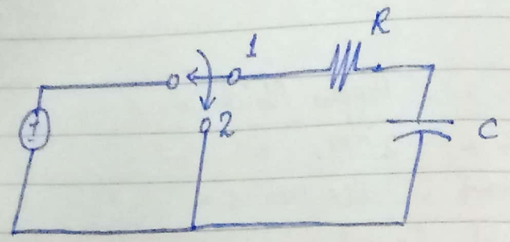
(1)

Source Free RC Circuit Analysis



Case 1:

$t < 0$ (switch at 1)

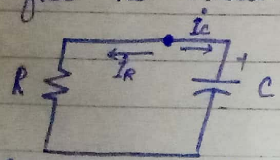
Case 2:

$t = 0$ (switch is at 2)

Case 3:

$t > 0$ (switch is at 2 for $t \to \infty$)

Source free RC circuit



Applying KCL $\quad i_R + i_C = 0$

$$i_R = -i_C$$

$$\because i_C = C\frac{dV_{(t)}}{dt}$$

$$\because i_R = \frac{V_{(t)}}{R}$$

$$\frac{V_{(t)}}{R} = -C\frac{dV_{(t)}}{dt}$$

2

(2)

$$-\frac{1}{RC}\,dt = \frac{1}{V_{(t)}}\,dV_{(t)}$$

Applying integral.

$$-\frac{1}{RC}\int_0^{t} dt = \int_{V_0}^{V_{c(t)}} \frac{1}{V_{(t)}}\,dV_{c(t)}$$

$$-\frac{t}{RC}\Big|_0^{t} = \ln V_{(t)}\Big|_{V_0}^{V_{c(t)}}$$

$$-\frac{\tau}{RC} = \ln V_{c(t)} - \ln V_0$$

$$-\frac{\tau}{RC} = \ln\left(\frac{V_{c(t)}}{V_0}\right)$$

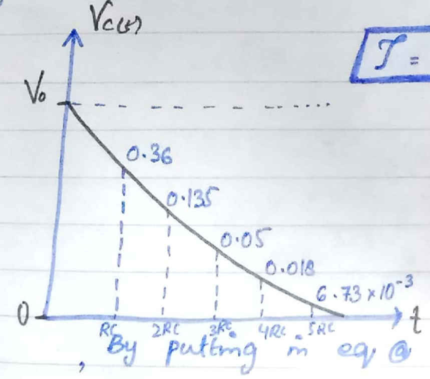
Applying Antilog.

$$e^{-\tau/RC} = \frac{V_{c(t)}}{V_0}$$

$$V_{c(t)} = V_0 e^{-\tau/RC} \quad \rightarrow (a)$$

↳ Natural Respone — without any source or external excitation

* Plotting graph b/w $V_{c(t)}$ and t



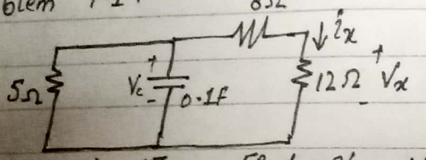
$\boxed{\tau = RC}$

① $t = 0$, By putting in eq (a) $\quad V_{c(t)} = V_0$

(3)

② $t = RC$ ③ $t = 2RC$ ④ $t = 3RC$ ⑤ $t = 4RC$
⑥ $t = 5RC$.

Problem 7.1.



Let $V_c(0) = 15V$ Find V_c, V_x and i_x for $t > 0$.

Solution.

$\tau = RC$

$\tau = R_{eq} C$

$R_{eq} = (8+12) \parallel 5$

$R_{eq} = (20) \parallel (5)$

$$R_{eq} = \frac{1}{\frac{1}{20} + \frac{1}{5}} = \frac{1}{5/20} = \frac{\cancel{20}\,4}{\cancel{5}}$$

$R_{eq} = 4\Omega$

$\tau = (4)(0.1) = 0.4\,sec$

Now

$V_{c(t)} = V(0)e^{-t/\tau}$

$V_{c(t)} = 15e^{-t/0.4}\,V$

$\boxed{V_{c(t)} = 15e^{-2.5t}\ \text{volts.}}$

4

$V(t)$ $V(0)$
$V(3)$ (4) $V(1)$

Now for 'V_x'.

By voltage divison rule

$$V_x = \frac{12}{(12+8)} \times V_{c(t)}$$

$$V_x = \frac{\cancel{12}\,3}{\cancel{20}\,5}\,15e^{-2.5t}$$

$\boxed{V_x = 9e^{-2.5t}\ \text{volts.}}$

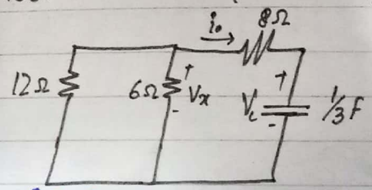
Now for i_x.

$$i_x = \frac{V_x}{R} = \frac{9e^{-2.5t}}{12}$$

$\boxed{i_x = 0.75e^{-2.5t}\ \text{Amps.}}$

Ans

Problem 7.1 c- (Practice Question).



Let $V_{c(0)} = 30v$ Find $V_{c(t)}$, V_x and i_o $t \geq 0$

Solution.

$R_{eq} = (12\Omega \parallel 6\Omega) + 8\Omega$

$$R_{eq} = \left(\frac{1}{1/12 + 1/6}\right) + 8$$

$R_{eq} = (4) + (8) = 12\Omega$

$\tau = RC = (12)(1/3) = 4\,secs$

Now.

$V_{c(t)} = V_0 e^{-t/\tau} = 30e^{-t/4}$

$\boxed{V_{c(t)} = 30e^{-0.25t}\ \text{volts.}}$

For V_x :

Applying voltage divider rule.

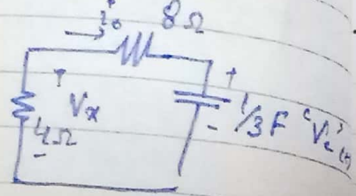


Fig ①

$V_x = \left(\frac{4}{12}\right) V_{c(t)}$

$V_x = \left(\frac{1}{3}\right) 30 e^{-0.25t}$

$\boxed{V_x = 10 e^{-0.25t}}$

Now for i_o :

As 4Ω and 8Ω are in series so the current through them is equal. Therefore

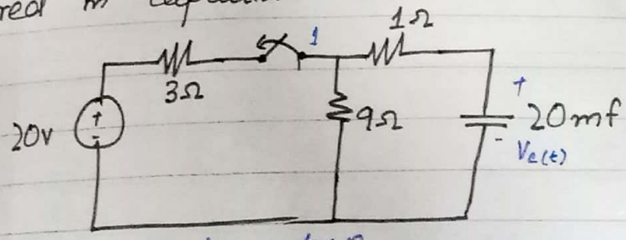
$i_o = \frac{V_x}{4} = \frac{10}{4} e^{-0.25t}$

$\boxed{i_o = 2.5 e^{-0.25t}}$

Ans

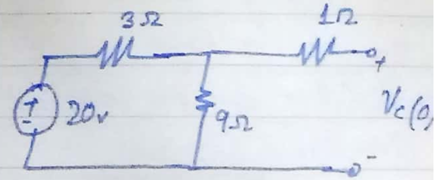
Example 7.2 :

The switch in the circuit has been closed for a long time and it is opened at $t=0$. Find $V_{c(t)}$ for $t \geq 0$. Calculate initial energy stored in capacitor.



For $V_c(0)$: when $t<0$

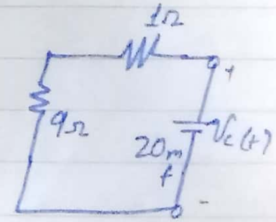
By voltage divider rule



$V_c(0) = \left(\frac{9}{9+3}\right) \times 20$

$\therefore V_c(0) = 15$ volts.

Now For $V_{(t)}$: $t \geq 0$



$V_{(t)} = V_o e^{-t/\tau}$

$\tau = RC = (10)(20\times10^{-3})$

$\tau = 200\times10^{-3}$

$\tau = 0.2$ secs

$V_{c(t)} = 15 e^{-t/0.2}$ volts

$V_{c(t)} = 15 e^{-5t}$ volts.

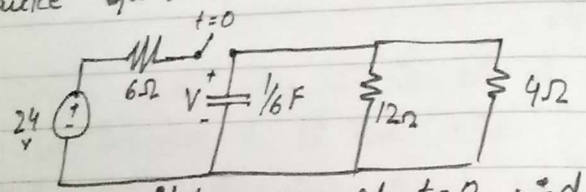
Now For energy stored in capacitor initially.

$E_c(0) = \frac{1}{2} C[V_c(0)]^2 = \frac{1}{2}(20\times10^{-3})(15)^2$

$E_c(0) = 2.25\ J$

Ans

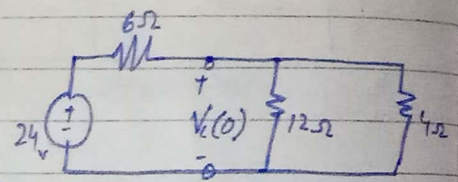
Practice Questions.



If the switch open at $t=0$ find $V_c(t)$ for $t \geq 0$ and $w_c(0)$.

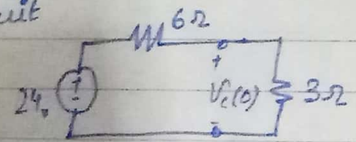
Solution.

For $V_c(0)$.



By voltage divider

By as voltage divider can be applied in series simplifying circuit



$$V_c(0) = \frac{3}{6+3} \times 24 = \frac{\cancel{3}}{\cancel{9}\,3} \times \cancel{24}\,8$$

$$V_c(0) = 8 \text{ volts}$$

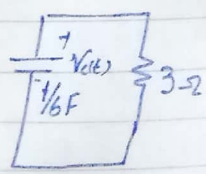
Now $V_c(t)$.

$$V_c(t) = V_c(0)e^{-t/\tau}$$

$$\tau = RC = R_{eq}C$$

$$R_{eq} = (12 \parallel 4)$$

$$R_{eq} = 3\Omega$$



$$\tau = (3)(1/6) = 0.5 \text{ sec}$$

$$V_c(t) = 8e^{-t/0.5} = V_c(t) = 8e^{-2t}$$

$$W_c(0) = \frac{1}{2}C(V_c(0))^2 = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)(8)^2$$

$$W_c(0) = \frac{64}{12}$$

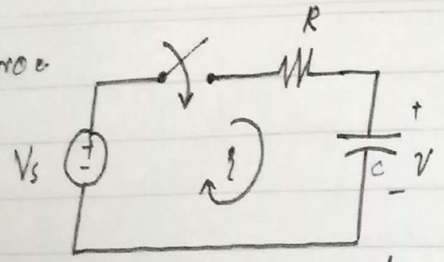
$$W_c(0) = 5.33\,J$$

Ans.

$$\frac{V_c(t)}{R} = \frac{V_0}{R}e^{-t/\tau}$$

$$i_c(t) = i_0 e^{-t/\tau}$$

Q no.



Determine Response at $t > 0$
→ behaviour.

Applying KVL assume that

$t=0$ $\quad V(0) = V_0$, $V(t)$ at $t > 0$

$-V_s + iR + V = 0$

$-V_s + RC\frac{dV}{dt} + V = 0 \quad \because i = i_c = C\frac{dV}{dt}$

$RC\frac{dV}{dt} = V_s - V$

$RC\frac{dV}{dt} = -(V - V_s)$

$$\boxed{\frac{dV}{V - V_s} = \frac{-1}{RC}dt}$$

Applying integral.

$$\int_{V_0}^{V(t)} \frac{dV}{V - V_s} = \int_0^t \frac{-1}{RC} dt$$

$$\ln[V - V_s] \Big|_{V_0}^{V(t)} = -\frac{t}{RC}\Big|_0^t$$

$$\ln[V_{(t)} - V_s] - \ln[V_0 - V_s] = \frac{-t}{RC}$$

$$\ln\left[\frac{V_{(t)} - V_s}{V_0 - V_s}\right] = \frac{-t}{RC}$$

Applying Antilog

$$\frac{V_{(t)} - V_s}{V_0 - V_s} = e^{-t/RC}$$

$$V_{(t)} - V_s = [V_0 - V_s]e^{-t/RC}$$

$$\boxed{V_{(t)} = V_s + \{(V_0 - V_s)e^{-t/RC}\}}$$

Steady state Response
for every long time

Transistent Response. → Temporary
For every short time
Its a temporary response that will die out with time.

$t \to \infty$
The steady state response is the behavior of the circuit a long time after an external excitation is applied.

