Electrical Signals Propagating Through a Transmission Line

Hamza Akmal
24100232@lums.edu.pk
LUMS School of Science and Engineering

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1 Abstract

This experiment aims to explore the fundamental principles of electrical signal propagation through transmission lines, which is a pivotal concept in the realm of electrical and communication engineering. By employing a pulse generator and an oscilloscope, along with essential components like coaxial cables, inductors, capacitors, and resistors, we delve into the behavior of electrical signals as they traverse through these mediums. The setup is designed to mimic real-world transmission systems, offering insights into signal behavior in practical applications and to provide a comprehensive understanding of the behavior of electrical signals in transmission lines, shedding light on important concepts like impedance matching, signal reflection, and the effects of line capacitance and inductance. The outcomes of this study are expected to have significant implications in optimizing transmission line designs and improving communication systems.

2 Introduction

In the realm of electrical engineering, the transmission of signals through various mediums is a fundamental and critical aspect of communication systems. The efficient and accurate propagation of electrical signals is pivotal for the reliable functioning of a wide array of electronic devices and systems. Transmission lines serve as conduits for the transfer of electrical energy from one point to another, and understanding their behavior is indispensable for designing robust and high-performance communication networks. Such lines use a fascinating amalgamation of principles from electromagnetic theory, wave propagation, and circuit analysis. Understanding the behavior of signals as they traverse these pathways is essential for comprehending the limitations, distortions, and optimizations that characterize signal transmission in real-world applications.

The two most important parameters of a transmission line are its inductance and capacitance. Current in the transmission line sets up a magnetic flux which induces a voltage in the conductors. The voltage is considered to be positive when the upper terminal/lead is at a more positive potential than the lower lead. When a force is applied to such a line, incident and reflected waves are present. The incident wave propagates from source to the receiving end and vice versa for the reflected wave. Impedance on the other hand is independent of line length or load and is a function of line parameters only. If the line is terminated with a load impedance equal to characteristic impedance of the line, the total voltage can be taken to be the incident voltage whereas if the load impedance is not equal to characteristic impedance then another wave must be set up so that Ohm's Law is obeyed.

3 Theoretical Background

In this study, we explore the propagation speed of a pulse in a transmission line and the nature of its reflection. The experimental setup involves applying pulses to a coaxial cable and terminating them with a variable resistor at the end. Special attention is given to the case of short pulses, where the pulse duration approaches zero.

What is the significance of horizontal sweep rate?

Using a cathode-ray oscilloscope (CRO), we can resolve the incident and reflected pulses. The speed of the pulse is calculated using the horizontal sweep rate of the oscilloscope and the length of the cable. In one instance, with a wavelength of 80 cm and a frequency of 200 kHz, the sweep rate/time is determined as follows:

$$\text{Sweep rate/time} = \frac{1}{f\lambda} = \frac{1}{200 \times 10^3 \times 80 \times 10^{-2}} = 6.3 \times 10^{-7} \text{ sec/cm.} \tag{1}$$

This calculation is based on data taken directly from Rank (1), as the resistors' values were unknown.

For a distance of 1.5 cm, the time is calculated as $6.3 \times 10^{-7} \times 1.5 = 9.45 \times 10^{-7}$ sec. Extending this to a cable length of 176 m, the pulse speed is found to be 2×10^8 m/sec.

For a coaxial cable, the pulse speed v is given by $v = \frac{c}{k}$, where c is the speed of light in vacuum, and k is the dielectric coefficient of the cable's insulating material.

The duty cycle of a signal is defined as the ratio of the time when the signal is on (t_{on}) to the total period of the signal ($t_{\text{on}} + t_{\text{off}}$), given by the equation:

$$\text{Duty cycle} = \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}}. \tag{2}$$

The period of the signal is the sum of the on and off times:

$$\text{Period} = t_{\text{on}} + t_{\text{off}}. \tag{3}$$

Expected interactions at specific Impedance values

Distortion in the waveform may occur if the reflected wave overlaps with the incident wave, which is particularly probable when the load impedance (Z_L) is greater than or equal to the characteristic impedance of the transmission line (Z_0). If $Z_L < Z_0$, the reflected wave could have a negative polarity.

