Matboard Project Report

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Author Note
Group 51 has decided to be considered for the Project Bonus.

Describing and Summarizing the Beams

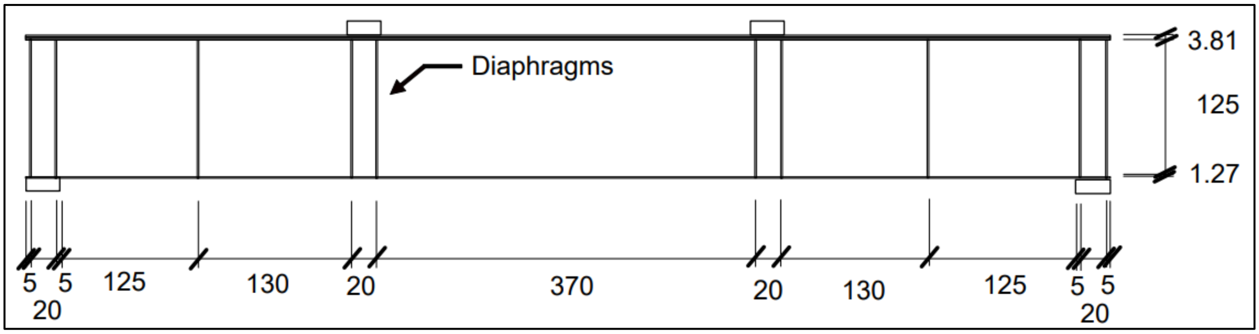

Figure 1. Longitudinal Cross Section of Concept I.

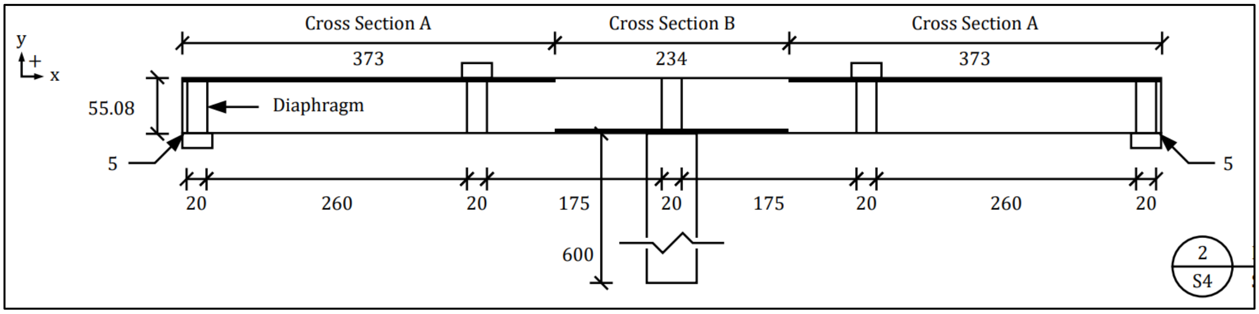

Figure 2. Longitudinal Cross Section of Concept II.

Property	Concept I	Concept II
Failure Load (N)	1953	1739
Failure Mode	Shear Buckling of Webs	Flexural Compression in Double Flanges
Dimensions (mm)	980 long by 125 tall by 100 wide	Beam: 980 long by 55.08 tall by 100 wide Column: 50 long by 600 tall by 50 wide
Strength/Weight	2.75 N/g	2.75 N/g

Design Process and Features

Optimization Process

Both designs are specifically chosen to be box girder cross-sections to maximize the k value in the plate buckling calculations, thus, decreasing the critical compressive stress for the design. Also, box girders tend to have higher I values compared to T beams. For diaphragms at points of interest (loads, supports), 20 mm spacing was elected (10 mm on each side of the point). This minimizes the a value for shear web buckling calculations as much as possible, but also provides enough space between the diaphragms and the edges of the platforms to ensure the design does not slip off the supports.

Concept I

Given the nature of the problem and the materials provided, a couple of aspects that needed to be designed around stood out. Firstly, increasing the I value of the cross-section would be a priority, as I is involved in almost every failure load equation, where failure load is proportional to second moment of area. Thus, the height of the box girder was increased as much as possible given the limited materials. Secondly, the compressive strength was identified to be much less than the tensile strength for the matboard, so, to optimize the design, more mass was placed on the top (the portion in compression according to the bending moment diagram). This decreases y_{top} and increases y_{bottom}, thus, reducing the difference between compressive and tensile failure loads. To do this with the limited material dimensions, the upper flange was tripled in thickness. This change has the added benefit of increasing the flange's buckling failure load due to a larger t. With these two principles in mind, as much material as possible was devoted to the beam span while leaving just enough material for the diaphragms.

Concept II

For design 2, calculating the shear and bending moment diagrams (using the 2nd moment-area theorem) revealed that the requirements for moment was reduced by a greater amount than the requirements for shear (67% vs 22% reduction). Since adding a support column would cause the I value of the cross section to decrease (same amount of material in both designs), the focus was to design specifically for shear in the cross section of the second design. The most effective way deduced was by doubling the thickness of the webs, sacrificing height and thickness of the flange under compression. Also, given that there are regions of negative moment in the second design, two different cross sections were used, where one is an inverted version of the other (so that the flange under compression is always two sheets thick). This also satisfied the BMD which assumed a uniform I value.

Matboard Building Instructions

Note that these instructions refer to their design's drawings and matboard usage diagrams. They are best understood with those two documents on hand.

Concept I

1. The drawings include figures of each matboard component, including cut and fold lines. Cut all sections and make folds where illustrated in drawings for design #1. (Thick lines for cutting, dotted lines are folding).

2. The base plate is the purple section in the matboard figure, which will be folded into the shape shown in the corresponding purple colour in the cross-section diagram. This shape will span the entire length, while other sections will be glued onto it. It mainly determines the entire shape of the bridge's construction.

3. The flange reinforcement section in the matboard figure is divided into three parts, two on the top and one on the bottom, which are three different colours in the figure (purple blue, pink, and cyan). Glue the bottom flange to the two 10 mm glue tabs on the base plate. Then glue two flanges to the top of the base plate. This will result in a cross-section that is triple layered at the top flange and single layered at the bottom flange.

4. Finally, the Diaphragms are glued into the cross section. Refer to the elevation view of the bridge for the exact locations. A total of 10 diaphragms are glued. Each one is 100 mm by 125 mm. 1.27 mm for each diaphragm is allocated for two small glue tabs on each end (0.635 mm on each side).

Concept II

1. The drawings include figures of each Matboard component, including cut and fold lines. Cut all sections and make folds where illustrated in drawings for design #2. (Illustrated in blue in the diagram).

2. The "base plate" in the diagrams dictates the shape of the cross section. Fold this base plate (BLUE in diagram) into the rough shape of the cross section. Be sure to note which side will be the flange and which sides will be the webs.

3. The web reinforcement section (highlighted in YELLOW in diagram) is to be cut and glued to both webs of the base plate (BLUE in diagram), to form a double layered web.

4. The Diaphragms should now be glued into the cross section. Refer to the elevation view of the bridge for the exact locations. Also note that 0.08 mm of each side of the diaphragm width is to be folded into a small glue tab. Glue each of the 10 diaphragms to the webs in the cross section.

5. The bottom flange reinforcement (RED in diagram) should then be glued to the 10 mm glue tabs of the base plate. This will "complete" the cross section and you should have a box by now.

6. Extra flange reinforcement is to be glued on the top flange or on the bottom flange. The two 373 mm sections are to be glued on the top flange near the two ends, while the 234 mm section is to be glued roughly in the middle of the bridge on the bottom flange.

7. The Vertical column should be made by folding the four 50 mm sections into a square cross section and then gluing the 10 mm tab to the fourth wall.

8. Finally, glue the cross section of the column to the 50 mm by 100 mm "column support plate" (to distribute the loading onto the support) – then glue the support plate to the underside of the bridge, as seen in the elevation view.

Recommending a Bridge to Build

Within the given design space (material properties, provided matboard, and beam dimensions), the first design is recommended to be built. Firstly, the strength to weight ratios of the two designs are nearly identical, meaning that this metric cannot differentiate the two designs. Therefore, the main objective shifts to purely increasing maximum load force due to the following assumptions: the material provided is within the means of the stakeholders, and the factor of safety for design 1 is greater (assuming 1200 N is the loading force being designed for). Secondly, design 1 does not require a column support, meaning that the complexity of the design, and by extension the difficulty of building it, is lower. Since the support does not increase the strength to weight ratio, the additional work is not useful.

Of course, neither of these designs are perfectly optimized in terms of dimensions and strength-to-weight ratios. Instead, the team's design process consisted in opting for "round" numbers. Also, there is room for more altered cross sections in both designs; for example, weaker cross sections could be used in areas with lesser moment or shear (the team only slightly delved into this). Given more time and resources, the team is unable to determine which basic design concept holds more merit. Regardless, given the two designs outlined above, concept 1 is recommended due to its larger failure load (higher factor of safety) and reduced complexity.

Matboard Project Calculations

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Units and Precision

All force units are in Newtons [N], all length units are in millimeters [mm], and all stress units are in megapascals [MPa]. Failure loads are rounded down in slide-precision to be conservative rather than overestimate the failure.

Concept I

Force Diagrams (I)

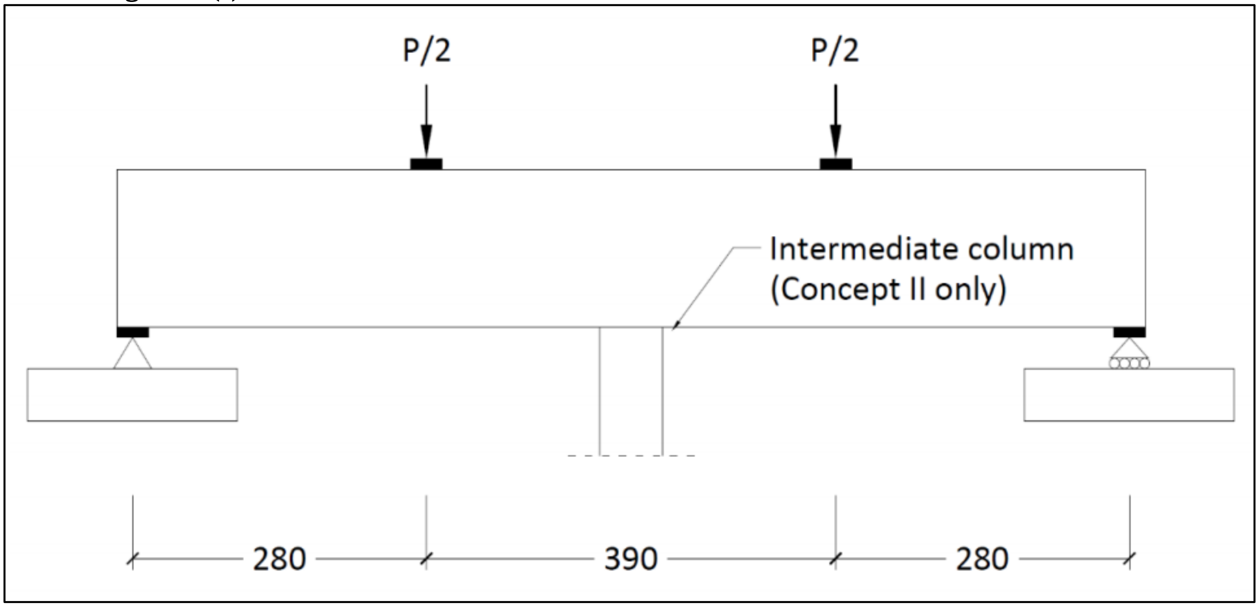


Figure 1. Given beam loading schematic (elevation).

