Group Assignment: Solving a Truss Bridge Using Systems of Equations

Alfred Xue, Arvin Bal, Benjamin Liang, Dhrumil Patel, Hamza Dugmag, Navin Vanderwert

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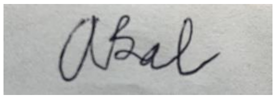
## Team Member Contributions

**Alfred Xue**

First and foremost, I would like to say how great of a team captain Ben was. I created the document with some of the formatting. I also formatted Questions 1 and 7, taking the code and the reasoning in our MATLAB file and transferring them into this report. Lastly, I wrote the small paragraph under each of our group members' contributions.

**Arvin Bal**

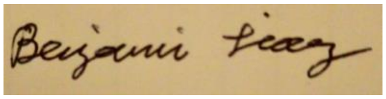
First and foremost, I would like to say how great of a team captain Ben was. I collaborated throughout the zoom calls and contributed ideas that Hamza recorded. I was responsible for question 3, so I created the vector x in the solution and confirmed that the values matched those on my CIV Assignment 5 which I provided in this document.



**Benjamin Liang**

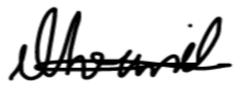
I wrote up question 2 and led the team in discussions and made sure they stayed on task as the designed team captain. Although I did not physically type the MATLAB code (Hamza and Navin were our designated screen sharers) I played a substantial role in figuring out how exactly we would write the code and troubleshooting issues that arose.

El capitano,



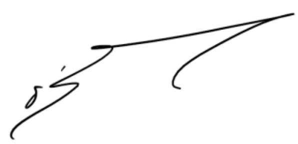
**Dhrumil Patel**

First and foremost, I would like to say how great of a team captain Ben was. I critiqued and helped fix mistakes while Hamza and Navin wrote the code. I was responsible for answering question 6 and helped create/facilitate the meetings on Zoom. I actively participated in the group discussions by sharing my ideas and input.



**Hamza Dugmag**

First and foremost, I would like to say how great of a team captain Ben was. I volunteered to share my screen and write some of the MATLAB code as we discussed what ESC103F concepts we should use to address the problems. I provided the equilibrium equations for the left joints of the bridge, wrote the solution to Question 5, and finalized the formatting of the report.



**Navin Vanderwert**

First and foremost, I would like to say how great of a team captain Ben was. I shared my screen and coded the first few questions, taking input from team members and facilitating its translation to MATLAB. I wrote Question 4 in the document after a lengthy discussion with my teammates in addition to proofreading.

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## Question 1: Expressing the System of Equations

Please note: all matrices are presented as two images: the left half above and the right have below.

We first assigned the truss parameters that define the applied forces and bridge geometry.

```
%% Question 1: Expressing the System of Equations
% Truss parameters
UDL = 70  % Uniform load [kN/m]
SPAN = 30  % Total length [m]
J = UDL * 5  % Joint load [kN] = 350
R = (5 * J) / 2  % Support reaction [kN] = 875

THETA = pi / 3  % Angle between members [rad] = 1.0472
s = sin(THETA)  % Member vertical component = 0.8660
c = cos(THETA)  % Member horizontal component = 0.5000
```

Since the bridge is symmetrically loaded and shaped, only the left half of the bridge needed to be inspected since the right half is a reflection. By manually examining the left joints, the equilibrium equations for the truss were produced. However, the middle joint, $G$, is unique and lies directly on the center line, so its equilibrium equations needed to be considered as well. Each joint had two equations, one for each axis ($x$ and $y$).

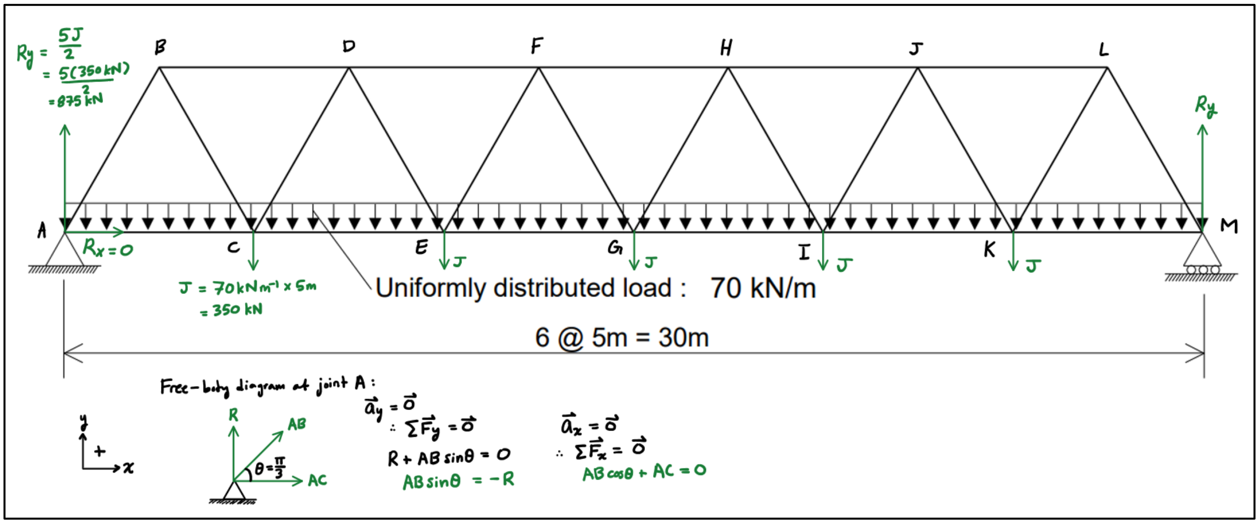



Figure 1. The Warren truss of interest and sample equilibrium equations for joint $A$.

