2

Frequency Distributions and Graphs

STATISTICS TODAY

How Your Identity Can Be Stolen

Identity fraud is a big business today—more than 12.7 million people were victims. The total amount of the fraud in 2014 was $16 billion. The average amount of the fraud for a victim is $1260, and the average time to correct the problem is 40 hours. The ways in which a person's identity can be stolen are presented in the following table:

Government documents or benefits fraud	38.7%
Credit card fraud	17.4
Phone or utilities fraud	12.5
Bank fraud	8.2
Attempted identity theft	4.8
Employment-related fraud	4.8
Loan fraud	4.4
Other identity theft	9.2

Source: Javelin Strategy & Research; Council of Better Business Bureau, Inc.

Looking at the numbers presented in a table does not have the same impact as presenting numbers in a well-drawn chart or graph. The article did not include any graphs. This chapter will show you how to construct appropriate graphs to represent data and help you to get your point across to your audience.

See Statistics Today—Revisited at the end of the chapter for some suggestions on how to represent the data graphically.

OUTLINE

OBJECTIVES

After completing this chapter, you should be able to

1. Organize data using a frequency distribution.
2. Represent data in frequency distributions graphically, using histograms, frequency polygons, and ogives.
3. Represent data using bar graphs, Pareto charts, time series graphs, pie graphs, and dotplots.

Introduction

When conducting a statistical study, the researcher must gather data for the particular variable under study. For example, if a researcher wishes to study the number of people who were bitten by poisonous snakes in a specific geographic area over the past several years, he or she has to gather the data from various doctors, hospitals, or health departments.

To describe situations, draw conclusions, or make inferences about events, the researcher must organize the data in some meaningful way. The most convenient method of organizing data is to construct a *frequency distribution.*

After organizing the data, the researcher must present them so they can be understood by those who will benefit from reading the study. The most useful method of presenting the data is by constructing *statistical charts* and *graphs.* There are many different types of charts and graphs, and each one has a specific purpose.

This chapter explains how to organize data by constructing frequency distributions and how to present the data by constructing charts and graphs. The charts and graphs illustrated here are histograms, frequency polygons, ogives, pie graphs, Pareto charts, and time series graphs. A graph that combines the characteristics of a frequency distribution and a histogram, called a stem and leaf plot, is also explained.

2–1 Organizing Data

OBJECTIVE 1

Organize data using a frequency distribution.

Suppose a researcher wished to do a study on the ages of the 50 wealthiest people in the world. The researcher first would have to get the data on the ages of the people. In this case, these ages are listed in *Forbes Magazine*. When the data are in original form, they are called **raw data** and are listed next.

45	46	64	57	85
92	51	71	54	48
27	66	76	55	69
54	44	54	75	46
61	68	78	61	83
88	45	89	67	56
81	58	55	62	38
55	56	64	81	38
49	68	91	56	68
46	47	83	71	62

Since little information can be obtained from looking at raw data, the researcher organizes the data into what is called a *frequency distribution.*

Unusual Stats

Of Americans 50 years old and over, 23% think their greatest achievements are still ahead of them.

A **frequency distribution** is the organization of raw data in table form, using classes and frequencies.

Each raw data value is placed into a quantitative or qualitative category called a **class.** The **frequency** of a class then is the number of data values contained in a specific class. A frequency distribution is shown for the preceding data set.

Class limits	Tally	Frequency
27–35	/	1
36–44	///	3
45–53	~~////~~ ////	9
54–62	~~////~~ ~~////~~ ~~////~~	15
63–71	~~////~~ ~~////~~	10
72–80	///	3
81–89	~~////~~ //	7
90–98	//	2
		50

Now some general observations can be made from looking at the frequency distribution. For example, it can be stated that the majority of the wealthy people in the study are 45 years old or older.

The classes in this distribution are 27–35, 36–44, etc. These values are called *class limits*. The data values 27, 28, 29, 30, 31, 32, 33, 34, 35 can be tallied in the first class; 36, 37, 38, 39, 40, 41, 42, 43, 44 in the second class; and so on.

Two types of frequency distributions that are most often used are the *categorical frequency distribution* and the *grouped frequency distribution*. The procedures for constructing these distributions are shown now.

Categorical Frequency Distributions

The **categorical frequency distribution** is used for data that can be placed in specific categories, such as nominal- or ordinal-level data. For example, data such as political affiliation, religious affiliation, or major field of study would use categorical frequency distributions.

EXAMPLE 2–1 Distribution of Blood Types

Twenty-five army inductees were given a blood test to determine their blood type. The data set is

A	B	B	AB	O
O	O	B	AB	B
B	B	O	A	O
A	O	O	O	AB
AB	A	O	B	A

Construct a frequency distribution for the data.

SOLUTION

Since the data are categorical, discrete classes can be used. There are four blood types: A, B, O, and AB. These types will be used as the classes for the distribution.

The procedure for constructing a frequency distribution for categorical data is given next.

Step 1 Make a table as shown.

A Class	B Tally	C Frequency	D Percent
A			
B			
O			
AB			

Step 2 Tally the data and place the results in column B.

Step 3 Count the tallies and place the results in column C.

Step 4 Find the percentage of values in each class by using the formula

$$\% = \frac{f}{n} \cdot 100$$

where f = frequency of the class and n = total number of values. For example, in the class of type A blood, the percentage is

$$\% = \frac{5}{25} \cdot 100 = 20\%$$

Percentages are not normally part of a frequency distribution, but they can be added since they are used in certain types of graphs such as pie graphs. Also, the decimal equivalent of a percent is called a *relative frequency.*

Step 5 Find the totals for columns C (frequency) and D (percent). The completed table is shown. It is a good idea to add the percent column to make sure it sums to 100%. This column won't always sum to 100% because of rounding.

A Class	B Tally	C Frequency	D Percent
A	~~////~~	5	20
B	~~////~~ //	7	28
O	~~////~~ ////	9	36
AB	////	4	16
		Total 25	100%

For the sample, more people have type O blood than any other type.

Grouped Frequency Distributions

When the range of the data is large, the data must be grouped into classes that are more than one unit in width, in what is called a **grouped frequency distribution.** For example, a distribution of the blood glucose levels in milligrams per deciliter (mg/dL) for 50 randomly selected college students is shown.

Unusual Stats

Six percent of Americans say they find life dull.

Class limits	Class boundaries	Tally	Frequency
58–64	57.5–64.5	/	1
65–71	64.5–71.5	~~////~~ /	6
72–78	71.5–78.5	~~////~~ ~~////~~	10
79–85	78.5–85.5	~~////~~ ~~////~~ ////	14
86–92	85.5–92.5	~~////~~ ~~////~~ //	12
93–99	92.5–99.5	~~////~~	5
100–106	99.5–106.5	//	2
			Total 50

The procedure for constructing the preceding frequency distribution is given in Example 2–2; however, several things should be noted. In this distribution, the values 58 and 64 of the first class are called *class limits.* The **lower class limit** is 58; it represents the smallest data value that can be included in the class. The **upper class limit** is 64; it represents the largest data value that can be included in the class. The numbers in the second column are called **class boundaries.** These numbers are used to separate the classes so that there are no gaps in the frequency distribution. The gaps are due to the limits; for example, there is a gap between 64 and 65.

Students sometimes have difficulty finding class boundaries when given the class limits. The basic rule of thumb is that *the class limits should have the same decimal place value as the data, but the class boundaries should have one additional place value and end in a 5.* For example, if the values in the data set are whole numbers, such as 59, 68, and 82, the limits for a class might be 58–64, and the boundaries are 57.5–64.5. Find the boundaries by subtracting 0.5 from 58 (the lower class limit) and adding 0.5 to 64 (the upper class limit).

$$\text{Lower limit} - 0.5 = 58 - 0.5 = 57.5 = \text{lower boundary}$$
$$\text{Upper limit} + 0.5 = 64 + 0.5 = 64.5 = \text{upper boundary}$$

Unusual Stats

One out of every hundred people in the United States is color-blind.

If the data are in tenths, such as 6.2, 7.8, and 12.6, the limits for a class hypothetically might be 7.8–8.8, and the boundaries for that class would be 7.75–8.85. Find these values by subtracting 0.05 from 7.8 and adding 0.05 to 8.8.

Class boundaries are not always included in frequency distributions; however, they give a more formal approach to the procedure of organizing data, including the fact that sometimes the data have been rounded. You should be familiar with boundaries since you may encounter them in a statistical study.

Finally, the **class width** for a class in a frequency distribution is found by subtracting the lower (or upper) class limit of one class from the lower (or upper) class limit of the next class. For example, the class width in the preceding distribution on the distribution of blood glucose levels is 7, found from $65 - 58 = 7$.

The class width can also be found by subtracting the lower boundary from the upper boundary for any given class. In this case, $64.5 - 57.5 = 7$.

Note: Do not subtract the limits of a single class. It will result in an incorrect answer.

