Q9) axiom 1:- $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix} = \begin{bmatrix} a+a' & 0 \\ 0 & b+b' \end{bmatrix}$

The sum of ~~th~~ 2 diagonal matrices is also diagonal

axiom 2:-

Since axiom ~~as~~ 1 holds ~~for~~, this one will also hold as the ~~ad axiom~~ axiom 2 holds for all ~~mat~~ matrices in R_2 with standard addition.

axiom 3:-

The same reason as axiom 2, since addition is the standard one in R_2 it follows that this property will hold for all matrices in R_2.

axiom 4:-

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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axiom 5:-
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} a' & 0 \\ 0 & b' \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a+a' & 0 \\ 0 & b+b' \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$a' = -a \ , \ b' = -b$$

$$-u = \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix}$$

axiom 6:- The scalar product of a diagonal matrix is a diagonal matrix.

axiom 7:- follows from the properties of matrices in R_2

axiom 8:- same reason as in axiom 7, it holds.

axiom 9:- same reason as ~~the~~ axiom 7 and 8.

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for 6, 7, 8, 9

6) $k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix}$

7) $k\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}\right) = k\begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} = \begin{bmatrix} k(a+c) & 0 \\ 0 & k(b+d) \end{bmatrix}$

$\begin{bmatrix} ka & 0 \\ 0 & kb \end{bmatrix} + \begin{bmatrix} kc & 0 \\ 0 & kd \end{bmatrix} = \begin{bmatrix} k(a+c) & 0 \\ 0 & k(b+d) \end{bmatrix}$

8) $(k+m)u = ku + mu$

$(k+m)\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} (k+m)a & 0 \\ 0 & (k+m)b \end{bmatrix}$

$k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + m\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} (k+m)a & 0 \\ 0 & (k+m)b \end{bmatrix}$

axiom 10 :-

$1\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Date:

Q11) $(1, y) + (1, y') = (1, y+y')$

$k(1, y) = (1, ky)$

axiom 1 :-

$(1, x) + (1, y) = (1, x+y)$

~~is true~~ holds

axiom 2 :-

$(1, x) + (1, y) = (1, y) + (1, x)$

$(1, x+y) = (1, x+y)$

Holds

axiom 3 :-

$(1, x) + [(1, y) + (1, z)] = [(1, x) + (1, y)] + (1, z)$

$(1, x) + (1, y+z) = (1, x+y) + (1, z)$

$(1, x+y+z) = (1, x+y+z)$

axiom 4 :- $(1, x) + (1, x') = (1, x)$

$(1, x+x') = (1, x)$

$x + x' = x$

$x' = 0$

$0 = (1, 0)$

Date:

axiom 5

$$-u = (1, y)$$

$$u + (-u) = 0$$

$$(1, x) + (1, y) = (1, 0)$$

$$(1, x+y) = (1, 0)$$

$$x + y = 0$$

$$y = -x$$

$$u = (1, x) \quad ; \quad -u = (1, -x)$$

axiom 6

$$k(1, y) = (1, ky)$$

holds

axiom 7 :-

$$k[(1, y) + (1, x)] = k(1, x) + k(1, y)$$

$$k(1, x+y) = (1, kx) + (1, ky)$$

$$(1, k(x+y)) = (1, k(x+y))$$

holds

axiom 8 :-

$$(k+m)u = ku + mu$$

$$(k+m)(1, x) = k(1, x) + m(1, x)$$

$$(1, (k+m)x) = (1, kx) + (1, mx)$$

$$(1, (k+m)x) = (1, (k+m)x)$$

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axiom 9:-

$$k(mu) = (km)u$$

$$k(m(1,x)) = (km)(1,x)$$

$$k(1, mx) = (1, (km)x)$$

$$(1, (km)x) = (1, (km)x)$$

holds.

axiom 10:-

$$1(1, x) = (1, x)$$

holds

Date:

Ex 4.2

Q4 (a) axiom 1:-

$$(A+B)^T = (A^T + B^T) = -A-B = \cancel{} -(A+B)$$

It holds

axiom 6:-

$$\cancel{}\ (kA)^T = kA^T = k(-A) = -(kA)$$

it holds

☆ It is a subspace

~~Q~~ b) $Ax = 0$ has only trivial solution

for there to be only the trivial solution, ~~A~~ $\det(A) \neq 0$

which means axiom 6 fails when $k=0$ as $0\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$

$$= \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$ for which $\det = 0$.