The case of $Z_L = Z_0$ is special; while we are still unsure about its precise effects, one might anticipate that increasing pulse width could increase the overlap between the incident and reflected waves.

Ideally, when $Z_L = Z_0$, we expect to observe no reflected wave, suggesting that we have matched the load to the characteristic impedance of the coaxial cable, effectively minimizing reflections and thus achieving $\Gamma = 0$ where Γ is the reflection coefficient.

Transients on a Lossless Transmission Line

Transients are temporary voltage and current waveforms that occur due to sudden changes in the transmission line, such as the switching of loads or the presence of pulses. In an ideal lossless transmission line, there is no attenuation of the signal as it travels along the line. The primary concern is with the impedance of the line and any mismatch at the line termination, which can lead to reflections.

The magnitude and phase of the reflection depend on the nature of the impedance mismatch. This can be quantified with the Voltage Standing Wave Ratio (VSWR), where a VSWR value of 1 indicates no reflections and higher values indicate greater mismatches.

A short-circuited line will reflect with the same amplitude but inverted phase. The reflection coefficient, Γ, is given by the formula:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{4}$$

where Z_L is the load impedance and Z_0 is the characteristic impedance of the transmission line.

Understanding the impedance of the transmission line is crucial to maintain signal integrity. Any discrepancy between the characteristic impedance of the line and the load impedance can result in reflections that degrade the quality of the signal.

The Time Domain Reflectometry (TDR) technique can be employed to diagnose such impedance mismatches. TDR is a valuable tool for analyzing the integrity of a transmission line. It involves sending a fast pulse through the line and observing the reflected signals to study discontinuities and impedance profiles along the line.

The relationship between the incident and reflected signals reveals much about the nature of the impedance irregularities. An ideal transmission line would exhibit no reflection at all, indicating perfect impedance matching along the entire length of the line, which is a rare but desired condition in practice.

Characteristics of Waves in a Transmission Line

The key quantities defining the characteristics of waves in a transmission line include inductance (L), velocity, capacitance (C), current, and voltage. As a current or voltage wave travels down a transmission line, a voltage drop occurs across the distributed capacitance. This process cannot be instantaneous; it requires time for the wave to propagate along the line.

A current traveling away from the generator is positive. The impedance depends on the size and spacing of the conductors and the type of insulation used.

Reflection Coefficient and Impedance Matching

For an incident voltage e^+ and incident current i^+, related by $i^+ = \frac{e^+}{Z_0}$, where Z_0 is the characteristic impedance of the line, we can define the reflection coefficient Γ when the load impedance Z_L is different from Z_0.

When $Z_L = Z_0$, the incident voltage over incident current $\frac{e^+}{i^+}$ equals $Z_R = Z_0$, adhering to Ohm's law. However, if a wave encounters a load impedance not equal to the characteristic impedance of the wire, a reflected wave is generated to satisfy Ohm's law. The total voltage e and total current i in the line are the sums of the incident and reflected waves, given by:

$$\begin{aligned} total\ e &= e^+ + e^-, \\ total\ i &= i^+ + i^- = \frac{e^+}{Z_0} - \frac{e^-}{Z_0}. \end{aligned} \tag{5}$$

The reflection coefficient Γ is then calculated using the equation:

$$\Gamma = \frac{e^-}{e^+} = \frac{Z_L - Z_0}{Z_L + Z_0}. \tag{6}$$

Γ is the voltage reflection coefficient, which indicates the ratio of the reflected voltage wave to the incident voltage wave.