Calculating the reaction forces for the pin (A_x, A_y) and roller (B_y):

$$\sum F_x = 0$$
$$\therefore A_x = 0$$

Since the loading and beam are symmetric, the vertical reaction forces are equal:

$$B_y = A_y$$

$$\sum F_y = 0$$
$$2A_y = \frac{2P}{2}$$
$$\therefore A_y = B_y = \frac{P}{2}$$

With all the forces in terms of P, the shear force diagram (SFD) can be constructed. The bending moment diagram (BMD) was produced by integrating the SFD along the span.

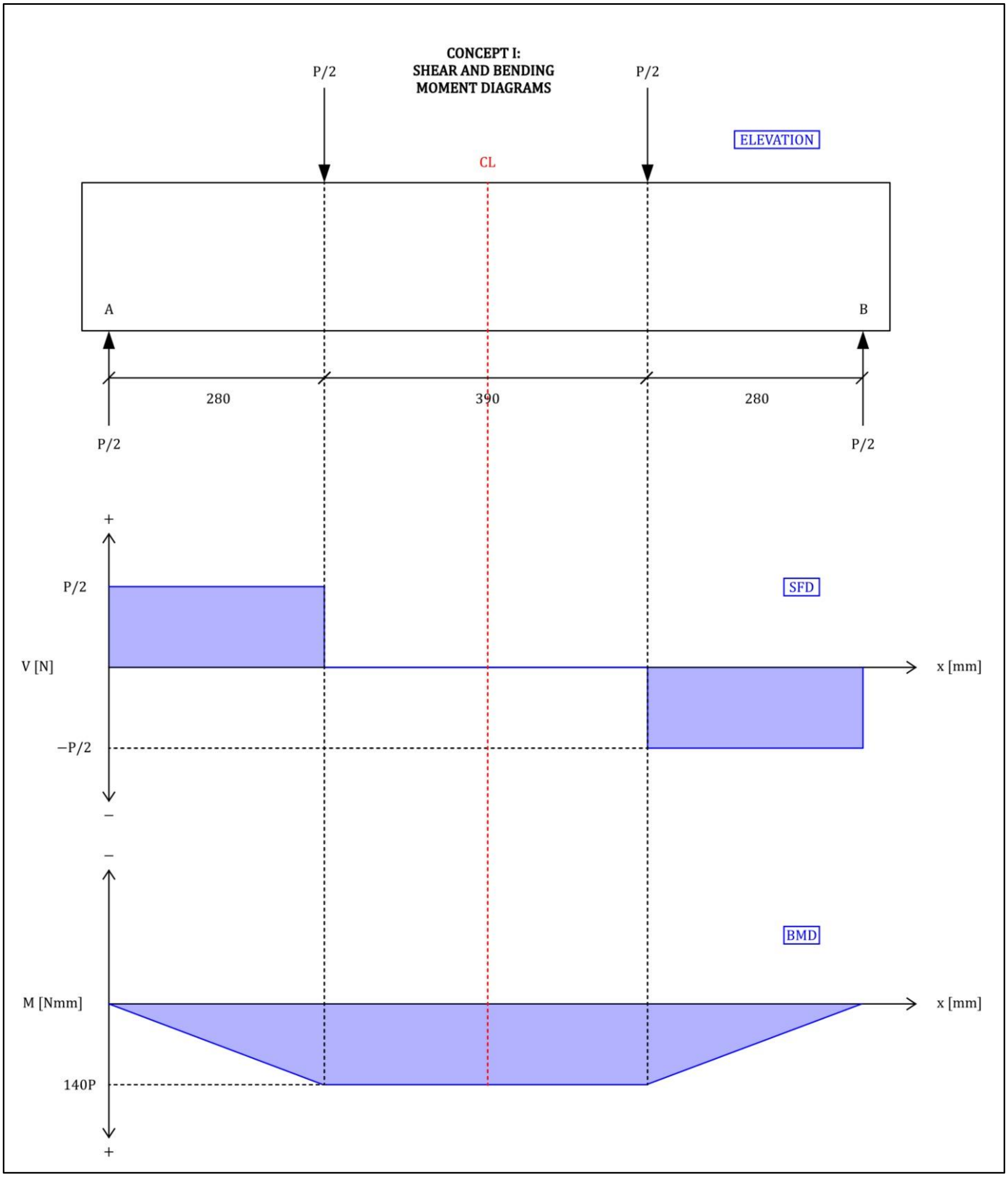
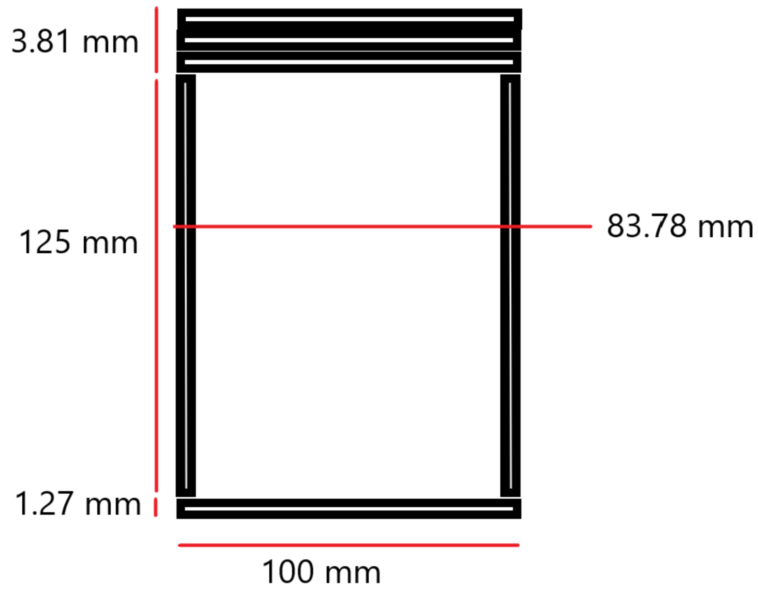


Figure 2. Labelled loading schematic, SFD, and BMD of Concept I.

Therefore, the maximum moment is $M_{\max} = 140P$, while the maximum shear force is $V_{\max} = \frac{P}{2}$.

Section Properties (I)

The following rough sketch was used to calculate the centroidal axis, second moment of area, and highest first moment of area of the cross-section:



Summary:

$$\bar{y} = 83.8 \text{ mm}$$
$$I = 2.17 \times 10^6 \text{ mm}^4$$
$$Q_{\max} = 19.21 \times 10^3 \text{ mm}^3$$

Calculating the location of the Centroidal Axis ($\bar{y}$) from bottom to top:

$$\bar{y} = \frac{\sum Ay}{\sum A}$$
$$= \frac{(100)(1.27)\left(\frac{1.27}{2}\right) + (2)(1.27)(125)\left(\frac{125}{2} + 1.27\right) + (100)(3.81)(\frac{3.81}{2} + 125 + 1.27)}{(100)(1.27) + (2)(1.27)(125) + (100)(3.81)}$$
$$= \frac{69162.295}{825.5}$$
$$= 83.8 \text{ mm}$$

$$y_{\text{bot}} = |0 - \bar{y}|$$
$$= 83.8 \text{ mm}$$

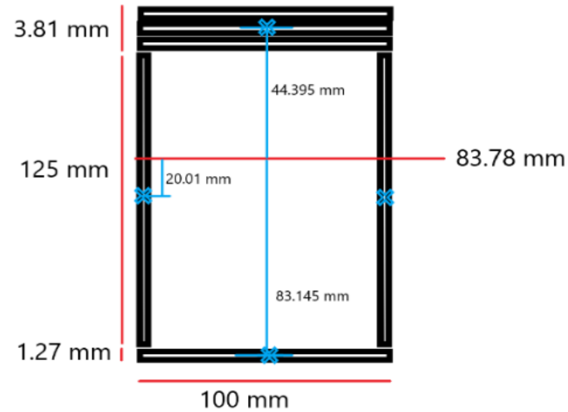
$$y_{\text{top}} = (1.27 + 125 + 3.81) - \bar{y}$$
$$= 46.3 \text{ mm}$$

Using the parallel axis theorem for the Second Moment of Area (I):

$$I = \sum \frac{bh^3}{12} + \sum Ad^2$$

$$= \frac{(100)(1.27)^3}{12} + (100)(1.27)\left(83.78 - \frac{1.27}{2}\right)^2 + \frac{(2)(1.27)(125)^3}{12} + (2)(1.27)(125)(20.01)^2 + \frac{(100)(3.81)^3}{12} + (100)(3.81)(44.395)^2$$

$$= 2.17 \times 10^6 \text{ mm}^4$$

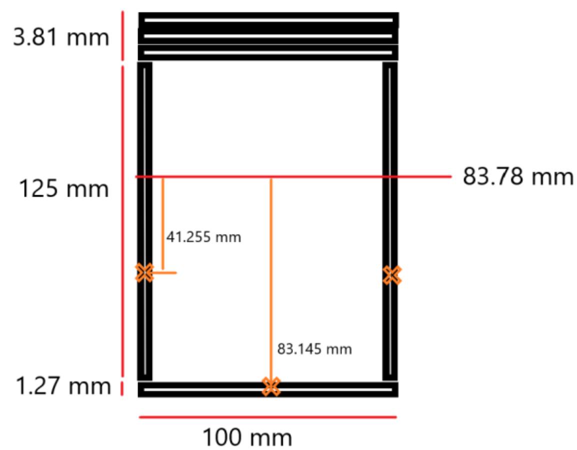


The maximum First Moment of Area (Q) is at the centroidal axis:

$$Q_{\text{max}} = \sum Ad^2$$

$$= (2)(82.51 \text{ mm})(1.27 \text{ mm})(41.255 \text{ mm}) + (1.27 \text{ mm})(100 \text{ mm})(83.145 \text{ mm})$$

$$= 19.21 \times 10^3 \text{ mm}^3$$



Failure Modes (I)

Under positive moment, the bottom experiences tension. Flexural Tensile Failure: $$\sigma_t = \frac{M_{\text{max}} y_{\text{bot}}}{I}$$ $$30\text{ MPa} = \frac{(140P)(83.8\text{ mm})}{2.17 \times 10^6\text{ mm}^4}$$ $$\therefore P_1 = 5550\text{ N}$$	Under positive moment, the top experiences tension. Flexural Compression Failure: $$\sigma_c = \frac{M_{\text{max}} y_{\text{top}}}{I}$$ $$6\text{ MPa} = \frac{(140P)(46.3\text{ mm})}{2.17 \times 10^6\text{ mm}^4}$$ $$\therefore P_2 = 2000\text{ N}$$

Web Shear Failure:

$$\tau_w = \frac{V_{\text{max}} Q_{\text{max}}}{I b_w}$$

$$4\text{ MPa} = \frac{\left(\frac{P}{2}\right)(19.21 \times 10^3\text{ mm}^3)}{(2.17 \times 10^6\text{ mm}^4)(2.54\text{ mm})}$$