☆ It is not a subspace.

Date:

(c) $AB = BA$

axiom 1 :- assuming A and C are in vector space.

$$AB + CB = BA + BC$$
$$(A+C)B = B(A+C)$$

so it hold under addition.

axiom 6 :-

$$k(AB) = k(BA)$$
$$(kA)B = B(kA)$$

so it holds under scalar multiplication

★ It is a subspace

(d) for all invertible matrices in $n \times n$, it is not closed under ~~addition~~ scalar multiplication as for them to be invertible, $\det(A) \neq 0$ and if A is multiplied by 0 the resulting matrice ~~does~~ has $\det = 0$.

★ It is not subspace.

Date:

Q5) $P_3 = a_0 + a_1x + a_2x^2 + a_3x^3$

(a) $(0 + a_1x + a_2x^2 + a_3x^3) + (0 + b_1x + b_2x^2 + b_3x^3)$

$= 0 + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3$

axiom 1 holds

$k(0 + a_1x + a_2x^2 + a_3x^3)$

$= 0 + ka_1x + ka_2x^2 + ka_3x^3$

axiom 6 hold.

☆ It is subspace of P_3.

(b) $a_0 + a_1 + a_2 + a_3 = 0$

$a_0 = -a_1 - a_2 - a_3$

$W = -a_1 - a_2 - a_3 + a_1x + a_2x^2 + a_3x^3$

$(-a_1 - a_2 - a_3 + a_1x + a_2x^2 + a_3x^3) + (-b_1 - b_2 - b_3 + b_1x + b_2x^2 + b_3x^3)$

$= -(a_1+b_1) - (a_2+b_2) - (a_3+b_3) + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3$

This belongs in W so it is closed under axiom 1.

$k(-a_1 - a_2 - a_3 + a_1x + a_2x^2 + a_3x^3) = -ka_1 - ka_2 - ka_3 + ka_1x + ka_2x^2 + ka_3x^3$

This belong in W so it is closed under axiom 6.

☆ This is a subspace of P_3

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Ex 4.3

Q6) $P_1 = 2 + x + 4x^2$, $P_2 = 1 - x + 3x^2$, $P_3 = 3 + 2x + 5x^2$

$aP_1 + bP_2 + cP_3 = -9 - 7x - 15x^2$

$(2a + ax + 4ax^2) + (b - bx + 3bx^2) + (3c + 2cx + 5cx^2)$

$= (2a + b + 3c) + (a - b + 2c)x + (4a + 3b + 5c)x^2$

$2a + b + 3c = -9$

$a - b + 2c = -7$

$4a + 3b + 5c = -15$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & -9 \\ 1 & -1 & 2 & -7 \\ 4 & 3 & 5 & -15 \end{array}\right]$$

$$R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 2 & 1 & 3 & -9 \\ 4 & 3 & 5 & -15 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 0 & 3 & -1 & 5 \\ 0 & 7 & -3 & 13 \end{array}\right]$$

$$\boxed{-9 - 7x - 15x^2 = -2P_1 + P_2 - 2P_3}$$

$$\begin{array}{c} \\ \frac{1}{3}R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 0 & 1 & -1/3 & 5/3 \\ 0 & 7 & -3 & 13 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - 7R_2 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 0 & 1 & -1/3 & 5/3 \\ 0 & 0 & -2/3 & 4/3 \end{array}\right]$$

$$\begin{array}{c} \\ \\ -\frac{3}{2}R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 0 & 1 & -1/3 & 5/3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\begin{array}{c} R_1 - 2R_3 \\ R_2 + \frac{1}{3}R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\begin{array}{c} R_1 + R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

(b) $\left[\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{array}\right]$

$$\begin{array}{c} \\ R_1 \leftrightarrow R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 2 & 1 & 3 & 6 \\ 4 & 3 & 5 & 6 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 7 & -3 & -38 \end{array}\right]$$

$$\begin{array}{c} \\ \frac{1}{3}R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 1 & -1/3 & -\frac{16}{3} \\ 0 & 7 & -3 & -38 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - 7R_2 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 0 & -2/3 & -2/3 \end{array}\right]$$

$$\begin{array}{c} \\ \\ -\frac{3}{2}R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{array}{c} R_1 - 2R_3 \\ R_2 + \frac{1}{3}R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 9 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{array}{c} R_1 + R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$6 + 11x + 6x^2 = 4p_1 - 5p_2 + p_3$$