4 Experimental Procedure

In the conducted experiment, two essential instruments were utilized: a TGP 110 10MHz Pulse Generator and a BK Precision 2534 60MHz Digital Storage Oscilloscope. The TGP 110 Pulse Generator, an electronic device capable of producing rectangular pulses, was employed to generate signals with controllable properties. Parameters such as period, frequency, and pulse width were adjusted by manipulating the instrument's knobs. The pulse generator was connected to the oscilloscope using a 1 m long BNC cable, with each end plugged into the respective channel one ports on both devices.

The oscilloscope's multiple menu settings facilitated the display of the generated pulse waveforms, allowing for measurements of maximum, minimum, and peak-to-peak values. The waveform was presented on a graph, with voltage values depicted along the y-axis and time on the x-axis. Adjustments to the graphical display were made using the oscilloscope's knobs, enabling changes to the axis increments and varying levels of zoom.

Initially, the pulse generator was set to Run and Pulse modes with a period of 1 ms and a pulse width of 50 ns. In Run mode, the waveforms were observed to be dynamic. The introduction of an equation to represent the pulse signal can be denoted as:

$$V(t) = V_0 \cdot \mathrm{rect}\left(\frac{t - t_0}{\tau}\right), \tag{7}$$

where $V(t)$ is the voltage as a function of time, V_0 is the amplitude, rect is the rectangular function, t is time, t_0 is the time offset, and τ is the pulse width.

The dynamic nature of the waveforms was stabilized by adjusting the trigger level to a point within the waveform's range, effectively 'freezing' the wave at a specific instance. Conversely, in Single mode, the oscilloscope displayed a static snapshot of the waveform, rendering the trigger adjustment unnecessary. When the trigger level was set outside the waveform's maximum-to-minimum amplitude range, the displayed signal devolved into noise. The waveform experienced scrambling and slight transposition along the x-axis upon the signal reaching a local extremum.

Looking at Figure (1) it is observed that the propagating waveform has a shape known as the ringing square wave. The ringing effect causes a small distortion and a slight downward dip right after the main body of the waveform. The lack of high order frequencies causes the ringing as it produces an incomplete set of functions; these can be used for the composition of the Fourier Transform.

4.1 Measurement of Characteristic Impedance

The characteristic impedance of the coaxial cable, denoted as Z_0, was measured in the subsequent part of the experiment. A variable resistor was utilized, with its resistance adjusted by a screwdriver, and connected to one end of a 25 m coaxial cable. The opposite end was linked to a BNC Tee, which was plugged into the pulse generator. A 1 m BNC cable, plugged into the third port on the tee, established the connection to the oscilloscope. The flow of current within the cable, and consequently the signal waveform, was influenced by the adjustment of the resistance. The resistance was varied until the waveform displayed a minimal downward dip, indicating negligible signal reflection. This resistance value was identified as the load impedance Z_L of the resistor. A multimeter was employed

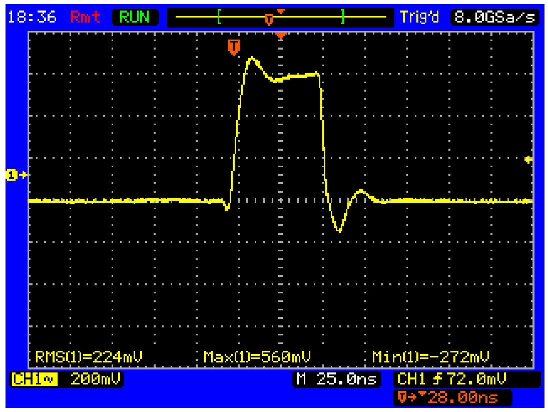

Figure 1: Waveform of the characteristic impedance

to measure Z_L, by connecting its probes to the terminals of the resistor. With the condition $Z_0 = Z_L$, the characteristic impedance of the cable was determined to be 100 Ohms, expressed as $Z_0 = 100\ \Omega$.

