For each problem, explain/justify how you obtained your answer in order to obtain full credit. In fact, most of the credit for each problem will be given for the derivation/model used as opposed to the final answer.

Question 1. Say we roll a fair 6-sided die six times. Using the fact that each roll is an independent random event, what is the probability that we roll:
   (a) The sequence 1,1,2,2,3,3 ?
   (b) A three exactly 4 times?
   (c) A single 1.
   (d) One or more 1's

Question 2. Suppose one 6-sided and one 20-sided die are rolled. Let A be the event that the first die comes up 1 and B that the sum of the dice is 2. Are these events independent ? Explain using the formal definition of independence.

Question 3. Say a hacker has a list of n distinct password candidates, only one of which will successfully log her into a secure system.
   (a) If she tries passwords from the list uniformly at random, deleting those passwords that do not work, what is the probability that her first successful login will be (exactly) on her k-th try?
   (b) When $n = 6$ and $k = 3$ what is the value of this probability ?
   (c) Now say the hacker tries passwords from the list at random, but does not delete previously tried passwords from the list. She stops after her first successful login attempt. What is the probability that her first successful login will be (exactly) on her k-th try?
   (d) When $n = 6$ and $k = 3$ what is the value of this probability ?
   Hint: use the fact that the outcome of each try is an independent random event (since passwords are selected uniformly at random at each attempt)

Question 4. A website wants to detect if a visitor is a robot. They decide to deploy three CAPTCHA tests that are hard for robots and if the visitor fails in one of the tests, they are flagged as a possible robot. The probability that a human succeeds at a single test is 0.95, while a robot only succeeds with probability 0.3. Assume all tests are independent.
   (a) If a visitor is actually a robot, what is the probability they get flagged?
   (b) If a visitor is human, what is the probability they get flagged?
   (c) The fraction of visitors on the site that are robots is $1/10$. Suppose a visitor gets flagged. What is the probability that visitor is a robot? Hint: use Bayes Rule.