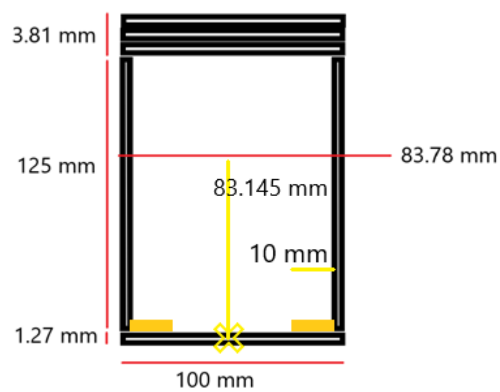
$$\therefore P_3 = 2290\text{ N}$$

Since the main sheet is bent into an “n” shape and the Q-value from the top and bottom are roughly equal due to a nearly symmetric cross-section, the 100 mm flange glue interfaces will resist more shear force than the 20 mm tabs. Thus, the governing Glue Shear Failure is at the tabs on the bottom flange:

$$\tau_g = \frac{V_{\text{max}} Q_{\text{glue}}}{I b_{\text{glue}}}$$

$$2\text{ MPa} = \frac{\left(\frac{P}{2}\right)(1.27\text{ mm} \times 100\text{ mm} \times 83.145\text{ mm})}{(2.17 \times 10^6\text{ mm}^4)(2 \times 10\text{ mm})}$$

$$\therefore P_4 = 16440\text{ N}$$

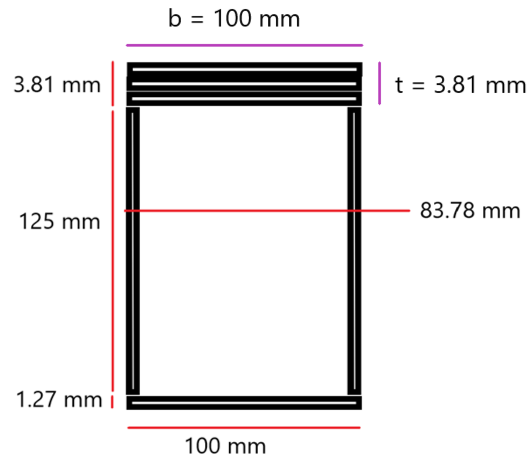


The triple flanges on the top will experience compression and they are restrained on both ends. Thus, $k = 0.0425$ is not applicable for this cross-section. Thus, <u>Top Flange Flexural Buckling ($k = 4$):</u>

$$\frac{M_{\text{max}} y_{\text{top}}}{I} = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

$$\frac{(140P)(46.3\text{ mm})}{2.17\times10^6\text{ mm}^4} = \frac{4\pi^2(4000\text{ MPa})}{12(1-0.2^2)}\left(\frac{3.81\text{ mm}}{100\text{ mm}}\right)^2$$

$$\therefore P_5 = 6660\text{ N}$$

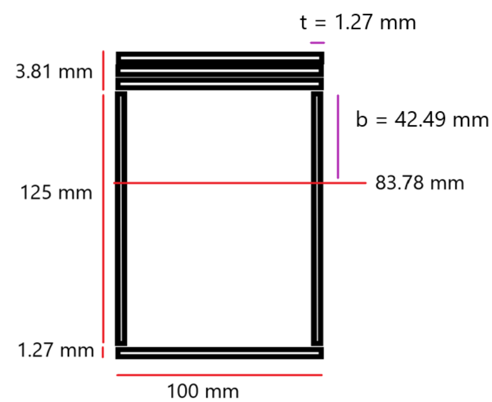


<u>Web Flexural Buckling ($k = 6$):</u>

$$\frac{My}{I} = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

$$\frac{(140P)(42.49\text{ mm})}{2.17\times10^6\text{ mm}^4} = \frac{6\pi^2(4000\text{ MPa})}{12(1-0.2^2)}\left(\frac{1.27\text{ mm}}{42.49\text{ mm}}\right)^2$$

$$\therefore P_6 = 6700\text{ N}$$



Web Shear Buckling:

$$\frac{V_{\max}Q_{\max}}{Ib} = \frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right]$$

$$\frac{\left(\frac{P}{2}\right)(19210\text{ mm}^3)}{(2.17\times 10^6\text{ mm}^4)(2.54\text{ mm})} = \frac{5\pi^2(4000\text{ MPa})}{12(1-0.2^2)}\left[\left(\frac{1.27\text{ mm}}{125\text{ mm}}\right)^2 + \left(\frac{1.27\text{ mm}}{130\text{ mm}}\right)^2\right]$$

$$\frac{\left(\frac{P}{2}\right)(19210\text{ mm}^3)}{(2.17\times 10^6\text{ mm}^4)(2.54\text{ mm})} = 3.40\ldots$$

$$\therefore P_7 = 1953\text{ N}$$

Summary (I)

Therefore, Concept I will fail at 1953 N due to shear buckling of the webs. Now, calculating the strength-to-weight ratio (χ):

$$\chi = \frac{P_{\max}}{A^{\%}m}$$

$A^{\%}$ is the percentage area of matboard used out of the given $1016\text{ mm}\times 813\text{ mm}$. dimensions. m is the total mass of the matboard. Thus, $A^{\%}m$ is the mass of the design.

$$\begin{aligned} A^{\%} &= \frac{980\times(100+100+100+100+2\times 10+2\times 125)+10\times 100\times 125)}{1016\times 813}\times 100\% \\ &= \frac{781600}{826008}\times 100\% \\ &= 94.6\% \end{aligned}$$

$$\begin{aligned} \chi &= \frac{1953\text{ N}}{0.946\times 750\text{ g}} \\ &= 2.75\text{ N}\cdot\text{g}^{-1} \end{aligned}$$

Concept II

Reaction Forces (II)

Since there are four reaction forces but only three equilibrium equations, the beam for Concept II is statically indeterminate. Fortunately, it is equivalent to a statically determinate system with no displacement at midspan. This system as the same SFD and BMD as Concept I.

However, since the curvature diagram (CD) depends on the beam's area moment of inertia, calculations will be written in terms of $P\ and\ I$. The curvature diagram is obtained by multiplying the BMD by $\frac{1}{EI}$, where $E = 4000$ MPa. I is assumed to be the smallest area moment of inertia of the beam cross-sections.

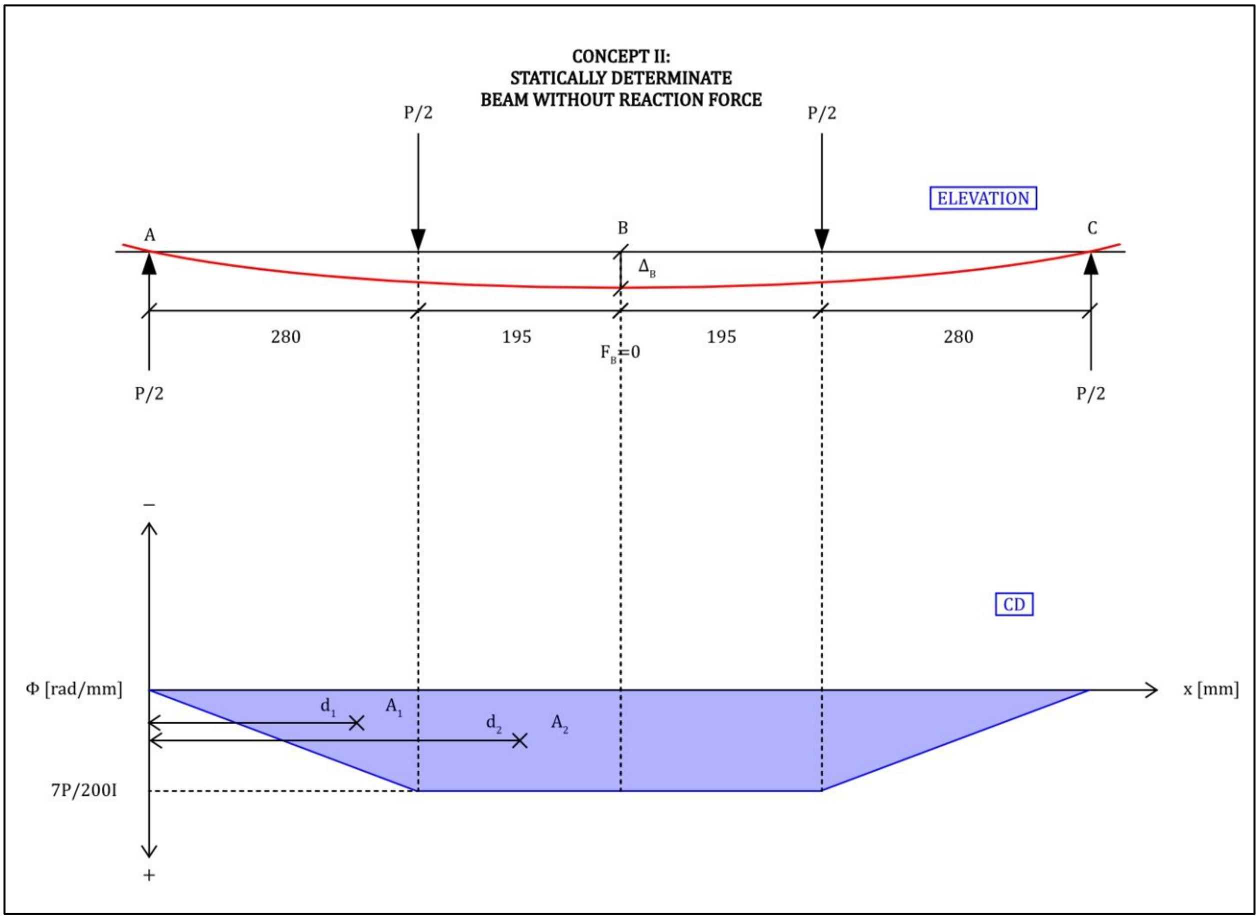


Figure 3. CD of statically determinate beam with no intermediate support.

$$
\begin{aligned}
\Delta_B &= \delta_{AB} \\
&= \int_A^B x\phi(x)dx \\
&= A_1 d_1 + A_2 d_2 \\
&= \left(\frac{1}{2}\cdot 280 \cdot \frac{7P}{200I}\right)\left(\frac{2}{3}\cdot 280\right) + \left(195 \cdot \frac{7P}{200I}\right)\left(280 + \frac{195}{2}\right) \\
&= \frac{167573P}{48I}
\end{aligned}
$$

Removing the two applied loads and adding the intermediate support reaction results in an equal but opposite displacement at midspan. Equating the two displacements yields the reaction force.