4.2 Observations with Different Loads

Subsequent to the impedance measurement, various loads were attached to the BNC cable, with the resulting waveforms observed. These loads were implemented using resistors, arranged similarly to the variable resistor to form a complete circuit. In certain cases, male-to-female adapters were necessary to connect the resistors to the cable. The resistances and corresponding waveforms are detailed in the results section. For the final load of 200 Ohms, the oscilloscope's timescale was set to 50 ns/division. An infinite load was simulated by leaving the circuit open, disconnected at the free end of the coaxial cable.

4.3 Effects of Capacitive and Inductive Loads

A capacitor and four inductors, each distinguished by color and inductance value, were provided to investigate their effects on the signal waveforms. The connections for these components were established in the same manner as the resistors, employing the BNC Tee and adapters. The outcomes of these connections are discussed in the subsequent section.

4.4 Data Acquisition and Equipment Challenges

Data from the oscilloscope were captured by interfacing one of its ports with a desktop computer via USB cable. The corresponding software enabled the oscilloscope's display to be viewed on the computer screen, allowing for waveforms to be saved to a desktop file. Moreover, the oscilloscope could be controlled through the software, which was necessary to access the measurement display menu, as physical adjustments were disabled while connected to the computer.

During the experimentation, issues arose with the pulse generator, which produced unintelligible signals with excessive noise levels. It was discovered that manual pressure had to be applied to the knobs to stabilize the signals for measurement and recording. This difficulty was encountered early in the experiment but was eventually overcome, allowing for the acquisition of accurate results.

4.5 Measurement of Signal Speed

The speed of the electrical signal was measured as it propagated through the transmission line. The distance covered by the signal, measured to be 51 m, was divided by the time taken for the signal to traverse from one end to the other, recorded as 200 ns. Hence, the speed v of the signal was calculated using the formula:

$$v = \frac{\Delta x}{\Delta t}, \tag{8}$$

where Δx is the distance and Δt is the time interval. The speed was determined to be approximately 2.55×10^8 m/s, with an uncertainty of ± 0.1.

5 Voltage in Transmission Lines

The total voltage in the circuit, or in this case, the transmission line, was found to be dependent on a number of factors as shown below:

$$V_{\text{tot}} = L\frac{di}{dt} + \frac{Q}{C} + iR, \tag{9}$$

where L represents the inductive effects, C the capacitive effects, and R the resistive effects of the transmission line.

5.1 Impedance and Resonance

The impedance for the resistive, capacitive, and inductive components of a circuit was expressed as:

$$Z_R = \frac{V}{I} = R, \quad Z_C = \frac{1}{i\omega C}, \quad Z_L = i\omega L. \tag{10}$$

It was observed that in practice, impedances are more easily measured than calculated.

5.2 Power and Attenuation

The experiment demonstrated that attenuation in signals is frequency dependent, and dispersion of the reflected pulse was delayed. With a shorted line, where $R_L = 0$, the voltage reflection V_r was the inverse of the incident voltage V_i. The power P delivered to the load was calculated using the equation:

$$P = \left(\frac{2V_i}{R_L + \frac{1}{Z_L}} \right)^2 . \tag{11}$$

Power was found to be maximized when the load resistance R_L matched the characteristic impedance Z_0, resulting in no reflection.

5.3 Reflection from Capacitive Load

Reflection from a capacitive load was studied, and it was established that:

$$C = \frac{I}{Z_0}. \tag{12}$$

The resulting voltage V_L was expected to follow the equation:

$$V_L = 1 - e^{-\frac{t}{\tau}}. \tag{13}$$

The waveform was anticipated to exhibit an exponential character, corresponding to the charging and discharging intervals of the capacitor, as confirmed by the oscilloscope display.

5.4 Reflection from Inductive Loads

Reflections from an inductive load were analyzed. Following the derivations made prior to the experiments, the current I as a function of time t was expressed by the equation:

$$I = I_0(1 - e^{-\frac{t}{\tau}}), \tag{14}$$

where τ, the time constant, is given by:

$$\tau = \frac{L}{Z_0}. \tag{15}$$

and L represents the inductance, while Z_0 is the characteristic impedance of the transmission line. The inductance L was set to 500 nH, as specified. A similar shape of the waveform was anticipated but with variations in phasing based on the magnitude of inductance. Waveforms for three different inductors were obtained and are shown in the results.

