Assignment 2

Group Members

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Ex 2.1

a)

$$X = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}$$

$$\mu = (2.5 \quad 1.25)$$

$$\bar{X} = X - \mu$$

$$\bar{X} = \begin{bmatrix} -1.5 & -0.25 \\ -0.5 & 0.75 \\ 0.5 & -0.25 \\ 1.5 & -0.25 \end{bmatrix}$$

$$sd = (1.118 \quad 0.4330)$$

$$X = \bar{X}/sd$$

Normalized X is

$$X = \begin{bmatrix} -1.34 & -0.577 \\ -0.447 & 1.732 \\ 0.447 & -0.5773 \\ 1.34 & -0.5773 \end{bmatrix}$$

$$D = X^T X$$

$$D = \begin{bmatrix} 3.990 & -1.0322 \\ -1.0322 & 3.999 \end{bmatrix}$$

$$\begin{vmatrix} 3.9908-\lambda & -1.0322 \\ -1.0322 & 3.999-\lambda \end{vmatrix} = 0$$

$$(3.99-\lambda)(3.999-\lambda) - (+1.0654) = 0$$

$$(4-\lambda)(4-\lambda) - 1 = 0$$

$$16 - 4\lambda - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda^2 - 5\lambda - 3\lambda + 15 = 0$$

$$\lambda(\lambda - 5) - 3(\lambda - 5) = 0$$

$$(\lambda - 3)(\lambda - 5) = 0$$

$$\boxed{\lambda = 3} \qquad \boxed{\lambda = 5}$$

take the largest λ (eigen value) $\lambda = 5$

$$\begin{bmatrix} 4-5 & -1.0322 \\ -1.0322 & 4-5 \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$-V_1 - V_2 = 0$$
$$-V_1 - V_2 = 0$$

$$V_1 = -V_2$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow$$

magnitude $= \sqrt{2}$

$$D = PC_1 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \Rightarrow \begin{pmatrix} 0.7071 \\ -0.7071 \end{pmatrix}$$

$$C = D^T x$$

$$C_1 = \begin{pmatrix} 0.7071 & -0.7071 \end{pmatrix} \begin{pmatrix} -1.34 \\ -0.577 \end{pmatrix}$$

$$\boxed{C_1 = \cancel{0.44}\ -0.5430}$$

$$C_2 = \begin{pmatrix} 0.7071 & -0.7071 \end{pmatrix} \begin{pmatrix} -0.447 \\ 1.732 \end{pmatrix}$$

$$\boxed{C_2 = -1.53}$$

$$C_3 = \begin{pmatrix} 0.7071 & -0.7071 \end{pmatrix} \begin{pmatrix} 0.447 \\ -0.5773 \end{pmatrix}$$

$$\boxed{C_3 = 0.7207}$$

$$C_4 = \begin{pmatrix} 0.7071 & -0.7071 \end{pmatrix} \begin{pmatrix} 1.34 \\ -0.5773 \end{pmatrix}$$

$$\boxed{C_4 = 1.352}$$

1c b)

$$X = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -1 & 3 \\ -1 & 4 \end{bmatrix}$$

$$\mu = \left(-5/4 \quad 10/4\right) \Rightarrow \left(-1.25 \quad 2.5\right)$$

$$\bar{X} = X - \mu$$

$$\bar{X} = \begin{bmatrix} +0.25 & -1.5 \\ -0.75 & -0.5 \\ +0.25 & 0.5 \\ +0.25 & 1.5 \end{bmatrix}$$

$$sd = \left(0.4330 \quad 1.118\right)$$

$$X = \bar{X}/sd$$

Normalized X is

$$X = \begin{bmatrix} 0.5773 & -1.34 \\ -1.732 & -0.447 \\ 0.5773 & 0.447 \\ 0.5773 & 1.34 \end{bmatrix}$$

$$= X^T X$$

$$\text{Covariance Matrix} = \begin{bmatrix} 3.99 & 1.03 \\ 1.03 & 3.99 \end{bmatrix}$$

1c PC, or eigen vector $= \begin{pmatrix} 1/\sqrt{2} \end{pmatrix}$ $\begin{pmatrix} 0.7071 \end{pmatrix}$

$$= \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

eigen value and vector

eigen value

$$0 = \begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix}$$

$$(4-\lambda)(4-\lambda) - 1 = 0$$

$$16 - 4\lambda - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\lambda^2 - 5\lambda - 3\lambda + 15 = 0$$

$$\lambda(\lambda-5) - 3(\lambda-5) = 0$$

$$\boxed{\lambda = 3} \quad \boxed{\lambda = 5}$$

eigen vector for $\lambda = 5$

$$\begin{bmatrix} 4-5 & 1 \\ 1 & 4-5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-v_1 + v_2 = 0 \quad - (1)$$