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Date:

(c) $0 = 0P_1 + 0P_2 + 0P_3$

(d) $\left[\begin{array}{ccc|c} 2 & 1 & 3 & 7 \\ 1 & -1 & 2 & 8 \\ 4 & 3 & 5 & 9 \end{array}\right]$

$R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|c} 1 & -1 & 2 & 8 \\ 2 & 1 & 3 & 7 \\ 4 & 3 & 5 & 9 \end{array}\right]$

$\begin{array}{c} \\ R_2-2R_1 \\ R_3-4R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 8 \\ 0 & 3 & -1 & -9 \\ 0 & 7 & -3 & -23 \end{array}\right]$

$\frac{1}{3}R_2 \left[\begin{array}{ccc|c} 1 & -1 & 2 & 8 \\ 0 & 1 & -1/3 & -3 \\ 0 & 7 & -3 & -23 \end{array}\right]$

$\begin{array}{c} \\ \\ R_3-7R_2 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 8 \\ 0 & 1 & -1/3 & -3 \\ 0 & 0 & -\frac{2}{3} & -2 \end{array}\right]$

$\begin{array}{c} \\ \\ \frac{-3}{2}R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 8 \\ 0 & 1 & -1/3 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$\begin{array}{c} R_1-2R_3 \\ R_2+\frac{1}{3}R_3 \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$R_1+R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$7+8x+9x^2 = 0P_1 - 2P_2 + 3P_3$

Date:

Q8) $V_1 = (2,1,0,3)$, $V_2 = (3,-1,5,2)$, $V_3 = (-1,0,2,1)$

(a)

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 2 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{array}\right]$$

$$\begin{array}{r} R_1 \times \frac{1}{2} \\ \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{array}\right]$$

$$\begin{array}{r} \\ R_2 - R_1 \\ \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1 \\ 0 & -5/2 & 1/2 & 2 \\ 0 & 5 & 2 & -7 \\ 0 & -5/2 & 5/2 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ -\frac{2}{5} R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 5 & 2 & -7 \\ 0 & -5/2 & 5/2 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ \\ R_3 - 5R_2 \\ R_4 + \frac{5}{2} R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 2 & -2 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{3} R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -2 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \\ R_4 - 2R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1 \\ 0 & 1 & -1/5 & -4/5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} R_1 + \frac{1}{2} R_3 \\ R_2 + \frac{1}{5} R_3 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & 0 & 1/2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} R_1 - \frac{3}{2} R_2 \\ \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

☆ The system is consistent so $(2,3,-7,3)$ is in ~~Span~~ $\mathrm{Span}(V_1, V_2, V_3)$

Date:

(b) $(0,0,0,0)$

Spans (V_1, V_2, V_3) as $0V_1 + 0V_2 + 0V_3 = (0,0,0,0)$

(c) $(1,1,1,1)$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 5 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{array}\right]$$

$$\begin{array}{c} \frac{1}{2}R_1 \\ \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1/2 \\ 1 & -1 & 0 & 1 \\ 0 & 5 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - R_1 \\ \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1/2 \\ 0 & -5/2 & 1/2 & 1/2 \\ 0 & 5 & 2 & 1 \\ 0 & -\frac{5}{2} & 5/2 & -1/2 \end{array}\right]$$

$$\begin{array}{c} \\ -\frac{2}{5}R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1/2 \\ 0 & 1 & -1/5 & -1/5 \\ 0 & 5 & 2 & 1 \\ 0 & -5/2 & 5/2 & -1/2 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - 5R_2 \\ R_4 + \frac{5}{2}R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1/2 \\ 0 & 1 & -1/5 & -1/5 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 2 & -1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ \frac{1}{3}R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & 1/2 \\ 0 & 1 & -1/5 & -1/5 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 2 & -1 \end{array}\right]$$

$$\begin{array}{c} R_1 + \frac{1}{2}R_3 \\ R_2 + \frac{1}{5}R_3 \\ \\ R_4 - 2R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & -7/3 \end{array}\right]$$

☆ The system is inconsistent so $(1,1,1,1)$ does not ~~span~~ span (V_1, V_2, V_3).

