Ex 1.1

a)

i) is Symmetric

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & y \\ x & -2 & -8 \end{bmatrix}$$

$$A = A^T$$

$$A^T = \begin{bmatrix} 4 & 2 & x \\ 2 & -1 & -2 \\ -1 & y & -8 \end{bmatrix} = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & y \\ x & -2 & -8 \end{bmatrix}$$

$\boxed{x = -1}$ $\boxed{y = -2}$

ii) is Orthogonal:

if the Matrix is orthogonal the all the vectors in the Matrix are orthonormal to each other. So just take the dot product of vectors.

$$\begin{pmatrix} 4 \\ 2 \\ x \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = 0 \qquad \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ y \\ -8 \end{pmatrix} = 0$$

$$8 - 2 - 2x = 0 \qquad -2 - y + 16 = 0$$

$$-2x = -6 \qquad \boxed{y = 14}$$

$$\boxed{x = 3}$$

iii) <u>has Rank 2</u>

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & y \\ x & -2 & -8 \end{bmatrix}$$

$$\begin{matrix} -4 & +2 & -2y \\ x & -2 & -8 \end{matrix}$$

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & y \\ x & -2 & -8 \end{bmatrix} \Rightarrow$$

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & y \\ x-4 & 0 & -2y-8 \end{bmatrix} \quad R_3 \leftarrow -2R_2 + R_3$$

So Rank is 2 the last Row of matrix should be 0 so

$$x - 4 = 0 \qquad -2y - 8 = 0$$

$$\boxed{x = 4} \qquad -2y = 8$$

$$\boxed{y = -4}$$

iv) Singular

$$|A| = 0$$

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & y \\ x & -2 & -8 \end{bmatrix}$$

$$|A| = \left[4(8+2y) - 2(-16 \mp xy) - 1(-4+x) \right]$$

$$0 = \left[16 + 8y + 32 + 2xy + 4 - x \right]$$

$$0 = (52 + 8y - x + 2xy)$$

So we have 2 variables and having one equation so we cannot compute the value of x and y

Ex 1.7

b)

$x = 4 \qquad y = 0$

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & 0 \\ 4 & -2 & -8 \end{bmatrix}$$

Eigen Decomposition

$$A = Q \Lambda Q^{-1}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 2 & -1 \\ 2 & -1-\lambda & 0 \\ 4 & -2 & -8-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)\left[(-8-\lambda)(-1-\lambda)\right] - 2\left[2(-8-\lambda)\right] - 1\left[-4-4(-1-\lambda)\right] = 0$$

$$(4-\lambda)(-8-\lambda)(-1-\lambda) - 4(-8-\lambda) - 1(\cancel{-4}+\cancel{4}+4\lambda) = 0$$

$$(-8-\lambda)\left[(4-\lambda)(-1-\lambda) - 4\right] - 4\lambda = 0$$

$$(-8-\lambda)\left[-4-4\lambda+\lambda+\lambda^2-4\right] = 0$$

$$-8-\lambda = 4\lambda$$

$$-8 = 5\lambda$$

$$\boxed{\lambda = -8/5}$$

$$\lambda^2 - 3\lambda - 8 = 4\lambda$$

$$\lambda^2 - 7\lambda - 8 = 0$$

$$(\lambda - 8)(\lambda + 1) = 0$$

$$\boxed{\lambda = -1} \qquad \boxed{\lambda = 8}$$

$\boxed{\lambda = -1}$

Find eigen vector

$$\begin{bmatrix} 5 & 2 & -1 \\ 2 & 0 & 0 \\ 4 & -2 & -7 \end{bmatrix} \begin{pmatrix} \vec{v}_1 \\ \vec{v}_2 \\ v_3 \end{pmatrix} = 0$$

$5v_1 + 2v_2 - v_3 = 0$

$2\vec{v}_1 = 0$

$4v_1 - 2v_2 - 7v_3 = 0$ — 3

Let $\boxed{v_3 = 1}$

$\boxed{v_1 = 0}$

$2v_2 = +v_3$

$\boxed{v_2 = \frac{+v_3}{2}}$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

Put in 3

$-v_3 - 7v_3 = 0$

$\boxed{v_3 = 0}$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda = 8$

$$\begin{bmatrix} -4 & 2 & -1 \\ 2 & -9 & 0 \\ 4 & -2 & -16 \end{bmatrix} \vec{v}_3$$