The researcher must decide how many classes to use and the width of each class. To construct a frequency distribution, follow these rules:

1. *There should be between 5 and 20 classes.* Although there is no hard-and-fast rule for the number of classes contained in a frequency distribution, it is of utmost importance to have enough classes to present a clear description of the collected data.
2. *It is preferable but not absolutely necessary that the class width be an odd number.* This ensures that the midpoint of each class has the same place value as the data. The **class midpoint** X_m is obtained by adding the lower and upper boundaries and dividing by 2, or adding the lower and upper limits and dividing by 2:

$$X_m = \frac{\text{lower boundary} + \text{upper boundary}}{2}$$

or

$$X_m = \frac{\text{lower limit} + \text{upper limit}}{2}$$

For example, the midpoint of the first class in the example with glucose levels is

$$\frac{57.5 + 64.5}{2} = 61 \qquad \text{or} \qquad \frac{58 + 64}{2} = 61$$

The midpoint is the numeric location of the center of the class. Midpoints are necessary for graphing (see Section 2–2). If the class width is an even number, the midpoint is in tenths. For example, if the class width is 6 and the boundaries are 5.5 and 11.5, the midpoint is

$$\frac{5.5 + 11.5}{2} = \frac{17}{2} = 8.5$$

Rule 2 is only a suggestion, and it is not rigorously followed, especially when a computer is used to group data.

3. *The classes must be mutually exclusive.* Mutually exclusive classes have nonoverlapping class limits so that data cannot be placed into two classes. Many times, frequency distributions such as this

Age
10–20
20–30
30–40
40–50

are found in the literature or in surveys. If a person is 40 years old, into which class should she or he be placed? A better way to construct a frequency distribution is to use classes such as

Age
10–20
21–31
32–42
43–53

Recall that boundaries are mutually exclusive. For example, when a class boundary is 5.5 to 10.5, the data values that are included in that class are values from 6 to 10. A data value of 5 goes into the previous class, and a data value of 11 goes into the next-higher class.

4. *The classes must be continuous.* Even if there are no values in a class, the class must be included in the frequency distribution. There should be no gaps in a frequency distribution. The only exception occurs when the class with a zero frequency is the first or last class. A class with a zero frequency at either end can be omitted without affecting the distribution.

5. *The classes must be exhaustive.* There should be enough classes to accommodate all the data.

6. *The classes must be equal in width.* This avoids a distorted view of the data.

   One exception occurs when a distribution has a class that is **open-ended.** That is, the first class has no specific lower limit, or the last class has no specific upper limit. A frequency distribution with an open-ended class is called an **open-ended distribution.** Here are two examples of distributions with open-ended classes.

Age	Frequency
10–20	3
21–31	6
32–42	4
43–53	10
54 and above	8

Minutes	Frequency
Below 110	16
110–114	24
115–119	38
120–124	14
125–129	5

The frequency distribution for age is open-ended for the last class, which means that anybody who is 54 years or older will be tallied in the last class. The distribution for minutes is open-ended for the first class, meaning that any minute values below 110 will be tallied in that class.

The steps for constructing a grouped frequency distribution are summarized in the following Procedure Table.

Procedure Table

Constructing a Grouped Frequency Distribution

Step 1 Determine the classes.

Find the highest and lowest values.

Find the range.

Select the number of classes desired.

Find the width by dividing the range by the number of classes and rounding up.

Select a starting point (usually the lowest value or any convenient number less than the lowest value); add the width to get the lower limits.

Find the upper class limits.

Find the boundaries.

Step 2 Tally the data.

Step 3 Find the numerical frequencies from the tallies, and find the cumulative frequencies.

Example 2–2 shows the procedure for constructing a grouped frequency distribution, i.e., when the classes contain more than one data value.

EXAMPLE 2–2 Record High Temperatures

These data represent the record high temperatures in degrees Fahrenheit (°F) for each of the 50 states. Construct a grouped frequency distribution for the data, using 7 classes.

112	100	127	120	134	118	105	110	109	112
110	118	117	116	118	122	114	114	105	109
107	112	114	115	118	117	118	122	106	110
116	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Source: The World Almanac and Book of Facts.

SOLUTION

The procedure for constructing a grouped frequency distribution for numerical data follows.

Unusual Stats

America's most popular beverages are soft drinks. It is estimated that, on average, each person drinks about 52 gallons of soft drinks per year, compared to 22 gallons of beer.

Step 1 Determine the classes.

Find the highest value and lowest value: $H = 134$ and $L = 100$.

Find the range: R = highest value − lowest value = $H - L$, so

$$R = 134 - 100 = 34$$

Select the number of classes desired (usually between 5 and 20). In this case, 7 is arbitrarily chosen.

Find the class width by dividing the range by the number of classes.

$$\text{Width} = \frac{R}{\text{number of classes}} = \frac{34}{7} = 4.9$$

Round the answer up to the nearest whole number if there is a remainder: 4.9 ≈ 5. (Rounding *up* is different from rounding *off.* A number is rounded up if there is any decimal remainder when dividing. For example, 85 ÷ 6 = 14.167 and is rounded up to 15. Also, 53 ÷ 4 = 13.25 and is rounded up to 14. (Also, after dividing, if there is no remainder, you will need to add an extra class to accommodate all the data.)

Select a starting point for the lowest class limit. This can be the smallest data value or any convenient number less than the smallest data value. In this case, 100 is used. Add the width to the lowest score taken as the starting point to get the lower limit of the next class. Keep adding until there are 7 classes, as shown, 100, 105, 110, etc.

Subtract one unit from the lower limit of the second class to get the upper limit of the first class. Then add the width to each upper limit to get all the upper limits.

$$105 - 1 = 104$$

The first class is 100–104, the second class is 105–109, etc.

Find the class boundaries by subtracting 0.5 from each lower class limit and adding 0.5 to each upper class limit:

99.5–104.5, 104.5–109.5, etc.

Historical Note

Florence Nightingale, a nurse in the Crimean War in 1854, used statistics to persuade government officials to improve hospital care of soldiers in order to reduce the death rate from unsanitary conditions in the military hospitals that cared for the wounded soldiers.

Step 2 Tally the data.

Step 3 Find the numerical frequencies from the tallies.

The completed frequency distribution is

Class limits	Class boundaries	Tally	Frequency
100–104	99.5–104.5	//	2
105–109	104.5–109.5	~~////~~ ///	8
110–114	109.5–114.5	~~////~~ ~~////~~ ~~////~~ ///	18
115–119	114.5–119.5	~~////~~ ~~////~~ ///	13
120–124	119.5–124.5	~~////~~ //	7
125–129	124.5–129.5	/	1
130–134	129.5–134.5	/	1
			Total 50

The frequency distribution shows that the class 109.5–114.5 contains the largest number of temperatures (18) followed by the class 114.5–119.5 with 13 temperatures. Hence, most of the temperatures (31) fall between 110 and 119°F.

Sometimes it is necessary to use a *cumulative frequency distribution*. A **cumulative frequency distribution** is a distribution that shows the number of data values less than or equal to a specific value (usually an upper boundary). The values are found by adding the frequencies of the classes less than or equal to the upper class boundary of a specific class. This gives an ascending cumulative frequency. In this example, the cumulative frequency for the first class is 0 + 2 = 2; for the second class it is 0 + 2 + 8 = 10; for the third class it is 0 + 2 + 8 + 18 = 28. Naturally, a shorter way to do this would be to just add the cumulative frequency of the class below to the frequency of the given class. For example, the cumulative frequency for the number of data values less than 114.5 can be found by adding 10 + 18 = 28. The cumulative frequency distribution for the data in this example is as follows:

	Cumulative frequency
Less than 99.5	0
Less than 104.5	2
Less than 109.5	10
Less than 114.5	28
Less than 119.5	41
Less than 124.5	48
Less than 129.5	49
Less than 134.5	50

Cumulative frequencies are used to show how many data values are accumulated up to and including a specific class. In Example 2–2, of the total record high temperatures 28 are less than or equal to 114°F. Forty-eight of the total record high temperatures are less than or equal to 124°F.

After the raw data have been organized into a frequency distribution, it will be analyzed by looking for peaks and extreme values. The peaks show which class or classes have the most data values compared to the other classes. Extreme values, called *outliers,* show large or small data values that are relative to other data values.

When the range of the data values is relatively small, a frequency distribution can be constructed using single data values for each class. This type of distribution is called an **ungrouped frequency distribution** and is shown next.

EXAMPLE 2–3 Hours of Sleep

The data shown represent the number of hours 30 college students said they sleep per night. Construct and analyze a frequency distribution.

8	6	6	8	5	7
7	8	7	6	6	7
9	7	7	6	8	10
6	7	6	7	8	7
7	8	7	8	9	8

SOLUTION

Step 1 Determine the number of classes. Since the range is small (10 − 5 = 5), classes consisting of a single data value can be used. They are 5, 6, 7, 8, 9, and 10.

Note: If the data are continuous, class boundaries can be used. Subtract 0.5 from each class value to get the lower class boundary, and add 0.5 to each class value to get the upper class boundary.

Step 2 Tally the data.

Step 3 From the tallies, find the numerical frequencies and cumulative frequencies. The completed ungrouped frequency distribution is shown.

Class limits	Class boundaries	Tally	Frequency
5	4.5–5.5	/	1
6	5.5–6.5	~~////~~ //	7
7	6.5–7.5	~~////~~ ~~////~~ /	11
8	7.5–8.5	~~////~~ ///	8
9	8.5–9.5	//	2
10	9.5–10.5	/	1

In this case, 11 students sleep 7 hours a night. Most of the students sleep between 5.5 and 8.5 hours.