(d) $(-4, 6, -13, 4)$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & -4 \\ 1 & -1 & 0 & 6 \\ 0 & 5 & 2 & -13 \\ 3 & 2 & 1 & 4 \end{array}\right]$$

$$\begin{array}{c} \frac{1}{2}R_1 \\ \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & -2 \\ 1 & -1 & 0 & 6 \\ 0 & 5 & 2 & -13 \\ 3 & 2 & 1 & 4 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - R_1 \\ \\ R_4 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & -2 \\ 0 & -5/2 & 1/2 & 8 \\ 0 & 5 & 2 & -13 \\ 0 & -\frac{5}{2} & 5/2 & 10 \end{array}\right]$$

$$\begin{array}{c} \\ -\frac{2}{5}R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & -2 \\ 0 & 1 & -\frac{1}{5} & -\frac{16}{5} \\ 0 & 5 & 2 & -13 \\ 0 & -5/2 & 5/2 & 10 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - 5R_2 \\ R_4 + \frac{5}{2}R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & -2 \\ 0 & 1 & -1/5 & -16/5 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

$$\begin{array}{c} \\ \\ \frac{1}{3}R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & -1/2 & -2 \\ 0 & 1 & -1/5 & -16/5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

$$\begin{array}{c} R_1 + \frac{1}{2}R_3 \\ R_2 + \frac{1}{5}R_3 \\ \\ R_4 - 2R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 3/2 & 0 & -3/2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} R_1 - \frac{3}{2}R_2 \\ \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

* The system is consistent so $(-4, 6, -13, 4)$ in span(v_1, v_2, v_3).

Date:

Q18 $u_1 = (0,1,1)$, $u_2 = (2,-1,1)$, $u_3 = (1,1,-2)$

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

$T_A(u_1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = (1, 0)$

$T_A(u_2) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = (1, -2)$

$T_A(u_3) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = (2, 3)$

$a(1,0) + b(1,-2) + c(2,3) = (b_1, b_2)$

$a + b + 2c = b_1$

$-2b + 3c = b_2$

$\left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 0 & -2 & 3 & b_2 \end{array}\right]$

$-\frac{1}{2}R_2 \left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 0 & 1 & -3/2 & -\frac{b_2}{2} \end{array}\right]$

$R_1 - R_2 \left[\begin{array}{ccc|c} 1 & 0 & 7/2 & b_1 \\ 0 & 1 & -3/2 & -\frac{b_2}{2} \end{array}\right]$

* The system is consistent for all vectors (b_1, b_2) so it is spanned.

Date:

b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix}$

$T(u_1) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = (1, -2)$

$T(u_2) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = (-1, \overset{-2}{\cancel{}})$

$T(u_3) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} = (1, 8)$

$\cancel{\begin{bmatrix} 1 & -1 & 1 \\ -2 & -2 & 8 \end{bmatrix}}$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & b_1 \\ -2 & -2 & 8 & b_2 \end{array}\right]$

$R_2 + 2R_1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & b_1 \\ 0 & -4 & 10 & b_2 + 2b_1 \end{array}\right]$

$-\frac{1}{4} R_2 \left[\begin{array}{ccc|c} 1 & -1 & 1 & b_1 \\ 0 & 1 & \cancel{}\ \frac{-5}{2} & \frac{b_2 + 2b_1}{-4} \end{array}\right]$

$R_1 + R_2 \left[\begin{array}{ccc|c} 1 & 0 & -3/2 & b_1 - \frac{b_2 + 2b_1}{4} \\ 0 & 1 & -5/2 & -\frac{b_2 + 2b_1}{4} \end{array}\right]$

* The system is consistent for all choices (b_1, b_2)

Ex 4.4

Q6) $\begin{bmatrix}1 & 0\\1 & k\end{bmatrix}, \begin{bmatrix}-1 & 0\\k & 1\end{bmatrix}, \begin{bmatrix}2 & 0\\1 & 3\end{bmatrix}$

$$a\begin{bmatrix}1 & 0\\1 & k\end{bmatrix}+b\begin{bmatrix}-1 & 0\\k & 1\end{bmatrix}+c\begin{bmatrix}2 & 0\\1 & 3\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$a - b + 2c = 0$