$-4v_1 + 2v_2 - v_3 = 0$

$2v_1 - 9v_2 = 0$

$4v_1 - 2v_2 - 16v_3 = 0$

$\boxed{v_1 = \frac{9v_2}{2}}$

$-\cancel{4}^{2}\left(\frac{9v_2}{\cancel{2}}\right) + 2v_2 - v_3 = 0$

$-16v_2 - v_3 = 0$

$\boxed{v_3 = -16v_2}$

Let $\boxed{v_2 = 1}$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 9/2 \\ 1 \\ -16 \end{pmatrix}$$

$\lambda = -8/5$

$$\begin{pmatrix} 4+8/5 & 2 & -1 \\ 2 & -1+8/5 & 0 \\ 4 & -2 & -8+\frac{8}{5} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} \frac{28}{5} & 2 & -1 \\ 2 & \frac{3}{5} & 0 \\ 4 & -2 & \frac{-32}{5} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$$

$$\frac{28}{5} v_1 + 2 v_2 - v_3 = 0$$

$$2 v_1 + \frac{3}{5} v_2 = 0$$

$$4 v_1 - 2 v_2 - \frac{32}{5} v_3 = 0$$

$$\boxed{v_1 = -\frac{3}{10} v_2}$$

$$\begin{array}{r} \overset{2}{2}8 \\ 3 \\ \hline 84 \end{array}$$

$$\frac{28}{5}\left(\frac{-3}{10} v_2\right) + 2 v_2 - v_3 = 0$$

Let $\boxed{v_2 = 1}$

$$\frac{-84 + 100 v_2}{50} = v_3$$

$$\boxed{\frac{16}{50} v_2 = v_3}$$

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \frac{-3}{10} \\ 1 \\ \frac{16}{50} \end{pmatrix}$$

eigen vectors is

$$B = \begin{bmatrix} -9/2 & 0 & -3/10 \\ 1 & 1/2 & 1 \\ -16 & 1 & 16/50 \end{bmatrix}$$

eigen value

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -8/5 \end{bmatrix}$$

Now take inverse of B Matrix and then put values in

$$\boxed{A = Q \Lambda Q^{-1}}$$

and Multiply Matrix

Ex 1.2

Eigen Values

a)

$$\lambda = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$V = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$A = V \lambda V^{-1}$$

$$V^{-1} = \frac{1}{3-2} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A = V \lambda V^{-1}$$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -4 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 3-8 & -6+24 \\ 1-4 & -2+12 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & +18 \\ -3 & 10 \end{bmatrix}$$

Ex 1.2

b)

Show that eigen values of AB is equals to eigen values of BA

Ans We will show that AB and BA have the same characteristics polynomial

$$P_{AB} = \det(AB - \lambda I)$$

$$= \det(ABAA^{-1} - \lambda AA^{-1})$$

$$= \det[A(BA - \lambda I)A^{-1}]$$

$$= \det(A)\det(BA - \lambda I)\det(A^{-1})$$

$$= \cancel{\det(A)}\det(BA - \lambda I)\frac{1}{\cancel{\det(A)}}$$

$$= \det(BA - \lambda I)$$

$$= P_{BA}$$ ~~[illegible]~~ re.

1.3. A).

$$v^T C v = v^T X^T X v$$

$$= (Xv)^T \cdot Xv$$

$$= \| X \cdot v \|^2 \geqslant 0$$

1.3. B)

$$\| U^T x \|_2^2 = (U^T x)^T \cdot U^T x$$

$$= x^T \cdot \underbrace{U \cdot U^T}_{I} \cdot x$$

$$= x^T \cdot I \cdot x$$

$$= x^T \cdot x$$

$$= \| x \|_2^2 = 1 .$$

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1.3. c) $\min_{\|v\|_2=1} v^T C v = \lambda_{min}$?

from the eigen decomposition property of c we can say that

$$C = Q^T \Lambda Q$$

then $v^T C v = v^T Q^T \Lambda Q v$

$$= (Qv)^T \Lambda Qv.$$ lets $Qv = w$.

$$= w^T \Lambda w = \lambda_1 w_1^2 + \dots + \lambda_n w_n^2$$

Obviously these values lie in $[\lambda_1 \dots \lambda_n]$ as we can see by setting $w = (1,0,0,\dots 0)$ and $w = (0,0,\dots,1)$ respectively.

then then minimum is obviously λ_1 which is the smallest eigenvalue. thus

$$\min_{\|v\|_2=1} w^T \Lambda w = \min \lambda$$