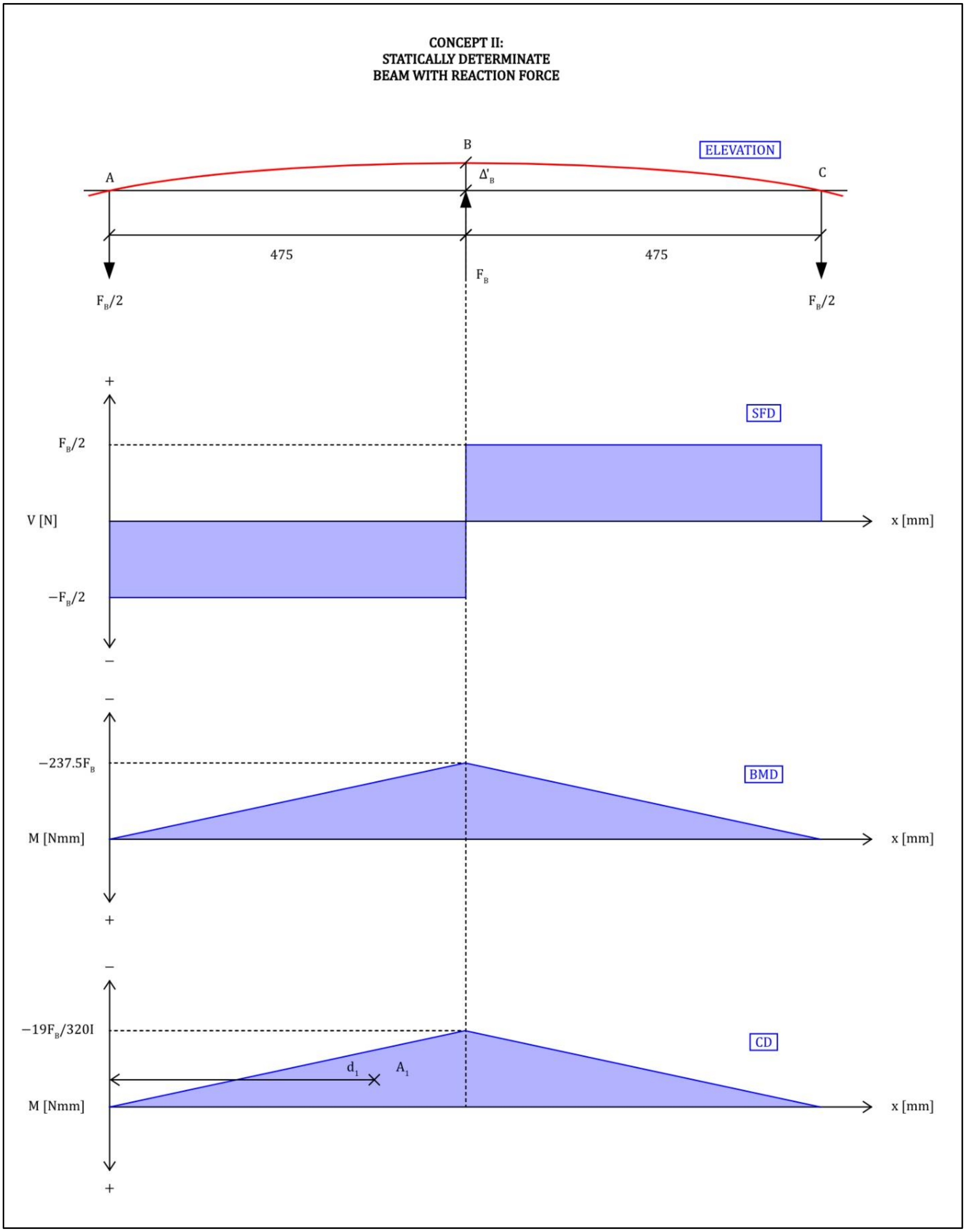


Figure 4. Analysis of beam deflection with all reaction forces but without applied loads.

$$\begin{aligned}\Delta_B' &= \delta'_{AB} \\ &= \int_A^B x\phi(x)dx \\ &= A_1 d_1 \\ &= \left(\frac{1}{2}\cdot 475\cdot\frac{-19F_B}{320I}\right)\left(\frac{2}{3}\cdot 475\right) \\ &= \frac{-857375F_B}{192I}\end{aligned}$$

$$\Delta_B = -\Delta'_B$$
$$\frac{167573P}{48I} = -\frac{-857375F_B}{192I}$$

$$\therefore F_B = \frac{670292P}{857375} \approx 0.7818P$$

Now, the SFD and BMD of Concept II can be drawn to find the maximum shear and moment. As before, the horizontal reaction forces are zero and $C_y = A_y$ due to symmetry.

$$\sum F_y = 0$$
$$2A_y + F_B = P$$
$$2A_y + \frac{670292P}{857375} = P$$

$$\therefore A_y = C_y = \frac{187083P}{1714750} \approx 0.1091P$$

Force Diagrams (II)

Refer to Figure 5 on the next page for the following calculations:

The cross-section changes at the zeros of the BMD since the location of tension and compression along the cross-section flip as the moment sign flips. The zeros can be calculated by determining the upper limit of integration of the SFD such that the area is zero.

$$\int_0^{\text{flip}} V(x)dx - \int_{\text{flip}}^{u} V(x)dx = 0$$

$$\text{flip} = 280\text{ mm}$$

On the left half of the beam span,

$$(280)(0.1091P) - (u_1 - 280)(-0.3909P) = 0$$
$$\therefore u_1 = 358\text{ mm}$$

By symmetry, the switching point on the right half of the beam span is simply,

$$\begin{aligned}\therefore u_2 &= L - u_1 \\ &= 950 - 358 \\ &= 592\text{ mm}\end{aligned}$$

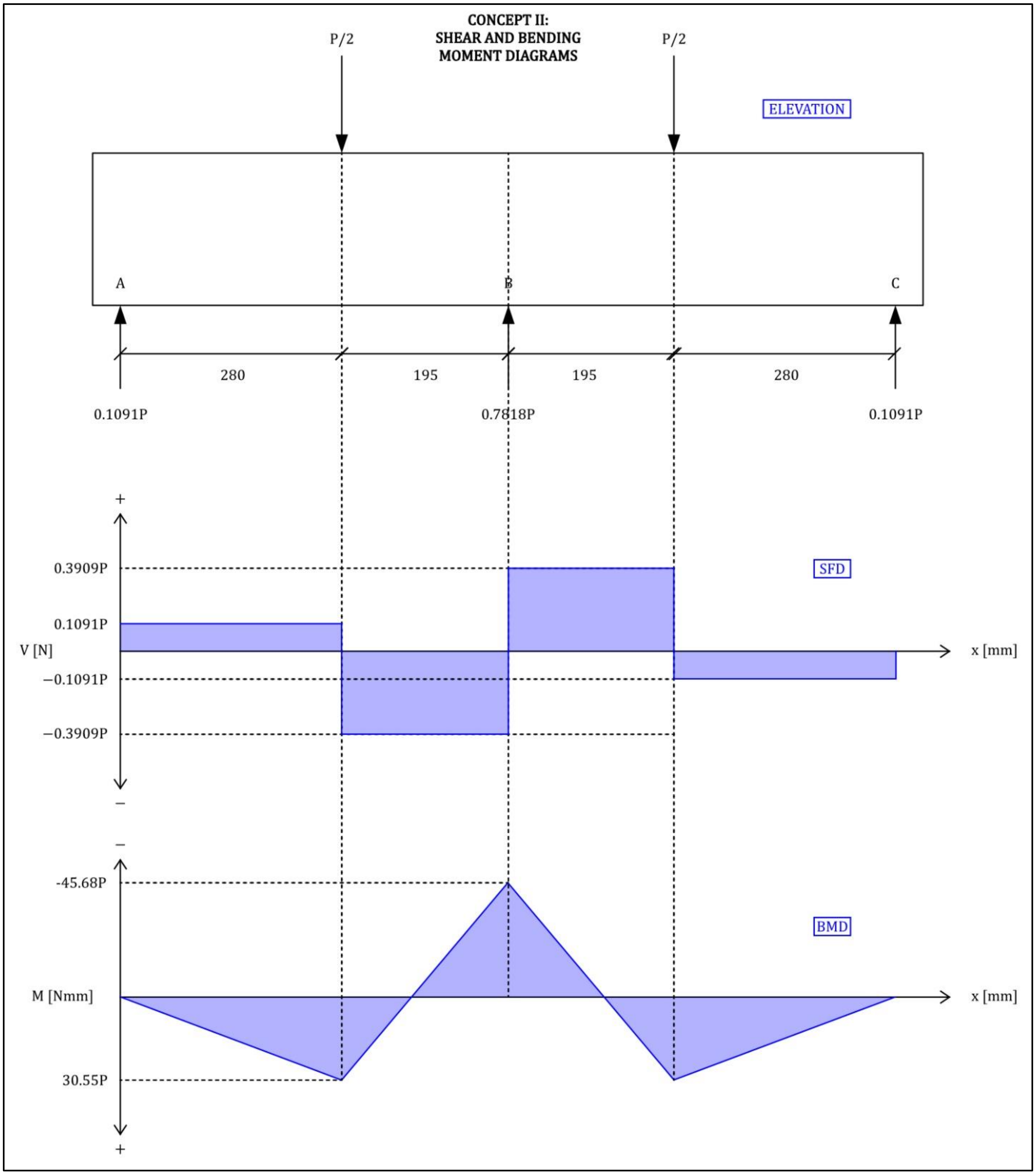


Figure 5. Labelled loading schematic, SFD, and BMD of Concept II.

Section Properties (II)

Refer to Sheet 5 in the Drawings document for the cross-section used in the calculations. Section A and Section B are vertical reflections of each other, so they share the same section properties. The only disparity is that the difference in $\bar{y}$ is the total height of the cross-section, thus, inverting y_{top} and y_{bottom}.

<u>Cross-Section A (inverted Cross-Section B):</u>

$$\bar{y} = \frac{A_1 d_1 + A_2 d_2 + A_3 d_3}{A_T}$$
$$= \frac{(100 \times 1.27) \times \frac{1.27}{2} + (5.08 \times 50) \times \left(1.27 + \frac{50}{2}\right) + (100 \times 2.54) \times (50 + 2.54)}{(100 \times 1.27) + (5.08 \times 50) + (100 \times 2.54)}$$
$$= 31.6 \text{ mm} = y_{\text{max}}$$

$$y_{\text{min}} = 50 + (3)(1.27) - \bar{y} = 22.2 \text{ mm}$$

$$I = I_0 + A_0 {d_0}^2 + I_1 + A_1 {d_1}^2 + I_2 + A_2 {d_2}^2$$
$$= \frac{100 \times 1.27^3}{12} + (127)(31.015)^2 + \frac{5.08 \times 50^3}{12} + (254)(5.38)^2 + \frac{100 \times 2.54^3}{12} + (254)(20.89)^2$$
$$= 0.293 \times 10^6 \text{ mm}^4$$

<u>Intermediate support cross-section:</u>

$$I = I_{\text{out}} - I_{\text{in}}$$
$$= \frac{50^4}{12} - \frac{(50 - (2)(1.27))^4}{12}$$
$$= 98000 \text{ mm}^4$$

Failure Modes (II)

Since the cross-section is inverted based on regions of positive/negative moment, they share the same section properties, so, calculations for the two different cross-sections/regions are identical. Therefore, the highest absolute value for M and V are used:

$$M = 45.68P \qquad V = 0.3909P$$

The double flanges are designed to always be in compression, so, the single flange is in tension.