Complete Respone

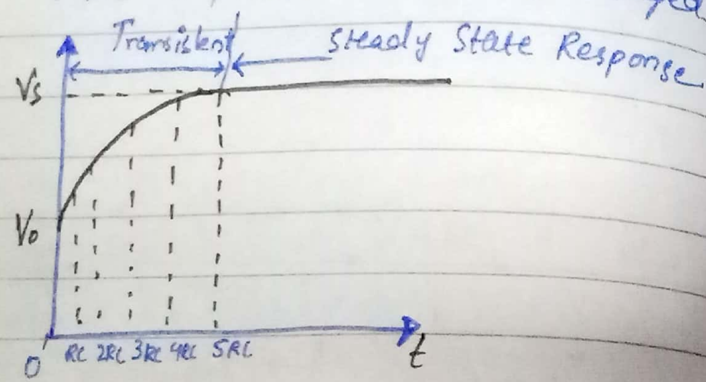
$$\boxed{V_{(t)} = V_0 e^{-t/RC} + V_s[1 - e^{-t/RC}]}$$

Natural Respone
Without any source or external excitation

Forced Resistor Response with some excitation source.

For $V_0 > 0$ any no.

when capacitor is initially charged.

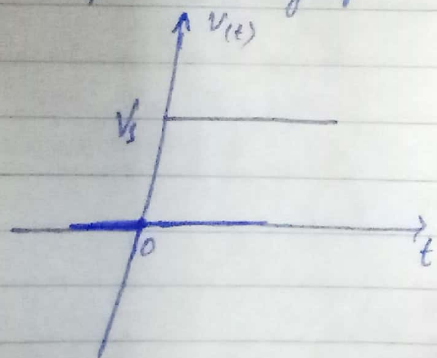


$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \text{Unit Step}$$

Unit Impulse $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{other value of } t \end{cases}$

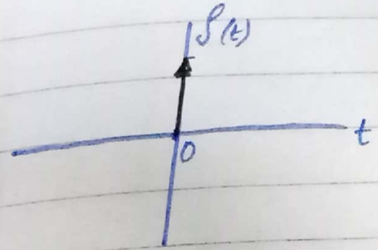
Unit Ramp $\begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases} = r(t)$

Step Input graph

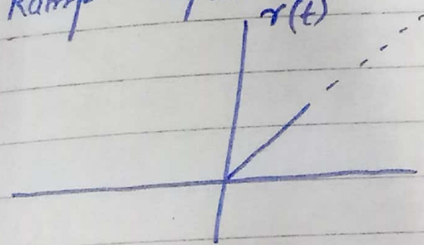


Unit

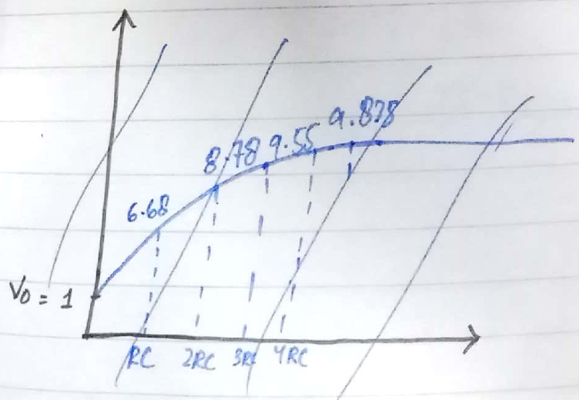
Impulse Input



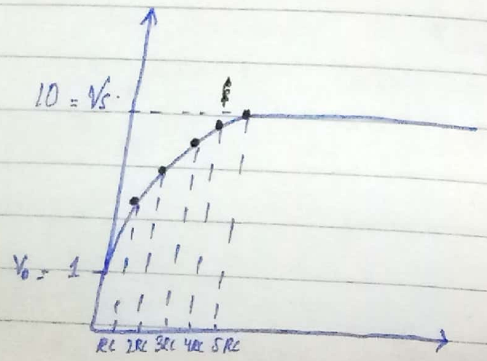
Ramp Input : $r(t)$



$V_0 = 1$, $RC = 1$, $V_s = 10$



(13)



At $t = 1$

$V_{(t)} = 6.68$

At $t = 2$

$V_{(t)} = 8.78$

At $t = 3$

$V_{(t)} = 9.55$

At $t = 4$

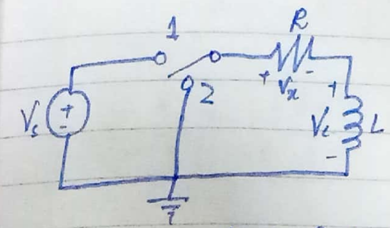
$V_{(t)} = 9.83$

At $t = 5$

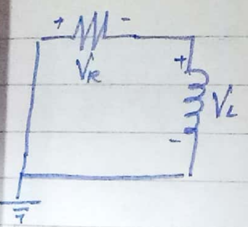
$V_{(t)} = 9.94$

(14)

TOPIC :- SOURCE FREE RESPONSE OF RL Network



Let assume that inductor of 'L' Henry, is attached with resistor 'R' and the after $t=0$ the switch contact moved towards '2' position and inductor discharges



Applying KVL.

$V_R + V_L = 0$

$\because V_R = iR$

$V_L = L\frac{di}{dt}$

$$iR + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = -Ri$$

$$\frac{di}{dt} = \frac{-R}{L} i$$

$$\frac{di}{i} = \frac{-R}{L} dt$$

$$\int_{I_0}^{i(t)} \frac{di}{i} = \frac{-R}{L}\int_0^t dt$$

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$$\ln(i)\Big|_{I_0}^{i(t)} = \frac{-R}{L} t \Big|_0^t$$

$$\ln(i_{(t)}) - \ln(I_0) = \frac{-Rt}{L} + \frac{R(0)}{L}$$

$$\ln\left(\frac{i_{(t)}}{I_0}\right) = \frac{-R}{L}t$$

Applying Antilog

$$\frac{i_{(t)}}{I_0} = e^{-R/L \times t}$$

$$i_{(t)} = I_0 \, e^{-\frac{R}{L} \times t}$$

$$i_{(t)} = I_0 \, e^{-t/(L/R)}$$

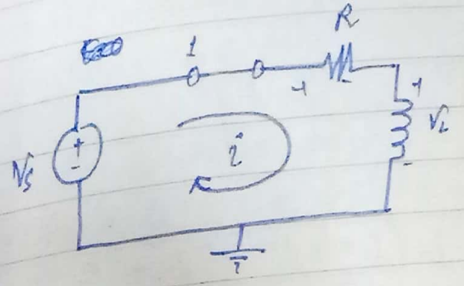
Here I_0 is intial current and $\left(\frac{L}{R}\right)$ is time constant

$$V_L = L\frac{di}{dt}$$

inductor oppose change of current
capacitor oppose change of voltage

* $i_C \propto dv$
* $V_L \propto di$

Source Response Of R_L Networks



$$I_S = \frac{V_S}{R}$$

Applying KVL

$$-V_S + V_R + V_L = 0$$

$$-V_S + iR + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = V_S - iR$$

$$L\frac{di}{dt} = I_S R - iR$$

$$L\frac{di}{dt} = R(I_S - i)$$

$$\frac{di}{dt} = \frac{R}{L}(I_S - i)$$

$$\frac{di}{dt} = \frac{-R}{L}(i - I_S)$$

$$\frac{di}{(i - I_S)} = \frac{-R}{L}dt$$

$$\int_{I_0}^{i(t)} \frac{di}{i - I_s} = \frac{-R}{L}\int_0^t dt$$

$$\ln(i - I_s)\Big|_0^{i(t)} = \frac{-R}{L}\, t\Big|_0^t$$

$$\ln(i(t) - I_s) - \ln(I_0 - I_s) = \frac{-R}{L}t + \frac{R}{L}(0)$$

$$\ln\left(\frac{i(t) - I_s}{I_0 - I_s}\right) = \frac{-R}{L}t$$

Applying Antilog

$$\frac{i(t) - I_s}{I_0 - I_s} = e^{-\frac{L}{R}\times t}$$

$$i(t) - I_s = (I_0 - I_s)e^{-\frac{L}{R}\times t}$$

$$i(t) = I_s + (I_0 - I_s)e^{-\frac{L}{R}\times t}.$$

$$i(t) = I_0 e^{-\frac{L}{R}\times t} + I_s\left(1 - e^{-\frac{L}{R}\times t}\right).$$

i) Natural Response part

$$i(t) = I_0 e^{-\frac{L}{R}\times t}$$

ii) Forced Response part

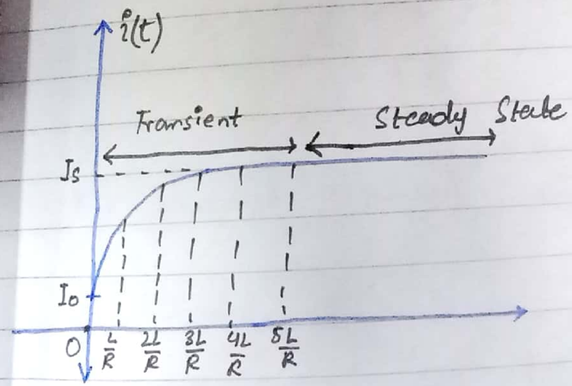
$$i(t) = I_s\left(1 - e^{-\frac{L}{R}\times t}\right)$$

iii) Transient Response part

$$i(t) = (I_0 - I_s)e^{-\frac{L}{R}\times t}$$

iv) Steady state Response part

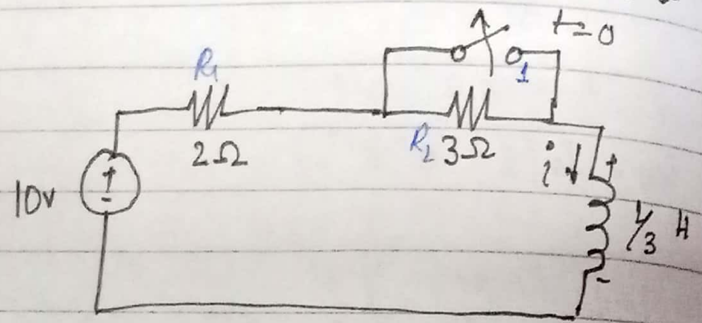
$$i(t) = I_s.$$



* For DC inductors behaves like short and capacitor behaves like open

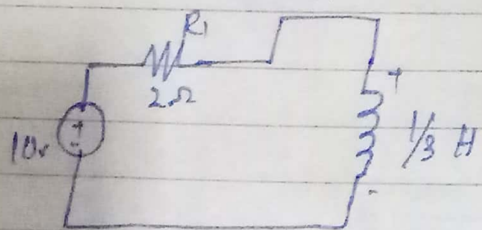
Topic :- RL Response :

Q. Problem related to RL Circuit :-

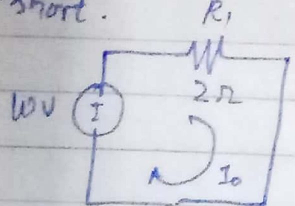


STEP # 01 :

At when $t<0$, we can calculate I_0 when switch is at 1



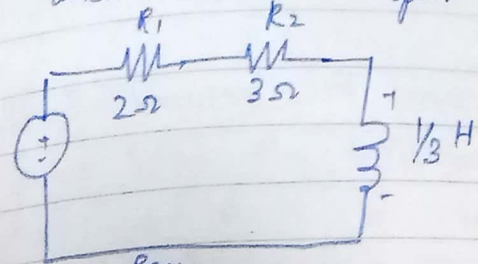
After some time inductor behaves like short.