Since matrix $A$ will be multiplied by the vector of unknown member forces, $\vec{x}$, $A$ contains the coefficients of the member forces in the equilibirium equations. Each row is an equation, while each column corresponds to an unknown member force. Thus, the vector of constants, $\vec{b}$ corresponds to the equilibrium equation constants isolated on one side of the equality.

```
% Create vector of equation constants
b_top = [-R 0 0 0 J 0 0 0 J 0 0 0]
b_G = [J 0]  % Middle joint [x constant, y constant]
b_bot = flip(b_top)

% Create matrix of coefficients according to equilibrium
A_top = [
    s 0 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    -c -1 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    s 0 s 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    c 0 -c -1 0 0 0 0 0 0 0 0 zeros(1, 11);
    0 0 s 0 s 0 0 0 0 0 0 0 zeros(1, 11);
    0 1 c 0 -c -1 0 0 0 0 0 0 zeros(1, 11);
    0 0 0 0 s 0 s 0 0 0 0 0 zeros(1, 11);
    0 0 0 1 c 0 -c -1 0 0 0 0 zeros(1, 11);
    0 0 0 0 0 0 s 0 s 0 0 0 zeros(1, 11);
    0 0 0 0 0 1 c 0 -c -1 0 0 zeros(1, 11);
    0 0 0 0 0 0 0 0 s 0 s 0 zeros(1, 11);
    0 0 0 0 0 0 0 1 c 0 -c -1 zeros(1, 11)
]
A_G = [
    0 0 0 0 0 0 0 0 0 0 s 0 s 0 0 0 0 0 0 0 0 0 0;  % y
    0 0 0 0 0 0 0 0 0 1 c 0 -c -1 0 0 0 0 0 0 0 0 0  % x
]  % Middle joint
A_bot = fliplr(flip(A_top))

% Construct the full matrix and vector
b = [b_top b_G b_bot].'
A = [A_top; A_G; A_bot]
```

The top half of matrix $A$ represents the coefficients for the left joints of the bridge. We flipped it horizontally and vertically to produce the bottom half of matrix $A$, representing the right joints. Alongside the coefficients for the middle joint, we combined these three submatrices to produce $A$.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 |

Figure 2. The coefficient matrix.

An interesting pattern in the entire matrix is that the rows alternate between containing $[s \quad 0 \quad s]$ and $[1 \quad c \quad 0 \quad -c \quad -1]$ surrounded by zeros, where $s = \sin\frac{\pi}{3}$ and $c = \cos\frac{\pi}{3}$. Each row shifts the alternating pattern to the right.

| | 1 |
|---|---|
| 1 | -875 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0 |
| 5 | 350 |
| 6 | 0 |
| 7 | 0 |
| 8 | 0 |
| 9 | 350 |
| 10 | 0 |
| 11 | 0 |
| 12 | 0 |
| 13 | 350 |
| 14 | 0 |
| 15 | 0 |
| 16 | 0 |
| 17 | 0 |
| 18 | 350 |
| 19 | 0 |
| 20 | 0 |
| 21 | 0 |
| 22 | 350 |
| 23 | 0 |
| 24 | 0 |
| 25 | 0 |
| 26 | -875 |

Figure 3. The vector of constants.

$$\therefore A\vec{x} = \vec{b}$$

## Question 2: Gaussian Elimination on the Original System

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -875.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 350.0000 |
| -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | -875.0000 |

Figure 4. The augmented matrix $[A|\vec{b}]$ is generated by appending the vector of constants to the matrix.

Using the built-in `rref` function in MATLAB, which brings $[A|\vec{b}]$ to its reduced normal form $[R|\vec{d}]$ using the Gaussian elimination algorithm, we find the true size of this matrix: $r = \text{rank}\, A$.

| $10^3$ × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0104 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5052 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0104 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0104 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.6062 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.3135 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.6062 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.6166 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.2021 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.7176 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.2021 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.8187 |
| 13 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.2021 |
| 14 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.7176 |
| 15 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.2021 |
| 16 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.6166 |
| 17 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0.6062 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 1.3135 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | 0 | -0.6062 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 0 | -1.0104 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0 | 1.0104 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0 | 0.5052 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | -1.0104 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