<u>Flexural Tensile Failure:</u>	<u>Flexural Compressive Failure:</u>
$\sigma_t = \frac{M y_{\text{max}}}{I}$	$\sigma_c = \frac{M y_{\text{min}}}{I}$
$30 = \frac{(45.68P) \times 31.65}{293431}$	$6 = \frac{(45.68P) \times 22.16}{293431}$
$\therefore P_1 = 6090 \text{ N}$	$\therefore P_2 = 1739 \text{ N}$

Calculating the first moments of area for the shear failures:

$$\begin{aligned} Q_{\text{mid}} &= A_1 d_1 + A_2 d_2 \\ &= (30.38 \times 5.08) \times 15.19 + (100 \times 1.27) \times 30.015 \\ &= 6160\ mm^3 \end{aligned}$$

$$\begin{aligned} Q_{\text{thin plate}} &= A_1 d_1 \\ &= (100 \times 1.27) \times 31.015 \\ &= 3940\ mm^3 \end{aligned}$$

The thick plate is folded, continuous/flush with the webs, <u>Shear Failure of Walls:</u> $$\tau_{\text{shear}} = \frac{VQ_{\text{mid}}}{Ib}$$ $$4 = \frac{(0.3909P) \times 6156}{293431 \times 4 \times 1.27}$$ $$\therefore P_3 = 2470 \text{ N}$$	<u>Glue Shear Failure:</u> $$\tau_{\text{glue}} = \frac{VQ_{\text{thin plate}}}{Ib}$$ $$2 = \frac{(0.3909P) \times 3939}{293431 \times 2 \times 10}$$ $$\therefore P_4 = 7620 \text{ N}$$

<u>Compressive Flange Buckling ($k = 4$):</u> $$\frac{My_{\text{min}}}{I} = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$ $$\frac{(45.68P)(22.16)}{293431} = \frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{100-5.08}\right)^2$$ $$\therefore P_5 = 2840 \text{ N}$$	<u>Flexural Web Buckling ($k = 6$):</u> $$\frac{My}{I} = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$ $$\frac{(45.68P)(19.62)}{293431} = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{2.54}{19.62}\right)^2$$ $$\therefore P_6 = 112800 \text{ N}$$

The largest distance between diaphragms minimizes the failure load for <u>Shear Web Buckling:</u>

$$\frac{VQ_{\text{mid}}}{Ib} = \frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right]$$

$$\frac{(0.3909P)(6156)}{(293431)(5.08)} = \frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{2.54}{50}\right)^2 + \left(\frac{2.54}{260}\right)^2\right]$$

$$\therefore P_7 = 28400 \text{ N}$$

The intermediate column support also experiences compression, and may fail by compression:

Crushing: $$\sigma_c = \frac{F}{A}$$ $$6 = \frac{0.7818P}{247.55}$$ $$\therefore P_8 = 1900\text{ N}$$
Global Euler Buckling: $$P_e = \frac{\pi^2 EI}{L^2}$$ $$0.7818P = \frac{\pi^2(4000)(98038)}{600^2}$$ $$\therefore P_9 = 13800\text{ N}$$
Thin-Plate Buckling of Restrained Edges ($k = 4$): $$\frac{F}{A} = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$ $$\frac{0.7818P}{247.55} = \frac{4\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{50-2.54}\right)^2$$ $$\therefore P_{10} = 3100\text{ N}$$

Summary (II)

Therefore, Concept II will fail at 1739 N due to flexural compressive failure. Now, calculating the strength-to-weight ratio (χ):

$$A^{\%} = \frac{(980 \times (225.08 + 200 + 100)) + (600 \times 210) + (1016 \times 50) + (50 \times 100)}{1016 \times 813}$$
$$= \frac{696378.4}{826008}$$
$$= 84.3\%$$

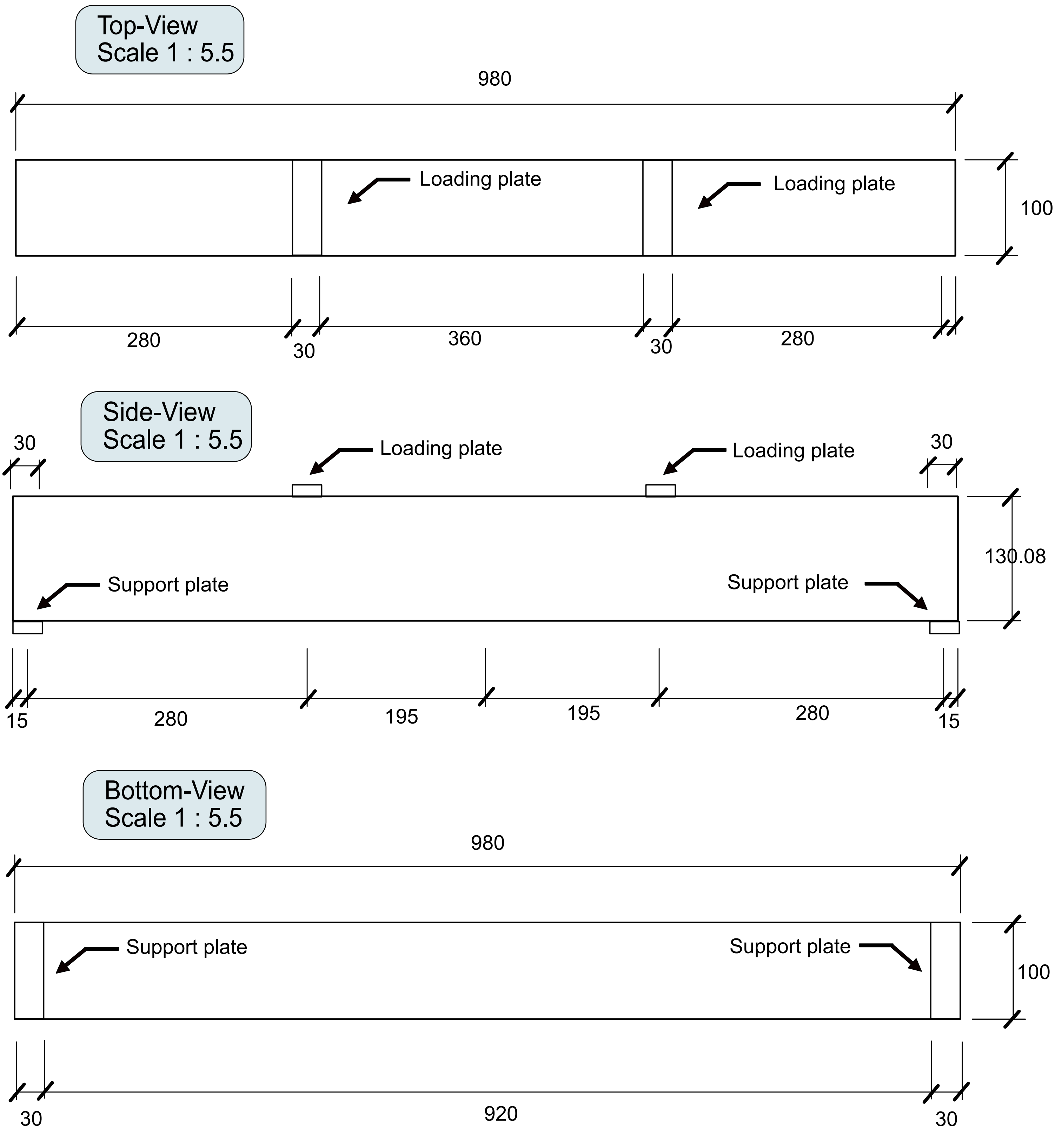
$$\chi = \frac{P_{\max}}{A^{\%} m}$$
$$= \frac{1739\text{ N}}{0.843 \times 750\text{ g}}$$
$$= \frac{1739\text{ N}}{0.946 \times 750\text{ g}}$$
$$= 2.75\text{ N}\cdot\text{g}^{-1}$$

```python
# Hamza Dugmag

# one web glued at mid-width
# [force] = N and [length] = mm

# Cross-section:
# ======================
# |         |          |
# |         |          |
# |         |          |
# |         |          |
# |         |          |
# |         |          |

"""PARAMETERS"""
thickness = 1.27
flange_width = 95
web_height = 127
glue_interface = 10

diaphragm_width = flange_width
diaphragm_height = web_height

assert flange_width <= 100
assert flange_width + 2*web_height + (web_height + glue_interface) + min(diaphragm_width, diaphragm_height) <= 813
assert 8*max(diaphragm_width, diaphragm_height) <= 1016

"""SECTION PROPERTIES"""
sum_A = flange_width*thickness + 3*(web_height*thickness)
y_bar = (flange_width*thickness)*web_height + 3*((web_height*thickness)*web_height/2)
y_bar /= sum_A

I_flange = (flange_width*thickness**3)/12 + (flange_width*thickness)*(y_bar - web_height)**2
I_web = 3 * (thickness*web_height**3)/12 + (thickness*web_height)*(y_bar - web_height/2)**2
I = I_flange + I_web

M_distance = 120  # M_max = 120P
y_top = web_height - y_bar
y_bot = y_bar

"""WALL FAILURES"""
# WALL TENSILE FAILURE
tensile_strength = 30
P1 = (tensile_strength*I) / (M_distance*y_bot)
print("Tensile:\t\t", int(round(P1, 0)))

# WALL COMPRESSIVE FAILURE
compressive_strength = 6
P2 = (compressive_strength*I) / (M_distance*y_top)
print("Compressive:\t\t", int(round(P2, 0)))

# WALL SHEAR FAILURE
V_coeff = 1/2  # V_max = P/2
b = 3*thickness  # mid-depth

Q_flange = (flange_width*thickness)*(web_height - y_bar)
```

```python
Q_webs = 3 * ((web_height - y_bar)*thickness)*((web_height - y_bar)/2)
Q = Q_flange + Q_webs

shear_strength = 4
P3 = (shear_strength*I*b)/(V_coeff*Q)
print("Wall Shear:\t\t", int(round(P3, 0)))

"""GLUE FAILURE"""
# GLUE SHEAR FAILURE
glue_strength = 2
P4 = (glue_strength*I*glue_interface)/(V_coeff*Q_flange)
print("Glue Shear:\t\t", int(round(P4, 0)))

"""THIN-PLATE BUCKLING"""
# FLANGE BETWEEN WEBS
flange_perp_width = flange_width/2
poisson = 0.2
elasticity = 4000
pi = 3.14159265359

flexural_compressive_stress = (M_distance*y_top)/I
flange_buckling_compressive_stress = (4*elasticity*pi**2)/(12*(1 - poisson**2)) * (thickness/flange_perp_width)**2
P5 = flange_buckling_compressive_stress/flexural_compressive_stress
print("Flange Buckling (k=4):\t", int(round(P5, 0)))

# FLANGE PERTRUDING WEBS
print("Flange Buckling (k=.4):\t n/a")

# WEB FLEXURAL BUCKLING
web_perp_width = web_height - y_bar
web_buckling_compressive_stress = (6*elasticity*pi**2)/(12*(1 - poisson**2)) * (thickness/web_perp_width)**2
P6 = web_buckling_compressive_stress/flexural_compressive_stress
print("Flexural Web Buckling:\t", int(round(P6, 0)))

# WEB SHEAR BUCKLING
span = 950
diaphragm_sep = 600

shear_stress = (V_coeff*Q)/(I*b)
shear_buckling_stress = (5*elasticity*pi**2)/(12*(1 - poisson**2)) * ((thickness/web_height)**2 + (thickness/diaphragm_sep)**2)
P7 = shear_buckling_stress/shear_stress
print("Shear Web Buckling:\t", int(round(P7, 0)))
```