The cumulative frequencies are

	Cumulative frequency
Less than 4.5	0
Less than 5.5	1
Less than 6.5	8
Less than 7.5	19
Less than 8.5	27
Less than 9.5	29
Less than 10.5	30

When you are constructing a frequency distribution, the guidelines presented in this section should be followed. However, you can construct several different but correct frequency distributions for the same data by using a different class width, a different number of classes, or a different starting point.

Furthermore, the method shown here for constructing a frequency distribution is not unique, and there are other ways of constructing one. Slight variations exist, especially in computer packages. But regardless of what methods are used, classes should be mutually exclusive, continuous, exhaustive, and of equal width.

Interesting Fact

Male dogs bite children more often than female dogs do; however, female cats bite children more often than male cats do.

In summary, the different types of frequency distributions were shown in this section. The first type, shown in Example 2–1, is used when the data are categorical (nominal), such as blood type or political affiliation. This type is called a categorical frequency distribution. The second type of distribution is used when the range is large and classes several units in width are needed. This type is called a grouped frequency distribution and is shown in Example 2–2. Another type of distribution is used for numerical data and when the range of data is small, as shown in Example 2–3. Since each class is only one unit, this distribution is called an ungrouped frequency distribution.

All the different types of distributions are used in statistics and are helpful when one is organizing and presenting data.

The reasons for constructing a frequency distribution are as follows:

1. To organize the data in a meaningful, intelligible way.
2. To enable the reader to determine the nature or shape of the distribution.
3. To facilitate computational procedures for measures of average and spread (shown in Sections 3–1 and 3–2).

4. To enable the researcher to draw charts and graphs for the presentation of data (shown in Section 2–2).
5. To enable the reader to make comparisons among different data sets.

The factors used to analyze a frequency distribution are essentially the same as those used to analyze histograms and frequency polygons, which are shown in Section 2–2.

Exercises 2–1

1. Eating at Fast Food Restaurants A survey was taken of 50 individuals. They were asked how many days per week they ate at a fast-food restaurant. Construct a frequency distribution using 8 classes (0–7). Based on the distribution, how often did most people eat at a fast-food restaurant?

1	3	4	0	4
5	2	2	3	1
2	2	2	2	2
2	2	2	2	3
2	2	5	2	4
2	4	5	2	1
4	1	3	2	2
2	0	7	2	3
2	2	2	5	2
3	3	4	1	3

2. Ages of Declaration of Independence Signers The ages of the signers of the Declaration of Independence are shown. (Age is approximate since only the birth year appeared in the source, and one has been omitted since his birth year is unknown.) Construct a grouped frequency distribution and a cumulative frequency distribution for the data, using 7 classes.

41	54	47	40	39	35	50	37	49	42	70	32
44	52	39	50	40	30	34	69	39	45	33	42
44	63	60	27	42	34	50	42	52	38	36	45
35	43	48	46	31	27	55	63	46	33	60	62
35	46	45	34	53	50	50					

3. Maximum Wind Speeds The data show the maximum wind speeds in miles per hour recorded for 40 states. Construct a frequency distribution using 7 classes.

59	78	62	72	67
76	92	77	64	83
64	70	67	75	75
78	75	71	72	93
68	69	76	72	85
64	70	77	74	72
53	67	48	76	59
87	53	77	70	63

4. Consumption of Natural Gas Construct a frequency distribution for the energy consumption of natural gas (in billions of Btu) by the 50 states and the District of Columbia. Use 9 classes.

474	475	205	639	197	344	3	409	247	66
377	87	747	1166	223	248	958	406	251	3462
2391	514	371	58	224	530	317	267	769	9
188	289	76	678	331	52	214	165	255	319
34	1300	284	834	114	1082	73	62	95	393
146									

5. Scores in the Rose Bowl The data show the scores of the winning teams in the Rose Bowl. Construct a frequency distribution for the data using a class width of 7.

24	20	45	21	26	38	49	32	41	38
28	34	37	34	17	38	21	20	41	38
21	38	34	46	17	22	20	22	45	20
45	24	28	23	17	17	27	14	23	18

2–2 Histograms, Frequency Polygons, and Ogives

OBJECTIVE 2

Represent data in frequency distributions graphically, using histograms, frequency polygons, and ogives.

After you have organized the data into a frequency distribution, you can present them in graphical form. The purpose of graphs in statistics is to convey the data to the viewers in pictorial form. It is easier for most people to comprehend the meaning of data presented graphically than data presented numerically in tables or frequency distributions. This is especially true if the users have little or no statistical knowledge.

Statistical graphs can be used to describe the data set or to analyze it. Graphs are also useful in getting the audience's attention in a publication or a speaking presentation. They can be used to discuss an issue, reinforce a critical point, or summarize a data set. They can also be used to discover a trend or pattern in a situation over a period of time.

The three most commonly used graphs in research are

1. The histogram.
2. The frequency polygon.
3. The cumulative frequency graph, or ogive (pronounced o-jive).

The steps for constructing the histogram, frequency polygon, and the ogive are summarized in the procedure table.

Procedure Table

Constructing a Histogram, Frequency Polygon, and Ogive

Step 1 Draw and label the x and y axes.

Step 2 On the x axis, label the class boundaries of the frequency distribution for the histogram and ogive. Label the midpoints for the frequency polygon.

Step 3 Plot the frequencies for each class, and draw the vertical bars for the histogram and the lines for the frequency polygon and ogive.

(*Note:* Remember that the lines for the frequency polygon begin and end on the x axis while the lines for the ogive begin on the x axis.)

Historical Note

Karl Pearson introduced the histogram in 1891. He used it to show time concepts of various reigns of Prime Ministers.

The Histogram

The **histogram** is a graph that displays the data by using contiguous vertical bars (unless the frequency of a class is 0) of various heights to represent the frequencies of the classes.

EXAMPLE 2–4 Record High Temperatures

Construct a histogram to represent the data shown for the record high temperatures for each of the 50 states (see Example 2–2).

Class boundaries	Frequency
99.5–104.5	2
104.5–109.5	8
109.5–114.5	18
114.5–119.5	13
119.5–124.5	7
124.5–129.5	1
129.5–134.5	1

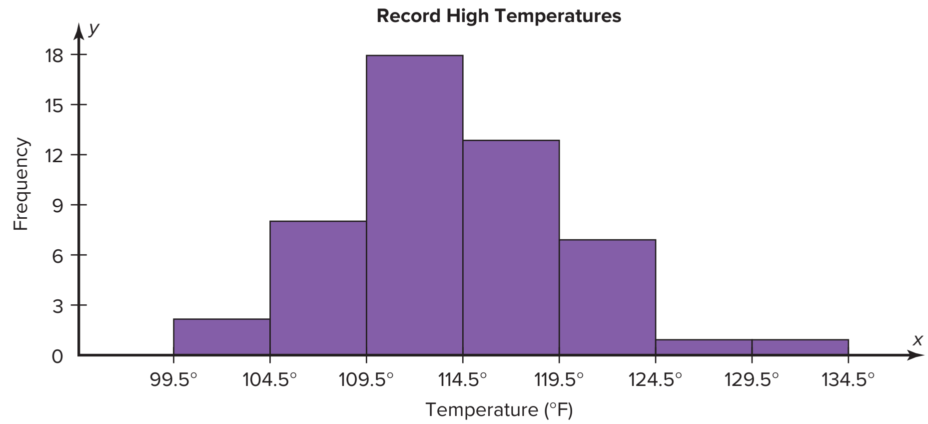
SOLUTION

Step 1 Draw and label the x and y axes. The x axis is always the horizontal axis, and the y axis is always the vertical axis.

Step 2 Represent the frequency on the y axis and the class boundaries on the x axis.

Step 3 Using the frequencies as the heights, draw vertical bars for each class. See Figure 2–1.

FIGURE 2–1 Histogram for Example 2–4



As the histogram shows, the class with the greatest number of data values (18) is 109.5–114.5, followed by 13 for 114.5–119.5. The graph also has one peak with the data clustering around it.

Historical Note

Graphs originated when ancient astronomers drew the position of the stars in the heavens. Roman surveyors also used coordinates to locate landmarks on their maps.

The development of statistical graphs can be traced to William Playfair (1759–1823), an engineer and drafter who used graphs to present economic data pictorially.

The Frequency Polygon

Another way to represent the same data set is by using a frequency polygon.

The **frequency polygon** is a graph that displays the data by using lines that connect points plotted for the frequencies at the midpoints of the classes. The frequencies are represented by the heights of the points.

Example 2–5 shows the procedure for constructing a frequency polygon. Be sure to begin and end on the x axis.

EXAMPLE 2–5 Record High Temperatures

Using the frequency distribution given in Example 2–4, construct a frequency polygon.