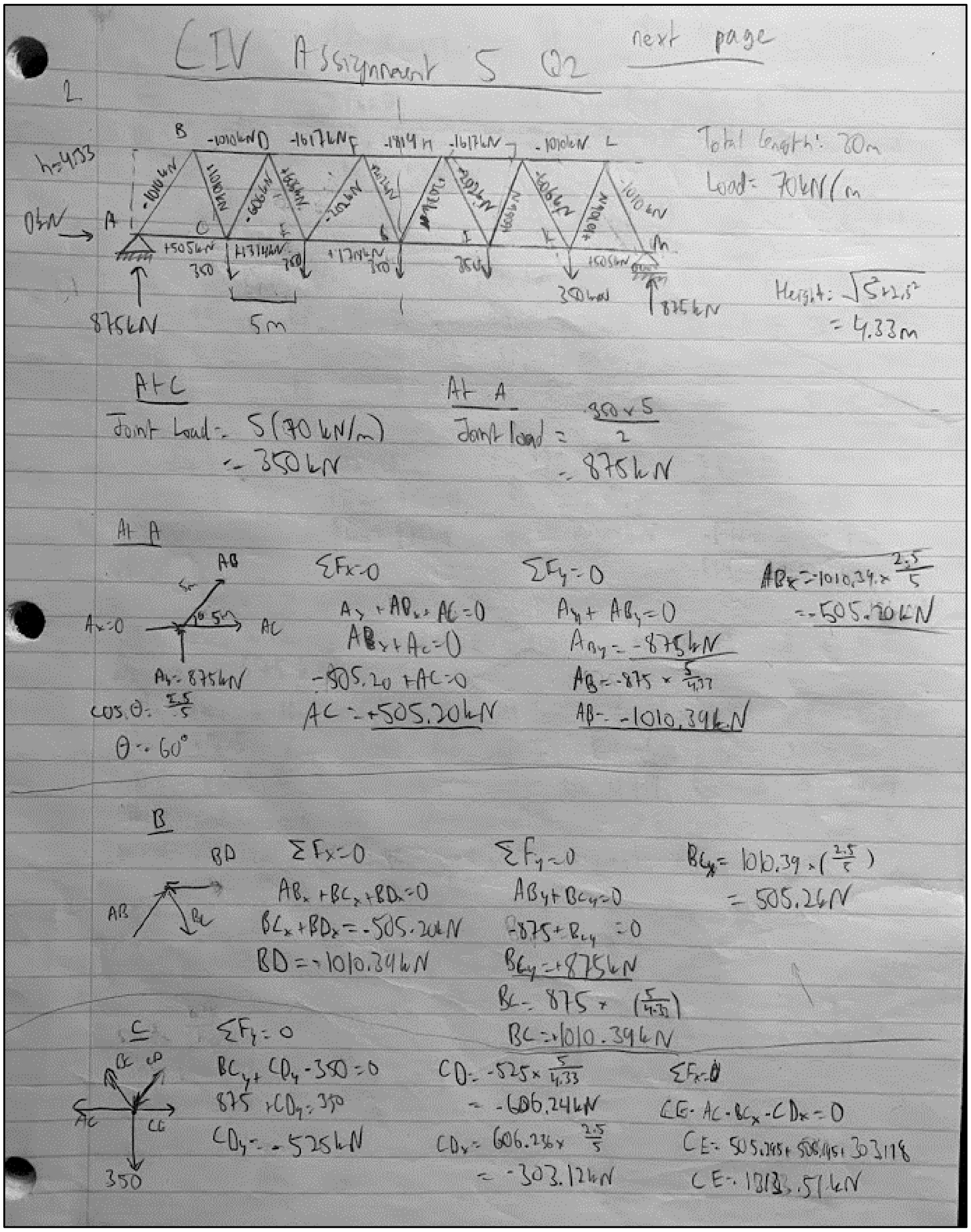
Figure 5. The reduced normal form $[R|d]$ of the augmented matrix $[A|b]$ is shown below.

Note that all values in the matrix are smaller than the actual values by a factor of $10^3$, as shown by the scaling factor in the top left corner. Thus, leading ones are displayed as $0.0010 \times 10^3$.

From the row reduced matrix above, we can see that $r = 23$ because there are 23 leading ones.

```
%% Question 2: GE on the Original System
% Create the augmented matrix and reduce it to RNF
aug = [A b]
rnf = rref(aug)
```

**Question 3: Presenting the Solution**


CIV Assignment 5 Q2 next page

Total Length: 30m

Load: 70kN/m

Height: $\sqrt{5^2 - 2.5^2}$

$= 4.33m$

At C

Joint Load = 5(70 kN/m)