Matboard Project Drawings

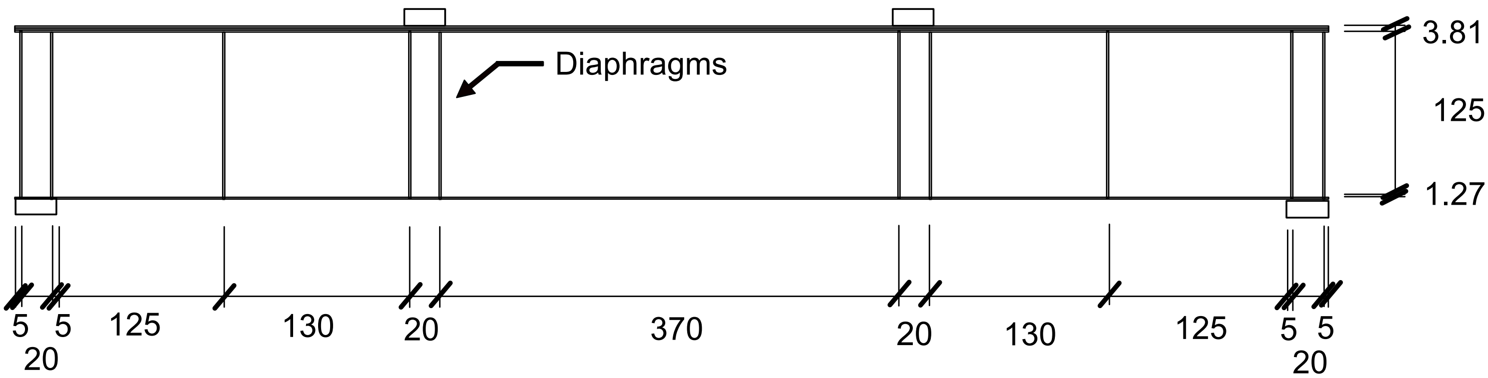
Group 51: Eric Su, Hamza Dugmag, Rory Gao, Victor Milne
TA: Michel Rahman
Engineering Science – CIV102
University of Toronto
December 8, 2020



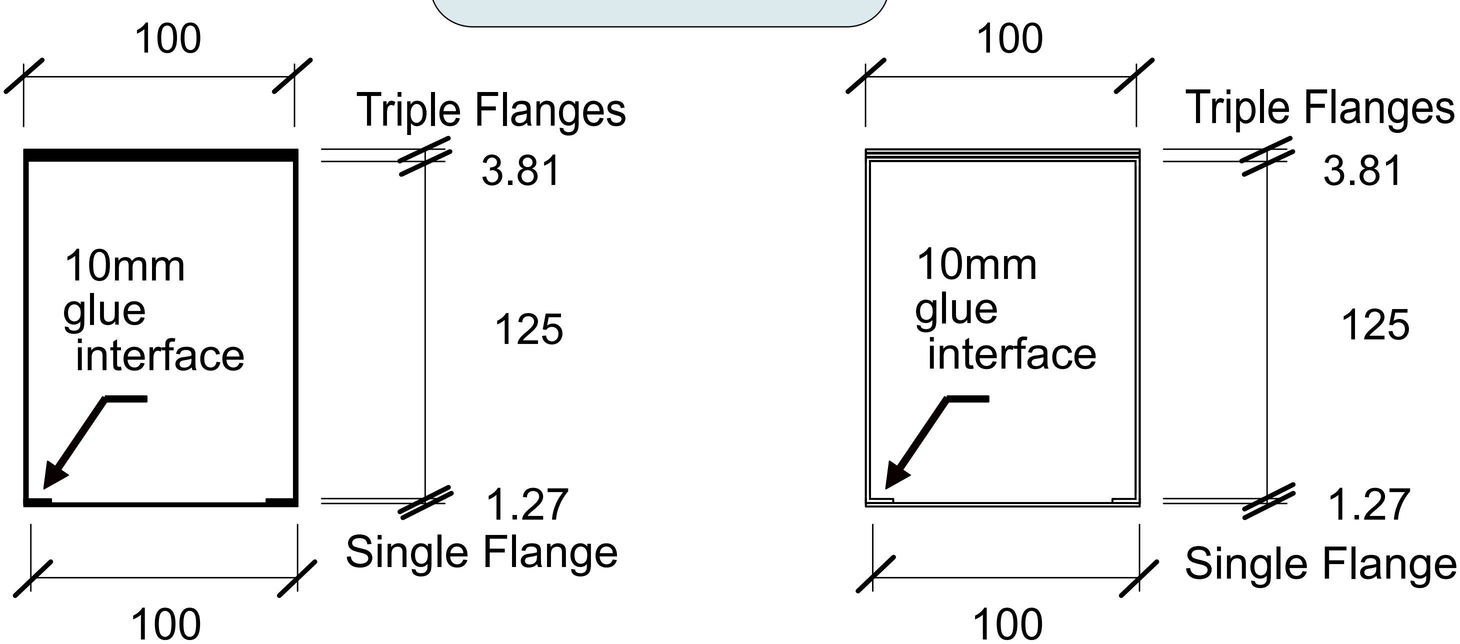
Dimension in millimeters (mm).

No.	Date.	Reason for issue	Design:	Team51	Concept #1: Elevation and Span Views (Sheet 1)
1	20-12-07	Issued for Submission	Drawn:	Eric Su	
			Checked:	Victor Milne	
				Rory Gao	
				Hamza Dugmag	Matboard Project (CIV102)

Longitudinal Cross Section
Scale 1 : 5.5



Cross Section
Scale 1 : 2.75

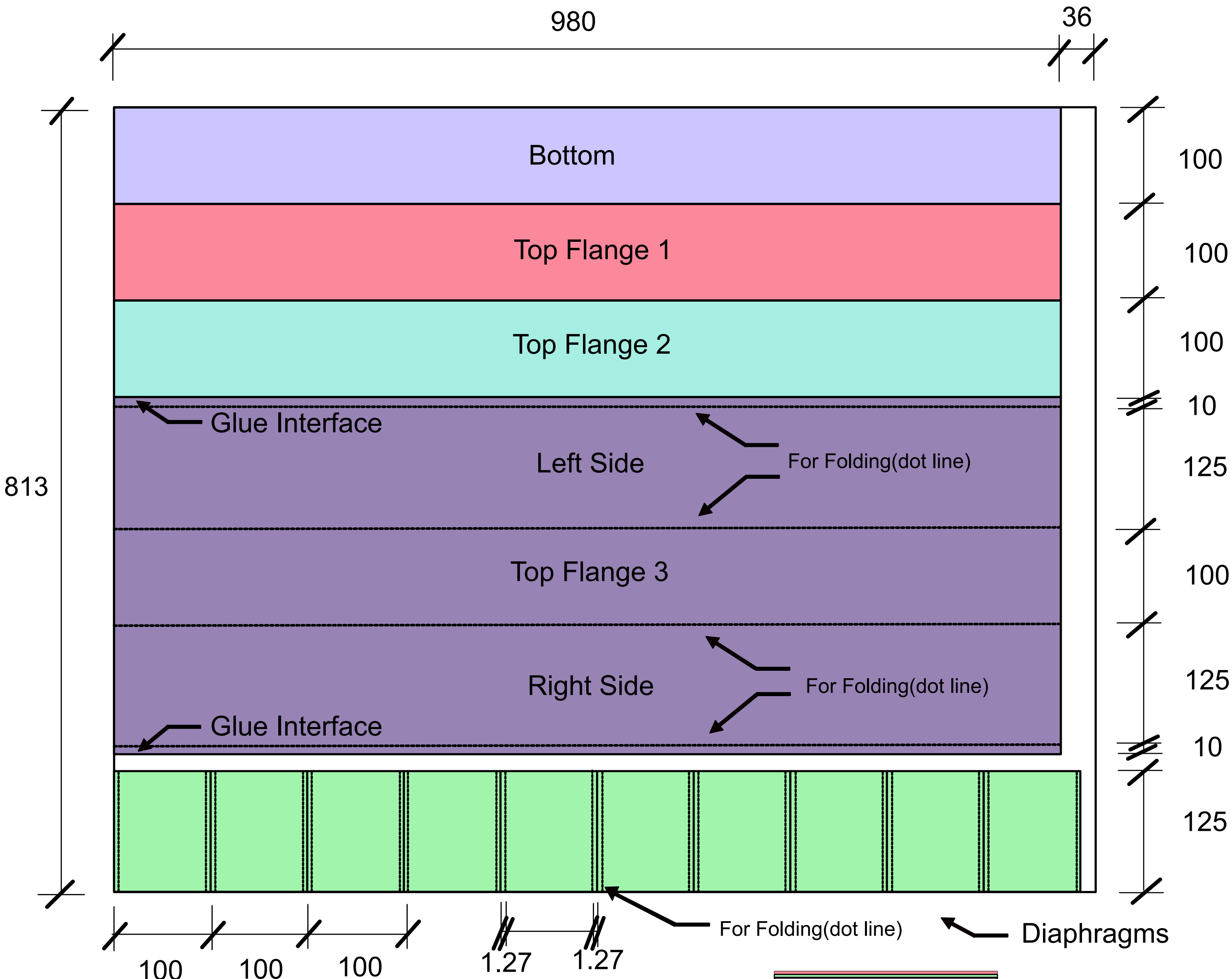


Dimension in millimeters (mm).

No.	Date.	Reason for issue	Design:	Team51
1	20-12-07	Issued for Review	Drawn:	Eric Su
2	20-12-08	Issued for Submission	Checked:	Victor Milne
				Rory Gao
				Hamza Dugmag

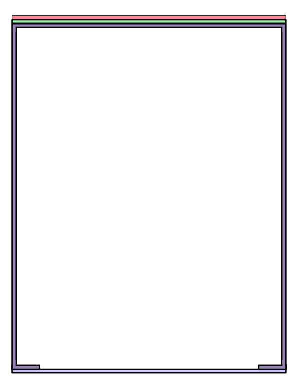
Concept #1: Cross Sections (Sheet 2)

Matboard Project (CIV102)