SOLUTION

Step 1 Find the midpoints of each class. Recall that midpoints are found by adding the upper and lower boundaries and dividing by 2:

$$\frac{99.5 + 104.5}{2} = 102 \qquad \frac{104.5 + 109.5}{2} = 107$$

and so on. The midpoints are

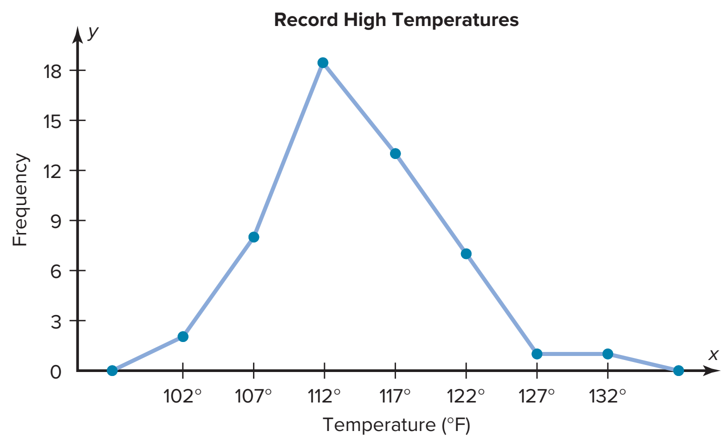
Class boundaries	Midpoints	Frequency
99.5–104.5	102	2
104.5–109.5	107	8
109.5–114.5	112	18
114.5–119.5	117	13
119.5–124.5	122	7
124.5–129.5	127	1
129.5–134.5	132	1

Step 2 Draw the x and y axes. Label the x axis with the midpoint of each class, and then use a suitable scale on the y axis for the frequencies.

Step 3 Using the midpoints for the x values and the frequencies as the y values, plot the points.

Step 4 Connect adjacent points with line segments. Draw a line back to the x axis at the beginning and end of the graph, at the same distance that the previous and next midpoints would be located, as shown in Figure 2–2.

FIGURE 2–2
Frequency Polygon for Example 2–5



The frequency polygon and the histogram are two different ways to represent the same data set. The choice of which one to use is left to the discretion of the researcher.

The Ogive

The third type of graph that can be used represents the cumulative frequencies for the classes. This type of graph is called the *cumulative frequency graph,* or *ogive.* The **cumulative frequency** is the sum of the frequencies accumulated up to the upper boundary of a class in the distribution.

The **ogive** is a graph that represents the cumulative frequencies for the classes in a frequency distribution.

Example 2–6 shows the procedure for constructing an ogive. Be sure to start on the x axis.

EXAMPLE 2–6 Record High Temperatures

Construct an ogive for the frequency distribution described in Example 2–4.

SOLUTION

Step 1 Find the cumulative frequency for each class.

	Cumulative frequency
Less than 99.5	0
Less than 104.5	2
Less than 109.5	10
Less than 114.5	28
Less than 119.5	41
Less than 124.5	48
Less than 129.5	49
Less than 134.5	50

Step 2 Draw the *x* and *y* axes. Label the *x* axis with the class boundaries. Use an appropriate scale for the *y* axis to represent the cumulative frequencies. (Depending on the numbers in the cumulative frequency columns, scales such as 0, 1, 2, 3, . . . , or 5, 10, 15, 20, . . . , or 1000, 2000, 3000, . . . can be used. Do *not* label the *y* axis with the numbers in the cumulative frequency column.) In this example, a scale of 0, 5, 10, 15, . . . will be used.

Step 3 Plot the cumulative frequency at each upper class boundary, as shown in Figure 2–3. Upper boundaries are used since the cumulative frequencies represent the number of data values accumulated up to the upper boundary of each class.

Step 4 Starting with the first upper class boundary, 104.5, connect adjacent points with line segments, as shown in Figure 2–4. Then extend the graph to the first lower class boundary, 99.5, on the *x* axis.

FIGURE 2–3
Plotting the Cumulative Frequency for Example 2–6

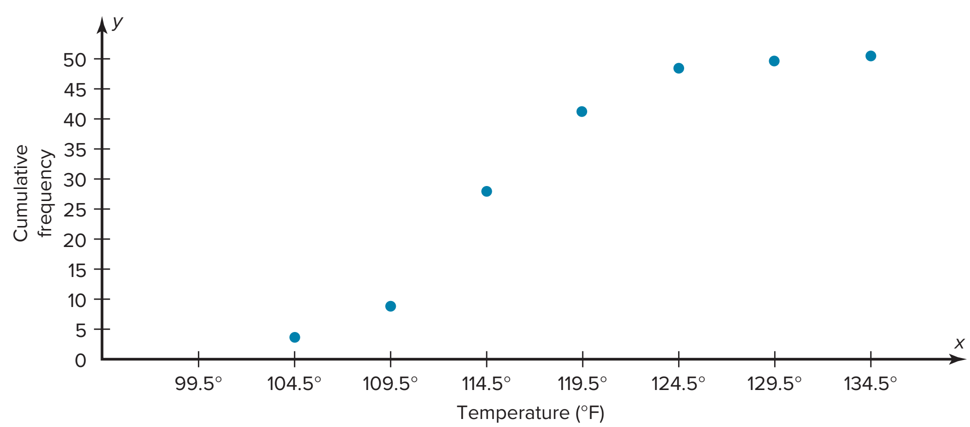


FIGURE 2–4
Ogive for Example 2–6

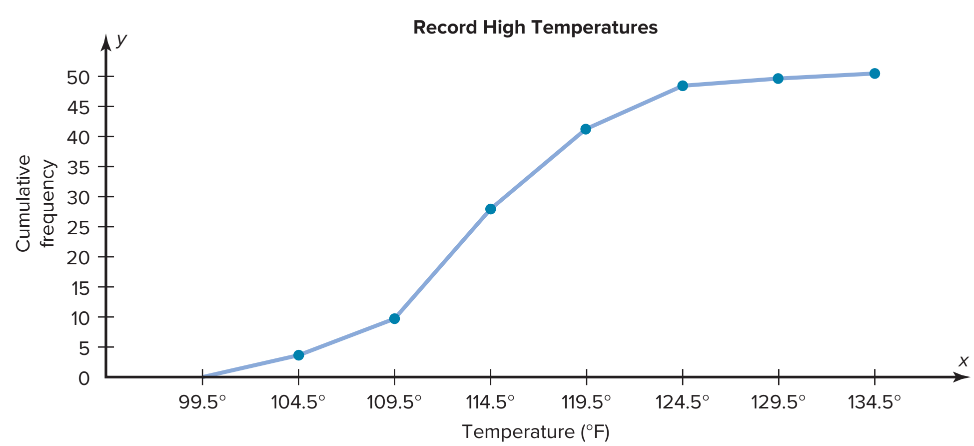
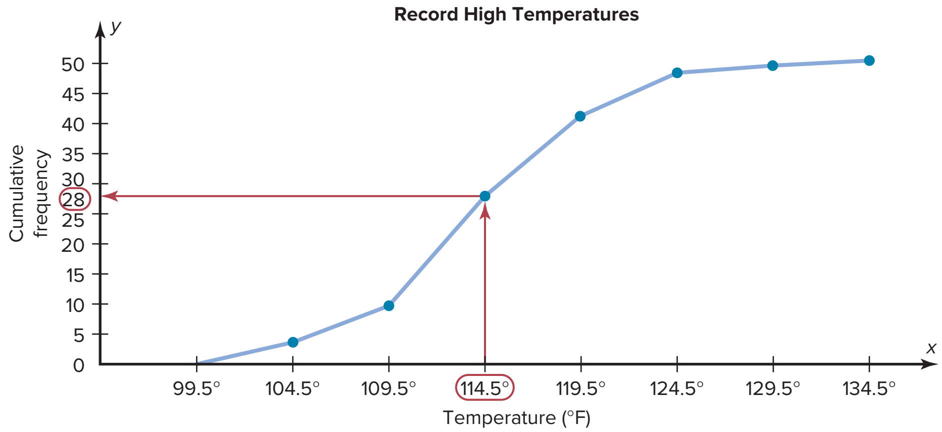


FIGURE 2–5
Finding a Specific Cumulative Frequency



Unusual Stats

Twenty-two percent of Americans sleep 6 hours a day or less.

Cumulative frequency graphs are used to visually represent how many values are below a certain upper class boundary. For example, to find out how many record high temperatures are less than 114.5°F, locate 114.5°F on the *x* axis, draw a vertical line up until it intersects the graph, and then draw a horizontal line at that point to the *y* axis. The *y* axis value is 28, as shown in Figure 2–5.

Relative Frequency Graphs

The histogram, the frequency polygon, and the ogive shown previously were constructed by using frequencies in terms of the raw data. These distributions can be converted to distributions using *proportions* instead of raw data as frequencies. These types of graphs are called **relative frequency graphs.**

Graphs of relative frequencies instead of frequencies are used when the proportion of data values that fall into a given class is more important than the actual number of data values that fall into that class. For example, if you wanted to compare the age distribution of adults in Philadelphia, Pennsylvania, with the age distribution of adults of Erie, Pennsylvania, you would use relative frequency distributions. The reason is that since the population of Philadelphia is 1,526,006 and the population of Erie is 101,786, the bars using the actual data values for Philadelphia would be much taller than those for the same classes for Erie.

To convert a frequency into a proportion or relative frequency, divide the frequency for each class by the total of the frequencies. The sum of the relative frequencies will always be 1. These graphs are similar to the ones that use raw data as frequencies, but the values on the *y* axis are in terms of proportions. Example 2–7 shows the three types of relative frequency graphs.

EXAMPLE 2–7 Ages of State Governors

Construct at histogram, frequency polygon, and ogive using relative frequencies for the distribution shown. This is a grouped frequency distribution using the ages (at the time of this writing) of the governors of the 50 states of the United States.