5.5 Power and Resonance in Inductive Loads

Power delivery and resonance phenomena in the presence of inductive loads were also studied. When the load resistance R_L is equal to the characteristic impedance Z_0, power is maximized, indicating no reflection. The reflection from a capacitive load was described as follows:

$$C = \frac{I}{Z_0}. \tag{16}$$

The voltage V_L across the load was expected to be:

$$V_L = 1 - e^{-\frac{t}{\tau}}, \tag{17}$$

which showcases the exponential character corresponding to the charging and discharging intervals of the capacitor. This behavior was observed in the oscilloscope's display, confirming the theoretical expectations.

For inductances of 500 nH and 470 nH, the waveforms were captured and analyzed. The results for the third inductor, with an unknown inductance, were also recorded for further analysis. The experimental observations with the oscilloscope are illustrated in the figures provided in the results section.

6 Results

The results obtained for the resistors at different values are as follows:

1. For the 10Ω resistor, the maximum value (amplitude in the positive y-axis) was measured to be 624 ± 11.5 mV, the minimum value (amplitude in the negative y-axis) was -312 ± 11.5 mV, and the peak-to-peak value was calculated as 936 ± 11.5 mV.

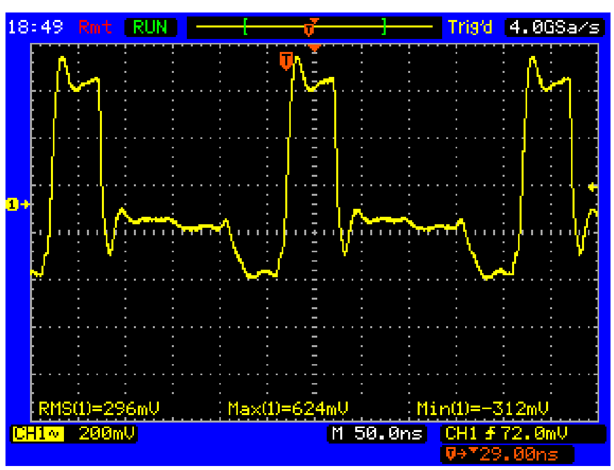

Figure 2: Waveform of the 10 ohm resistor

2. For the 25Ω resistor, the maximum value was 608 ± 11.5 mV, the minimum value was -216 ± 11.5 mV, and the peak-to-peak value was 824 ± 11.5 mV.

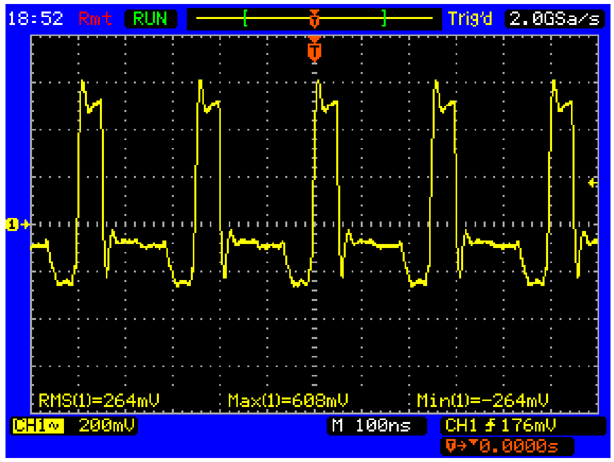

Figure 3: Waveform of the 25 ohm resistor

3. For the 50Ω resistor, the maximum value was 616 ± 11.5 mV, the minimum value was -256 ± 11.5 mV, and the peak-to-peak value was 872 ± 11.5 mV.