Matboard Construction
Scale 1 : 6.35

Shaded Cross-Section
Scale 1 : 2.75



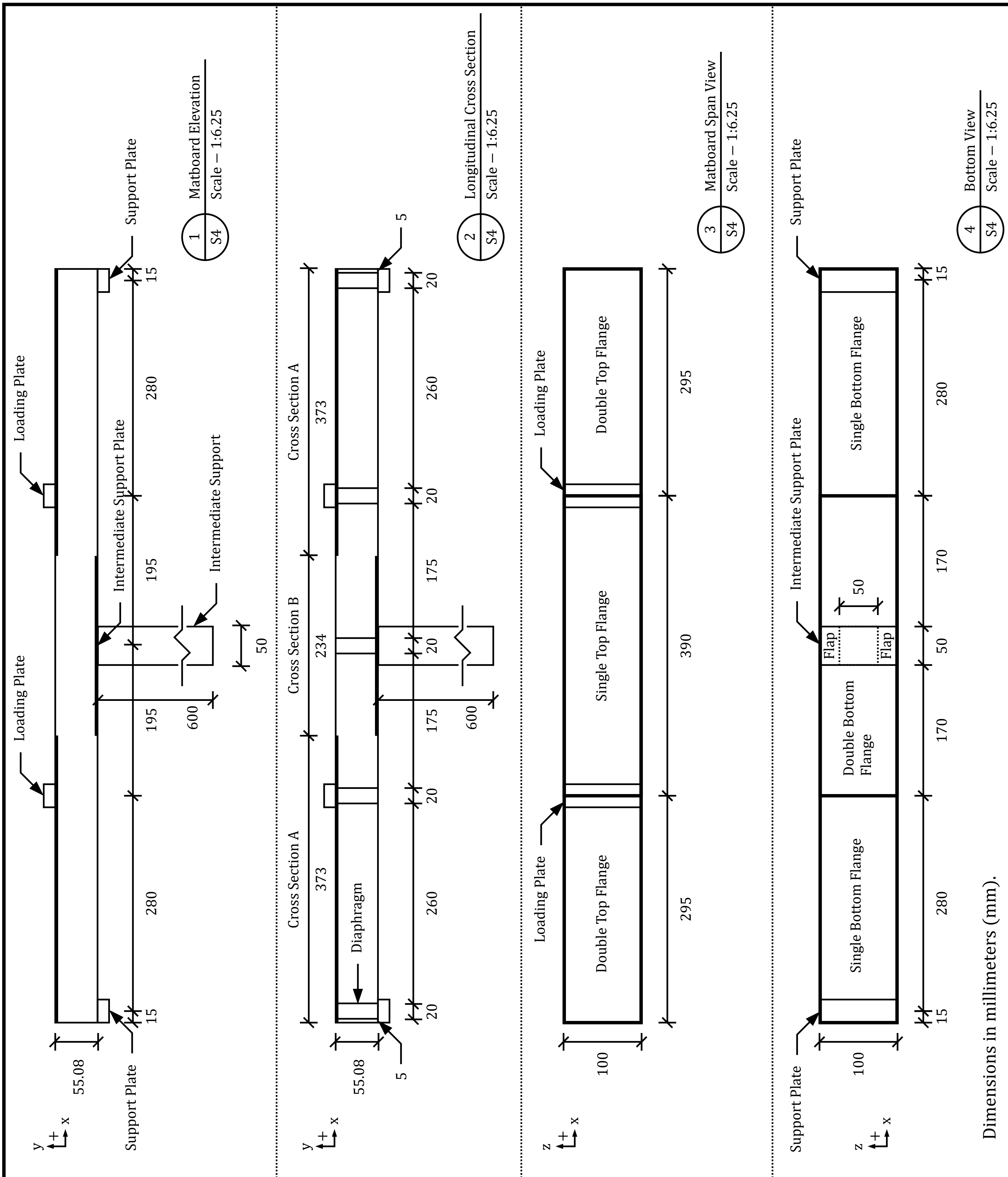
Refer to Sheet 2 for dimensions.

Dimensions in millimeters (mm).

No.	Date.	Reason for issue	Design:	Team51
1	20-12-07	Issued for Submission	Drawn:	Eric Su
			Checked:	Victor Milne
				Rory Gao
				Hamza Dugmag

Concept #1: Matboard Construction (Sheet 3)

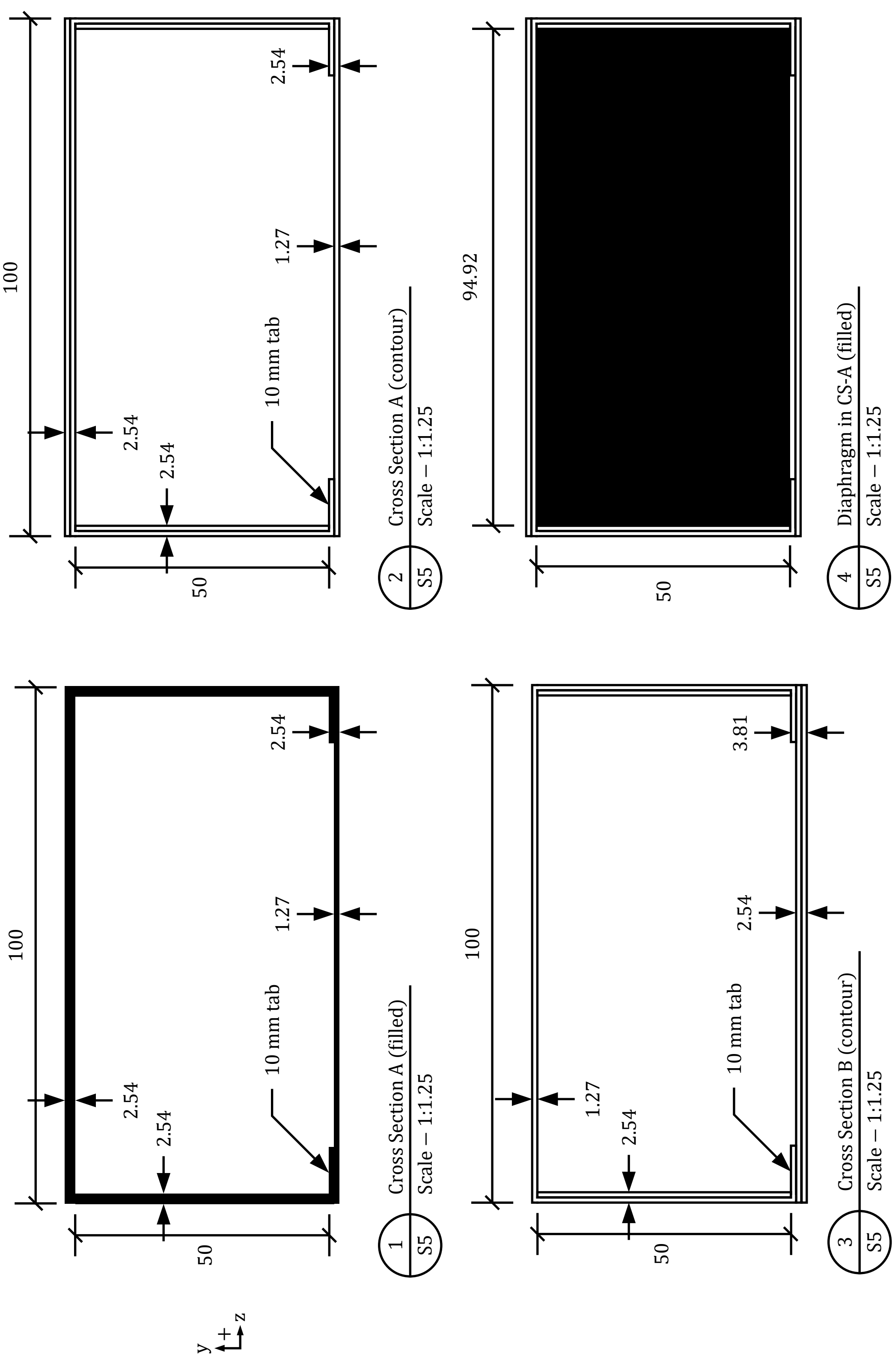
Matboard Project(CIV102)

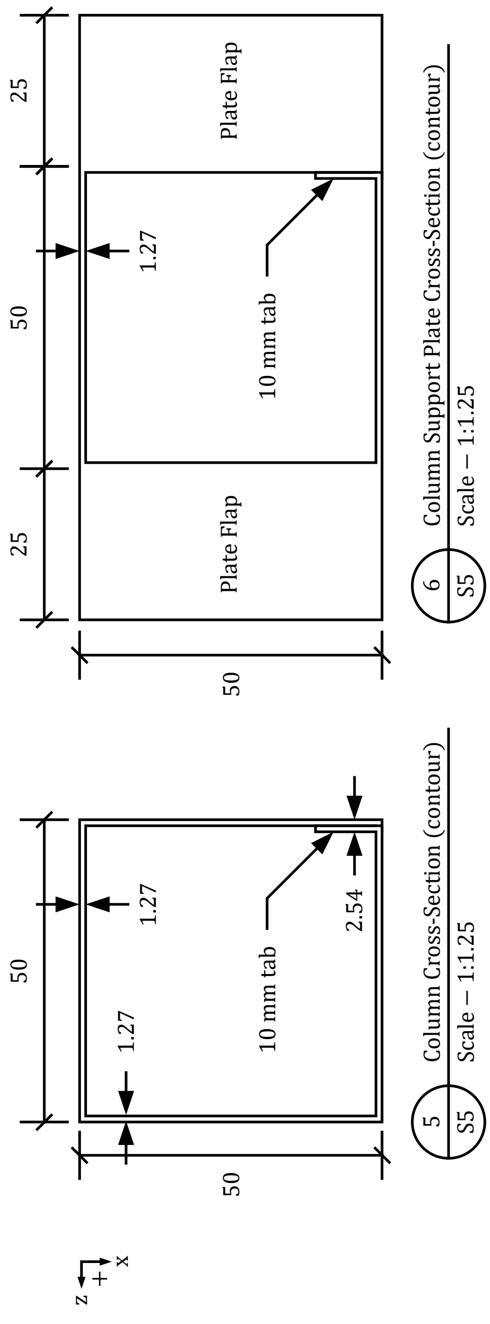


No.	Date	Reason for Issue	Design:	Team 51
1	2020-12-07	Issued for Review	Drawn:	Hamza Dugmag
2	2020-12-07	Issued for Submission	Checked:	Victor Milne
				Rory Gao
				Eric Su

Concept II: Elevation and Span Views

Matboard Project (CIV102) | Sheat S4



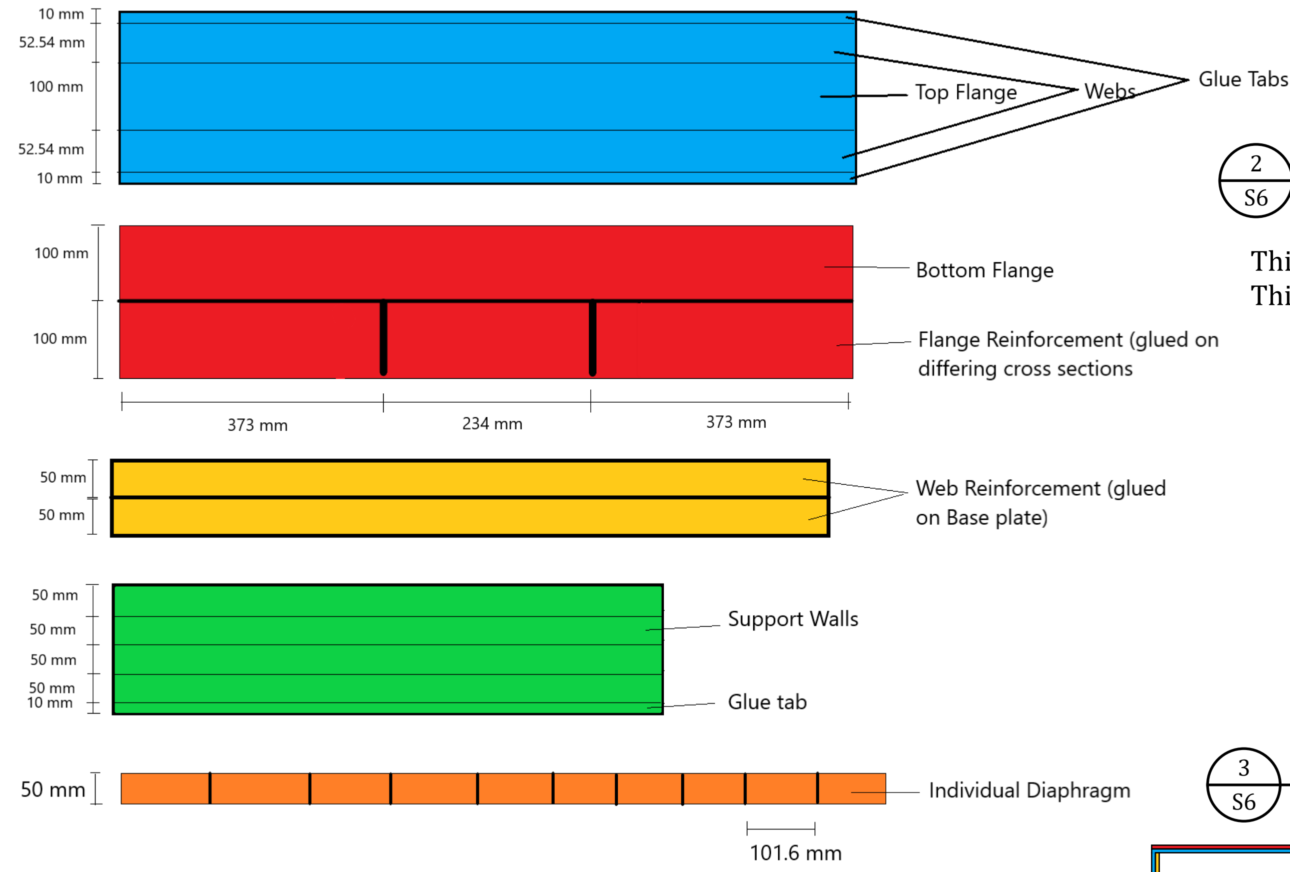


Dimensions in millimeters (mm).
CS-B is a vertical reflection of CS-A, so, filled and diaphragm drawings are similar.