$a + kb + c = 0$

$ka + b + 3c = 0$

$$\left[\begin{array}{ccc|c}1 & -1 & 2 & 0\\1 & k & 1 & 0\\k & 1 & 3 & 0\end{array}\right]$$

$$\begin{array}{l}\\R_2-R_1\\R_3-kR_1\end{array}\left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\0 & k+1 & \cancel{\ }-1 & 0\\0 & 1+k & 3-2k & 0\end{array}\right]$$

$$\begin{array}{l}\\\\R_3-R_2\end{array}\left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\0 & 1+k & -1 & 0\\0 & 0 & 4-2k & 0\end{array}\right]$$

☆ det $\neq 0$ for it to be linearly independent.

so $(1+k)(4-2k) \neq 0$

so The matrices are ~~all~~ linearly independent for all values of k except $(k=-1, k=2)$.

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Q11 $v_1 = (\lambda, -\frac{1}{2}, -\frac{1}{2})$, $v_2 = (-\frac{1}{2}, \lambda, -\frac{1}{2})$, $v_3 = (-\frac{1}{2}, -\frac{1}{2}, \lambda)$

$$a(\lambda, -\tfrac{1}{2}, -\tfrac{1}{2}) + b(-\tfrac{1}{2}, \lambda, -\tfrac{1}{2}) + c(-\tfrac{1}{2}, -\tfrac{1}{2}, \lambda) = (0,0,0)$$

$$\begin{bmatrix} \lambda & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \lambda & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \lambda \end{bmatrix}$$

$$\begin{aligned} \det &= \lambda\left(\lambda^2 - \tfrac{1}{4}\right) + \tfrac{1}{2}\left(-\tfrac{1}{2}\lambda - \tfrac{1}{4}\right) - \tfrac{1}{2}\left(\tfrac{1}{4} + \tfrac{1}{2}\lambda\right) \\ &= \lambda^3 - \tfrac{1}{4}\lambda - \tfrac{1}{2}\lambda - \tfrac{1}{4} \\ &= \lambda^3 - \tfrac{3}{4}\lambda - \tfrac{1}{4} \end{aligned}$$

for dependency det $= 0$

so $\lambda^3 - \frac{3}{4}\lambda - \frac{1}{4} = 0$

$\lambda = -\frac{1}{2}$ is a solution because then all rows would be same.

$$\begin{array}{r|l} & \lambda^2 - \tfrac{1}{2}\lambda - \tfrac{1}{2} \\ \lambda + \tfrac{1}{2} & \lambda^3 - \tfrac{3}{4}\lambda - \tfrac{1}{4} \\ & \lambda^3 + \tfrac{1}{2}\lambda^2 \\ \hline & -\tfrac{1}{2}\lambda^2 - \tfrac{3}{4}\lambda - \tfrac{1}{4} \\ & -\tfrac{1}{2}\lambda^2 - \tfrac{1}{4}\lambda \\ \hline & -\tfrac{1}{2}\lambda - \tfrac{1}{4} \\ & -\tfrac{1}{2}\lambda - \tfrac{1}{4} \\ \hline & 0 \end{array}$$

$(\lambda + \frac{1}{2})(\lambda^2 - \frac{1}{2}\lambda - \frac{1}{2})$

$(\lambda + \frac{1}{2})(\lambda + \frac{1}{2})(\lambda - 1)$

So it is dependent for $\lambda = 1$ and $\lambda = -\frac{1}{2}$

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Q13) $u_1 = (1,2)$, $u_2 = (-1,1)$

a) $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

$T(u_1) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = (-1, 4)$

$T(u_2) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = (-2, 2)$

$-a - 2b = 0$
$4a + 2b = 0$

$\det = \begin{vmatrix} -1 & -2 \\ 4 & 2 \end{vmatrix} = -2 + 8 = 6 \neq 0$

☆ since $\det \neq 0$ so this means only the trivial solution exists so they only form a linearly independent set.

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b) $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

$T(u_1) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = (-1, 2)$

$T(u_2) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = (-2, 4)$

since $T(u_2) = 2(T(u_1))$ it is a linearly dependent set.