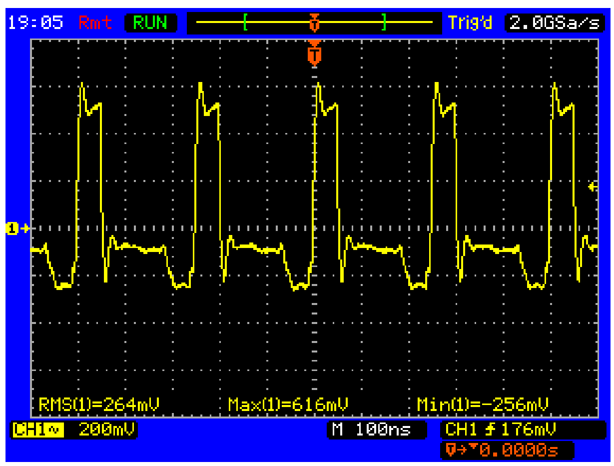

Figure 4: Waveform of the 50 ohm resistor

4. For the 75Ω resistor, the maximum value was 584 ± 11.5 mV, the minimum value was -264 ± 11.5 mV, and the peak-to-peak value was 848 ± 11.5 mV.

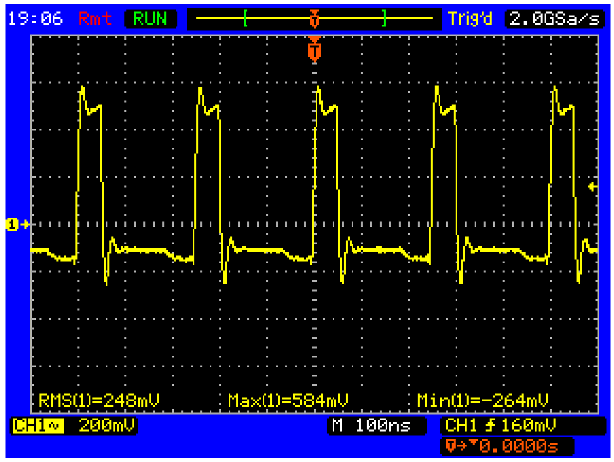

Figure 5: Waveform of the 75 ohm resistor

5. For the 200Ω resistor, the maximum value was 3.32 ± 0.3 V, the minimum value was -560 ± 11.5 mV, and the peak-to-peak value was 3880 ± 11.5 mV.

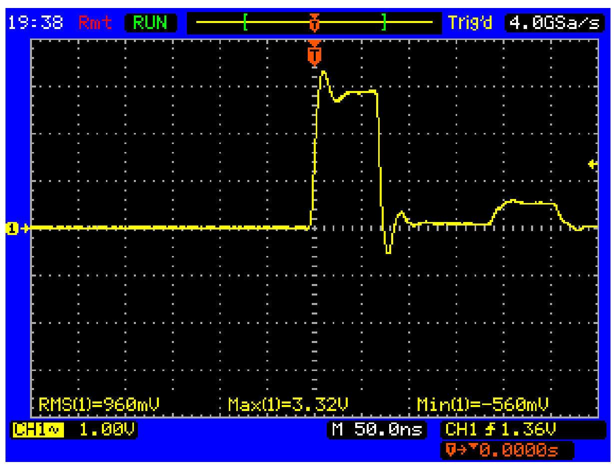

Figure 6: Waveform of the 200 ohm resistor

6. For the infinite load, the maximum value was 3.36 ± 0.3 V, the minimum value was -560 ± 11.5 mV, and the peak-to-peak value was 3920 ± 11.5 mV.