= 350 kN

At A

Joint load = $\frac{350 \times 5}{2}$

= 875 kN

At A

$\Sigma F_x = 0$

$A_x + AB_x + AC = 0$

$AB_x + A_c = 0$

$-505.20 + AC = 0$

$AC = +505.20kN$

$\Sigma F_y = 0$

$A_y + AB_y = 0$

$A_{By} = -875kN$

$AB = -875 \times \frac{5}{4.33}$

$AB = -1010.39kN$

$AB_x = -1010.39 \times \frac{2.5}{5}$

$= -505.20kN$

$\cos\theta = \frac{2.5}{5}$

$\theta = 60°$

B

$\Sigma F_x = 0$

$AB_x + BC_x + BD_x = 0$

$BC_x + BD_x = -505.20kN$

$BD = -1010.39kN$

$\Sigma F_y = 0$

$AB_y + BC_y = 0$

$-875 + B_{Cy} = 0$

$B_{Cy} = +875kN$

$BC = 875 \times (\frac{5}{4.33})$

$BC = +1010.39kN$

$BC_x = 1010.39 \times (\frac{2.5}{5})$

$= 505.2kN$

C

$\Sigma F_y = 0$

$BC_y + CD_y - 350 = 0$

$875 + CD_y = 350$

$CD_y = -525kN$

$CD = -525 \times \frac{5}{4.33}$

$= -606.24kN$

$CD_x = 606.236 \times \frac{2.5}{5}$

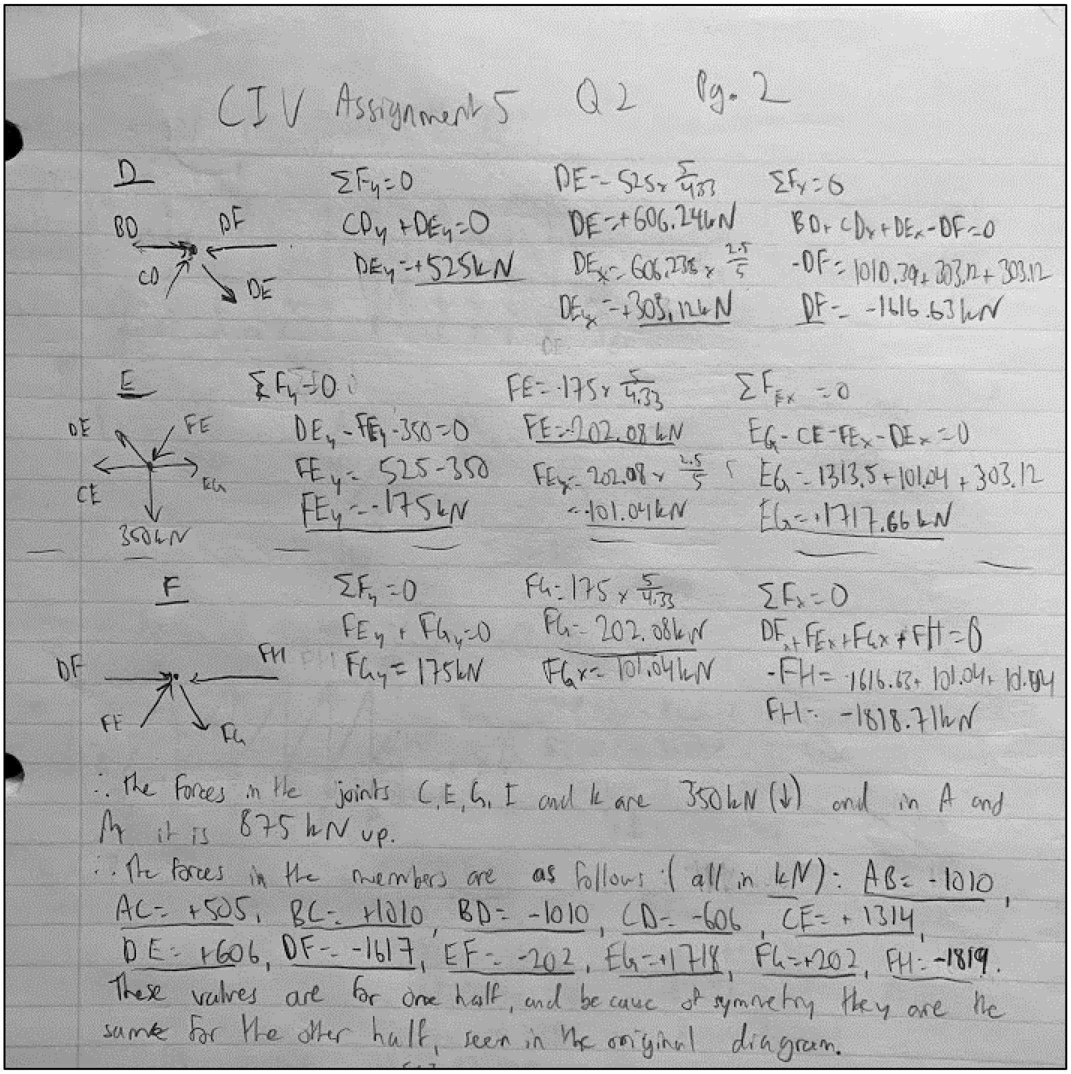
$= -303.12kN$

$\Sigma F_x = 0$

$CE - AC - BC_x - CD_x = 0$

$CE = 505.203 + 505.195 + 303.118$

$CE = 1313.51kN$


CIV Assignment 5 Q2 Pg. 2

D

$\Sigma F_y = 0$
$CD_y + DE_y = 0$
$DE_y = +525kN$

$DE = -525 \times \frac{5}{4.33}$
$DE = +606.246kN$
$DE_x = 606.238 \times \frac{2.5}{5}$
$DE_x = +303.124kN$

$\Sigma F_x = 0$
$BD + CD_x + DE_x - DF = 0$
$-DF = 1010.39 + 303.12 + 303.12$
$DF = -1616.63kN$

