# Assignment

# Group Members

**Muhammad Hamza jamil 2572890 s8mujami@stud.uni-saarland.de**

**Hacane Hechehouche 2571617 S8hahech@stud.uni-Saarland.de**

Q no 1)

a)

$$f(x) = \langle w, x \rangle$$

$$= w^t \cdot x$$

$$= (w_1, w_2 \dots w_n) \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$f(x) = (w_1 x_1 + w_2 x_2 + \dots + w_n x_n)$$

$$\nabla_x (f(x)) = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$$

$$\boxed{\nabla_x (f(x)) = w}$$

---

b)

$$f(x) = \langle x, Ax \rangle$$

$$= x^T A x$$

$$Ax = \begin{bmatrix} \sum_{j=1}^{n} a_{1j} x_j \\ \sum_{j=1}^{n} a_{2j} x_j \\ \vdots \\ \sum_{j=1}^{n} a_{nj} x_j \end{bmatrix}$$

$$x^T A x = \sum_{i=1}^{n} \sum_{j=1}^{n} x_k a_{ij} x_k$$

$$\frac{\partial}{x_k} f(x) = \begin{bmatrix} \sum_{i=1}^{n} x_i a_{ik} + \sum_{j=1}^{n} a_{kj} x_j \\ \vdots \\ \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{n} x_i a_{ik} \\ \vdots \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{n} a_{kj} x_j \\ \vdots \end{bmatrix}$$

$$= Ax + A^T x$$

c)

$$f(x) = \|Bx\|_2^2$$

$$f(x) = (Bx)^T (Bx)$$

$$f(x) = \left(\sum_{k=1}^{n} \sum_{j=1}^{n} B_{ji} x_k\right)\left(\sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} x_k\right)$$

$$f(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ji} B_{ij} x_k^2$$

$$\nabla_x f(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ji} B_{ij} 2x_k$$

$$= 2 * \left(B^T B\right) x_k$$

$$\boxed{\nabla_x f(x) = 2B^T Bx}$$

d)

$$f(x) = \|Bx - c\|_2^2$$

$$= (Bx - c)^T (Bx - c)$$

$$= ((Bx)^T - (c)^T)(Bx - c)$$

$$= (Bx)^T (Bx) - c(Bx)^T - c^T Bx - c \quad - (1)$$

$(Bx)^T (Bx) = 2B^T Bx$ — From Part c

$\therefore c(Bx)^T$

$= c(x^T B^T)$

$= (x^T (B^T c))$

$= ((B^T c) x^T)$

Form Part a

$\boxed{c(Bx)^T = B^T c}$

$\therefore c^T Bx$

$= \left[(c^T B)^T\right] x$

$= (B^T c)^T x$

$= \langle B^T c, x \rangle$

From Part a

$\boxed{c^T Bx = B^T c}$

Put values in (1)

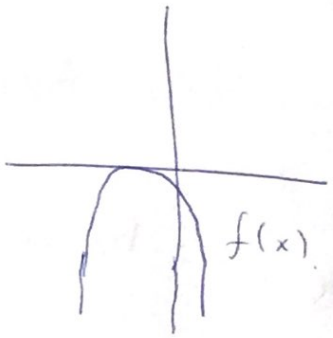
$$\nabla_x f(x) = 2B^T Bx - B^T c - B^T c - 0$$

$$= 2B^T Bx - 2B^T c$$

$$\boxed{\nabla_x f(x) = 2B^T (Bx - c)}$$

3.3

lets take $-(x+1)^2$ as $f(x)$




let's start at $x = -3$

with step = 1 and $\varepsilon = 0,5$

$$x_1 = x_0 + \varepsilon f'(x_0)$$

$$x_1 = -3 + 0,5 \cdot f'(-3)$$

$$= -3 + 0,5 \cdot (\overset{(-3)}{-2x} - 2)$$

$$= -3 + 0,5 \cdot 4 = \boxed{-1}$$

$$x_2 = x_1 + \varepsilon f'(x_1)$$

$$= -1 + 0,5 \cdot f'(-1)$$

$$= -1 + 0 = -1$$ the same. then we stop here

So $\arg\max f(x) = -1$