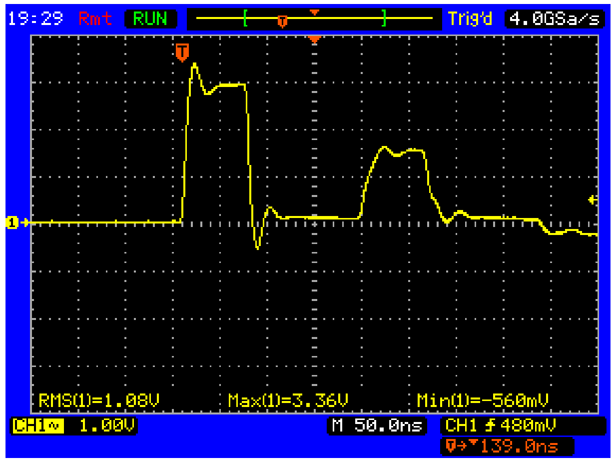

Figure 7: Waveform of the infinite load

7. For the 500 nH inductor:

Figure 8: Waveform of the 500 nH inductor

8. For the 47 μH inductor:

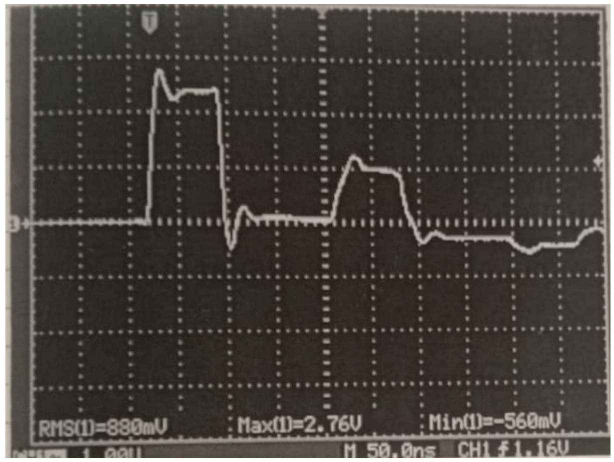

Figure 9: Waveform of the 47 μH inductor

9. For the unknown inductor:

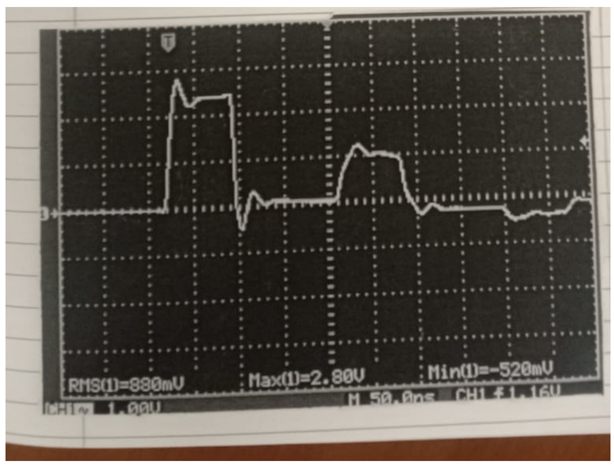

Figure 10: Waveform of the unknown inductor

10. For the 220 pF capacitor:

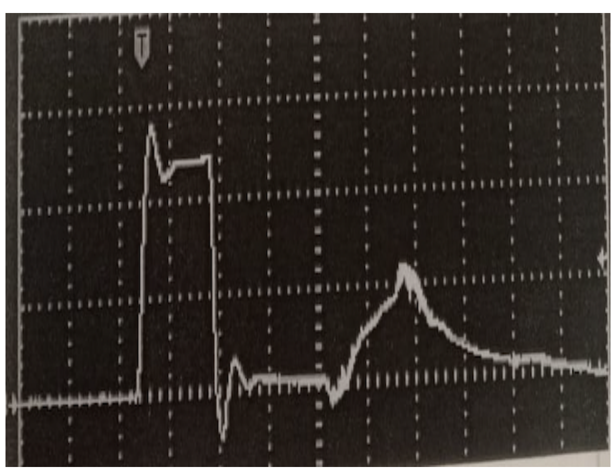

Figure 11: Waveform of the 220 pF capacitor

We observe that for our loads where $Z_L > Z_0$, the polarity of the reflected signal is negative. As the resistance value increases, the minimum values become less negative; the amplitude on the negative y-axis continues to rise as Z_L increases. When $Z_0 < Z_L$, the polarity becomes positive. However, since the amplitude also increases, the peak-to-peak values correspondingly increase.