$$v_1 - v_2 = 0 \quad - (2)$$

$$\boxed{v_2 = v_1}$$

1c PC, or eigen vector $= \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0.7071 \\ 0.7071 \end{pmatrix}$

f

$C_1 = D^T x$

$C_1 = (0.7071 \quad 0.7071) \begin{pmatrix} 0.25 \\ -1.5 \end{pmatrix}$

$\boxed{C_1 = -0.8838}$

$C_2 = (0.7071 \quad 0.7071) \begin{pmatrix} -0.75 \\ -0.5 \end{pmatrix}$

$\boxed{C_2 = -0.8838}$

$C_3 = (0.7071 \quad 0.7071) \begin{pmatrix} +0.25 \\ +0.5 \end{pmatrix}$

$\boxed{C_3 = 0.5303}$

$C_4 = (0.7071 \quad 0.7071) \begin{pmatrix} 0.25 \\ 1.5 \end{pmatrix}$

$\boxed{C_4 = 1.237}$

1(c) $\hat{x} = z v^T$ so it scales the eigen vector by the point value.

<u>First</u>

$x_1 = -0,543 \cdot [0,7071 \;, -0,7071]$

$= [-0,3839 \;, +0,3839]$

$x_2 = -1,53 \cdot [0,7071 \;, -0,7071]$

$= [-1,08 \;, +1,08]$

$x_3 = 0,7207 \cdot [0,7071 \;, -0,7071]$

$= [0,5096 \;, -0,5096]$

$x_4 = 1,352 \cdot [0,7021 \;, -0,7071]$

$= [0,955 \;, -0,955]$

<u>Second</u>

$x_1 = -0,8838 \cdot [0,7071 \;, 0,7071]$

$= [-0,6249 \;, -0,6249]$

$x_2 = [-0,6249 \;, -0,6249]$

$x_3 = [0,3749 \;, 0,3749]$

$x_4 = [0,8746 \;, 0,8746]$

1. d in image recognition we can extract many Features for our data, then using PCA we can pick the most Variant ~~feature~~ direction in our data ~~fea~~ then we compress it in that direction then we make the Image recognition on the new dimension.

1. e unsupervised. the parameter is how many eigen Values to work with (how many dimensions).

- the is no ~~presice~~ precise answer, but we take enough components to explaine enough of the Variance

1. f linear because it maps the values lineerly.

$$F(ax_1 + bx_2) = aF(x_1) + bF(x_2).$$

non linear : Isomap
Fast manifold learning
Laplacian eigen map.

a)

$$f(x) = \sin(\pi e^x)$$

$$u = \pi e^x$$

$$v = \sin u$$

$$\frac{du}{dx} = \pi e^x$$

$$\frac{dv}{du} = \cos u$$

$$\frac{df}{dx} = \frac{du}{dx} \times \frac{dv}{du}$$

$$= \pi e^x \cos u$$

$$\boxed{\text{Put } u = \pi e^x}$$

$$\boxed{\frac{df}{dx} = \pi e^x \cos(\pi e^x)}$$

$$f'(0) = \pi e^0 \cos(\pi e^0)$$

$$\boxed{f'(0) = -\pi}$$

$$f' = \pi e^x \cos \pi e^x$$

$$f'' = (\pi e^x)' \cos(\pi e^x) + (\cos(\pi e^x))' \pi e^x$$

$$v = \cos(\pi e^x)$$

$$z = \pi e^x$$

$$w = \cos(z)$$

$$\frac{dz}{dx} = \pi e^x$$

$$\frac{dw}{dz} = \cancel{\cos} -\sin z$$

$$\frac{dv}{dx} = -\pi e^x \sin z$$

$$\text{Put } z = \pi e^x$$

$$\boxed{\frac{dv}{dx} = -\pi e^x \sin(\pi e^x)}$$

$$f''(x) = \pi e^x \cos \pi e^x - (\pi e^x \sin(\pi e^x)) \pi e^x$$

$$f''(x) = \pi e^x \cos \pi e^x - \pi^2 (e^x)^2 \sin(\pi e^x)$$

$$f''(0) = \pi e^0 \cos(\pi e^0) - \pi^2 (e^0)^2 \sin(\pi e^0)$$

$$\boxed{f''(0) = -\pi}$$

(2)

(a) $f'(x) = \pi e^{x} . \cos(\pi e^{x})$

$f''(x) = \pi\left(e^{x} \cos(\pi e^{x}) - e^{x} . \sin(\pi e^{x})\right)$

$= \pi e^{x}\left(\cos(\pi e^{x}) - \sin(\pi e^{x})\right)$

$f'(0) = \pi e^{0} . \cos(\pi . e^{0}) = -\pi .$

$f''(0) = \pi e^{0}\left(\cos(\pi e^{0}) - \sin(\pi e^{0})\right)$

$= -\pi$

(b)

$$\begin{cases} f'(x) = 18x - 9x^2 \\ f''(x) = 18 - 18x \end{cases}$$

$\cancel{f'(x) = 0}$

$f'(x) = 0$

$18x - 9x^2 = 0$

$x(18 - 9x) = 0 \longrightarrow \boxed{x = 0}$

$\boxed{x = 2}$

for $\underline{x = 0}$

$f''(x=0) = 18$

for $x = 2$

$f''(x=2) = -18$

so at $x = 0$ we have minimum.

at $x = 2$ we have maximum.