E

$\Sigma F_y = 0$
$DE_y - FE_y - 350 = 0$
$FE_y = 525 - 350$
$FE_y = -175kN$

$FE = -175 \times \frac{5}{4.33}$
$FE = -202.08kN$
$FE_x = 202.08 \times \frac{2.5}{5}$
$= -101.04kN$

$\Sigma F_{Ex} = 0$
$EG - CE - FE_x - DE_x = 0$
$EG = 1313.5 + 101.04 + 303.12$
$EG = +1717.66kN$

F

$\Sigma F_y = 0$
$FE_y + FG_y = 0$
$FG_y = 175kN$

$FG = 175 \times \frac{5}{4.33}$
$FG = 202.08kN$
$FG_x = 101.04kN$

$\Sigma F_x = 0$
$DF_x + FE_x + FG_x + FH = 0$
$-FH = 1616.63 + 101.04 + 101.04$
$FH = -1818.71kN$

∴ The forces in the joints C, E, G, I and K are 350kN (↓) and in A and M it is 875 kN up.
∴ The forces in the members are as follows (all in kN): AB = -1010, AC = +505, BC = +1010, BD = -1010, CD = -606, CE = +1314, DE = +606, DF = -1617, EF = -202, EG = +1718, FG = +202, FH = -1819.
These values are for one half, and because of symmetry they are the same for the other half, seen in the original diagram.

Figures 6-7. The written solution to CIV102 Assignment 5 Question 2

| $10^3$ x | 1 |
|---|---|
| 1 | -1.0104 |
| 2 | 0.5052 |
| 3 | 1.0104 |
| 4 | -1.0104 |
| 5 | -0.6062 |
| 6 | 1.3135 |
| 7 | 0.6062 |
| 8 | -1.6166 |
| 9 | -0.2021 |
| 10 | 1.7176 |
| 11 | 0.2021 |
| 12 | -1.8187 |
| 13 | 0.2021 |
| 14 | 1.7176 |
| 15 | -0.2021 |
| 16 | -1.6166 |
| 17 | 0.6062 |
| 18 | 1.3135 |
| 19 | -0.6062 |
| 20 | -1.0104 |
| 21 | 1.0104 |
| 22 | 0.5052 |
| 23 | -1.0104 |

Figure 8. The $\vec{x}$ vector from the reduced normal form of matrix $A$.

The vector form of the solution, $\vec{x}$, was obtained by creating a new array with the contents of the last column of the reduced normal form matrix. It only contains rows 0 to 23 since 24-26 are zero rows, which are redundant.

The vector solution shows the loads in each member with negative values being members in compression and positive being in tension. The solutions in $\vec{x}$ correspond with the list of the forces on the second page of the written solution. The diagram on the first page with the labelled forces shows the symmetry of the truss, as does $\vec{x}$ since the values are flipped about row 12. Since the values in the written work match those in the $\vec{x}$ from the reduced normal form, we can verify that our $\vec{x}$ vector is correct.

```
%% Question 3: Presenting the Solution
% Obtain the solutions column from the RNF
X = rnf(1:23, 24)
```

## Question 4: Simplifying the Linear System

By inspection, the equilibrium equations for joint G (both $x$ and $y$) as well as the $x$ equation for joint H are redundant and produce the zero rows in the RNF of $[R|\vec{d}]$.

The G equations are redundant since joints A to F (or M to H if starting from the other side) reveal the forces in all members leading up to G, so the forces acting on joint G need not be known.

Member FH and joint G pass through the centre line, which is shared by both halves of the truss. So, joints F and H both calculate for FH in their $x$ equations. When members on the left extend into the right half, and vice versa, redundancy occurs.

After removing the three redundant rows from the coefficient matrix, it is now $23 \times 23$. The bottom half of the matrix is a vertical and horizontal reflection *about* the last row of the top half since reflecting the last row horizontally would produce the redundant $x$ equation for H that was detected in Question 4. Similarly, the vector of constants is $13 \times 1$ after removing the associated redundant constants. The bottom half is a vertical reflection about the last row of the top half.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.0000 | 0.5000 | 0 | -0.5000 | -1.0000 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -875.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 1.0000 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 | 0 | 350.0000 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 | 0.5000 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | 0 | 0.8660 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5000 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.8660 | -875.0000 |

Figure 9. The modified augmented matrix $[A_r|\overrightarrow{b_r}]$.

```
%% Question 4: Simplifying the Linear System
% Create the modified b vector
br_top = [-R 0 0 0 J 0 0 0 J 0 0]
br_CL = [0]  % Axis of reflection (FH at centre line)
br_bot = flip(br_top)  % Reflected top half
br = [br_top br_CL br_bot].'

% Create the modified A matrix
Ar_top = [
    s 0 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    -c -1 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    s 0 s 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    c 0 -c -1 0 0 0 0 0 0 0 0 zeros(1, 11);
    0 0 s 0 s 0 0 0 0 0 0 0 zeros(1, 11);
    0 1 c 0 -c -1 0 0 0 0 0 0 zeros(1, 11);
    0 0 0 0 s 0 s 0 0 0 0 0 zeros(1, 11);
    0 0 0 1 c 0 -c -1 0 0 0 0 zeros(1, 11);
    0 0 0 0 0 0 s 0 s 0 0 0 zeros(1, 11);
    0 0 0 0 0 1 c 0 -c -1 0 0 zeros(1, 11);
    0 0 0 0 0 0 0 0 s 0 s 0 zeros(1, 11);
]
Ar_CL = [0 0 0 0 0 0 0 1 c 0 -c -1 zeros(1, 11)]
Ar_bot = fliplr(flip(Ar_top))
Ar = [Ar_top; Ar_CL; Ar_bot]

augr = [Ar br]  % Modified augmented matrix
```