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Ex 4.5

Q15) $A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$; $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$k_1 A_1 + k_2 A_2 + k_3 A_3 + k_4 A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$k_1 = 0$

$k_1 + k_2 = 0$

$k_1 + k_2 + k_3 = 0$

$k_1 + k_2 + k_3 + k_4 = 0$

$k_1 A_1 + k_2 A_2 + k_3 A_3 + k_4 A_4 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$k_1 = a$

$k_1 + k_2 = b$

$k_1 + k_2 + k_3 = c$

$k_1 + k_2 + k_3 + k_4 = d$

1

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & a \\ 1 & 1 & 0 & 0 & b \\ 1 & 1 & 1 & 0 & c \\ 1 & 1 & 1 & 1 & d \end{array}\right]$$

☆ Since the coefficient matrix det $\neq 0$, then the 1 system has only the trivial solution and 2 system is consistent for all values of a, b, c, d.

$$k_1 A_1 + k_2 A_2 + k_3 A_3 + k_4 A_4 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$k_1 = 1$$
$$k_1 + k_2 = 0$$
$$k_1 + k_2 + k_3 = 1$$
$$k_1 + k_2 + k_3 + k_4 = 0$$

$$\begin{cases} k_1 = 1 \\ k_2 = -1 \\ k_3 = 1 \\ k_4 = -1 \end{cases}$$

$A = A_1 - A_2 + A_3 - A_4$

$(A)_s = (1, -1, 1, -1)$

Date:

Q18) $P_1 = 1+2x+x^2$, $P_2 = \cancel{2+x}\ 2+9x$, $P_3 = 3+3x+4x^2$

$P = 2+17x-3x^2$

$a P_1 + b P_2 + c P_3 = 0$

$a P_1 + b P_2 + c P_3 = c_0 + c_1 x + c_2 x^2$

$(a+2b+3c) + (2a+9b+3c)x + (a+4c)x^2 = 0, P$

$a + 2b + 3c = 0$, c_0

$2a + 9b + 3c = 0$, c_1

$a + 0 + 4c = 0$, c_2

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{bmatrix}\right) = 36 - 10 - 27 = -1 \neq 0$$

☆ since $\det \neq 0$ then it is linearly ~~independently~~ independent and also spans P_2 as such is a basis for P_2.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 2 & 9 & 3 & 17 \\ 1 & 0 & 4 & -3 \end{array}\right]$$

$$\begin{array}{c} \\ R_2-2R_1 \\ R_3-R_1 \end{array}\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 5 & -3 & 13 \\ 0 & -2 & 1 & -5 \end{array}\right]$$

$$\begin{array}{c} \\ \frac{1}{5}R_2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -3/5 & 13/5 \\ 0 & -2 & 1 & -5 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3+2R_2 \end{array}\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -3/5 & 13/5 \\ 0 & 0 & -1/5 & 1/5 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -3/5 & 13/5 \\ 0 & 0 & -1/5 & 1/5 \end{array}\right]$$

$$-5R_3 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -3/5 & 13/5 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\begin{array}{l} R_1-3R_3 \\ R_2+\frac{3}{5}R_3 \\ \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$R_1-2R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$P = P_1 + 2P_2 - P_3$

$(P)_S = (1, 2, -1)$

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Ex 4.6 (6, 18, 19c)

Q6) $x+y+z=0$

$3x+2y-2z=0$

$4x+3y-z=0$

$6x+5y+z=0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 3 & 2 & -2 & 0 \\ 4 & 3 & -1 & 0 \\ 6 & 5 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2-3R_1 \\ R_3-4R_1 \\ R_4-6R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ \\ R_3-R_2 \\ R_4-R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$z=t$

$y=-5t$

$x=4t$

$$(x,y,z)=t(4,-5,1)$$

The solution space is spanned by $(4,-5,1)$. This vector is linearly independent and this also means it is a basis for the solution space. and it's dimension is 1.

Date:

Q19) c)

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{c} \\ R_2+R_1 \\ R_3-R_1 \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{c} \\ \\ R_3-R_2 \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_1 = 0$, $x_2 = 0$, $x_3 = 0$

The zero vector is the only basis of the solution space, so it is a 0 vector space so

Dim = 0 .