Class boundaries	Frequency
42.5–47.5	4
47.5–52.5	4
52.5–57.5	11
57.5–62.5	14
62.5–67.5	9
67.5–72.5	5
72.5–77.5	3
	Total 50

SOLUTION

Step 1 Convert each frequency to a proportion or relative frequency by dividing the frequency for each class by the total number of observations.

For the class 42.5–47.5 the relative frequency $= \frac{4}{50} = 0.08$; for the class 47.5–52.5, the relative frequency is $\frac{4}{50} = 0.08$; for the class 52.5–57.5, the relative frequency is $\frac{11}{50} = 0.22$, and so on.

Place these values in the column labeled Relative Frequency. Also, find the midpoints, as shown in Example 2-5, for each class and place them in the midpoint column

Class boundaries	Midpoints	Relative frequency
42.5–47.5	45	0.08
47.5–52.5	50	0.08
52.5–57.5	55	0.22
57.5–62.5	60	0.28
62.5–67.5	65	0.18
67.5–72.5	70	0.10
72.5–77.5	75	0.06

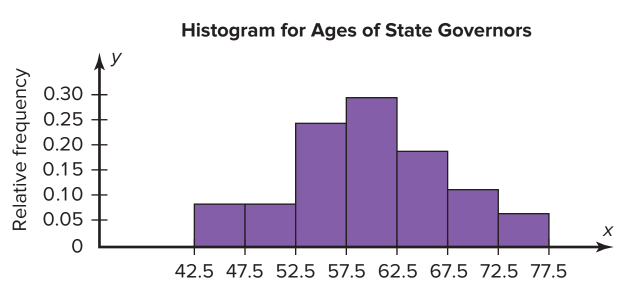
Step 2 Find the cumulative relative frequencies. To do this, add the frequency in each class to the total frequency of the preceding class. In this case, 0.00 + 0.08 = 0.08, 0.08 + 0.08 = 0.16, 0.16 + 0.22 = 0.38, 0.28 + 0.38 = 0.66, etc. Place these values in a column labeled Cumulative relative frequency.

An alternative method would be to change the cumulative frequencies for the classes to relative frequencies. (Divide each by the total).

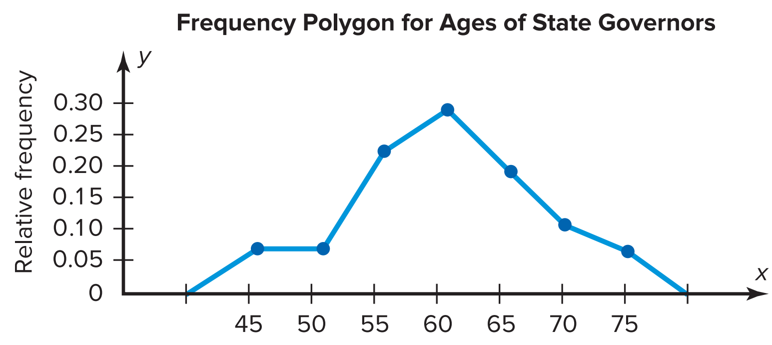
	Cumulative frequency	Cumulative relative frequency
Less than 42.5	0	0.00
Less than 47.5	4	0.08
Less than 52.5	8	0.16
Less than 57.5	19	0.38
Less than 62.5	33	0.66
Less than 67.5	42	0.84
Less than 72.5	47	0.94
Less than 77.5	50	1.00

Step 3 Draw each graph as shown in Figure 2-6. For the histogram and ogive, use the class boundaries along the x axis. For the frequency, use the midpoints on the x axis. For the scale on the y axis, use proportions.

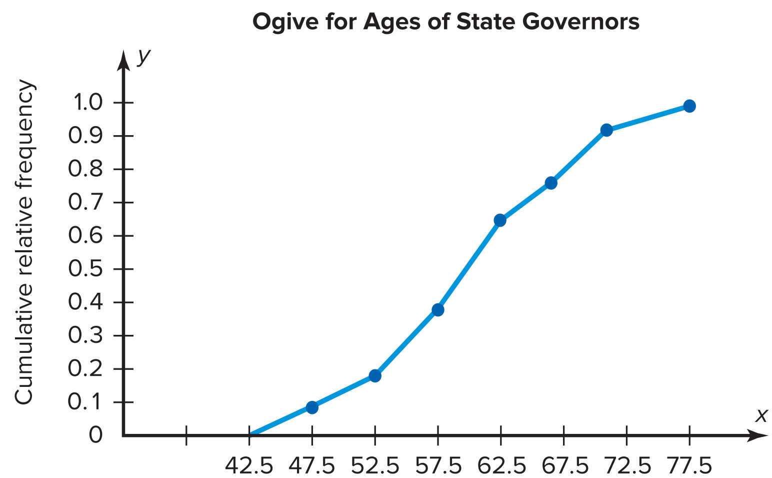
FIGURE 2–6
Graphs for Example 2–7



(a) Histogram



(b) Frequency polygon



(c) Ogive

Distribution Shapes

When one is describing data, it is important to be able to recognize the shapes of the distribution values. In later chapters, you will see that the shape of a distribution also determines the appropriate statistical methods used to analyze the data.

A distribution can have many shapes, and one method of analyzing a distribution is to draw a histogram or frequency polygon for the distribution. Several of the most common shapes are shown in Figure 2–7: *the bell-shaped or mound-shaped, the uniform-shaped, the J-shaped, the reverse J-shaped, the positively or right-skewed shape, the negatively or left-skewed shape, the bimodal-shaped, and the U-shaped.*

Distributions are most often not perfectly shaped, so it is not necessary to have an exact shape but rather to identify an overall pattern.

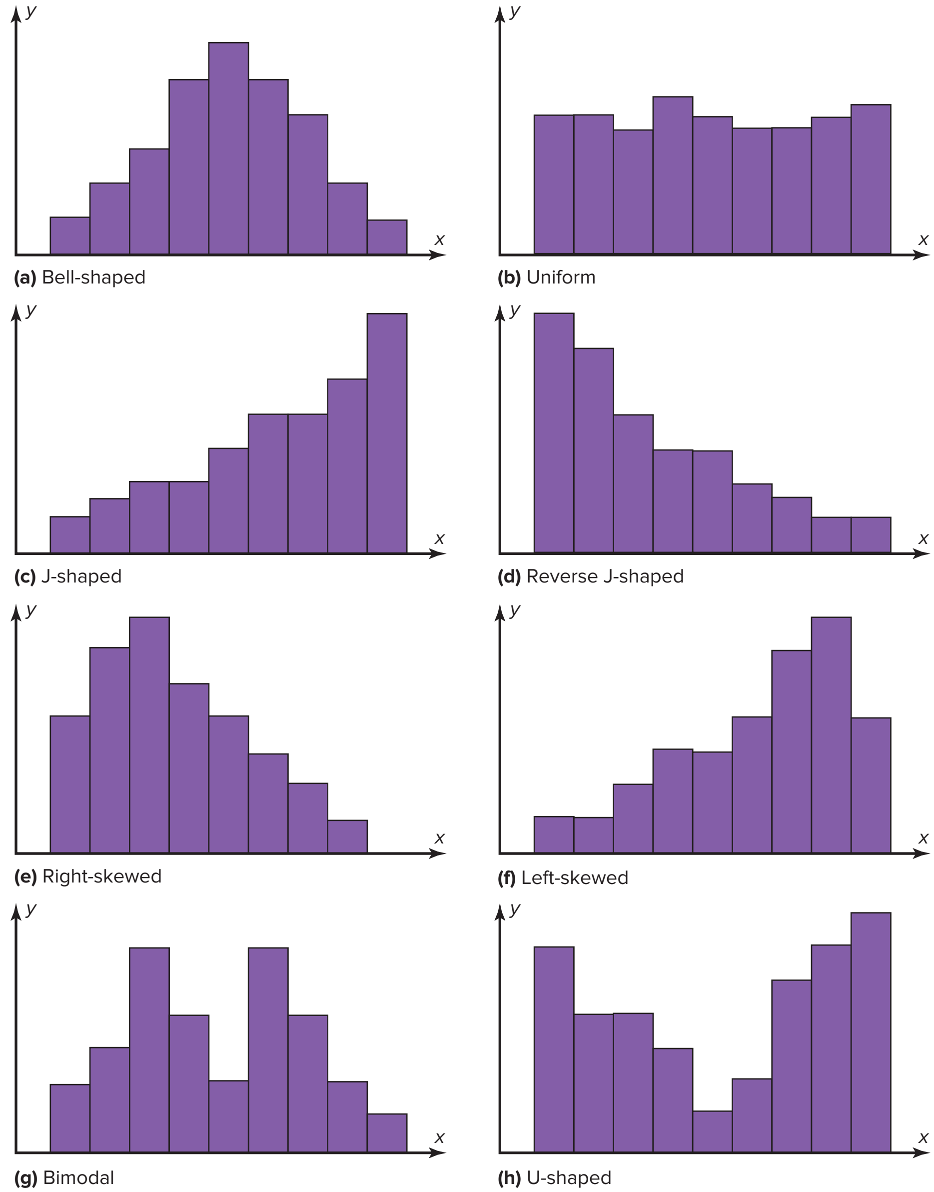
A *bell-shaped distribution* shown in Figure 2–7(a) has a single peak and tapers off at either end. It is approximately symmetric; i.e., it is roughly the same on both sides of a line running through the center.