## Question 5: Matrix Inverse and Verification

Given the modified augmented matrix $[A_r|b_r]$, $A_r$ is a square matrix. Thus, $A_r^{-1}$ may be computed using MATLAB's built-in `inv` function. The solution is obtained by multiplying the matrix by the vector of constants.

```
%% Question 5: Matrix Inverse and Verification

% Find the inverse of matrix A to obtain member forces

Ar_inv = inv(Ar)

x = Ar_inv * br
```

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1547 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | -0.5774 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | -1.1547 | 0 | 1.1547 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 1.1547 | 0 | -0.5774 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | -1.7321 | -1.0000 | 1.1547 | 0 | -0.5774 | -1.0000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | 0 | 0 | 0 | 0 |
| 8 | 2.3094 | 0 | -1.7321 | -1.0000 | 1.1547 | 0 | -0.5774 | -1.0000 | 0 | 0 | 0 | 0 |
| 9 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | 0 | 0 |
| 10 | -2.8868 | -1.0000 | 2.3094 | 0 | -1.7321 | -1.0000 | 1.1547 | 0 | -0.5774 | -1.0000 | 0 | 0 |
| 11 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 |
| 12 | 3.4641 | 0 | -2.8868 | -1.0000 | 2.3094 | 0 | -1.7321 | -1.0000 | 1.1547 | 0 | -0.5774 | -1.0000 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 |
| 0 | -1.0000 | -0.5774 | 0 | 1.1547 | -1.0000 | -1.7321 | 0 | 2.3094 | -1.0000 | -2.8868 |
| 0 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 |
| 0 | 0 | 0 | -1.0000 | -0.5774 | 0 | 1.1547 | -1.0000 | -1.7321 | 0 | 2.3094 |
| 0 | 0 | 0 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 | 0 | -1.1547 |
| 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5774 | 0 | 1.1547 | -1.0000 | -1.7321 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1.1547 | 0 | -1.1547 | 0 | 1.1547 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5774 | 0 | 1.1547 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.1547 | 0 | -1.1547 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.0000 | -0.5774 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.1547 |

Figure 10. The inverse of the square matrix.

| $10^3$ × | 1 |
|---|---|
| 1 | -1.0104 |
| 2 | 0.5052 |
| 3 | 1.0104 |
| 4 | -1.0104 |
| 5 | -0.6062 |
| 6 | 1.3135 |
| 7 | 0.6062 |
| 8 | -1.6166 |
| 9 | -0.2021 |
| 10 | 1.7176 |
| 11 | 0.2021 |
| 12 | -1.8187 |
| 13 | 0.2021 |
| 14 | 1.7176 |
| 15 | -0.2021 |
| 16 | -1.6166 |
| 17 | 0.6062 |
| 18 | 1.3135 |
| 19 | -0.6062 |
| 20 | -1.0104 |
| 21 | 1.0104 |
| 22 | 0.5052 |
| 23 | -1.0104 |

Figure 11. The solution vector produced by $\vec{x} = A_r^{-1}\overrightarrow{b_r}$. It matches the solution obtained in Question 3.

**Question 6: Modifying the Uniformly Distributed Load**

In the augmented matrix $[A_r|\overrightarrow{b_r}]$ the value of each element of $\overrightarrow{b_r}$ increases by a factor of $\frac{90}{70}$, but matrix $A_r$ remains constant. Vector $\overrightarrow{b_r}$ is consists of the joint loads and reaction forces, which are linearly proportional to the uniformly distributed load. Hence, the elements of scalar factor of $\overrightarrow{b_r}$ is also linearly proportional to the uniform distributed load. $A_r$ remains constant as it is only dependent on the geometry of the truss bridge, which remains the same under both $90\ \mathrm{kN/m}$ and $70\ \mathrm{kN/m}$ loads.

Given $\overrightarrow{A_r}\vec{x} = \overrightarrow{b_r}$, $A_r$ is constant while $\overrightarrow{b_r}$ is scaled by $\frac{90}{70}$. Thus, the left-hand side of the equation must also be scaled by $\frac{90}{70}$. Since the matrix is independent of the factor, we scale $\vec{x}$ by $\frac{90}{70}$ to achieve equality and produce the member forces under the new load, instead of using Gaussian Elimination on the new augmented matrix.

```
%% Question 6: Modifying the Uniformly Distributed Load
% Scale forces and update constants
udl_new = 90
factor = udl_new / UDL  % 90/70
br_new = br .* factor  % J and R are proportional to udl

% Update the augmented matrix and solve for new forces
augr_new = [Ar br_new]  % Ar is constant
rnf_new = rref(augr_new)
x_new = rnf_new(1:23, 24)
```