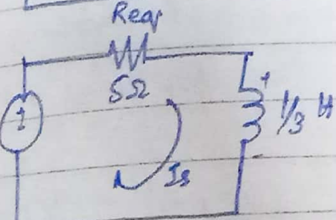


$$I_0 = \frac{V_s}{R_1} = \frac{10}{2} = 5$$

$I_0 = 5$ amps

Now when switch is open





$$I_s = \frac{V_s}{R_e} = \frac{10}{5}$$

$I_s = 2$ Amps

$$i(t) = I_0 e^{-t/(L/R)} + I_s\left(1 - e^{-t/(L/R)}\right)$$

$$i(t) = 5e^{-t/((1/3)/5)} + 2\left(1 - e^{-t/((1/3)/5)}\right)$$

$$i(t) = 5e^{-15t} + 2\left(1 - e^{-15t}\right)$$

$$i(t) = 5e^{-15t} - 2e^{-15t} + 2$$

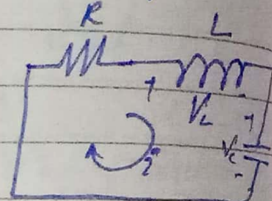
$$i(t) = 2 + 3e^{-15t}$$

Ans

TOPIC :- RLC RESPONSE :-

Let assume that RLC network having inductor and capacitor with initial value of voltages as V_0 & I_0 the voltage of capacitor current through inductor at $t = 0$. Now Applying KVL

$$iR + V_L + V_C = 0$$



$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$L\frac{di}{dt} + iR + \frac{1}{C}\int i\,dt = 0 \qquad \because V_L = L\frac{di}{dt}$$

$$V_C = \frac{1}{C}\int i\,dt$$

Dividing by 'L'

$$\frac{di}{dt} + i\frac{R}{L} + \frac{1}{LC}\int i\,dt = 0 \qquad i = C\frac{dV}{dt}$$

=

$$\frac{di}{dt} + \frac{Ri}{L} + \frac{1}{LC}\int i\,dt = 0$$

Differentiate again w.r.t 't'

$$\frac{d}{dt}\left(\frac{di}{dt}\right) + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\frac{d}{dt}\left(\int i\,dt\right) = 0$$

$$\boxed{\frac{d^2}{dt^2}i + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0}$$

The 2nd order differential equation

Now solution of differential Equation are Exponential in Nature
Here - Expected solutions can be obtained by placing $i = Ae^{st}$ where A & S are constant in eq (i)

$$\frac{d^2}{dt^2}(Ae^{st}) + \frac{R}{L}\frac{d}{dt}(Ae^{st}) + \frac{1}{LC}(Ae^{st}) = 0$$

$$As^2e^{st} + \frac{R}{L}(Ase^{st}) + \frac{1}{LC}Ae^{st} = 0$$

$$Ae^{st}\left(s^2 + \frac{R}{L}(s) + \frac{1}{LC}\right) = 0$$

Now

$$\boxed{s^2 + \frac{R}{L}(s) + \frac{1}{LC} = 0}$$ Characteristic Equation of system.

Applying Quadratic formula

$$S = \frac{-(R/L) \pm \sqrt{(R/L)^2 - 4(1)(1/LC)}}{2(1)}$$

$$S = \frac{-R}{2L} \pm \frac{\sqrt{(R/L)^2 - 4/LC}}{2}$$

$$S = \frac{-R}{2L} \pm \sqrt{\frac{\left(\frac{R}{L}\right)^2 - (4/LC)}{4}}$$

$$S = \frac{-R}{2L} \pm \sqrt{\frac{\left(\frac{R}{L}\right)^2}{4} - \frac{\cancel{4}}{\cancel{4}LC}}$$

$$S = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Let

$$\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

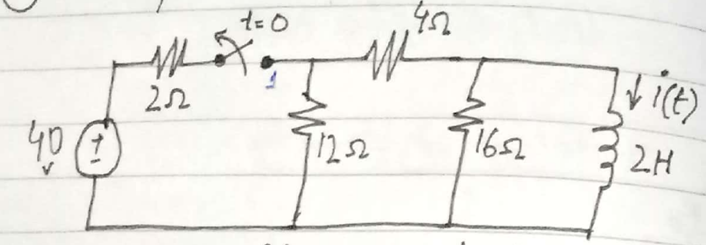
Now here

$S =$ is called the Natural Frequency (Neper/s)
$\omega_0 =$ is called undamping natural frequency (Rad/s)
α is called damping factor (

Decaying of energy → Damping.

Source Free RL Circuits Problems

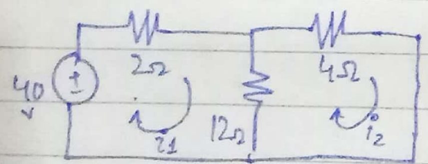
① Example 7.4



Calculate $i(t)$ for $t > 0$

Solution

When t is < 0 and switch is at 1 we have circuit



Here after some times inductor behave like short, and current always choose less resistive path therefore current will always choose short instead of path with 16 Ω resistor therefore discarding 16 Ω resistor

By mesh Analysis.

In mesh 1.

$-40 + 2i + 12i_1 - 12i_2 = 0$

$14i_1 - 12i_2 = 40$

$7i_1 - 6i_2 = 20 \longrightarrow (i)$

In Mesh 2

$12i_2 - 12i_1 + 4i_2 = 0$

$-12i_1 + 16i_2 = 0$

$$\begin{bmatrix} 7 & -6 \\ -12 & 16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$D = (7)(16) - (-6)(-12)$

$D = 40$

$D_1 = \begin{bmatrix} 20 & -6 \\ 0 & 16 \end{bmatrix} = (20)(16) - (0)(-6)$

$D_1 = 320$

$D_2 = \begin{bmatrix} 7 & 20 \\ -12 & 0 \end{bmatrix} = (7)(0) - (20)(-12)$

$D_2 = 240$

$i_1 = \frac{320}{40}$, $i_2 = \frac{240}{40}$

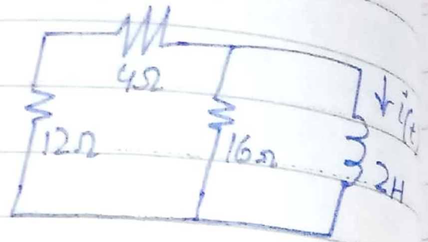
$i_1 = 8 \text{ Amps}$, $i_2 = 6 \text{ Amps}$

The current through inductor is $i_2 = 6 \text{Amps}$ and it cannot change instantaneously

$I_0 = i(0) = i_2$

$I_0 = 6 \text{Amps}.$

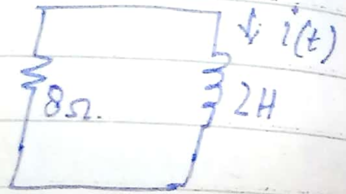
Now when $t>0$ and switch is open we have circuit



$Req = (12+4) \parallel 16$

$Req = (16) \parallel (16)$

$Req = 8\Omega$



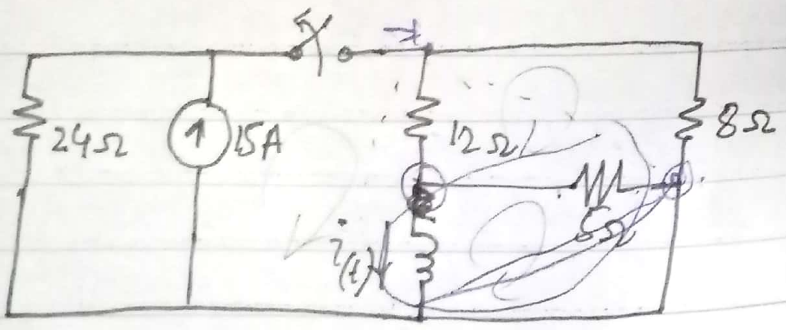
Now time constant

$\tau = \left(\frac{L}{R}\right) = \frac{\cancel{2}}{\cancel{8}4} = \frac{1}{4} s$

$i(t) = I_0 e^{-t/\tau} = 6 e^{-t/(1/4)}$

$i(t) = 6 e^{-4t}$ Amps.

<u>7.4</u> practice problems.



Topic: Source Free Response of Series RLC Circuit: (28)

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\boxed{S^2 + \frac{R}{L}S + \frac{1}{LC} = 0}$$

Characterstic Equation of system.

Depending on Nature of Root

(i) If Roots are Real and Unequal. $\boxed{\alpha > \omega_0}$

(ii) If Roots are Real and equal $\boxed{\alpha = \omega_0}$

(iii) If roots are Complex. $\boxed{\alpha < \omega_0}$

(i) $\alpha > \omega_0$ (this reponse is called over damped)

(ii) $\alpha = \omega_0$ (this reponse is called critically damped) (Most desirable condition).

(iii) $\alpha < \omega_0$ (this response is called under damped)

Case 1: $(\alpha > \omega_0)$

$\because$ 2nd order system must have 2 initial condition

$$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

1st $i(0)$ I_0, V_0

2nd $\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$

$$i(t) = A_1 e^{(-\alpha + \sqrt{\alpha^2 - \omega_0^2})t} + A_2 e^{(-\alpha - \sqrt{\alpha^2 - \omega_0^2})t}$$

$$i(t) = A_1 e^{-\alpha t} \cdot e^{(\sqrt{\alpha^2 - \omega_0^2})t} + A_2 e^{-\alpha t} \cdot e^{(-\sqrt{\alpha^2 - \omega_0^2})t}$$

$$i(t) = e^{-\alpha t}\left[A_1 e^{(\sqrt{\alpha^2 - \omega_0^2})t} + A_2 e^{(-\sqrt{\alpha^2 - \omega_0^2})t}\right]$$

ye parallel say agy pura karna hay idr pura nahi hay

Case 3rd ($\alpha < \omega_0$)

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$
$$S_1, S_2 = -\alpha \pm \sqrt{-\omega_0^2 + \alpha^2}$$
$$S_1, S_2 = -\alpha \pm \sqrt{-(\omega_0^2 - \alpha^2)}$$
$$S_1, S_2 = -\alpha \pm \sqrt{-1(\omega_0^2 - \alpha^2)} \qquad \because \sqrt{-1} = i$$
$$S_1, S_2 = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ called ~~damping~~ damp Natural frequ

$$S_1, S_2 = -\alpha \pm j\omega_d$$

Case 2nd ($\alpha = \omega_0$) critical damp

$$S_1, S_2 = -\alpha$$
$$\alpha = \omega_0$$

The equation

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0 \longrightarrow (1)$$

Actually

$$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

If we have $S_1 = S_2 = -\alpha$

$$i(t) = A_1 e^{-\alpha t} + A_2 e^{-\alpha t}$$
$$i(t) = e^{-\alpha t}(A_1 + A_2)$$
$$i(t) = A e^{-\alpha t}$$

Now equation has single initial condition (single Constant) which is not justify the second order system. It should ... constant term (initial condition).

Now putting in eq (1)

$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$
$$2\alpha = \frac{R}{L}$$

putting all values in eq (1)

$$\frac{d^2 i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_0^2 i = 0 \qquad \because \alpha = \omega_0$$
$$\frac{d^2 i}{dt^2} + 2\alpha\frac{di}{dt} + \alpha^2 i = 0$$
$$\frac{d^2 i}{dt^2} + \alpha\frac{di}{dt} + \alpha\frac{di}{dt} + \alpha^2 i = 0$$
$$\frac{d}{dt}\left(\frac{di}{dt} + \alpha i\right) + \alpha\left(\frac{di}{dt} + \alpha i\right) = 0$$

let

$$f = \frac{di}{dt} + \alpha i$$
$$\frac{d}{dt}f + \alpha f = 0$$

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Now as the 1st order differential eqn

$$f = Ae^{-\alpha t}$$

$$\frac{di}{dt} + \alpha i = Ae^{-\alpha t}$$

$$e^{\alpha t}\left[\frac{di}{dt} + \alpha i\right] = A$$

$$e^{\alpha t} \cdot \frac{di}{dt} + i \cdot \alpha e^{\alpha t} = A$$

Now we can suppressed if

$$\frac{d}{dt}\left(i \cdot e^{\alpha t}\right) = A$$

$$d\left(i \cdot e^{\alpha t}\right) = A\,dt$$

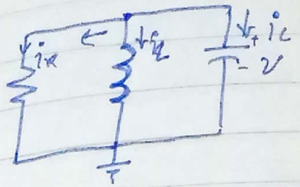
$$\int d\left(i \cdot e^{\alpha t}\right) = A\int dt$$

$$i \cdot e^{\alpha t} = At + C$$

$$i = \left(At + C\right)e^{-\alpha t}$$

$$\boxed{i(t) = A_1 t e^{-\alpha t} + A_2 e^{-\alpha t}.}$$

TOPIC :- Parallel RLC :-



Applying KCL at node a

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} + \frac{1}{LC}\int v\,dt = 0$$

$$\frac{d}{dt}\left(\frac{dv}{dt} + \frac{v}{RC} + \frac{1}{LC}\int v\,dt\right) = 0$$

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0}$$

$$v = Ae^{st}$$

$$\frac{d^2 Ae^{st}}{dt} + \frac{1}{RC}\frac{dAe^{st}}{dt} + \frac{1}{LC}Ae^{st} = 0$$

$$As^2e^{st} + \frac{1}{RC}Ase^{st} + \frac{1}{LC}Ae^{st} = 0$$

$$Ae^{st}\left(s^2 + \frac{1(s)}{RC} + \frac{1}{LC}\right) = 0$$

Now characteristics equation.