A *uniform distribution* is basically flat or rectangular. See Figure 2–7(b).

A *J-shaped distribution* is shown in Figure 2–7(c), and it has a few data values on the left side and increases as one moves to the right. A *reverse J-shaped distribution* is the opposite of the J-shaped distribution. See Figure 2–7(d).

When the peak of a distribution is to the left and the data values taper off to the right, a distribution is said to be *positively or right-skewed.* See Figure 2–7(e). When

FIGURE 2–7
Distribution Shapes



Exercises 2–2

1. **Number of College Faculty** The number of faculty listed for a sample of private colleges that offer only bachelor's degrees is listed below. Use these data to construct a frequency distribution with 7 classes, a histogram, a frequency polygon, and an ogive. Discuss the shape of this distribution. What proportion of schools have 180 or more faculty?

165	221	218	206	138	135	224	204
70	210	207	154	155	82	120	116
176	162	225	214	93	389	77	135
221	161	128	310				

Source: World Almanac and Book of Facts.

2. **Railroad Crossing Accidents** The data show the number of railroad crossing accidents for the 50 states of the United States for a specific year. Construct a histogram, frequency polygon, and ogive for the data. Comment on the skewness of the distribution. (The data in this exercise will be used for Exercise 15 in this section.)

Class limits	Frequency
1–43	24
44–86	17
87–129	3
130–172	4
173–215	1
216–258	0
259–301	0
302–344	1

3. **Pupils Per Teacher** The average number of pupils per teacher in each state is shown. Construct a grouped frequency distribution with 6 classes. Draw a histogram, frequency polygon, and ogive. Analyze the distribution.

16	16	15	12	14
13	16	14	15	14
18	18	18	12	15
15	16	16	15	15
25	19	15	12	22
18	14	13	17	9
13	14	13	16	12
14	16	10	22	20
12	14	18	15	14
16	12	12	13	15

4. **Home Runs** The data show the most number of home runs hit by a batter in the American League over the last 30 seasons. Construct a frequency distribution using 5 classes. Draw a histogram, a frequency polygon, and an ogive for the date, using relative frequencies. Describe the shape of the histogram.

40	43	40
53	47	46
44	57	43
43	52	44
54	47	51
39	48	36
37	56	42
54	56	49
54	52	40
48	50	40

5. **Protein Grams in Fast Food** The amount of protein (in grams) for a variety of fast-food sandwiches is reported here. Construct a frequency distribution, using 6 classes. Draw a histogram, a frequency polygon, and an ogive for the data, using relative frequencies. Describe the shape of the histogram.

23	30	20	27	44	26	35	20	29	29
25	15	18	27	19	22	12	26	34	15
27	35	26	43	35	14	24	12	23	31
40	35	38	57	22	42	24	21	27	33

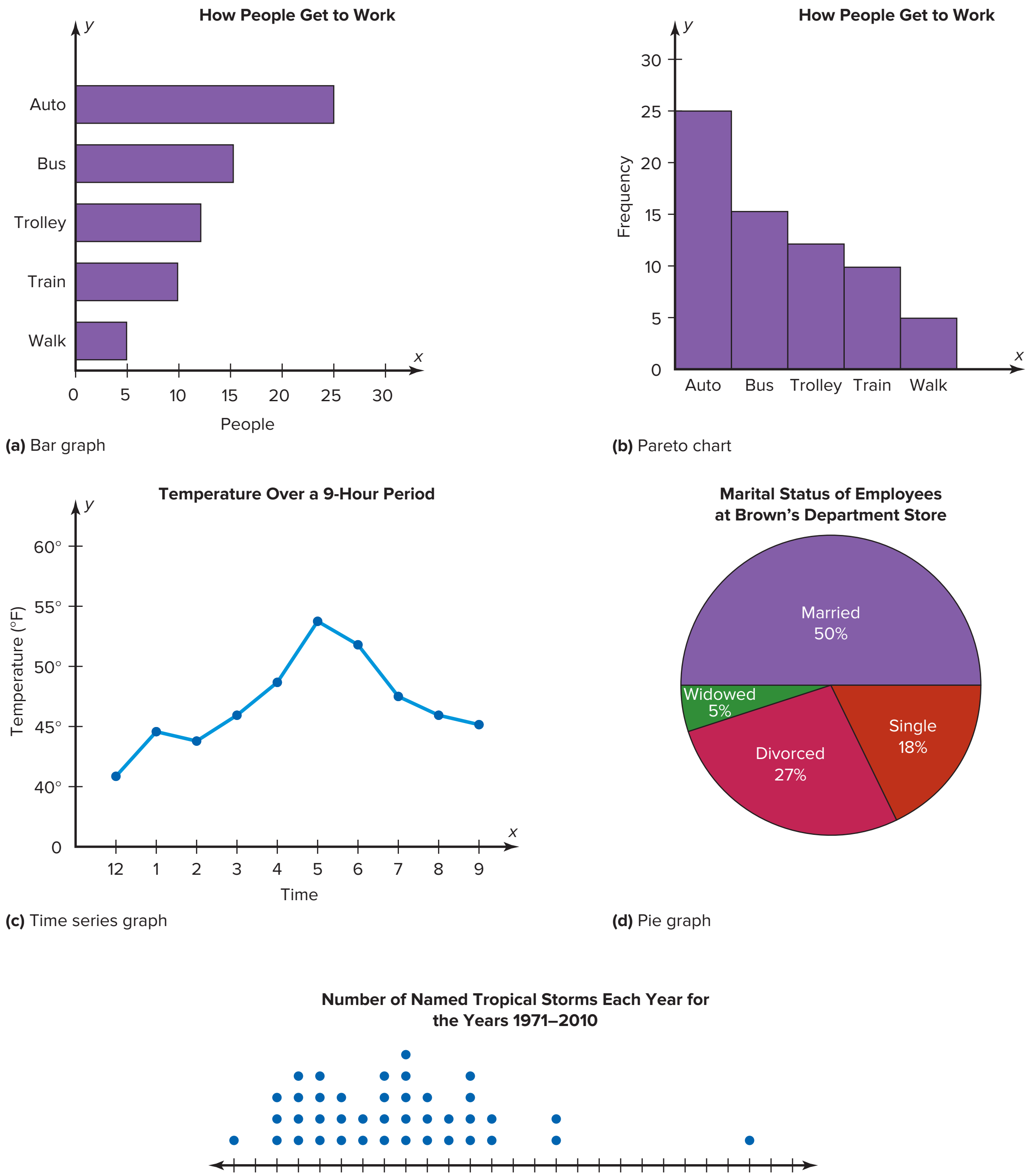
6. **Blood Glucose Levels** The frequency distribution shows the blood glucose levels (in milligrams per deciliter) for 50 patients at a medical facility. Construct a histogram, frequency polygon, and ogive for the data. Comment on the shape of the distribution. What range of glucose levels did most patients fall into?

Class limits	Frequency
60–64	2
65–69	1
70–74	5
75–79	12
80–84	18
85–89	6
90–94	5
95–99	1

2–3 Other Types of Graphs

In addition to the histogram, the frequency polygon, and the ogive, several other types of graphs are often used in statistics. They are the bar graph, Pareto chart, time series graph, pie graph, and the dotplot. Figure 2–8 shows an example of each type of graph.

FIGURE 2–8 Other Types of Graphs Used in Statistics



(a) Bar graph

(b) Pareto chart

(c) Time series graph

(d) Pie graph

(e) Dotplot

OBJECTIVE 3

Represent data using bar graphs, Pareto charts, time series graphs, pie graphs, and dotplots.

Bar Graphs

When the data are qualitative or categorical, bar graphs can be used to represent the data. A bar graph can be drawn using either horizontal or vertical bars.

A **bar graph** represents the data by using vertical or horizontal bars whose heights or lengths represent the frequencies of the data.

EXAMPLE 2–8 College Spending for First-Year Students

The table shows the average money spent by first-year college students. Draw a horizontal and vertical bar graph for the data.

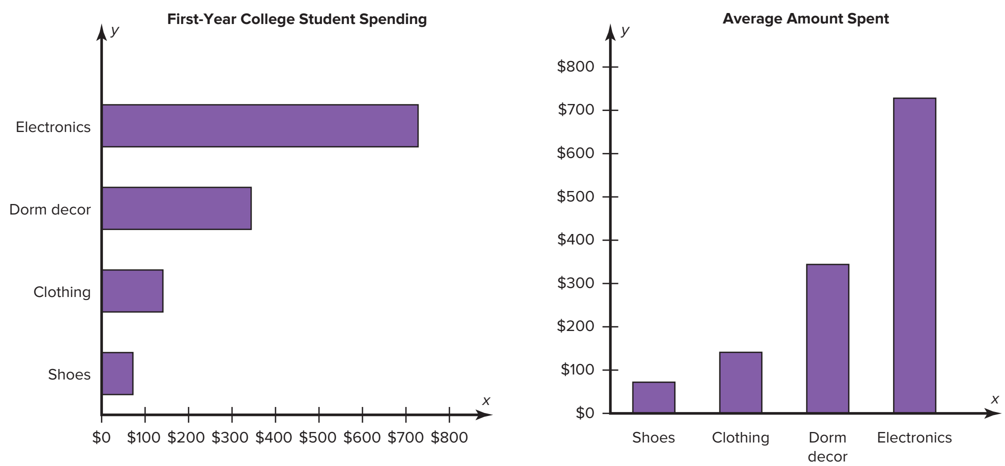
Electronics	$728
Dorm decor	344
Clothing	141
Shoes	72

Source: The National Retail Federation.