The uncertainty in time was calculated as follows: On the x-axis, each division represents 10 mV, being $\frac{50}{5}$. The formula for uncertainty in a digital measurement is:

$$\Delta t = \frac{0.5 \cdot \Delta x}{\sqrt{3}}. \tag{18}$$

Applying the values, we get:

$$\Delta t = \frac{0.5 \cdot 10}{\sqrt{3}} = 2.88 \text{ ns} \approx 2.9 \text{ ns}. \tag{19}$$

The uncertainty in the voltage was similarly calculated: each division on the y-axis corresponds to 40 mV, which is $\frac{200}{5}$. Using the same formula for digital measurement uncertainty, we have:

$$\Delta V = \frac{0.5 \cdot \Delta y}{\sqrt{3}} = 11.54 \text{ mV} \approx 11.5 \text{ mV}. \tag{20}$$

These uncertainties were taken into account in all voltage measurements.

7 Conclusions and Discussions

The experimental observations confirmed that resistance levels below the characteristic impedance resulted in reflected waves with negative polarity. The amplitude of these reflected waves decreased and became less negative as resistance approached the characteristic impedance of 100 Ohms, where no reflected wave was apparent. Increasing resistance beyond this point inverted the polarity of the reflected wave, which then increased in amplitude with further resistance increments.

It was also consistently observed that the amplitude of the incident wave surpassed that of the reflected wave, with the maximum reflected wave amplitude occurring at an infinite resistance condition, which is indicative of an open circuit.

Furthermore, the inclusion of capacitors and inductors in the circuit introduced observable effects on signal propagation. Capacitors caused the voltage across the cable to increase and decrease gradually, in line with their energy storage capabilities. Inductors affected the amplitude of the reflected wave, with lower inductance leading to more negative amplitude. These components also showed characteristic charging and discharging patterns that affected the voltage across the transmission line.

It is important to acknowledge the presence of uncertainties and noise within the experiment. These elements could have influenced the measurements and the resultant interpretations of signal behavior. Future studies would benefit from a more detailed analysis of these factors to enhance the precision of the experimental conclusions.

In summary, this experiment addressed the key questions raised in the abstract by demonstrating the effects of impedance matching, signal reflection, and the influence of line capacitance and inductance on signal propagation through transmission lines. The insights gained from this study provide a basis for optimizing transmission line designs and can potentially contribute to advancements in communication systems, reaffirming the fundamental principles of electrical signal behavior in transmission line applications.

8 Future course of action and improvements on the experimental setup

1. It was observed that the pulse generator generated waveforms with characteristics warranting further investigation. Repeating the experiment with a more sophisticated generator would enable a more detailed study of the behavior and subtle variations in the pulse waveforms.

2. A 25-meter-long coaxial cable was utilized for the experiment. The reflected pulse's amplitude was influenced by cable attenuation, and it was hypothesized that high cable attenuation was a contributing factor. An experiment employing a longer cable with a different dielectric constant and lower capacitance per meter would facilitate a better understanding of the relationship between the transmitted and reflected pulses using VSWR.

3. The experiment was conducted using only a single pulse mode on the pulse generator. Employing different modes, such as square pulse, double pulse, and delayed pulse, would allow for the identification of any potential relationships between the pulse width, time period, and frequency of the reflected wave.

4. The unknown inductor's inductance was suggested to be measured. A series of inductors and capacitors, encompassing a broad range of values, should be assembled and then utilized for the experiment. This approach would permit the modeling of the behavior of the terminating load when exposed to different capacitances and inductances.

References

[1] Rank, R. I. (1969). *Apparatus for Teaching Physics: Speed of a Pulse in a Transmission Line.* The Physics Teacher, 7, 344. `https://doi.org/10.1119/1.2351396`