$$s^2 + \frac{1}{RC} + \frac{1}{LC} = 0$$

By quadratic formula

$$S = \frac{(-1/RC) \pm \sqrt{(1/RC)^2 - 4(1)(1/LC)}}{2}$$

$$S = \frac{-1}{2RC} \pm \sqrt{\frac{\frac{1}{(RC)^2} - (4/LC)}{4}}$$

$$S = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

$$S = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{(\sqrt{LC})^2}}$$

Let;

$$\alpha = \frac{1}{2RC}, \quad \omega_0 \frac{1}{\sqrt{LC}}$$

$$S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

① Case (i) Over damped response $\alpha > \omega_0$

$$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

② Case (ii) Critically damped response $\alpha = \omega_0$

$$V(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$$

③ Under damped Response $\alpha < \omega_0$

$$V(t) = (B_1 \cos\omega_d t + B_2 \sin\omega_d t) e^{-\alpha t}$$

∴ From pervious

$$(A_1 + A_2) = B_1$$

$$j(A_1 - A_2) = B_2$$

x —— x - - - - x

SERIERS RESPONSE RLC c-

CASE (iii) $S_1, S_2 = -\alpha \pm j\omega_d$

$$\cancel{i(t) = A_1 e^{(-\alpha t + j\omega_d t)} + A_2 e^{(-\alpha t - j\omega_d t)}}$$

$$\cancel{i(t) = (A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}) e^{-\alpha t}}$$

$$\cancel{i(t) = [A_1 (}$$

$$i(t) = A_1 e^{(-\alpha t + j\omega_d t)} + A_2 e^{(-\alpha t - j\omega_d t)}$$

$$i(t) = A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} \cdot e^{-j\omega_d t}$$

$$i(t) = (A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}) e^{-\alpha t}.$$

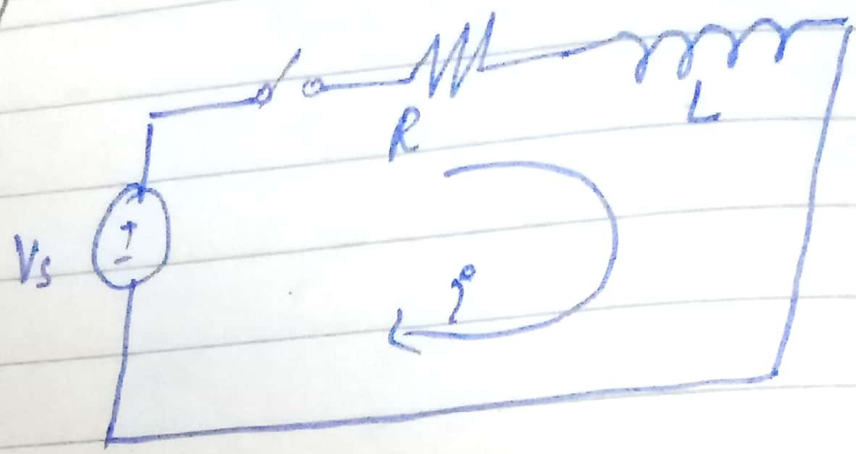
$$i(t) = [A_1(\cos\omega_d t + j\sin\omega_d t) + A_2(\cos\omega_d t - j\sin\omega_d t)] \times e^{-\alpha t}$$

$$i(t) = [A_1 \cos\omega_d t + A_1 j\sin\omega_d t + A_2 \cos\omega_d t - A_2 j\sin\omega_d t) \times e^{-\alpha t}$$

$$i(t) = [(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t] \times e^{-\alpha t}$$

$$i(t) = [B_1 \cos\omega_d t + B_2 \sin\omega_d t] \times e^{-\alpha t}$$

Topic :- Step Response Of Series RLC



t

Using KVL.

$$iR + L\frac{di}{dt} + v = V_s$$

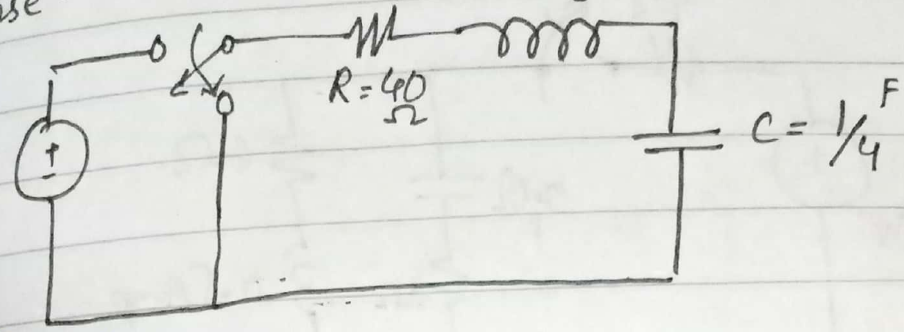
$$\because i = C\frac{dv}{dt}$$

$$RC\frac{dv}{dt} + L\frac{d}{dt}\left[C\frac{dv}{dt}\right] + v = V_s$$

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = V_s$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

Q- Determine the characterstics Equation of Network show & find roots and Nature of Response.



Solution:

$\alpha = \frac{R}{2L}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

$\alpha = \frac{40}{2(4)} = 5$

$\boxed{\alpha = 5}$

$\omega_0 = \frac{1}{\sqrt{(4)(1/4)}} = 1$

$\boxed{\omega_0 = 1}$

$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$S_1 = -5 + \sqrt{(5)^2 - (1)^2} = -5 + \sqrt{25 - 1}$

$\boxed{S_1 = -5 + \sqrt{24}}$

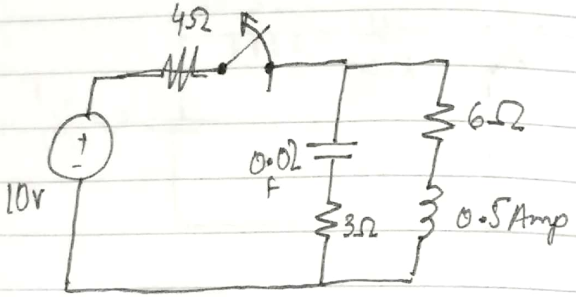
$S_2 = -5 - \sqrt{(25)^2 - (1)^2} = -5 - \sqrt{25 - 1}$

$\boxed{S_2 = -5 - \sqrt{24}}$

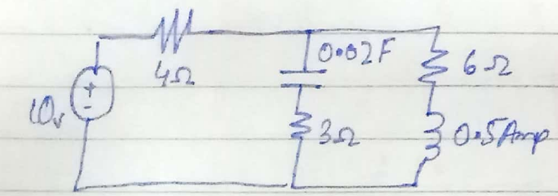
As here $\alpha = 5$ and $\omega_0 = 1$ $\because \alpha > \omega_0$

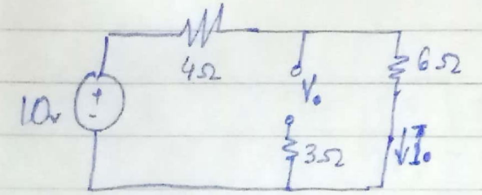
Therefore here is condition of overdamped

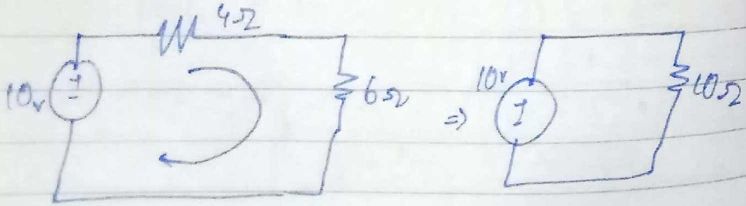
Q. Determine the response of Network shown below



At $t < 0$ assume for very long duration of time.







By ohm law

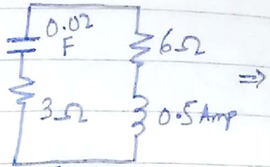
$I_0 = V/R = 10/10 = 1$ Amps

Now for V_0 we apply voltage divider

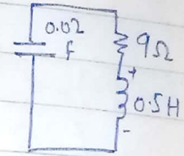
$V_0 = \left(\frac{R_2}{R_1+R_2}\right) \times V_s = \left(\frac{6}{10}\right) \times 10$

$\boxed{V_0 = 6 \text{volts}}$

Now for $t = 0$



Fig(2)



Fig(3)

$\alpha = \frac{R}{2L}$

$\alpha = \frac{9}{2(0.5)} = 9$

$\boxed{\alpha = 9}$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.5)(0.02)}} = \frac{1}{\sqrt{1/100}}$

$\boxed{\omega_0 = 10}$

Nature of Response.

$\boxed{\alpha < \omega_0}$

Therefore its underdamped.

$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$S_1, S_2 = -9 \pm \sqrt{(9)^2 - (10)^2}$

$S_1, S_2 = -9 \pm \sqrt{81 - 100}$

$S_1, S_2 = -9 \pm \sqrt{-19}$

$\boxed{S_1, S_2 = -9 \pm j(4.359)}$

here

$\omega_d = 4.359$

Now the Response.

$i(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

$i(t) = e^{-9t}(A_1 \cos 4.359 t + A_2 \sin 4.359 t)$

Now we need to determine A_1 & A_2 for complete solution.

$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) \rightarrow ①$

as $t = 0$

$i(0) = e^{-9(0)}(A_1 \cos 4.359(0) + A_2 \sin 4.359(0))$

$i(0) = (1)(A_1(1) + A_2(0))$

$\boxed{i(0) = A_1}$

$\because i(0) = I_0 = 1$ Amps

$\boxed{A_1 = 1}$

Now the second condition can be obtain by differentiating Eq ①.

$\frac{d}{dt} i(t) = \frac{d}{dt}\left[e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)\right]$

$\frac{d}{dt} i(t) = -9e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$

$+(-4.359 A_1 \sin 4.359t + 4.359 A_2 \cos 4.359t$

Now at $t = 0$

$\frac{di(0)}{dt} = -9(A_1(1) + A_2(0)) + (-4.359 A_1(0) + 4.359 A_2(1))$

$\boxed{\frac{di(0)}{dt} = -9A_1 + 4.359 A_2} \rightarrow ②$

from Figure (3)

Applying KVL:

$-V_c + V_R + V_L = 0$

$-V + iR + L\frac{di(t)}{dt} = 0$

$L\frac{di(t)}{dt} = V - iR$

$\frac{di(t)}{dt} = \frac{1}{L}[V - iR]$

Now at $t = 0$

$\frac{di(0)}{dt} = \frac{1}{L}[V(0) - i(0)R]$

$\frac{di(0)}{dt} = \frac{1}{L}[V_0 - I_0 R]$

$\frac{di(0)}{dt} = \frac{1}{(1/2)}[6 - (1)(9)]$

$\frac{di(0)}{dt} = \frac{-3}{1/2} = -6$

Now putting value in eq ②

$-6 = -9A_1 + 4.359 A_2$

$\boxed{\because A_1 = 1}$

$$-6 = -9 + 4.359A_2$$

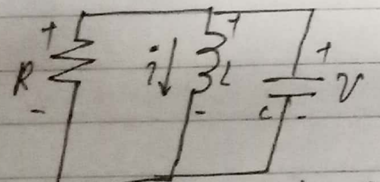
$$\boxed{A_2 = 0.688}$$

Now;

$$i(t) = e^{-9t}(\cos 4.359t + 0.688 \sin 4.359t)$$

Ans)

Q: Determine Response for Parallel RLC in the following condition.



where $i(0) = 0$, $v(0) = 5V$, $L = 1\,H$
$C = 10mf$, (a) $R = 1.923\,\Omega$
(b) $R = 5\Omega$ (c) $R = 6.25\Omega$

Solution:

for $t = 0$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(1.923)(10\times10^{-3})}$$

$$\alpha = 26$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5)(10\times10^{-3})}} = 10$$

Now S_1 and S_2

$$S_1, S_2 = -\alpha \pm \sqrt{(\alpha)^2 - (\omega_0)^2}$$

$$S_1, S_2 = -26 \pm \sqrt{(26)^2 - (10)^2}$$

$$S_1, S_2 = -26 \pm \sqrt{576}$$

$$S_1, S_2 = -26 \pm 24$$

$$\boxed{S_1 = -2}, \quad \boxed{S_2 = -50}$$

∴ Response is overdamped because $\boxed{\alpha > \omega_0}$

$$v(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$$v(t) = A_1 e^{-2t} + A_2 e^{-50t}$$