SOLUTION

1. Draw and label the *x* and *y* axes. For the horizontal bar graph place the frequency scale on the *x* axis, and for the vertical bar graph place the frequency scale on the *y* axis.
2. Draw the bars corresponding to the frequencies. See Figure 2–9.

FIGURE 2–9 Bar Graphs for Example 2–8



The graphs show that first-year college students spend the most on electronic equipment.

Bar graphs can also be used to compare data for two or more groups. These types of bar graphs are called *compound bar graphs.* Consider the following data for the number (in millions) of never married adults in the United States.

Year	Males	Females
1960	15.3	12.3
1980	24.2	20.2
2000	32.3	27.8
2010	40.2	34.0

Source: U.S. Census Bureau.

Figure 2–10 shows a bar graph that compares the number of never married males with the number of never married females for the years shown. The comparison is made by placing the bars next to each other for the specific years. The heights of the bars can be compared. This graph shows that there have consistently been more never married

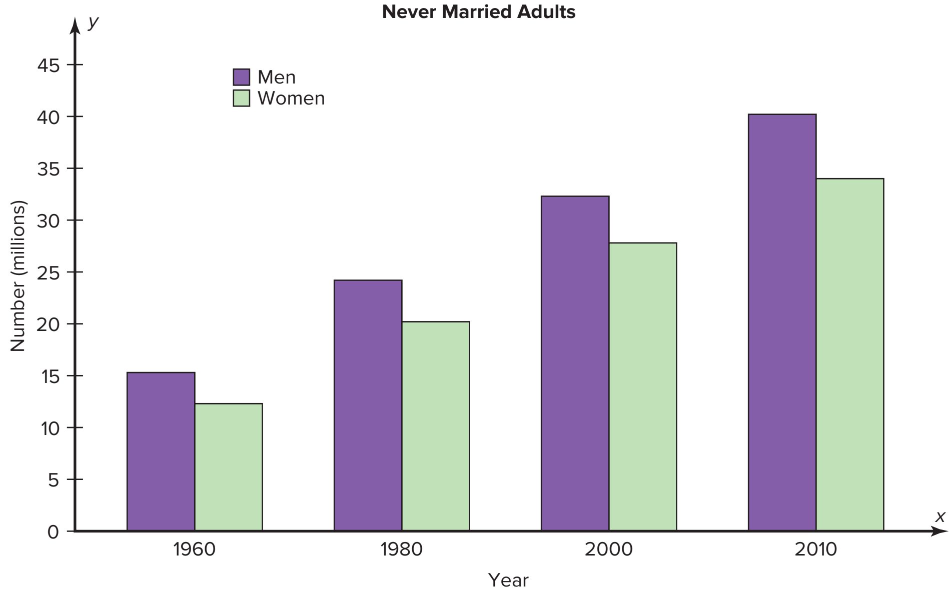


FIGURE 2–10
Example of a Compound Bar Graph

males than never married females and that the difference in the two groups has increased slightly over the last 50 years.

Pareto Charts

When the variable displayed on the horizontal axis is qualitative or categorical, a *Pareto chart* can also be used to represent the data.

A **Pareto chart** is used to represent a frequency distribution for a categorical variable, and the frequencies are displayed by the heights of vertical bars, which are arranged in order from highest to lowest.

EXAMPLE 2–9 Traffic Congestion

The data shown consist of the average number of hours that a commuter spends in traffic congestion per year in each city. Draw and analyze a Pareto chart for the data.

City	Hours
Atlanta	52
Boston	64
Chicago	61
New York	74
Washington, D. C.	82

Source: 2015 Urban Mobility Scorecard

SOLUTION

Step 1 Arrange the data from the largest to the smallest according to the number of hours.

City	Hours
Washington, D.C.	82
New York	74
Boston	64
Chicago	61
Atlanta	52

Historical Note

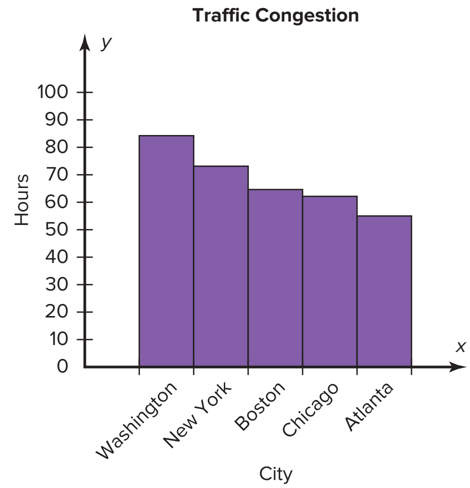
Vilfredo Pareto (1848–1923) was an Italian scholar who developed theories in economics, statistics, and the social sciences. His contributions to statistics include the development of a mathematical function used in economics. This function has many statistical applications and is called the Pareto distribution. In addition, he researched income distribution, and his findings became known as Pareto's law.

Step 2 Draw and label the x and y axes.

Step 3 Draw the vertical bars according to the number of hours (large to small).

The graph shown in Figure 2–11 shows that Washington, D.C. has the longest congestion times and Atlanta has the shortest times for the selection of cities.

FIGURE 2–11 Pareto Chart for Example 2–9



Suggestions for Drawing Pareto Charts

1. Make the bars the same width.
2. Arrange the data from largest to smallest according to frequency.
3. Make the units that are used for the frequency equal in size.

When you analyze a Pareto chart, make comparisons by looking at the heights of the bars.

The Time Series Graph

When data are collected over a period of time, they can be represented by a time series graph.

A **time series graph** represents data that occur over a specific period of time.

Example 2–10 shows the procedure for constructing a time series graph.

EXAMPLE 2–10 Price of an Advertisement for the Academy Awards Show

The data show the average cost (in millions of dollars) of a 30-second television ad on the Academy Awards show. Draw and analyze a time series graph for the data.

Year	2010	2011	2012	2013	2014	2015
Cost	1.40	1.55	1.61	1.65	1.78	1.90

Source: Kantar Media, *USA TODAY* RESEARCH

Historical Note

Time series graphs are over 1000 years old. The first ones were used to chart the movements of the planets and the sun.

SOLUTION

Step 1 Draw and label the *x* and *y* axes.

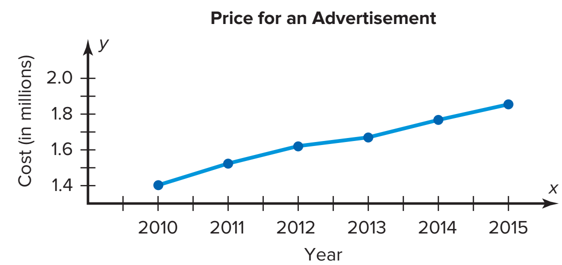
Step 2 Label the *x* axis for years and label the *y* axis for cost.

Step 3 Plot each point for the values shown in the table.

Step 4 Draw line segments connecting adjacent points. Do not try to fit a smooth curve through the data points. See Figure 2–12.

The data show that there has been an increase every year. The largest increase (shown by the steepest line segment) occurred for the year 2011 compared to 2010. The increases for the years 2011, 2012, and 2013 were relatively small compared to the increases from 2010 to 2014 and 2014 to 2015.

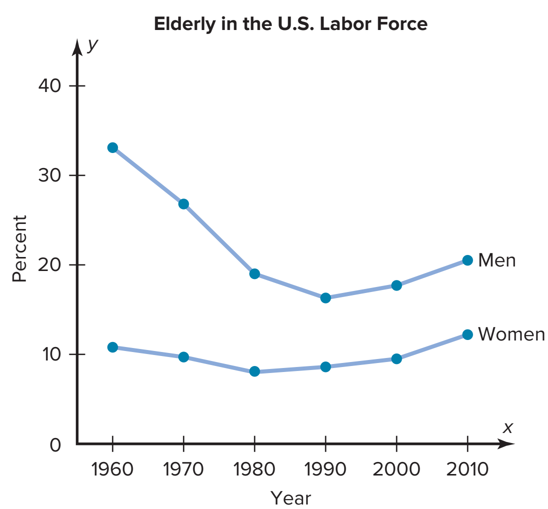
FIGURE 2–12 Figure for Example 2–10



When you analyze a time series graph, look for a trend or pattern that occurs over the time period. For example, is the line ascending (indicating an increase over time) or descending (indicating a decrease over time)? Another thing to look for is the slope, or steepness, of the line. A line that is steep over a specific time period indicates a rapid increase or decrease over that period.

Two or more data sets can be compared on the same graph called a *compound time series graph* if two or more lines are used, as shown in Figure 2–13. This graph shows

FIGURE 2–13 Two Time Series Graphs for Comparison



Source: Bureau of Census, U.S. Department of Commerce.

the percentage of elderly males and females in the U.S. labor force from 1960 to 2010. It shows that the percentage of elderly men decreased significantly from 1960 to 1990 and then increased slightly after that. For the elderly females, the percentage decreased slightly from 1960 to 1980 and then increased from 1980 to 2010.