| $10^3$ × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0009 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | -0.0005 | -0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.0009 | 0 | 0.0009 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0.0005 | 0 | -0.0005 | -0.0010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0.0009 | 0 | 0.0009 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0.0010 | 0.0005 | 0 | -0.0005 | -0.0010 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0.0009 | 0 | 0.0009 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0.0010 | 0.0005 | 0 | -0.0005 | -0.0010 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0009 | 0 | 0.0009 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0.0005 | 0 | -0.0005 | -0.0010 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0009 | 0 | 0.0009 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0010 | 0.0005 | 0 | -0.0005 | -0.0010 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1.1250 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.4500 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.4500 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.0009 | 0 | 0.0009 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -0.0010 | -0.0005 | 0 | 0.0005 | 0.0010 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.0009 | 0 | 0.0009 | 0 | 0 | 0 | 0 | 0 | 0 | 0.4500 |
| 0 | 0 | 0 | -0.0010 | -0.0005 | 0 | 0.0005 | 0.0010 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.0009 | 0 | 0.0009 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | -0.0010 | -0.0005 | 0 | 0.0005 | 0.0010 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.0009 | 0 | 0.0009 | 0 | 0 | 0.4500 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.0010 | -0.0005 | 0 | 0.0005 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0009 | 0 | 0.0009 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.0010 | -0.0005 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0009 | -1.1250 |

Figure 12. The modified augmented matrix. Only the last column is scaled by the specified factor.

## Question 7: Adapting to Bridge Constraints

Since the distributed load is proportional to the joint load and hence the reaction forces, $\overrightarrow{b_r}$ scales proportional to the distributed load. Since $A$ is only dependent on the bridge geometry (not the forces), $A_r^{-1}$ remains constant. Thus, when calculating $\overrightarrow{x_{r'}} = A_r^{-1}\left(\overrightarrow{b_{r'}}\right) = A_r^{-1}\left(c\overrightarrow{b_r}\right) = c\left(A_r^{-1}\overrightarrow{b_r}\right) = c\overrightarrow{x_r}$, the new forces are obtained by scaling the original forces by $c$: the scaling factor of the distributed load.

From the matrix, the force in member FH is $1818$ kN (compressive) due to a uniformly distributed load of $70$ kN/m. Since we know that this force will always be the largest, and that member forces scale proportionally to this uniformly distributed load, we can multiply the original force, $-1818$, by the UDL scaling factor to produce $2000$ kN for the maximum force.

```
%% Question 7: Adapting to Bridge Constraints

% FH * MAX_UDL / UDL = MAX_FH

% -1818 * MAX_UDL / 70 = -2000

MAX_UDL = (-2000 * UDL) / x(12)  % 12th row
```

```
MAX_UDL = 76.9800
```

Figure 13. The output for the maximum uniformly distributed load.

We changed the initial parameter to this maximum value ($76.98$ kN/m) to verify our solution using $[A_r|\overrightarrow{b_r}]$. Row 12 is the force in member FH, which correctly gives $-2000$ kN.

| $10^3$ × | 1 |
|---|---|
| 1 | -1.1111 |
| 2 | 0.5556 |
| 3 | 1.1111 |
| 4 | -1.1111 |
| 5 | -0.6667 |
| 6 | 1.4444 |
| 7 | 0.6667 |
| 8 | -1.7778 |
| 9 | -0.2222 |
| 10 | 1.8889 |
| 11 | 0.2222 |
| 12 | -2.0000 |
| 13 | 0.2222 |
| 14 | 1.8889 |
| 15 | -0.2222 |
| 16 | -1.7778 |
| 17 | 0.6667 |
| 18 | 1.4444 |
| 19 | -0.6667 |
| 20 | -1.1111 |
| 21 | 1.1111 |
| 22 | 0.5556 |
| 23 | -1.1111 |

Figure 14. The scaled solution vector.

## Reflection

Through this assignment, we have gained practical experience working on an engineering problem in MATLAB. We learned to make connections between two useful courses: ESC103 and CIV102. By using theoretical knowledge from ESC103, we were able to apply in a more practical way via CIV102 to solve a real-world engineering problem.

We learned that communication and collaboration is crucial in solving engineering problems. Everyone provided different insights, and we listened carefully to each other's perspectives to approach the problem in the most efficient way possible.

Something we learned was that MATLAB is very useful for creating equations and vectors which are repetitive. Since the matrices and vectors had large dimensions, calculating everything manually would have taken too much time and is prone to errors which would be difficult to identify and fix. Using computational methods saved us time and demonstrated the usefulness of computation in engineering.

Additionally, we noticed a pattern in the matrix after completing the assignment. We should have automated it in MATLAB instead of manually creating the equations. To do that, we could look for an underlying pattern in the general Warren truss to understand how forces are distributed. We would then be able to extend the matrix to an arbitrary number of joints and members, and even arbitrary angles.