Applying KVL:

$$i_R + i_L + i_C = 0$$

$$\frac{v}{R} + i + C\frac{dv}{dt} = 0$$

$$C\frac{dv}{dt} = -\left(i + \frac{v}{R}\right)$$

$$\frac{dv}{dt} = -\frac{1}{C}\left(i + \frac{v}{R}\right)$$

for $t = 0$

$$\frac{dv(0)}{dt} = -\frac{1}{C}\left(i(0) + \frac{v(0)}{R}\right)$$

$$\frac{dv(0)}{dt} = \frac{-(5)}{1.923 \times (10\times10^{-3})} =$$

$$\frac{dv(0)}{dt} = -260.01$$

for $t = 0$

$$v(0) = A_1 + A_2$$

$$\boxed{5 = A_1 + A_2} \longrightarrow (1)$$

Now for Second condition.

$$\frac{d}{dt} v(t) = \frac{d}{dt}\left(A_1 e^{-2t} + A_2 e^{-50t}\right)$$

$$\frac{dv(t)}{dt} = -2A_1 e^{-2t} - 50A_2 e^{-50t}$$

for $t = 0$

$$\frac{dv(0)}{dt} = -2A_1 - 50A_2$$

$$\boxed{-260 = -2A_1 - 50A_2} \longrightarrow (2)$$

Solving Eq (1) & (2) we have

$$A_1 = 5 - A_2$$

put in eq (2)

$$-260 = -2(5 - A_2) - 50A_2$$

$$-260 = -10 + 2A_2 - 50A_2$$

$$=$$

$$-250 = -48A_2$$

$$A_2 = \frac{-250}{-48}$$

$$\boxed{A_2 = 5.2}$$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\boxed{v(t) = -0.2e^{-2t} + 5.2e^{-50t}}$$

Ans.

TOPIC :- AC CIRCUIT :-

$v(t) = V_m \sin(\omega t + \phi)$ OR $v(t) = V_m \cos(\omega t + \phi)$

$i(t) = I_m \sin(\omega t + \phi)$ OR $i(t) = I_m \cos(\omega t + \phi)$.

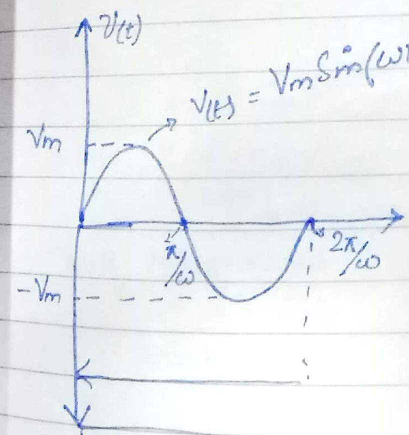
where

$v(t)$ is instantenous values of voltage.

V_m is the maximum values of voltage

ω is the angular frequency of Sinosoid

ϕ is the angular phase shift.



$$\boxed{\omega = 2\pi f}$$

Q:-
Find the amplitude and frequency of signal when

$$v(t) = 20\sin(211t + 20)$$

Sol:

$$v(t) = V_m \sin(\omega t + \phi)$$

here
By comparing given eq we have

$V_m = 20 \rightarrow$ Amplitude.

$\omega = 211 \rightarrow$ Angular frequency.

$\phi = 20 \rightarrow$ Angular phase shift

& $\omega = 2\pi f$

$$f = \frac{\omega}{2\pi} = \frac{211}{2\pi}$$

$$\boxed{f = 33.58\ Hz}$$

PHASOR :-

Phasor is a complex number that represents amplitude and phase of the sinosoidal.

$Z = x + jy$ Rectangular form

$Z = r\angle\theta$ Polar form

$Z = re^{j\theta}$ Exponential form

$$Z = r(\cos\theta + j\sin\theta)$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$\boxed{v(t) = V_m \cos(\omega t + \phi)} \rightarrow$ Time domain

Now.

$$V_m \,\mathbb{R}\, e^{j(\omega t + \phi)}$$

$$v(t) = V_m \,\mathbb{R}\, e^{j\omega t + j\phi}$$

$$v(t) = V_m \,\mathbb{R}\, e^{j\phi} \cdot e^{j\omega t}$$

$$v(t) = \mathbb{R}\,(V_m e^{j\phi}) \cdot (e^{j\omega t})$$

If we suppressed time part

$$v(t) = \mathbb{R}\, V_m e^{j\phi}$$

$$v(t) = V_m e^{j\phi}$$

$$V = V_m e^{j\phi}$$

$\boxed{V = V_m \angle\theta} \Rightarrow$ PHASOR DOMAIN.

Topic :- Description of AC (RC & RL) cir...

- Phase Relation in passive devices

R $\longrightarrow V = iR$

L $\longrightarrow V = I(\underbrace{j\omega L}_{X_L})$ $\longrightarrow \boxed{V = I X_L}$

C $\longrightarrow V = I\left(\underbrace{\frac{1}{j\omega C}}_{X_C}\right)$ $\longrightarrow \boxed{V = I X_C}$

In phasor domain

$\boxed{\bar{V} = \bar{I} X_L}$

$\boxed{\bar{V} = \bar{I}_c X_c}$

IMPEDENCE :-

The overall effect of Resistance and Reactance in the flow of current

$\boxed{Z = R + jX}$

R = Resistance (independent of frequency of Si...

X = Reactance (depends on frequency of signal) also (Nature of device Inductance or Capacitance).

Inductive Reactance $= X_L$

Capacitive Reactance $= X_C$

As we know that

$V_L = L\frac{di}{dt}$ if $i = I_m \cos(\omega t + \phi)$

$V_L = L\frac{d}{dt} I_m(\cos(\omega t + \phi))$

$V_L = I_m L\,[(-\sin(\omega t + \phi)\cdot\omega)]$

$V_L = I_m \omega L\,[\cos(\omega t + \phi + 90°)]$

$V_L = I_m \omega L\, \mathbb{R}e^{j(\omega t + \phi + 90°)}$

$V_L = I_m \omega L\, \mathbb{R}\, e^{j\omega t}\cdot e^{j\phi}\cdot e^{j90°}$

$V_L = I_m \omega L\, \mathbb{R}\, e^{j\omega t}\cdot e^{j\phi}\cdot j$

$V_L = I_m (j\omega L)\, \mathbb{R}\, e^{j\omega t}\, e^{j\phi}$

In phasor domain

$\bar{V}_L = \bar{I}_m (j\omega L) \angle \phi + 90°$

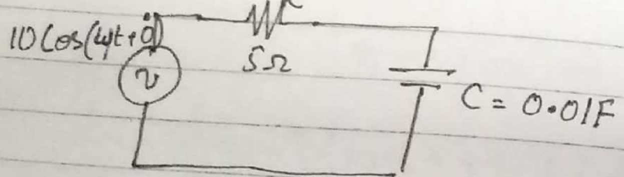
$\boxed{X_L = j\omega L = |X_L| \angle \phi_{X_L} + 90°.}$

$\boxed{X_C = \frac{1}{j\omega C} \Rightarrow |X_C| \angle \phi_{X_C} - 90°}$

Determine Impedance, Total current and voltage across each element and draw phasor diagram

$\bar{Z} = ?$ $\quad \bar{V}_R = ?$

$\bar{I} = ?$ $\quad \bar{V}_c = ?$



Solution:

we have

$V_s = 10 \cos(\omega t + 0^\circ) \Rightarrow 10 \cos(4t + 0^\circ)$

In phasor domain

$\bar{V}_s = 10 \angle 0^\circ$

let;

$Z_1 = R$ $\qquad Z_2 = X_c$

$Z = Z_1 + Z_2$

$Z = R + X_c$

$X_c = \dfrac{1}{j\omega C}$

$\omega = 4$

$C = 0.01F$

$\boxed{X_c = -j2.5}$ $\quad |X_c| = \sqrt{(2.5)^2} = 2.5$

$\boxed{X_c = 2.5 \angle -90^\circ}$ $\quad \phi = \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) = \tan^{-1}$

$\phi = -90^\circ$

$\boxed{Z = 5 - 2.5j\ \Omega}$

we know that

$\bar{Z} = |Z| \angle \phi_z \rightarrow (1)$

$|Z| = \sqrt{(5)^2 + (2.5)^2}$

$\boxed{|Z| = 5.59\Omega}$

$\phi_z = \tan^{-1}\left(\dfrac{Imaginary}{Real}\right) = \tan^{-1}\left(\dfrac{-2.5}{5}\right)$

Imaginary part

Real part

$\boxed{\phi_z = -26.5^\circ}$

putting in eqn (1)

$\bar{Z} = 5.59 \angle -26.5^\circ \Omega$

By ohm's law

$V = IZ$

$I = \dfrac{V}{Z} = \dfrac{10 \angle 0^\circ}{5.59 \angle -26.5^\circ}$

$\because r_1\angle\phi_1 \times r_2\angle\phi_2 = r_1 \times r_2 \angle \phi_1 + \phi_2$

$\dfrac{r_1\angle\phi_1}{r_2\angle\phi_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$

$I = \dfrac{10}{5.59} \angle 0^\circ - (-26.5^\circ)$

$\boxed{I = 1.79 \angle 26.5^\circ}$

As we have series circuit.

Voltage across resistor $\bar{V}_R$

$\bar{V}_R = \bar{I}\bar{R}$

$\bar{V}_R = (1.79\angle 26.5^\circ)(5\angle 0^\circ)$

$\boxed{\bar{V}_R = 8.95\angle 26.5^\circ \text{ V}}$

Voltage across capacitor $\bar{V}_c$

$\bar{V}_c = \bar{I}\bar{X}_c$

$\bar{V}_c = (1.79\angle 26.5^\circ)(2.5\angle -90^\circ)$

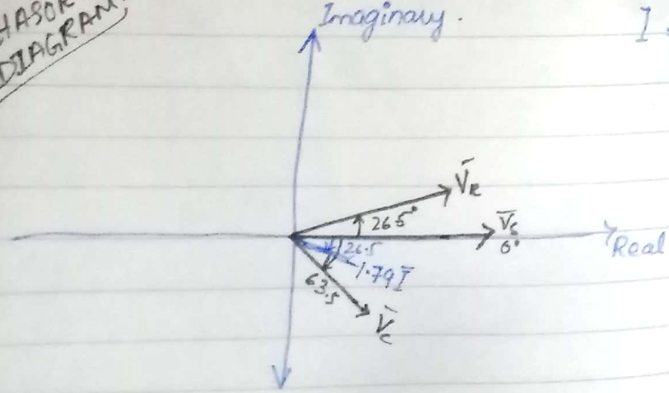
$\boxed{\bar{V}_c = 4.47\angle -63.5^\circ \text{ V}}$

Ans

x-axis → Real

y-axis → Imaginary

PHASOR DIAGRAM:



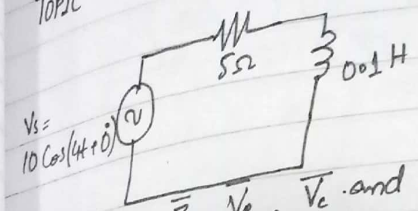
$V_s = 10\angle 0^\circ$

$V_R = 8.95\angle 26.5^\circ$

$V_c = 4.47\angle -63.5^\circ$

$I = 1.79\angle 26.5^\circ$

TOPIC :- AC Analysis of RL circuit :-



Determine $\bar{Z}$, $\bar{V}_R$, $\bar{V}_c$ and $\bar{I}$

Draw phasor diagram.

we have

$V_s = 10\cos(4t + 0^\circ)$

In phasor domain

$\bar{V}_s = 10\angle 0^\circ$

$X_L = j\omega L$

$X_L = j4(0.1)$

$X_L = 0.4j\ \Omega$

$|X_L| = 0.4\ \Omega$

$\phi_2 = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{0.4}{0}\right)$

$\boxed{\phi_2 = 90^\circ}$

$Z = R + jX_L$

$Z = 5 + 0.4j$

$|Z| = 5.01$

$\phi = \tan^{-1}(0.4/5)$

$\phi = 4.57^\circ$

$\boxed{\bar{Z} = 5.01\angle 4.57^\circ}$

$I = \frac{V_s}{Z} = \frac{10\angle 0^\circ}{5.01\angle 4.57^\circ}$

$I = 1.99\angle -4.57^\circ$

$\bar{V}_R = \bar{I}\bar{R}$

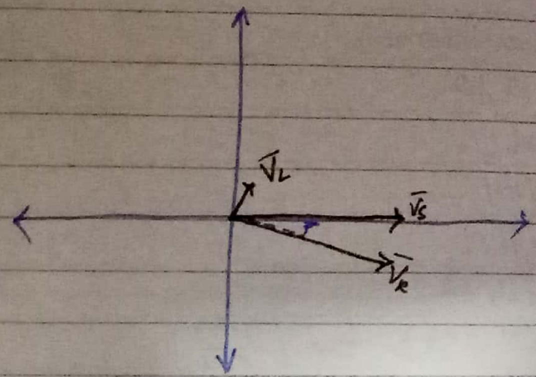
$\bar{V}_R = \left((1.99)\angle 4.57^\circ\right)(5\angle 0^\circ)$

$\boxed{\bar{V}_R = 9.95 \angle -4.57^\circ}$

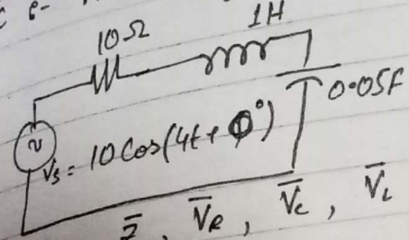
$\bar{V}_L = \bar{I}\bar{X}_L$

$\bar{V}_L = (1.99\angle -4.57^\circ)(0.4\angle 90^\circ)$

$\boxed{\bar{V}_L = 0.796 \angle -85.43}$



Topic :- AC Analysis of RLC circuit :-



Determine $\bar{Z}$, $\bar{V}_R$, $\bar{V}_C$, $\bar{V}_L$, $\bar{I}$ and draw phasor diagram

• Find the relation of current (Lead or Lag) with respect to voltage (Source).