The Pie Graph

Pie graphs are used extensively in statistics. The purpose of the pie graph is to show the relationship of the parts to the whole by visually comparing the sizes of the sections. Percentages or proportions can be used. The variable is nominal or categorical.

A **pie graph** is a circle that is divided into sections or wedges according to the percentage of frequencies in each category of the distribution.

Example 2–11 shows the procedure for constructing a pie graph.

EXAMPLE 2–11 Super Bowl Snack Foods

This frequency distribution shows the number of pounds of each snack food eaten during the Super Bowl. Construct a pie graph for the data.

Snack	Pounds (frequency)
Potato chips	11.2 million
Tortilla chips	8.2 million
Pretzels	4.3 million
Popcorn	3.8 million
Snack nuts	2.5 million
	Total $n = 30.0$ million

Source: *USA TODAY Weekend.*

SOLUTION

Step 1 Since there are 360° in a circle, the frequency for each class must be converted to a proportional part of the circle. This conversion is done by using the formula

$$\text{Degrees} = \frac{f}{n} \cdot 360°$$

where f = frequency for each class and n = sum of the frequencies. Hence, the following conversions are obtained. The degrees should sum to 360°.[1]

Potato chips	$\frac{11.2}{30} \cdot 360° =$	134°
Tortilla chips	$\frac{8.2}{30} \cdot 360° =$	98°
Pretzels	$\frac{4.3}{30} \cdot 360° =$	52°
Popcorn	$\frac{3.8}{30} \cdot 360° =$	46°
Snack nuts	$\frac{2.5}{30} \cdot 360° =$	30°
Total		360°

[1]*Note:* The degrees column does not always sum to 360° due to rounding.

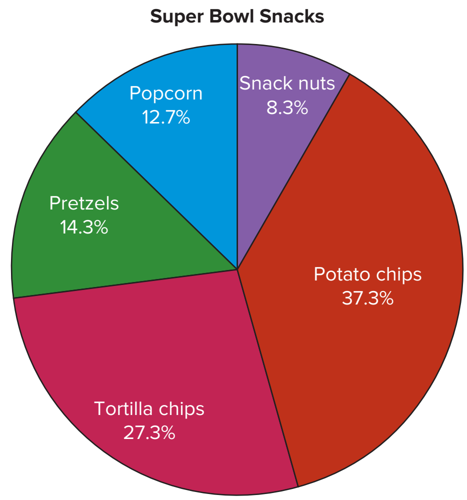
Step 2 Each frequency must also be converted to a percentage. Recall from Example 2–1 that this conversion is done by using the formula

$$\% = \frac{f}{n} \cdot 100$$

Hence, the following percentages are obtained. The percentages should sum to 100%.[2]

Potato chips	$\frac{11.2}{30} \cdot 100 = 37.3\%$
Tortilla chips	$\frac{8.2}{30} \cdot 100 = 27.3\%$
Pretzels	$\frac{4.3}{30} \cdot 100 = 14.3\%$
Popcorn	$\frac{3.8}{30} \cdot 100 = 12.7\%$
Snack nuts	$\frac{2.5}{30} \cdot 100 = 8.3\%$
Total	99.9%

FIGURE 2–14 Pie Graph for Example 2–11



Step 3 Next, using a protractor and a compass, draw the graph, using the appropriate degree measures found in Step 1, and label each section with the name and percentages, as shown in Figure 2–14.

[2]*Note:* The percent column does not always sum to 100% due to rounding.

EXAMPLE 2–12 Police Calls

Construct and analyze a pie graph for the calls received each shift by a local municipality for a recent year. (Data obtained by author.)

Shift	Frequency
1. Day	2594
2. Evening	2800
3. Night	2436
	7830

SOLUTION

Step 1 Find the number of degrees for each shift, using the formula:

$$\text{Degrees} = \frac{f}{n} \cdot 360°$$

For each shift, the following results are obtained:

Day: $\dfrac{2594}{7830} \cdot 360° = 119°$

Evening: $\dfrac{2800}{7830} \cdot 360° = 129°$

Night: $\dfrac{2436}{7830} \cdot 360° = 112°$

Step 2 Find the percentages:

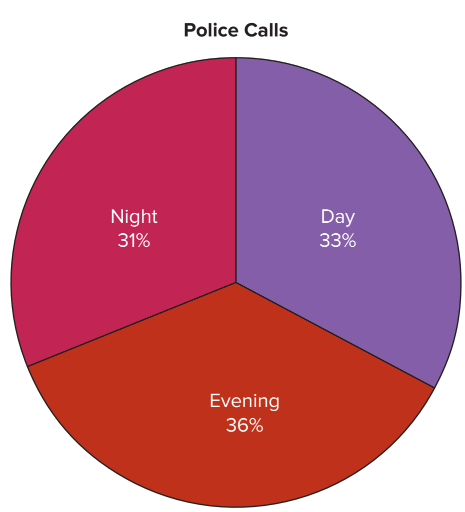
Day: $\dfrac{2594}{7830} \cdot 100 = 33\%$

Evening: $\dfrac{2800}{7830} \cdot 100 = 36\%$

Night: $\dfrac{2436}{7830} \cdot 100 = 31\%$

Step 3 Using a protractor, graph each section and write its name and corresponding percentage as shown in Figure 2–15.

FIGURE 2–15
Figure for Example 2–12



To analyze the nature of the data shown in the pie graph, look at the size of the sections in the pie graph. For example, are any sections relatively large compared to the rest? Figure 2–15 shows that the number of calls for the three shifts are about equal, although slightly more calls were received on the evening shift.

Note: Computer programs can construct pie graphs easily, so the mathematics shown here would only be used if those programs were not available.

Dotplots

A dotplot uses points or dots to represent the data values. If the data values occur more than once, the corresponding points are plotted above one another.

A **dotplot** is a statistical graph in which each data value is plotted as a point (dot) above the horizontal axis.

Dotplots are used to show how the data values are distributed and to see if there are any extremely high or low data values.

EXAMPLE 2–13 Named Storms

The data show the number of named storms each year for the last 40 years. Construct and analyze a dotplot for the data.

19	15	14	7	6	11	11
9	16	8	8	11	9	8
16	12	13	14	13	12	7
15	15	19	11	4	6	13
10	15	7	12	6	10	
28	12	8	7	12	9	

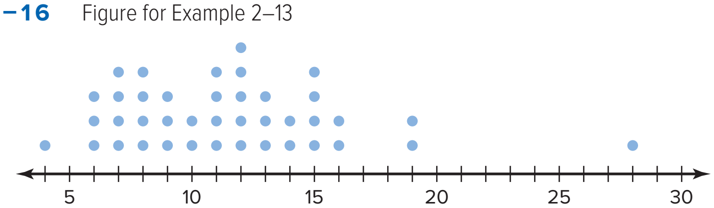
Source: NOAA.

Step 1 Find the lowest and highest data values, and decide what scale to use on the horizontal axis. The lowest data value is 4 and the highest data value is 28, so a scale from 4 to 28 is needed.

Step 2 Draw a horizontal line, and draw the scale on the line.

Step 3 Plot each data value above the line. If the value occurs more than once, plot the other point above the first point. See Figure 2–16.

FIGURE 2–16 Figure for Example 2–13



The graph shows that the majority of the named storms occur with frequency between 6 and 16 per year. There are only 3 years when there were 19 or more named storms per year.

Exercises 2–3

1. **Tech Company Employees** Construct a vertical and horizontal bar graph for the number of employees (in thousands) of a sample of the largest tech companies as of 2014.

Company	Employees
IBM	380
Hewlett Packard	302
Xerox	147
Microsoft	128
Intel	107

2. **Online Ad Spending** The amount spent (in billions of dollars) for ads online is shown. (The numbers for 2016 through 2019 are projected numbers.) Draw a time series graph and comment on the trend.

Year	2014	2015	2016	2017	2018	2019
Amount	$19.72	$31.53	$43.83	$53.29	$61.14	$69.04

3. **Roller Coaster Mania** The World Roller Coaster Census Report lists the following numbers of roller coasters on each continent. Represent the data graphically, using a Pareto chart.

Africa	17
Asia	315
Australia	22
Europe	413
North America	643
South America	45

4. **Years of Experience** The data show the number of years of experience the players on the Pittsburgh Steelers football team have at the beginning of the season. Draw and analyze a dot plot for the data.

4	4	2	9	7	3	7	12	6
5	1	4	5	2	7	6	12	3
12	4	0	4	0	0	0	2	9
2	6	7	13	4	2	6	9	4
4	0	3	5	4	2	6	9	4
4	0	3	5	3	11	1	4	2
3	15	1	6	0	11	3	10	3

5. **Worldwide Sales of Fast Foods** The worldwide sales (in billions of dollars) for several fast-food franchises for a specific year are shown. Construct a vertical bar graph and a horizontal bar graph for the data.

Wendy's	$ 8.7
KFC	14.2
Pizza Hut	9.3
Burger King	12.7
Subway	10.0

6. **Kids and Guns** The following data show where children obtain guns for committing crimes. Draw and analyze a pie graph for the data.

Source	Friend	Family	Street	Gun or Pawn Shop	Other
Number	24	15	9	9	6

7. **Colors of Automobiles** The popular car colors are shown. Construct a pie graph for the data.

White	19%
Silver	18
Black	16
Red	13
Blue	12
Gray	12
Other	10