Date:

Ex 4.8 (7b, 13, 26)

Q7 (b)

$x_1 + x_2 + 2x_3 = 5$

$x_1 + x_3 = -2$

$2x_1 + x_2 + 3x_3 = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 1 & 0 & 1 & -2 \\ 2 & 1 & 3 & 3 \end{array}\right]$$

$$\begin{array}{c} \\ R_2 - R_1 \\ R_3 - 2R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & -1 & -1 & -7 \\ 0 & -1 & -1 & -7 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & -1 & -1 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ -1R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3 = t$

$x_2 = 7 - t$

$x_1 = -2 - t$

$(x_1, x_2, x_3) = (\overset{-2-t}{\cancel{t}}, 7-t, t)$

$= (-2, 7, 0) + t(-1, -1, 1)$

★ The solution vector form is

$(-2, 7, 0) + t(-1, -1, 1)$

Date:

Q13 (a) $A = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$

$$\begin{array}{c} \\ R_2+2R_1 \\ R_3+R_1 \\ R_4+3R_1 \end{array} \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 2 & 6 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{c} \\ \\ R_3-R_2 \\ R_4-2R_2 \end{array} \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{c} \\ \\ \\ R_4-R_3 \end{array} \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c} R_1+2R_2 \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 11 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

basis of row space A

$r_1 = [1 \; 0 \; 11 \; 0 \; 3]$

$r_2 = [0 \; 1 \; 3 \; 0 \; 0]$

$r_3 = [0 \; 0 \; 0 \; 1 \; 0]$

basis of column space A

$c_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ -3 \end{bmatrix}, c_2 = \begin{bmatrix} -2 \\ 5 \\ 3 \\ 8 \end{bmatrix}$

$c_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

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(b) $A^T = \begin{bmatrix} 1 & -2 & -1 & -3 \\ -2 & 5 & 3 & 8 \\ 5 & -7 & -2 & -9 \\ 0 & 0 & 1 & 1 \\ 3 & -6 & -3 & -9 \end{bmatrix}$

~~RREF~~

$\begin{array}{l} \\ R_2+2R_1 \\ R_3-5R_1 \\ \\ R_5-3R_1 \end{array} \begin{bmatrix} 1 & -2 & -1 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 3 & 3 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{array}{l} \\ \\ R_3-3R_2 \\ \\ \\ \end{array} \begin{bmatrix} 1 & -2 & -1 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{array}{l} \\ \\ \\ R_4 \leftrightarrow R_3 \\ \\ \end{array} \begin{bmatrix} 1 & -2 & -1 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

R_1+R_3
R_2-R_3

$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

R_1+2R_2

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Basis for column space of A^T are C_1, C_2, C_3 and as rows of A are columns in A^T, The row space basis of A are

$r_1 = [1 \; -2 \; 5 \; 0 \; 3]$

$r_2 = [-2 \; 5 \; -7 \; 0 \; 6]$

$r_3 = [-1 \; 3 \; -2 \; 1 \; -3]$

Date:

Q26 $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 1 & -7 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(a)
$$\begin{array}{r} \\ R_2-2R_1 \\ R_3-R_1 \end{array} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 5 & -2 \\ 0 & -5 & 2 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ R_3+R_2 \end{array} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 5 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{5}R_2 \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -2/5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1+2R_2 \left[\begin{array}{ccc|c} 1 & 0 & 11/5 & 0 \\ 0 & 1 & -2/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3 = t$, $x_2 = \frac{2}{5}t$, $x_1 = -\frac{11}{5}t$

Date:

(b) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 1 & -7 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ -1 \end{bmatrix}$

$x_1 = x_2 = x_3 = 1$ is a solution

(c) $(x_1, x_2, x_3) = (1, 1, 1) + t\left(-\frac{11}{5}, \frac{2}{5}, 1\right)$

(d) $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 2 & 1 & 4 & 7 \\ 1 & -7 & 5 & -1 \end{array}\right]$

$$\begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 5 & -2 & 3 \\ 0 & -5 & 2 & -3 \end{array}\right]$$

$$\begin{array}{c} \\ \\ R_3 - R_2 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 5 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ \frac{1}{5}R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & \cancel{5}\,1 & -2/5 & 3/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} R_1 + 2R_2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & \cancel{8/5}\ 11/5 & 16/5 \\ 0 & 1 & -2/5 & 3/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = \frac{16}{5} - \frac{11}{5}s$

$x_2 = \frac{3}{5} + \frac{2}{5}s$

$x_3 = s$

if s is let equal to $t+1$, then the solution in c holds.