Sol:

$V_s = 10\cos(4t + 0^\circ)$

$\bar{V}_s = 10\angle 0^\circ$

$Z_1 = R \qquad Z_2 = X_L \quad , \quad Z_3 = X_C$

$Z = Z_1 + Z_2 + Z_3 \longrightarrow (1)$

$X_L = j\omega L = 4j$

$|X_L| = 4$

$X_C = \dfrac{1}{j\omega C} = \dfrac{1}{(0.05)(4)j}$

$X_C = \dfrac{1}{0.2j} = -5j$

$|X_C| = 5$

$\phi_L = \tan^{-1}(y/x) = \tan^{-1}(4/0)$

$\phi_L = 90^\circ$

$\phi_c = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{-5}{0}\right) = \tan^{-1}(-\infty)$

$\phi_c = -90^\circ$

$\bar{X}_L = 4\angle 90^\circ$

$\bar{X}_C = .5\angle -90^\circ$

Now putting values in eq ①

$Z = 10 + 4j - 5j$

$Z = 10 - j$

$|Z| = \sqrt{(10)^2 + (1)^2}$

$|Z| = 10.04$

$\phi_z = \tan^{-1}(y/x) = \tan^{-1}(-1/10)$

$\phi_z = -5.7^\circ$

$\bar{Z} = 10.04\angle -5.71^\circ$

$\bar{I} = \frac{V_s}{Z} = \frac{10\angle 0^\circ}{10.04\angle -5.71}$

$\bar{I} = 0.995\angle 5.71^\circ$ Amps

$\bar{V}_R = \bar{I}\bar{R} = (0.995\angle 5.71^\circ)(10\angle 0^\circ)$

$\bar{V}_R = 9.95\angle 5.71^\circ$ volts

$\bar{V}_L = \bar{I}X_L$

$\bar{V}_L = (0.995\angle 5.71)(4\angle 90^\circ)$

$\bar{V}_L = 3.98\angle 95.71^\circ$ volts

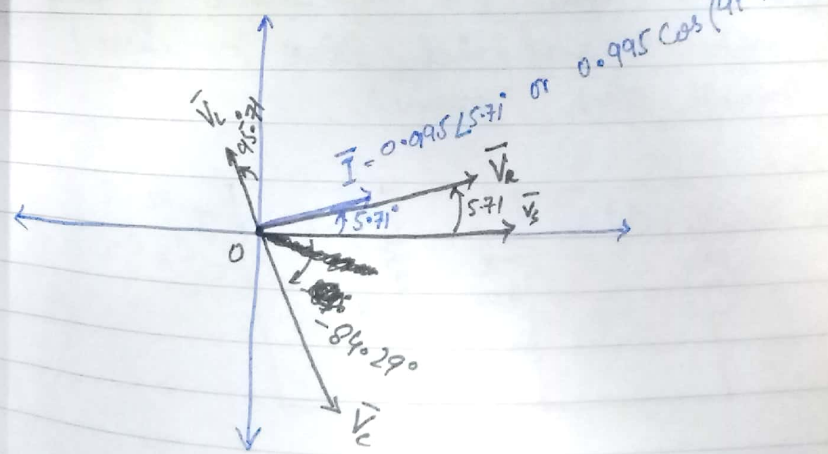
$\bar{V}_c = \bar{I}X_c$

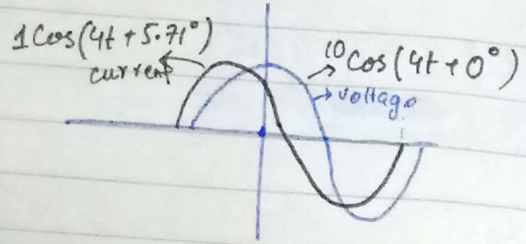
$\bar{V}_c = (0.995\angle 5.71)(5\angle 90^\circ)$

$\bar{V}_c = 4.975\angle -84.29^\circ$ volts

~~In inductor current will always Lag~~

* In capacitor current will lead voltage 90° (pure capacitor) (ideal)
* In inductor current will Lag voltage 90° (ideal).





The current is leading 5.71° to voltage

X -- X --- X ---X

$G = \frac{1}{R}$ (conductance) unit ℧ mho

R → resistance)

$B = \frac{1}{X}$ (Susceptance)

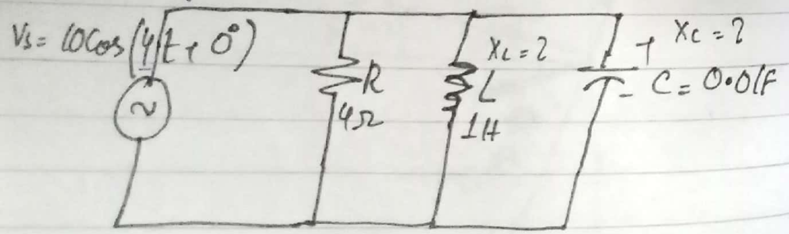
X → (Reactance)

$Y = \frac{1}{Z}$ (Admitance)

Z → (Impedance)

Q. Determine the impedance of ~~above~~ network and find ~~~~ the current through each Branch

$V_s = \cos(\omega t + \phi)$



Solution:

$V_s = 10\cos(4t + 0°)$

In phasor domain $\bar{V_s} = 10\angle 0°$

$Z_1 = R$ $\quad Z_2 = X_L$ $\quad Z_3 = X_C$

$X_L = j\omega L = 4j$

$|X_L| = 4$

$\phi_L = \tan^{-1}(4/0) = \phi_L = 90°$

$\bar{X_L} = 4\angle 90°$

$X_C = 1/j\omega C = \frac{1}{0.04j}$

$X_C = -25j$

$|X_C| = 25$

$\phi_C = -90°$

$\bar{X_C} = 25\angle -90°$

As in parallel.

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\frac{1}{Z_T} = \frac{1}{4} + \frac{1}{4j} + \frac{1}{(-25j)}$$

$$\frac{1}{Z_T} = 0.25 - 0.25j + 0.04j$$

$$Z_T = \frac{1}{0.25 - 0.21j}$$

Rationalize we get

$$Z_1 = \frac{0.25 + 0.21j}{(0.25)^2 - (0.21)^2}$$

$$\boxed{Z_1 = 2.4 + 2j}\ \Omega$$

$$|Z_T| = 3.12$$

$$\phi_T = \tan^{-1}(y/x) = \tan^{-1}(2/2.4)$$

$$\phi_T = 39.8^\circ$$

$$\bar{Z}_T = 3.12 \angle 39.8^\circ$$

Now, current through branch

$$I_R = \frac{\bar{V}_s}{R} = \frac{10\angle 0^\circ}{4\angle 0^\circ}$$

$$\boxed{I_R = 2.5\angle 0^\circ}\ \text{Amps}$$

$$I_L = \frac{\bar{V}_s}{X_L} = \frac{10\angle 0^\circ}{4\angle 90^\circ}$$

$$\boxed{I_L = 2.5\angle -90^\circ}\ \text{Amps}$$

$$I_C = \frac{V_s}{X_C} = \frac{10\angle 0^\circ}{25\angle -90^\circ}$$

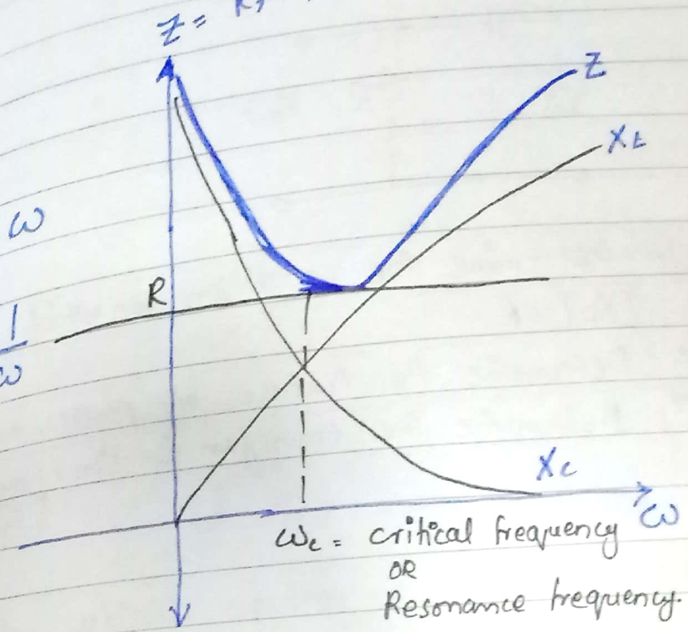
$$\boxed{I_C = 2.5\angle 90^\circ\ \text{Amps}}$$

Ans

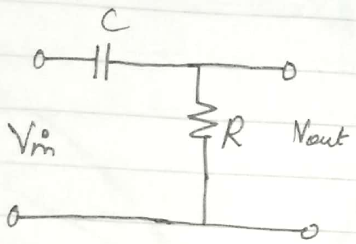
Relation b/w Impedance & Frequency

$X_L \propto \omega$

$X_C \propto \frac{1}{\omega}$



Topic :- Design of RC Network :.



