Statistical Methods for Computer Science

Assignment 2 — STU33009

1. (a) The sample space is the set of all possible outcomes. A six-sided die has a sample space of 6 elements. A six-sided die rolled three times would have a sample space of size $6 \cdot 6 \cdot 6 = 6^3$:

$$6^3 = 216$$

   (b) In order to calculate this, we can first calculate the number of outcomes in which no 2 is rolled. We can now only pick from 5 possibilities in each of the three rolls:

$$5^3 = 125$$

   We can now subtract this quantity from the total size of the sample space to get the number of outcomes in which at least a 2 is rolled:

$$216 - 125 = 91$$

   The probability that at least one 2 is rolled is the following:

$$\frac{91}{216} = 0.4212962963$$

   (c)
```
iters = 100000;
outcomes = randi([1,6],[1,3,iters]);
count = 0;

for i = 1:iters
    if any(outcomes(:,:,i) == 2)
        count = count + 1;
    end
end

probability = count/iters

>>
Probability =
    0.4222
```

   (d) If all of the rolls resulted in a 6, the sum would be 18. Since we want the sum to be 17, a single roll must be a 5 and the rest 6. However, this 5 could be outcome of any one of the three rolls:

$$\binom{1}{3} = 3$$

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Therefore, there are 3 possible outcomes in which the sum can be 17. The probability of one of these outcomes occurring would be:

$$\frac{3}{216} = 0.01388888889$$

(e) We can calculate this by calculating the probability that the remaining die rolls add up to 11. There are two remaining die rolls, therefore the sample space now becomes:

$$6^2 = 36$$

In order for 2 die rolls to add up to an 11, one of the die rolls should be a 5, there are 2 ways this can happen. Therefore, the probability that the three rolls sum to 12 given that the first roll is a 1 would be:

$$\frac{2}{36} = 0.05555555556$$

2. (a) There are two ways in which the second die can roll a 5. In one case, the first die could roll a 1, which would result in a second six-sided die to be rolled, which can roll a 5. The probability of the first die rolling a 1 would be $\frac{1}{6}$ and the probability of the second die rolling a 5 would also be $\frac{1}{6}$. The probability of this case would be:

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

In the second case, the fist die could roll a number other than 1. The probability of this would be $\frac{5}{6}$. Hence, the subsequent throw would be of a 20-sided die which would need to roll a 5. The probability of this happening would be $\frac{1}{20}$. The probability of this case would be:

$$\frac{5}{6} \cdot \frac{1}{20} = \frac{5}{120} = \frac{1}{24}$$

We can then add the probability of these cases to obtain the probability of any of them happening:

$$\frac{1}{36} + \frac{1}{24} = \frac{5}{72} = 0.06944444444$$

(b) In order for the second roll to be a 15, the first roll mustn't be a 1. the probability of the first roll not being a 1 is $\frac{5}{6}$. Subsequently, the probability of the second roll being a 15 is $\frac{1}{20}$. The probability of both happening would be:

$$\frac{5}{6} \cdot \frac{1}{20} = \frac{5}{120} = \frac{1}{24} = 0.04166666667$$

3. Let E be the event that the suspect is guilty.
   Let F be the event that the suspect possesses the found characteristic.

   We know that the probability of the suspect being guilty is 0.6, (i.e. $P(E) = 0.6$). We also know that the probability that a person from the general population, including a suspect that isn't guilty, possesses the characteristic is 0.2, (i.e. $P(F|E^c) = 0.2$). We also know that the probability that a guilty suspect possesses the characteristic is 1, (i.e. $P(F|E) = 1$).

   Using Bayes' Rule, we get:

$$P(E|F) = \frac{P(F|E)P(E)}{P(F)} = \frac{P(F|E)P(E)}{P(F|E)P(E) + P(F|E^c)P(E^c)}$$

$$P(E|F) = \frac{1 \cdot 0.6}{(1 \cdot 0.6) + (0.2 \cdot 0.4)} = 0.882353$$

4. Let E_i be the probability that the user is in cell i.
   Let F be the probability of observing two bars from the cell tower
   Let n be the number of cells in the map.

   We know that the probability that the user is in cell i before observing a signal from the cell tower is 0.05, (i.e. $P(E_i) = 0.05$). We also know that the probability of observing a signal from the cell tower given that the user is in cell i is 0.75, (i.e. $P(F|E_i) = 0.75$).

   Bayes' Rule tells us the following:

$$P(E_i|F) = \frac{P(F|E_i)P(E_i)}{P(F)}$$

   We can calculate P(F) using marginalisation. The probability of observing a signal from the cell tower would be equal to the sum of the probabilities of observing a signal at each cell i. From this we get:

$$P(F) = \sum_{i=0}^{n} P(F|E_i)P(E_i)$$

Plugging in the values, we get:

$$\begin{aligned}
P(F) = {} & (0.75)(0.05) + (0.95)(0.1) + (0.75)(0.05) + (0.05)(0.05) \\
& +(0.05)(0.05) + (0.75)(0.1) + (0.95)(0.05) + (0.75)(0.05) \\
& +(0.01)(0.05) + (0.05)(0.05) + (0.75)(0.1) + (0.95)(0.05) \\
& +(0.01)(0.05) + (0.01)(0.05) + (0.05)(0.1) + (0.75)(0.05) \\
& = 0.504
\end{aligned}$$

Plugging each cell into the formula, we can form the following matrix:

$$\begin{matrix}
\frac{(0.75)(0.05)}{0.504} & \frac{(0.95)(0.1)}{0.504} & \frac{(0.75)(0.05)}{0.504} & \frac{(0.05)(0.05)}{0.504} \\
\frac{(0.05)(0.05)}{0.504} & \frac{(0.75)(0.1)}{0.504} & \frac{(0.95)(0.05)}{0.504} & \frac{(0.75)(0.05)}{0.504} \\
\frac{(0.01)(0.05)}{0.504} & \frac{(0.05)(0.05)}{0.504} & \frac{(0.75)(0.1)}{0.504} & \frac{(0.95)(0.05)}{0.504} \\
\frac{(0.01)(0.05)}{0.504} & \frac{(0.01)(0.05)}{0.504} & \frac{(0.05)(0.1)}{0.504} & \frac{(0.75)(0.05)}{0.504}
\end{matrix}$$

This would simplify to:

$$\begin{matrix}
0.0744 & 0.1885 & 0.0744 & 0.0050 \\
0.0050 & 0.1488 & 0.0942 & 0.0744 \\
0.0010 & 0.0050 & 0.1488 & 0.0942 \\
0.0010 & 0.0010 & 0.0099 & 0.0744
\end{matrix}$$

Here is the Matlab program:

```
P_Es = [
    0.05 0.1 0.05 0.05;
    0.05 0.1 0.05 0.05;
    0.05 0.05 0.1 0.05;
    0.05 0.05 0.1 0.05;
];

P_F_given_Es = [
    0.75 0.95 0.75 0.05;
    0.05 0.75 0.95 0.75;
    0.01 0.05 0.75 0.95;
    0.01 0.01 0.05 0.75;
];

products = P_F_given_Es.*P_Es;
P_F = sum(products, 'all');
P_Es_given_F = products/P_F
```

```
>>
P_Es_given_F =
    0.0744    0.1885    0.0744    0.0050
    0.0050    0.1488    0.0942    0.0744
    0.0010    0.0050    0.1488    0.0942
    0.0010    0.0010    0.0099    0.0744
```