No.	Date	Reason for Issue	Design:	Team 51
1	2020-12-07	Issued for Review	Drawn:	Hamza Dugmag
2	2020-12-07	Issued for Submission	Checked:	Victor Milne
				Rory Gao
				Eric Su

Concept II: Beam and Column Cross-Sections

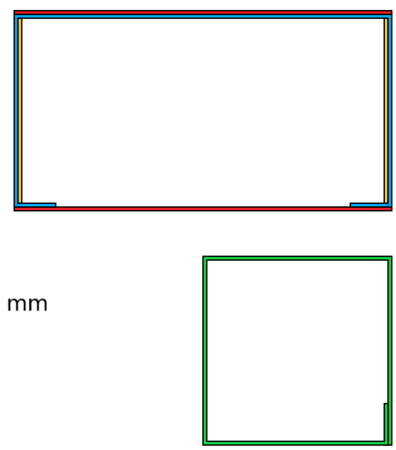
Matboard Project (CIV102) | Sheat S5



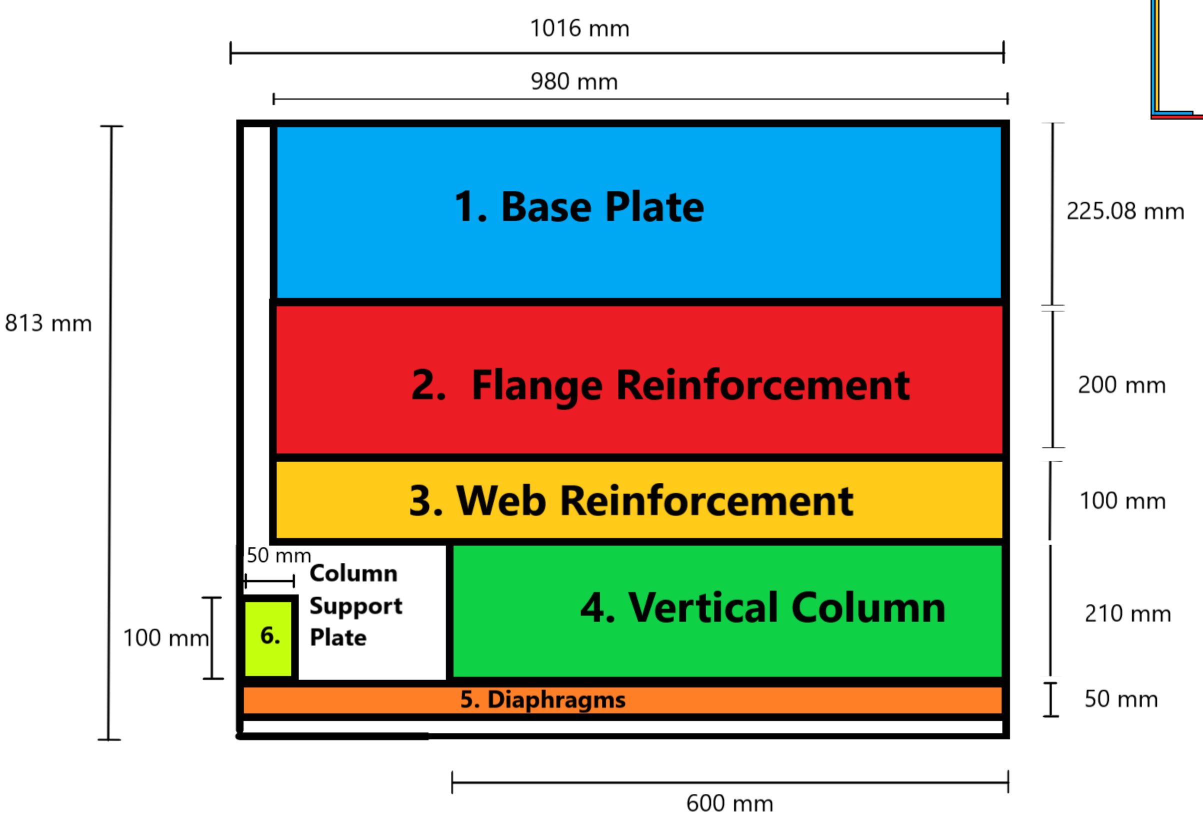
2 / S6 — Matboard Piece Cuts
Scale – 1:10

Thick lines for cut lines.
Thin lines for fold lines.

3 / S6 — Cross-Section Shading
Scale – 1:2



Refer to S5 for dimensions.



1 / S6 — Matboard Piece Division
Scale – 1:10

Dimensions in millimeters (mm).

No.	Date	Reason for Issue		
1	2020-12-07	Issued for Review	Design:	Team 51
2	2020-12-08	Issued for Submission	Drawn:	Rory Gao
			Checked:	Victor Milne
				Hamza Dugmag
				Eric Su

Concept II: Matboard Usage and Construction

Matboard Project (CIV102) | Sheat S6

Matboard Project Timelog

Group 51: Eric Su, Hamza Dugmag, Rory Gao, Victor Milne
TA: Michel Rahman
Engineering Science – CIV102
University of Toronto
December 8, 2020

2020-11-26
@7:00-7:45 PM
Everyone:
- Discussed how to start out project.
- Read through/understood/clarified the assignment.
- Considered automating calculations using Google Sheets or Python.
- Agreed on communication method (Discord).
- Decided to go for the bonus.
- Next steps: Each one comes up with 1-2 basic cross-section design, we collaborate on improving the "best" design. Create spreadsheet template to automate cross-section/force calculations.

2020-11-30
@3:30 - 4:30 PM
Rory: Came up with candidate cross section template, still need to do calculations.

2020-12-01
@12:00 - 3:00 PM
Hamza: Explored bent-N beams with webs at mid-width (2 flanges, 3 flanges, and 4 flanges). Also, researched L-beams, however, this did not satisfy flange buckling loads ($k = 0.0425$).

@3:00 - 4:15 PM
Hamza: Parametrized n-beam calculations and explored different dimensions of a 2-flange n beam (programmed Python script). But, noticed that only 5 diaphragms fit, which violates the constraint. Will discuss this design with the team.

2020-12-02
@7:00 - 8:00 PM
Victor: Reviewed the material properties and required dimensions, theory-crafted various designs, did some basic optimization of a box girder design.

2020-12-03
@2:00 - 4:45 PM
Everyone:
- Reviewed Rory's design and double-checked section properties.
- Reviewed the 3 other candidate designs and discussed values and limitations.
- Found that Hamza's design could not accommodate for 8 diaphragms without sacrificing compression load capacity.
- Discussed our intuition behind the property sections to make sure we know what we are doing.
- Rechecked calculations for Victor's design.
- Decided to meet again on Saturday to discuss Concept II.

Victor: Identified how he optimized his design: increasing I and decreasing y_{top} with as few materials.
Hamza: Shared screen and presented rough sketches of diagrams and compiled hand-written calculations from the team.
Rory: Created template for compiling the calculations for Concept I, started to work on said calculations.

2020-12-05

@2:30-3:00 PM

Hamza: Calculated Concept I reaction forces and constructed SFD and BMD.

@7:00-7:30 PM

Hamza, Rory, Victor:

- Discussed next steps.
- Checked on each other's work.
- Discussed how to approach Concept II.
- Decided to meet up tomorrow.

@8:00-9:15 PM

Hamza: Solved statically indeterminate beam: calculated midspan deflection, intermediate support reaction. Drew BMD and SFD of Concept II.
Rory: Calculated failure loads and modes for Concept I as well as section properties.
Victor: Drafted second design, focusing on shear based on BMD and SFD results.

2020-12-06

@12:00-1:00 PM

Everyone:

- Discussed monosymmetric I-beam to maximize I and accommodate for column materials but found that longer flange buckles easily.

Hamza, Rory, Eric: Discussed column design: property section and failure loads/modes.
Victor: Optimized/designed design #2 based on theory, intuition, and the BMD/SFD.
Rory: Sketched matboard construction and laid out some guidelines for Concept II.

@1:00-2:00 PM

Hamza: suggested two different cross-sections for the beam depending on the sign of the moment (while maintaining I to maintain the support force).
Victor: Calculated section properties and loads. Discussed distributing the support reaction force onto the beam by folding the matboard.

@2:00-2:45 PM

Everyone:

- Double checked calculations together and finalized the design.
- Delegated Concept II tasks and decided to meet tomorrow at 3pm to finalize the report.

Hamza, Victor: Sketched final beam and support cross-sections.
Rory, Eric: Sketched final matboard construction for concepts 1 and 2.

@9:30-11:00 PM

Hamza: Drew concept II cross-sections.

@9:00-12:00 PM

Eric: Drew concept I elevation and cross section view.
Rory: Verified calculations for concept 1 matboard usage.

2020-12-07
@11:00 PM-12:30 AM
Hamza: Drew concept II elevation and cross-section view

@1:00-2:30 AM
Hamza: Drew concept II top and bottom views.
Hamza, Victor: Discussed methodology for calculating midspan deflection and intermediate support reaction.
Eric: Modified concept I cross-section drawings and Matboard construction.

@11:00-12:00 PM
Hamza: drew column support cross-section.

@3:00-3:30 PM
Everyone:

- Went through Calculations and Drawings together.
- Set out next steps before working on the report together.
- Thought of what our recommended design would be and thought of a few arguments.

Rory: Started on matboard drawings for concept 2.

@7:30-8:30 PM
Rory: Updated calculations for concept 1.

@11:00-11:30 PM
Rory: Finished matboard usage calculations and drawings for concept 2.

2020-12-08
@11:00 AM-12:00 PM
Rory: Typed out final report + summary of bridges.
Eric: Finished Drawings for concept I.
Eric: Finished Matboard usage calculation and instruction for concept I.

@5:00-8:30 PM
Hamza: Compiled, formatted, and organized all calculations into a single PDF for submission.

@7:30 - 8:30 PM
Rory: Finished instructions for design #2, changed matboard usage drawings for #2.
Victor: Wrote rationale for design choices segment.
Victor: Wrote recommendation segment.
Eric: Wrote instructions for matboard construction in design #1.

@8:30 - 10:00 PM
Hamza: Compiled and organized all drawings into a single PDF for submission.

@10:00-11:30 PM
Hamza: Formatted timelog and report for submission.

Signatures

Eric Su

Hamza Dugmag

Rory Gao

Victor Milne