Q. (i) Determine the condition for which the $|X_c| = R$.

(ii) Determine the Nature of phase angle

(iii) Determine the transfer function = $\left(\frac{\text{output}}{\text{Input}}\right)$

As we have by voltage divider rule

$$V_{out} = \frac{R}{X_c + R} \times V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{X_c + R} = \frac{R}{\frac{1}{j\omega C} + R}$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{\frac{1}{j}|X_c| + R}$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{R - j|X_c|}$$

$$\frac{V_{out}}{V_{in}} = \frac{\cancel{R}}{\cancel{R}\left(1 - j\frac{|X_c|}{R}\right)}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{\left(1 - j\frac{|X_c|}{R}\right)}} \rightarrow \text{Transfer function}$$

Rationalizing.

$$\frac{V_{out}}{V_{in}} = \frac{1 + j\frac{|X_c|}{R}}{\left(1 + \left(\frac{|X_c|}{R}\right)^2\right)}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{1 + \left(\frac{|X_c|}{R}\right)^2} + \frac{j\,|X_c|/R}{1 + \left(\frac{|X_c|}{R}\right)^2}}$$

→ Iska mode. Transfer function ~~koga~~ ka magnitude hoga

$$\phi = \tan^{-1}\left(\frac{\frac{|X_c|/R}{1 + \left(\frac{|X_c|}{R}\right)^2}}{\frac{1}{1 + \left(\frac{|X_c|}{R}\right)^2}}\right)$$

$$\phi = \tan^{-1}\left(\frac{|X_c|}{R}\right)$$

$$\tan\phi = \frac{|X_c|}{R}$$

$$R\tan\phi = |X_c|$$

when $\phi = 45$

$\tan 45 = 1$

Therefore

$R = |X_c|$

So when we have $\phi = 45$ therefore we get

$$\boxed{R = |X_c|}$$

Q. Design RC phase shifter network to obtain 60° phase shift make necessary assumption if required.

let;

$R = 1K\Omega$, $F = 100KHz$

Now for Capacitence. C.

$$|X_c| = \frac{1}{2\pi f C} \rightarrow (a)$$

$$|X_c| = R \tan\phi$$

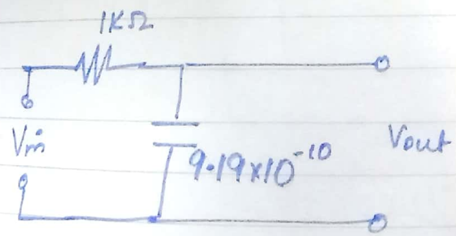
$$|X_c| = (1\times10^3)\tan 60° = 1\times10^3(\sqrt{3})$$

$$|X_c| = 1730$$

By eqn (a)

$$C = \frac{1}{2\pi f |X_c|} = \frac{1}{2\pi(100\times10^3)(1730)}$$

$$C = 9.19\times10^{-10} f$$

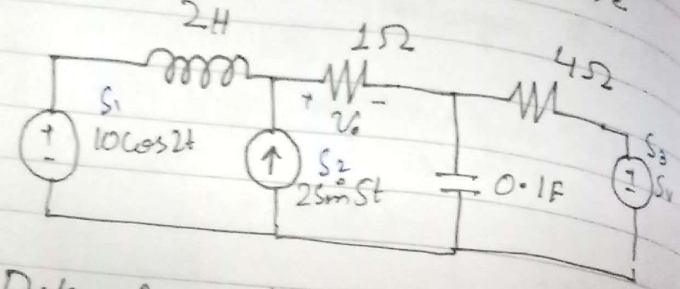


The output will be 60° phase ~~shift~~ shifts.

(65)

Topic: Sinosoidial & Steady State Anal

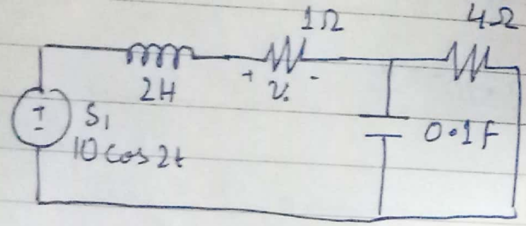
Topic - Super Position Theorem.



Determine V_o.

Solution:

Step 01. Considering S_1 as a single active source and eliminating S_2 & S_3 we have circuit



$\bar{S_1} = 10\angle 0^\circ$

$X_L = j\omega L = j(2)(2)$

$X_L = 4j$

$\bar{X_L} = 4\angle 90^\circ$

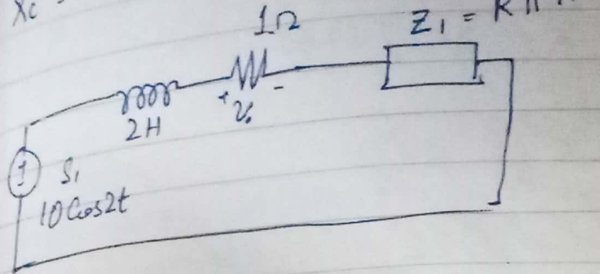
$X_C = \dfrac{1}{j\omega C} = \dfrac{1}{j(2)(0.2)}$

$X_C = -5j$

(66)

$\bar{X_C} = 5\angle -90^\circ$

Now



$Z_1 = \dfrac{R X_C}{R + X_C}$

$Z_1 = \dfrac{(4)(-5j)}{(4)+(-5j)} = \dfrac{-20j}{4-5j}$

$Z_1 = \dfrac{-20j(4+5j)}{(4)^2+(5)^2}$

$Z_1 = \dfrac{-20j(4+5j)}{41}$

$Z_1 = \dfrac{-20(4j-5)}{41}$

$Z_1 = \dfrac{20}{41}(5-4j)$

$Z_1 = 2.44 - 2j$

$\bar{Z_1} = |Z|\angle\phi$

$|Z| = \sqrt{(2.44)^2 + (2)^2}$

$|Z| = 3.15$

$\phi = \tan^{-1}\left(\dfrac{-2}{2.44}\right)$

$\phi = -39.80$

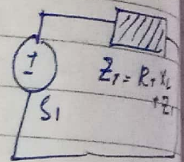
$\bar{Z}_1 = 3.15 \angle -39.80$.

Now for total impedance $\bar{Z}_T$ as resistor, inductor and Z_1 are in se

$Z_T = R + X_L + Z_1$

$Z_T = 1 + 4j + 2.44 - 2j$

$Z_T = 3.44 + 2j$



$|Z_T| = \sqrt{(3.44)^2 + (2)^2}$

$|Z_T| = 3.97$

$\phi = \tan^{-1}\left(\dfrac{2}{3.44}\right)$

$\phi = 30.17°$

$\bar{Z}_T = 3.97 \angle 30.17$

$\bar{I}_T = \dfrac{\bar{S}_1}{\bar{Z}_T} = \dfrac{10\angle 0°}{3.97\angle 30.17°}$

$\bar{I}_T = 2.51 \angle -30.17$

Now we want V_o as in series current is same

$\bar{V}_o = (\bar{I}_T)(\bar{R})$

$\bar{V}_o = (2.51 \angle -30.17)(1 \angle 0°)$

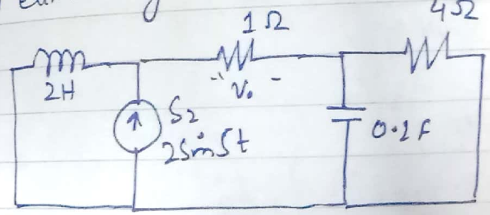
$\bar{V}_o = 2.51 \angle -30.17°$

or in time domain

$V_o = 2.51 \cos(2t - 30.17)$.

STEP 02:

Now considering S_2 as a active source and eliminating S_1 & S_3.



$S_2 = 2\sin 5t$

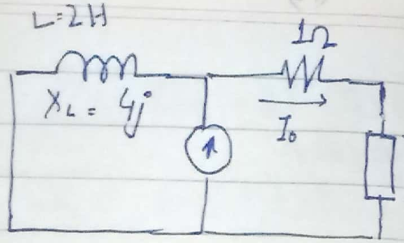
$\bar{S}_2 = 2 \angle -90°$

$\omega = 5$

$V_m \cos(\omega t + \phi) \Leftrightarrow V_m \angle \phi$

$V_m \sin(\omega t + \phi) \Rightarrow V_m \cos(\omega t + \phi - 90)$

$\Rightarrow V_m \angle \phi - 90°$



$\bar{Z}_1 = R \parallel X_c$

$\bar{Z}_1 = 3.15\angle{-39.80^\circ}$

$Z_1 = 2.44 - 2j$

~~Now R = 1Ω and Z_1 in series~~

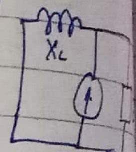
~~$\bar{Z}_2 = \bar{R} + \bar{Z}_1 = 1\angle 0^\circ + 3.15\angle{-39.80^\circ}$~~

~~$\bar{Z}_2 = 4.15$~~

As $R = 1\Omega$ and Z_1 are in series

$Z_2 = R + Z_1 = 1 + 2.44 - 2j$

$Z_2 = 3.44 - 2j$



By current divider rule

$$I_o = \left(\frac{X_L}{X_L + Z_2}\right) \times I_m \qquad \because I_m = S_2$$

$$I_o = \left(\frac{4j}{4j + 3.44 - 2j}\right) \times I_m$$

$$I_o = \left(\frac{4j}{3.44 + 2j}\right) \times I_m$$

$$I_o = \left[\frac{4j(3.44 - 2j)}{(3.44)^2 + (2)^2}\right] \times I_m$$

$$I_o = \frac{4(3.44j + 2)}{15.83}$$

$I_o = 0.25(2 + 3.44j) \times I_m$

$I_o = (0.50 + 0.86j) \times I_m$

$\bar{I}_o = 0.99\angle{59.8} \times 2\angle{-90^\circ}$

$\bar{I}_o = 1.98\angle{-30.17^\circ}$ Amps

Now for 'V_o' across $R = 1\Omega$

$\bar{V}_o = \bar{I}_o(\bar{R})$

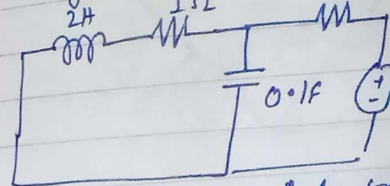
$\bar{V}_o = 1.98\angle{-30.17^\circ} \times 1\angle 0^\circ$

$\bar{V}_o = 1.98\angle{-30.17^\circ}$

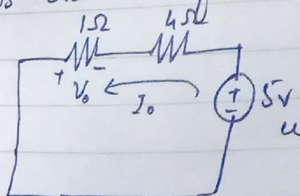
$V_o = 1.98\cos(5t - 30.17^\circ)$ in time domain

STEP 03

Consider S_3 as an active source eliminating S_1 and S_2



As dc therefore inductor short & capacitor open



$\omega = 0$

in dc frequency is zero

here

$X_L = j\omega L$

$\because \omega = 0$

$X_L = 0$

$X_C = 1/j\omega C$

$\because \omega = 0$

$X_C = \infty$.

By voltage divider rule

$$V_o = \frac{1}{5} \times S_3 = \frac{1}{5} \times 5$$

$V_o = 1$ volt.

Due to opposite polarity

$V_o = -1$ volts

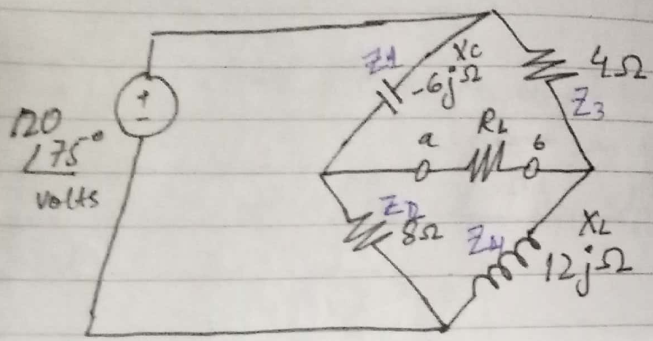
Now

Adds all three V_o to find total V_o

$$V_o = 2.51\cos(2t-30.17) + 1.98\cos(5t-30.17) - 1$$

Ans

TOPIC :- Thevenin Equivalent circuit.



Determine thevenin equivalent circuit.

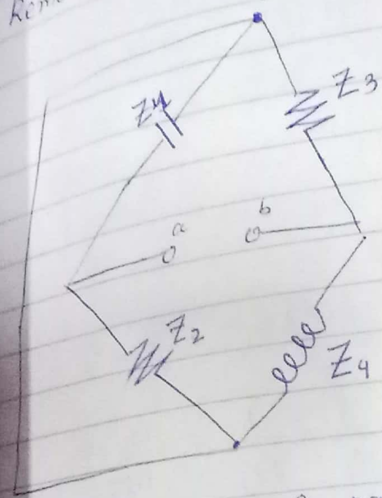
Solution.

$Z_{TH} = ?$

$V_{TH} = ? = V_{ab} = ?$

For Z_{TH}:

Remove source voltage and load resistor 'R_L'



∴ Z_1 & Z_2 are in parallel.

∴ Z_3 & Z_4 are in parallel.

So we find $Z_{12} = Z_1 \parallel Z_2$ & $Z_{34} = Z_3 \parallel Z_4$.

$$\frac{1}{Z_{12}} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{-6j} + \frac{1}{8}$$

$$\frac{1}{Z_{12}} = \frac{8-6j}{(-48j)}$$

$$Z_{12} = \frac{-48j}{8-6j} = \frac{-24j}{4-3j}$$

$$Z_{12} = \frac{-24j}{4-3j} \times \frac{(4+3j)}{(4+3j)}$$

$$Z_{12} = \frac{24j(3j-4)}{(4)^2+(3)^2}$$

$$Z_{12} = \frac{24}{25}(3-4j)$$

$$Z_{12} = 0.96(3-4j)$$

$$Z_{12} = 2.88 - 3.84j \ \Omega$$

$$\frac{1}{Z_{34}} = \frac{1}{Z_3} + \frac{1}{Z_4} = \frac{1}{4} + \frac{1}{12j} = \frac{12j+4}{48j}$$

$$Z_{34} = \frac{48j}{12j+4} = \frac{12j}{3j+1}$$

$$Z_{34} = \frac{12j(-3j+1)}{(3j+1)\times(-3j+1)}$$

$$Z_{34} = \frac{12(3+j)}{(3)^2+(1)^2}$$

$$Z_{34} \ \frac{12}{10}(3+j)$$

$$Z_{34} = 1.2(3+j)$$

$$Z_{34} = 3.6 + 1.2j \ \Omega$$

As Z_{12} and Z_{34} are in series.