**Appendix: MATLAB Code**

```
%%% Solving a Truss Bridge Using Systems of Equations
%%% Alfred Xue, Arvin Bal, Benjamin Liang
%%% Dhrumil Patel, Hamza Dugmag, Navin Vanderwert
%%% TUT0101 Group 3
%%% Updated November 24, 2020
%%% Engineering Science – ESC103F
%%% University of Toronto

clear all

%% Question 1: Expressing the System of Equations
% Truss parameters
UDL = 70  % Uniform load [kN/m]
SPAN = 30  % Total length [m]
J = UDL * 5  % Joint load [kN] = 350
R = (5 * J) / 2  % Support reaction [kN] = 875

THETA = pi / 3  % Angle between members [rad] = 1.0472
s = sin(THETA)  % Member vertical component = 0.8660
c = cos(THETA)  % Member horizontal component = 0.5000
%% Question 7: Adapting to Bridge Constraints
% FH * MAX_UDL / UDL = MAX_FH
% -1818 * MAX_UDL / 70 = -2000
MAX_UDL = (-2000 * UDL) / x(12)  % 12th row
```

```
% Create vector of equation constants
b_top = [-R 0 0 0 J 0 0 0 J 0 0 0]
b_G = [J 0]  % Middle joint [x constant, y constant]
b_bot = flip(b_top)

% Create matrix of coefficients according to equilibrium
A_top = [
    s 0 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    -c -1 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    s 0 s 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    c 0 -c -1 0 0 0 0 0 0 0 0 zeros(1, 11);
    0 0 s 0 s 0 0 0 0 0 0 0 zeros(1, 11);
    0 1 c 0 -c -1 0 0 0 0 0 0 zeros(1, 11);
    0 0 0 0 s 0 s 0 0 0 0 0 zeros(1, 11);
    0 0 0 1 c 0 -c -1 0 0 0 0 zeros(1, 11);
    0 0 0 0 0 0 s 0 s 0 0 0 zeros(1, 11);
    0 0 0 0 0 1 c 0 -c -1 0 0 zeros(1, 11);
    0 0 0 0 0 0 0 0 s 0 s 0 zeros(1, 11);
    0 0 0 0 0 0 0 1 c 0 -c -1 zeros(1, 11)
]
A_G = [
    0 0 0 0 0 0 0 0 0 0 s 0 s 0 0 0 0 0 0 0 0 0 0;  % y
    0 0 0 0 0 0 0 0 0 1 c 0 -c -1 0 0 0 0 0 0 0 0 0  % x
]  % Middle joint
A_bot = fliplr(flip(A_top))
```

```
% Construct the full matrix and vector
b = [b_top b_G b_bot].'
A = [A_top; A_G; A_bot]


%% Question 2: GE on the Original System
% Create the augmented matrix and reduce it to RNF
aug = [A b]
rnf = rref(aug)
% r = rank(A)


%% Question 3: Presenting the Solution
% Obtain the solutions column from the RNF
X = rnf(1:23, 24)


%% Question 4: Simplifying the Linear System
% Create the modified b vector
br_top = [-R 0 0 0 J 0 0 0 J 0 0]
br_CL = [0]  % Axis of reflection (FH at centre line)
br_bot = flip(br_top)  % Reflected top half
br = [br_top br_CL br_bot].'

% Create the modified A matrix
Ar_top = [
```

```
    s 0 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    -c -1 0 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    s 0 s 0 0 0 0 0 0 0 0 0 zeros(1, 11);
    c 0 -c -1 0 0 0 0 0 0 0 0 zeros(1, 11);
    0 0 s 0 s 0 0 0 0 0 0 0 zeros(1, 11);
    0 1 c 0 -c -1 0 0 0 0 0 0 zeros(1, 11);
    0 0 0 0 s 0 s 0 0 0 0 0 zeros(1, 11);
    0 0 0 1 c 0 -c -1 0 0 0 0 zeros(1, 11);
    0 0 0 0 0 0 s 0 s 0 0 0 zeros(1, 11);
    0 0 0 0 0 1 c 0 -c -1 0 0 zeros(1, 11);
    0 0 0 0 0 0 0 0 s 0 s 0 zeros(1, 11);
]
Ar_CL = [0 0 0 0 0 0 0 1 c 0 -c -1 zeros(1, 11)]
Ar_bot = fliplr(flip(Ar_top))
Ar = [Ar_top; Ar_CL; Ar_bot]

augr = [Ar br]  % Modified augmented matrix



%% Question 5: Matrix Inverse and Verification
% Find the inverse of matrix A to obtain member forces
Ar_inv = inv(Ar)
x = Ar_inv * br



%% Question 6: Modifying the Uniformly Distributed Load
```

```
% Scale forces and update constants
udl_new = 90
factor = udl_new / UDL  % 90/70
br_new = br .* factor  % J and R are proportional to udl

% Update the augmented matrix and solve for new forces
augr_new = [Ar br_new]  % Ar is constant
rnf_new = rref(augr_new)
x_new = rnf_new(1:23, 24)


%% Question 7: Adapting to Bridge Constraints
% FH * MAX_UDL / UDL = MAX_FH
% -1818 * MAX_UDL / 70 = -2000
MAX_UDL = (-2000 * UDL) / x(12)  % 12th row
% Change UDL = 90 in Question 1 and inspect x
```