Name : Hamza Bilal

FA23 - BCS- 233

Assignment #1

Statistics and Probability

Metric Ch # 6 (Basic Statistics).

This chapter explains frequency distribution (how to organize raw data into tables and graphs), both discreate and continous forms. and how to represent them with histograms and frequency polygons.

It covers the class limits, class boundaries, midpoints, step-by-step construction of tables and graphs, examples.

Key definitions.

Frequency distribution: Tabular arrangement classifying data into groups with counts (frequencies) for each group.

Hamza Bilal - 233

Grouped data: Data presented in class intervals (groups)

Discrete frequency: distribution: Values are distinct (e.g number of heads). use exact values as classes

Continous frequency distribution: Data measured on a scale and grouped in intervals (eg marks 10- 19).

Class limits: Minimum and maximum values defined for a class (eg 10-19)

Class boundries: Real boundaries of a class used in continous data- found by averaging successive class limits (eg boundary between 9 and 10 is 9.5).

class 0–9 has boundaries 0.5 to 9.5 when unit is 1.

- class mark / mid point :

$$\frac{(\text{lower limit} + \text{upper limit})}{2}$$

- class width / size (h): width of the class interval

- Range: $X_{max} - X_{min}$.

- Commulative frequency: Total frequency up to an upper class limit.

- Tally marks: Quick way to record observations (𝍸 for 5)

Important formulas.

Class size = $h = \frac{\text{Range}}{k}$ where

k = number of groups (commonly 5–20).

Hamza Bilal 233

- Class boundary between two adjacent classes = average of adjacent class limits (or add/subtract half the unit).

- for unequal class intervals (histogram): use height = frequency/ class width so bar area ∝ frequency.

Class boundaries & midpoints.

- Boundary between 9 and 10:

$$\frac{9+10}{2} = 9.5$$

so class 0-9 becomes boundary -0.5 to 9.5.

- midpoint for class 10-19 =

$$\frac{10+19}{2} = 14.5$$

- use boundaries when plotting histograms

Hamza Bilal 233

Commulative frequency:

total of frequencies up to and including a class.

- Useful for drawing ogive (commulative frequency curves) and finding medians / percentiles.

Frequency polygon:

- A polygon joining midpoints of class tops with straight lines ; closed at ends by joining to zero - frequency points if required.
- Construction : plot midpoints on x-axis, frequency on y, join the successive points.
- Useful for comparing distributions.

Unequal interval histograms

If classes have different widths, do not draw bars with heights = frequency.

Hamza Bilal 233.

$$\text{height} = \frac{\text{frequency}}{\text{class width}}$$

1st year chapter # 7.

Fundamental Principles.

- Fundamental Principle of counting: This principle is used to find the total number of possible outcomes when an event involves multiple stages. If event A can happen in 'm' ways and event B can happen in 'n' ways, the total number of ways for both to occur is $m \times n$. This can be extended to three or more events.

- Factorial (!): The factorial notation was introduced by Christian Kramp in 1808. It is the product of all natural numbers from 1 to 'n' and is denoted by n! or n. For example.

  4! [cite start] = 4 · 3 · 2 · 1 = 24.

Hamza Bilal 233

- Definition: A permutation is an arrangement of all or part of a set of objects order.

- Formula: the number of permutations of r objects taken from a set of n objects is given by the formula.

$$P(n,r) = \frac{n!}{(n-r)!}$$

- Permutations with Repetition: If a set of 'n' objects contains n_1 identical objects of one kind, n_2 identical objects of a second kind, and so on, the number of permutations is given by

$$\frac{n!}{n_1!\, n_2!\, [\text{cite start}] \cdots n_k!}$$

Circular Permutations

Definition: circular permutations are arrangements of objects in a circular order.

Hamza Bilal 233

- Case 1 (clockwise and Anticlockwise orders are different): The number of distinct circular permutations for 'n' objects is $(n-1)!$

- Case 2 (clockwise and Anticlockwise orders are identical): In situations where an arrangement and its mirror image are considered the same, the number of permutations is $\frac{(n-1)!}{2}$.

An example of this is arranging beads on a ring.

Combinations.

Definition: A combination of 'r' objects taken from a set of 'n' objects is a subset of 'r' objects where the order does not matter.

Formula: The number of combinations of 'n' different objects taken 'r' at a time is denoted by $C(n,r)$ or $\binom{n}{r}$ and is given by

Hamza Bilal 233

the formula : $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Book : Probability and Basic Probabity for computer scientists

Chapter 1: Introduction and Basic Probability.

Probability and statistics provide a language to measure and manage uncertainty: the book teaches how to evaluate probabilities of coutcomes, and make optimal decisions under uncertainty.

Detailed Concepts:

Random Experiment

- An experiment whose outcome cannot be predicted with certainty
  Example : Rolling a die.

Sample Space (S)

- The set of all possible outcomes
  Example: For a coin toss → $S = \{H, T\}$

Hamza Bilal 233

Event (E):

- A subset of the sample space.

Example: Getting an even number on a die → $E = \{2, 4, 6\}$

Set Operations on Events

- Union ($A \cup B$): Either event occurs.
- Intersection ($A \cap B$): Both occurs
- Complement (A^c): Event does not occur
- Venn diagrams are used to visualize

Axioms of Probability (Kolmogorov)

$P(E) \geq 0$

$P(S) = 1$

For disjoint events A, B:

$$P(A \cup B) = P(A) + P(B)$$

Rules

Addition Rule

- for mutually exclusive events:

$$P(A \cup B) = P(A) + P(B)$$

- General: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Hamza Bilal 233

Multiplication Rule :

- If independent : $P(A \cap B) = P(A) P(B)$.

Discrete sample spaces:

- finite or countable outcomes (like dice, coins).
- probability of each outcome = $\frac{1}{n}$ if equally likely.

Chapter # 2:

In this chapter, the high-level overview is

- Provides tools to calculate probabilities when experiments involve large outcome sets
- Focuses on counting methods: Permutations, combinations, binomial coefficients.
- Applications in computer science: algorithm analysis, cryptography, randomized algorithms.

Detailed concepts.

- **fundamental counting principle:**

If one task can be done in m ways and another in n ways, then together → m x n ways

Permutations

- Arrangements of objects.
- without repitition.

$$P(n,r) = \frac{n!}{(n-r)!}$$

with repetition: n^r

Example How many 3-letter "words" from 26 English letters? → 26^3

Combinations (order does not matter)

- selecting items without caring about order.
- Formula:

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Example: choosing 3 students from a class of 10 → $\binom{10}{3} = 120$.

Binomial coefficients

Properties: symmetry $\binom{n}{r} = \binom{n}{n-r}$

Binomial theorem.

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

Applications in probability:

Probability of exactly r successes in n Bernoulli trials.

$$P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}.$$

Example: flipping a coin 5 times, propability of exactly 2 heads.

Hamza Bilal 233