IMPLICIT DIFFERENTIATION

3.1.1 DEFINITION We will say that a given equation in x and y defines the function f ***implicitly*** if the graph of $y = f(x)$ coincides with a portion of the graph of the equation.

Question: Find dy/dx by implicit differentiation.

$$x^3y^2 - 5x^2y + x = 1$$

Answer:

$$x^3(2y)\frac{dy}{dx} + 3x^2y^2 - 5x^2\frac{dy}{dx} - 10xy + 1 = 0,\ (2x^3y - 5x^2)\frac{dy}{dx} = 10xy - 3x^2y^2 - 1,\ \text{so}\ \frac{dy}{dx} = \frac{10xy - 3x^2y^2 - 1}{2x^3y - 5x^2}.$$

Question:

$$\tan^3(xy^2 + y) = x$$

Answer:

$$3\tan^2(xy^2 + y)\sec^2(xy^2 + y)\left(2xy\frac{dy}{dx} + y^2 + \frac{dy}{dx}\right) = 1,\ \text{so}\ \frac{dy}{dx} = \frac{1 - 3y^2\tan^2(xy^2 + y)\sec^2(xy^2 + y)}{3(2xy + 1)\tan^2(xy^2 + y)\sec^2(xy^2 + y)}.$$

Question: Find d^2y/dx^2 by implicit differentiation.

$$x^3y^3 - 4 = 0$$

Answer:

$$\frac{dy}{dx} = -\frac{y}{x},\ \frac{d^2y}{dx^2} = -\frac{x(dy/dx) - y(1)}{x^2} = -\frac{x(-y/x) - y}{x^2} = \frac{2y}{x^2}.$$

Question:

25–28 Use implicit differentiation to find the slope of the tangent line to the curve at the specified point, and check that your answer is consistent with the accompanying graph on the next page. ■

28. $x^{2/3} + y^{2/3} = 4;\ (-1, 3\sqrt{3})$

Answer:

$$\frac{2}{3}\left(x^{-1/3} + y^{-1/3}\frac{dy}{dx}\right) = 0,\ \frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}} = \sqrt{3} \text{ at } (-1, 3\sqrt{3}).$$