Statistical Methods for Computer Science

Assignment 1 — STU33009

1. (a) For the selection of the first letter, we would have 10 choices. After selecting this letter, we would now have 9 choices for the second letter, of which each choice can be matched with 1 of the 10 choices made for the first letter $(10 \cdot 9)$. This pattern would continue until we have used up all 10 letters. Therefore:

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$10! = 3628800$$

   (b) Since the restriction imposes that E and F must be next to each other, we can, at first, treat them as one:

$$9! = 362880$$

   However, since the E and the F can be in any order, we must multiply this quantity by 2:

$$362880 \cdot 2 = 725760$$

   (c) We take the factorial of the total number of letters. We divide this by the factorial of the occurrence of each letter in the word (ignoring the letters that appear only once):

$$\frac{6!}{3! \cdot 2!} = 60$$

   (d) We would calculate the total number of ways 3 letters can be chosen from 5:

$$\binom{5}{3} = 10$$

2. (a) If we roll 2 six-sided dice, each outcome from dice 1 can be paired with all of the outcomes from dice 2. Therefore, the total outcomes would be $6 \cdot 6 = 6^2 = 36$. For n dice rolls, the total number of outcomes would be 6^n. For 4 dice rolls:

$$6^4 = 1296$$

   (b) If exactly two dice roll a 3, the remaining two only have 5 options to roll from, as opposed to 6, since they mustn't be 3. There are two remaining dice:

$$5^2 = 25$$

   We must now multiply this by the number of ways the 2 dice that roll the 3's, can be chosen from 4:

$$25 \cdot \binom{4}{2} = 25 \cdot 6 = 150$$

(c) From part (b), we know that the number of ways to obtain exactly two 3's is 150. We also know that the number of ways to obtain exactly four 3's is, of course, 1. What's remaining is to calculate the number of ways to obtain exactly three 3's.

If we have exactly three dice rolling a 3, the remaining die has only 5 numbers to roll from, since it mustn't roll a 3. However, since three out of the four dice roll 3's, the remaining die can be any 1 of the 4 dice. Therefore, we must multiply the possible outcomes of the remaining die, 5, by the number of possible dice that can become the remaining die, which would be $\binom{4}{1}$:

$$\binom{4}{1} = 4$$

$$5 \cdot 4 = 20$$

Now we can simply add up all of these possible outcomes to get our final answer:

$$150 + 20 + 1 = 171$$

3. (a) We have 8 cards, containing 2 cards from each of the 4 suits:

$$\frac{8!}{2! \cdot 2! \cdot 2! \cdot 2!} = \frac{40320}{16} = 2520$$

(b) There are 4 ways in which a card can be dealt with its duplicate from the other deck. In order to calculate this, we ignore the 4 duplicates from the set of 8 cards, and calculate the number of ways in which 2 cards can be chosen from the remaining 4:

$$\binom{4}{2} = 6$$

(c) There are a total of 4 "good" cards in the set of 8. There are two ways to deal two duplicate "good" cards out of the set of "good" cards. Using the same logic as in the previous part with, the addition of another step, we can ignore the duplicate scenarios, and calculate the number of ways to select 2 cards from the remaining 2 "good" cards:

$$\binom{2}{2} = 1$$

We can now add this quantity to the number of ways to deal 2 duplicates, which is 2:

$$2 + 1 = 3$$