Ex 2.3

Prove

A is a symmetric matrix, so the eigen value of the matrix A is.

$$Av = \lambda v$$

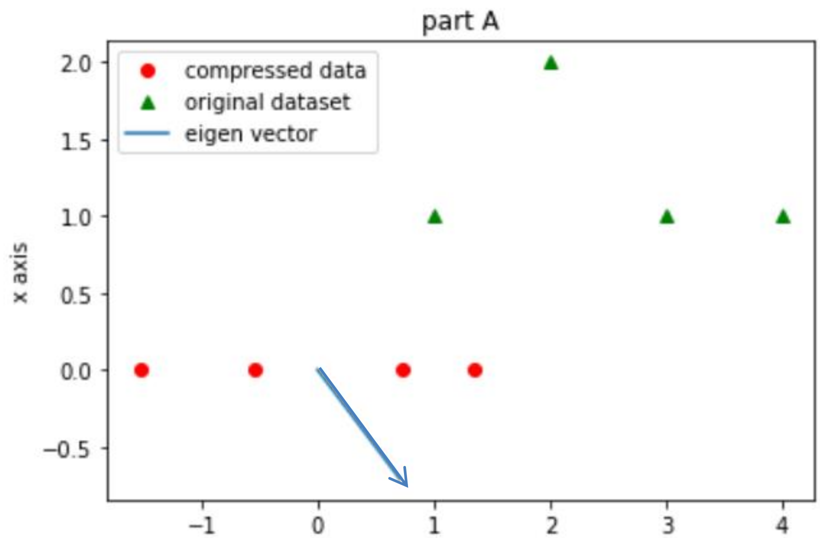
multiply A^{K-1} on both sides

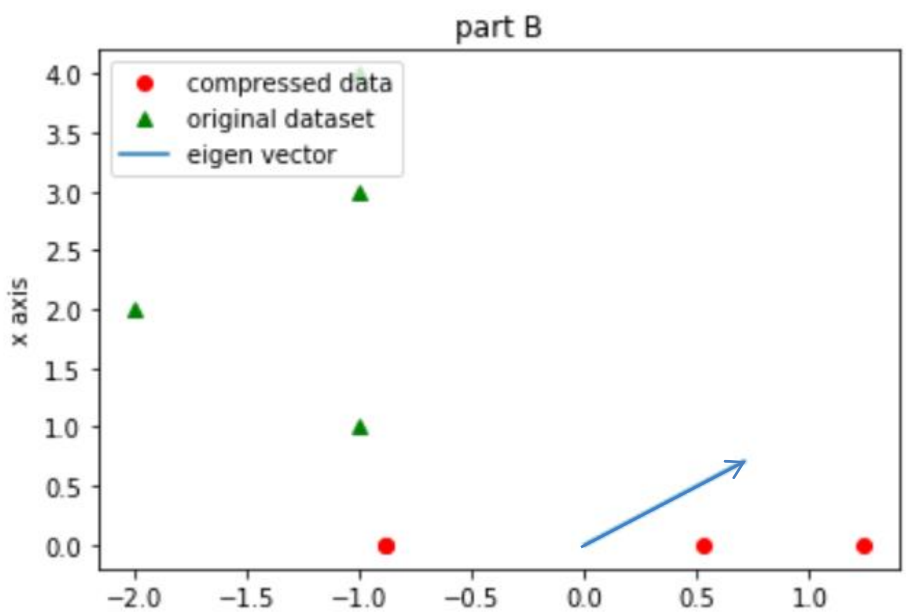
$$A^{K-1}Av = A^{K-1}\lambda v$$

$$A^{K}v = \lambda(A^{K-1}v)$$

$$A^{K}v = \lambda(\lambda^{K-1}v) \qquad \because Av = \lambda v$$

$$\boxed{A^{K}v = \lambda^{K}v}$$


part A
compressed data
original dataset
eigen vector
x axis
2.0
1.5
1.0
0.5
0.0
−0.5
−1
0
1
2
3
4


part B
compressed data
original dataset
eigen vector
x axis
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0