$Z_{TH} = Z_{12} + Z_{34} = (2.88 - 3.88j) + (3.6 + 1.2j)$

$Z_{TH} = 6.48 - 2.68j$

In phasor

$|Z_{TH}| = \sqrt{(6.48)^2 + (-2.68)^2}$

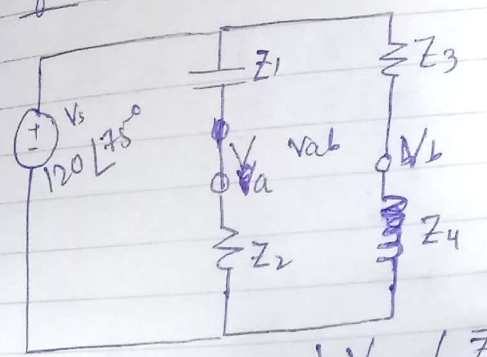
$|Z_{TH}| = 7$

$\phi = \tan^{-1}\left(\frac{\text{imaginary}}{\text{Real}}\right) = \tan^{-1}\left(\frac{-2.68}{6.48}\right)$

$\phi = -22.46°$

$\bar{Z}_{TH} = 7 \angle 22.46°$

Now for Vab:



$\bar{Z}_1 = 6\angle -90°$

$\bar{Z}_2 = 8\angle 0°$

$\bar{Z}_3 = 4\angle 0°$

$\bar{Z}_4 = 12\angle 90°$

$V_a = \left(\frac{\bar{Z}_2}{\bar{Z}_2 + \bar{Z}_1}\right) \times V_s$

$V_a = \left(\frac{8}{-6j+8}\right) V_s$

$V_a = \frac{4}{(-3j+4)} V_s$

$V_b = \left(\frac{\bar{Z}_4}{\bar{Z}_3 + \bar{Z}_4}\right) \times V_s$

(75)

$$V_a = \frac{4(4+3j)}{(4-3j)(4+3j)} \times V_s$$

$$V_a = \frac{4(4+3j)}{(4)^2+(3)^2} \times V_s$$

$$V_a = \frac{4}{25}(4+3j) \times V_s$$

$$V_a = 0.16(4+3j) \times V_s$$

$$V_a = (0.64 + 0.48j) \times V_s$$

$$V_a = 0.8\angle 36.86 \times 120\angle 75^\circ$$

$$V_a = \underbrace{96}_{r}\ \underbrace{\angle 111.86^\circ}_{\theta}$$

$x = r\cos\theta$

$y = r\sin\theta$

$Z = x + jy$

$$V_a = 96(\cos 111.86^\circ + j\sin 111.86)$$

$$\boxed{V_a = -35.74 + j\,89.09}$$

$$V_b = \left(\frac{Z_4}{Z_3+Z_4}\right) \times V_s$$

$$V_b = \frac{12j}{12j+4} \times V_s$$

$$V_b = \frac{12j(4-12j)}{(4+12j)(4-12j)} \times V_s$$

$$V_b = \frac{12(4j+12)}{(4)^2+(12)^2} \times V_s$$

(76)

$$V_b = \frac{12}{160}(12+4j) \times V_s$$

$$V_b = 0.075(12+4j) \times V_s$$

$$V_b = (0.9 + 0.3j) \times V_s$$

$$V_b = 0.94\angle 18.43 \times 120\angle 75^\circ$$

$$V_b = 112.8\angle 93.43$$

$$V_b = 112.8(\cos 93.43 + j\sin 93.43)$$

$$V_b = -6.748 + j\,112.59$$

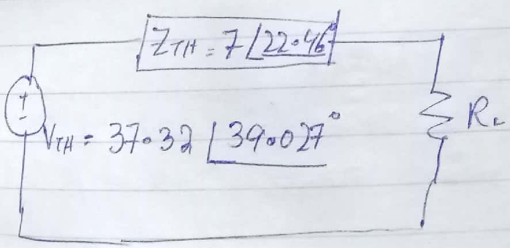
$V_{ab} = V_b - V_a = ?$

$$V_{ab} = -6.748 + 112.59j + 35.74 - 89.09j$$

$$V_{ab} = 28.992 + 23.50j$$

$$V_{ab} = 37.32\angle 39.027^\circ \text{ volts}$$

$V_{TH} = V_{ab}$



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Topic :- AC power.

* **Instantaneous power.**

Power in watts at any instant of time.

$$P(t) = v(t) \times i(t)$$

when

$$v(t) = V_m \cos(\omega t + \phi_v)$$

$$i(t) = I_m \cos(\omega t + \phi_i)$$

Now

$$P(t) = V_m \cos(\omega t + \phi_v) \times I_m \cos(\omega t + \phi_i)$$

$$P(t) = V_m I_m [\cos(\omega t + \phi_v) \cdot \cos(\omega t + \phi_i)]$$

$$\therefore \cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$P(t) = V_m I_m \cdot \frac{1}{2} [\cos(\phi_v - \phi_i) + \cos(2\omega t + \ldots$$

$$\boxed{P(t) = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i) - \frac{1}{2} V_m I_m \cos(2\omega t + \ldots}$$

* **Average Power.**

The average power (in watts) is the instantaneous power over a period of time

$$\boxed{P_{av} = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)}$$

$$P_{av} = \frac{1}{2} \bar{V}_m \bar{I}_m^{*} \cos(\phi_v - \phi_i).$$

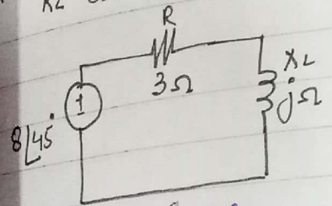
If $\bar{V} = V_m \angle \phi_v$

$\bar{I} = I_m \angle \phi_i$ (Phasor form)

$$\bar{V} \times \bar{I} = V_m I_m \angle \phi_v + \phi_i .$$

$$\boxed{\frac{1}{2} \bar{V} \times \bar{I} = \frac{1}{2} V_m I_m \angle \phi_v \times \phi_i}$$ → Average Power in Polar form.

Q. Determine the average power drawn by RL circuit ; Power drawn by resistor & inductor



Solution :

~~$Z = R + X_L$~~

~~$Z = 3 + j$~~

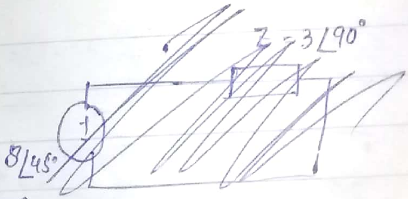
~~$|Z| = 3.162$~~

~~$\phi = \tan^{-1}(1/3)$~~

~~$Z = 3.162 \angle 18.43°$~~

~~$\bar{I} = \frac{8\angle 45°}{3.162\angle 18.43}$~~

$\bar{I} = 2 \angle 26.57$ Amps



~~$P_{av} = \frac{1}{2}(8\angle 45°)(2\angle 26.57)$~~

Sol:

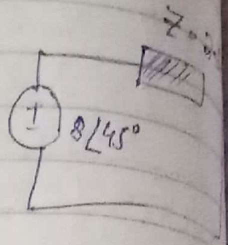
$Z = R + X_L$

$\bar{Z} = 3 + j$

$|Z| = \sqrt{(3)^2 + (1)^2}$

$|Z| = 3.162$

$\phi_z = \tan^{-1}(1/3) = 18.43°$



$\bar{Z} = 3.162 \angle 18.43$

$\bar{I}_m = \dfrac{8\angle 45°}{3.162\angle 18.43}$

$\bar{I}_m = 2.53 \angle 26.57°$

$\bar{P}_{av} = \frac{1}{2} \bar{V}_m \bar{I}_m^* \angle \phi_v - \phi_i$

$\bar{P}_{av} = \frac{1}{2} (8)(3.162) \angle 45 - 26.57$

$\bar{P}_{av} = 12.648 \angle 18.43°$ watts

Now Power across branches

$\bar{P}_R = \frac{1}{2} (\bar{I}_m)^2 R = \frac{1}{2} (2.53)^2 (3) \angle 26.53$

$P_R = 9.60 \angle 26.53$

$\bar{P}_I = \frac{1}{2} (I_m)^2 X_L = \frac{1}{2} (2.53)^2 \times (1) \angle 26.53° + 90°$

$\bar{P}_I = 3.20 \angle 116.53°$ Ans

$\because P = VI$

$P = I^2 R$

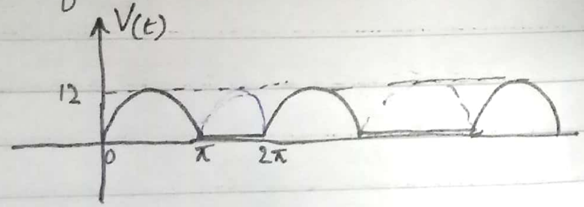
Effective OR RMS value of Period Circuit :

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_{(t)}^2\, dt}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i_{(t)}^2\, dt}$$

The effective value of periodic current the value of DC current that produces same amount of heat in a resistor as produces by periodic current, also known as Root mean square value of current.

Q. Determine the RMS value of voltage wave form shown in figure



$$V(t) = \begin{cases} 12\sin t & \text{when } 0 \le t \le \pi \\ 0 & \text{when } \pi \le t \le 2\pi \end{cases}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_{(t)}^2\, dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (12\sin t)\, dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{\pi}(12\sin t)^2 dt + \int_{\pi}^{2\pi}(0)^2\right]}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{\pi}(12\sin t)^2 dt}$$

$$V_{rms} = \sqrt{\frac{144}{T}\int_0^{\pi}\sin^2 t\, dt.}$$

$$V_{rms} = \sqrt{\frac{144}{2\pi}\int_0^{\pi}\frac{1-\cos 2t}{2}\, dt}$$

$$V_{rms} = \sqrt{\frac{144}{2\pi}\left[\int_0^{\pi}\frac{1}{2}\, dt - \frac{1}{2}\int_0^{\pi}\cos 2t\right]}$$

$$V_{rms} = \sqrt{\frac{144}{2\pi}\left[\frac{1}{2}\right]\left[t - \frac{1}{2}\sin 2t\right]_0^{\pi}}$$

$$V_{rms} = \sqrt{\frac{144}{4\pi}\left(\pi - \frac{1}{2}\sin 2\pi\right)}$$

$$V_{rms} = \sqrt{\frac{144}{4\pi}(\pi - 0)}$$

$$V_{rms} = \sqrt{\frac{144}{4\cancel{\pi}}(\cancel{\pi})}$$

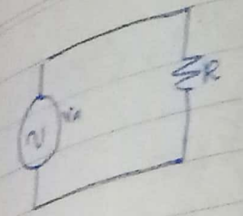
$V_{rms} =$ 6 volts.

Ans

(i) Real Power :-

Actual power consumed by load also called Active power measured in watts.

$$P = V_{rms} I_{rms} \underbrace{\cos\phi}_{\text{power factor } 0-1}.$$



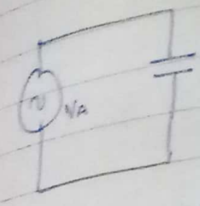
V_A = watts

$\phi_v = \phi_i$

ii) Reactive Power :-

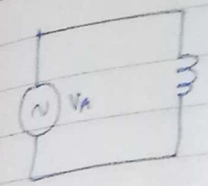
Amount of power that move back and forth from source to load but not consumed measured in VAR.

$$Q = V_{rms} I_{rms} \sin\phi.$$



$\boxed{S = Q}$

$\phi_v - \phi_i = 90^\circ$



$\boxed{S = Q}$

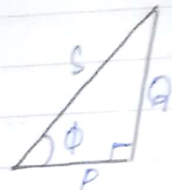
$\phi_v - \phi_i = 90^\circ.$

iii) Apparent Power :-

Total power drawn by load from a source including active and reactive component measured in VA.

$$S = V_{rms} \times I_{rms}.$$

Power Triangle :-



$P = S\cos\phi$

$Q = S\sin\phi$

Relation B/w Real, Reactive & Apparent Power :-

$S^2 = P^2 + Q^2$ Magnitude

$S^* = P + jQ$ complex form