Experimental Techniques in Physics Supported with AI/ML:

Fuzzy Logic

Lecture 10

Fuzzy logic. Basic concepts.

Why fuzzy logic?

- Complex problems are difficult to analyze precisely
- Expert knowledge in complex cases can be described in a fuzzy way, for example:

If the wind is very strong and the table is very light and the table is attached weakly then the table will fly away into the distance.

Fuzzy logic/systems include:

- Mathematics of sets and fuzzy logic,
- Fuzzy representation and knowledge processing for classification, regression and clustering,
- Learning membership functions and logic rules from data.
- Fuzzy control methods.

Fuzzy logic. Types of uncertainty.

- **Stochastic uncertainty**:
  E.g. dice throw, accident, insurance risk - probability calculus.
- **Measurement uncertainty**
  About 3 cm; 20 points - statistics.
- **Informational uncertainty** :
  Reliable borrower, meeting conditions - data mining.
- **Linguistic uncertainty**
  E.g. small, fast, low price - fuzzy logic.

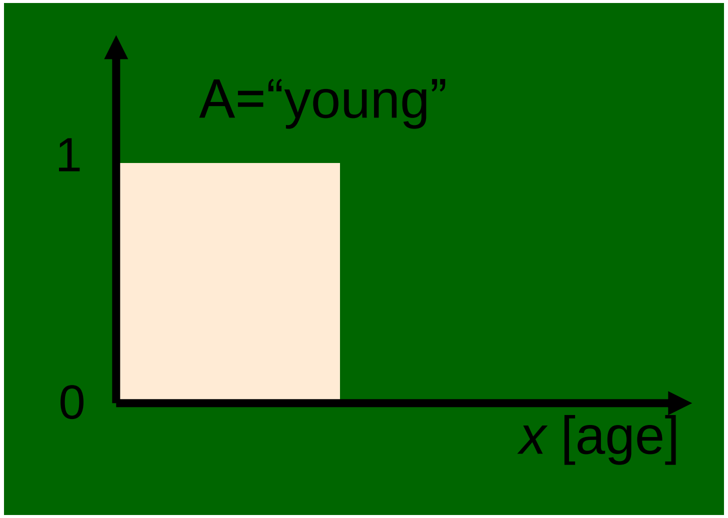
Fuzzy logic. Classical sets.

young = $\{ x \in M \mid \text{age}(x) \leq 20 \}$

Characteristic function

$$\mu_{young}(x) = \begin{cases} 1 : \text{age}(x) \leq 20 \\ 0 : \text{age}(x) > 20 \end{cases}$$

$\boxed{\mu_{young}(x)}$



Fuzzy logic. Classical sets.

X – universe, universal set, space; $x \in X$

A – linguistic variable, concept, fuzzy set.

The membership function determines the degree to which *x* belongs to *A*.

$$\mu_A : X \to [0,1]$$

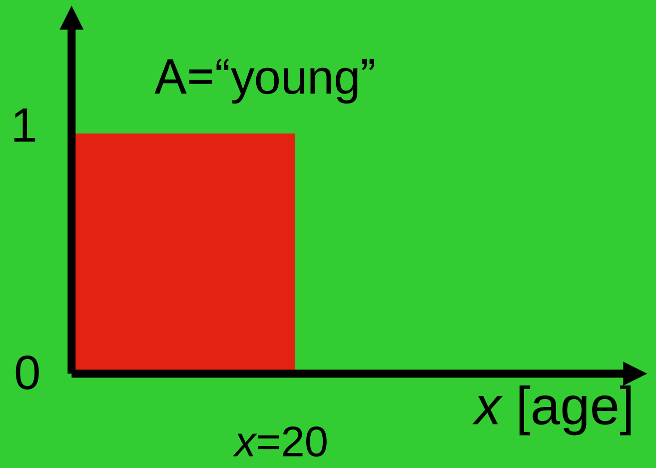
Degree of membership is not probability - bald 80% of the time is not bald 1 in 5 times.

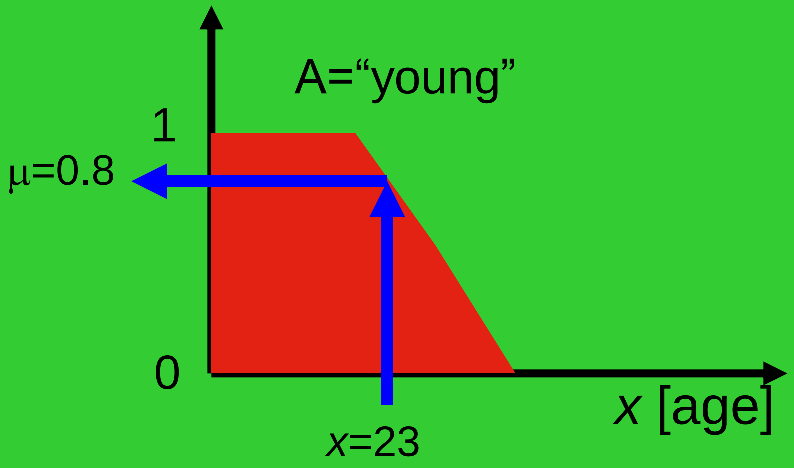
Probability is normalized to one, the membership function is not.

Fuzzy expressions/concepts are subjective and context-dependent.

Fuzzy logic. Examples.

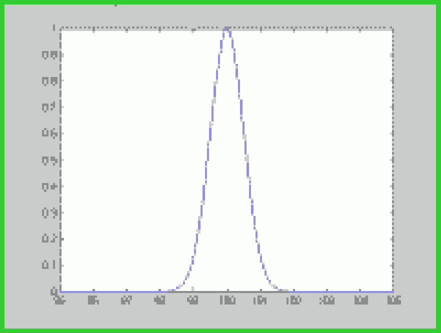
The classic and fuzzy concept of "young man"





The "boiling point" is about 100 degrees
(pressure, chemical composition).

$$\mu_W\left(T\right) = e^{-2\left(T-100\right)^2}$$



Fuzzy logic. Definitions.

Support (base) of the fuzzy set A:

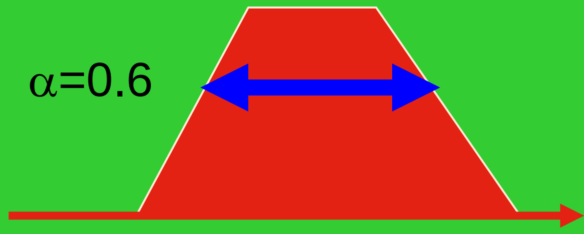
$\mathrm{supp}(A) = \{ x \in X : \mu_A(x) > 0 \}$

Core of the fuzzy set A:

$\mathrm{core}(A) = \{ x \in X : \mu_A(x) = 1 \}$

α-cut of the fuzzy set A:

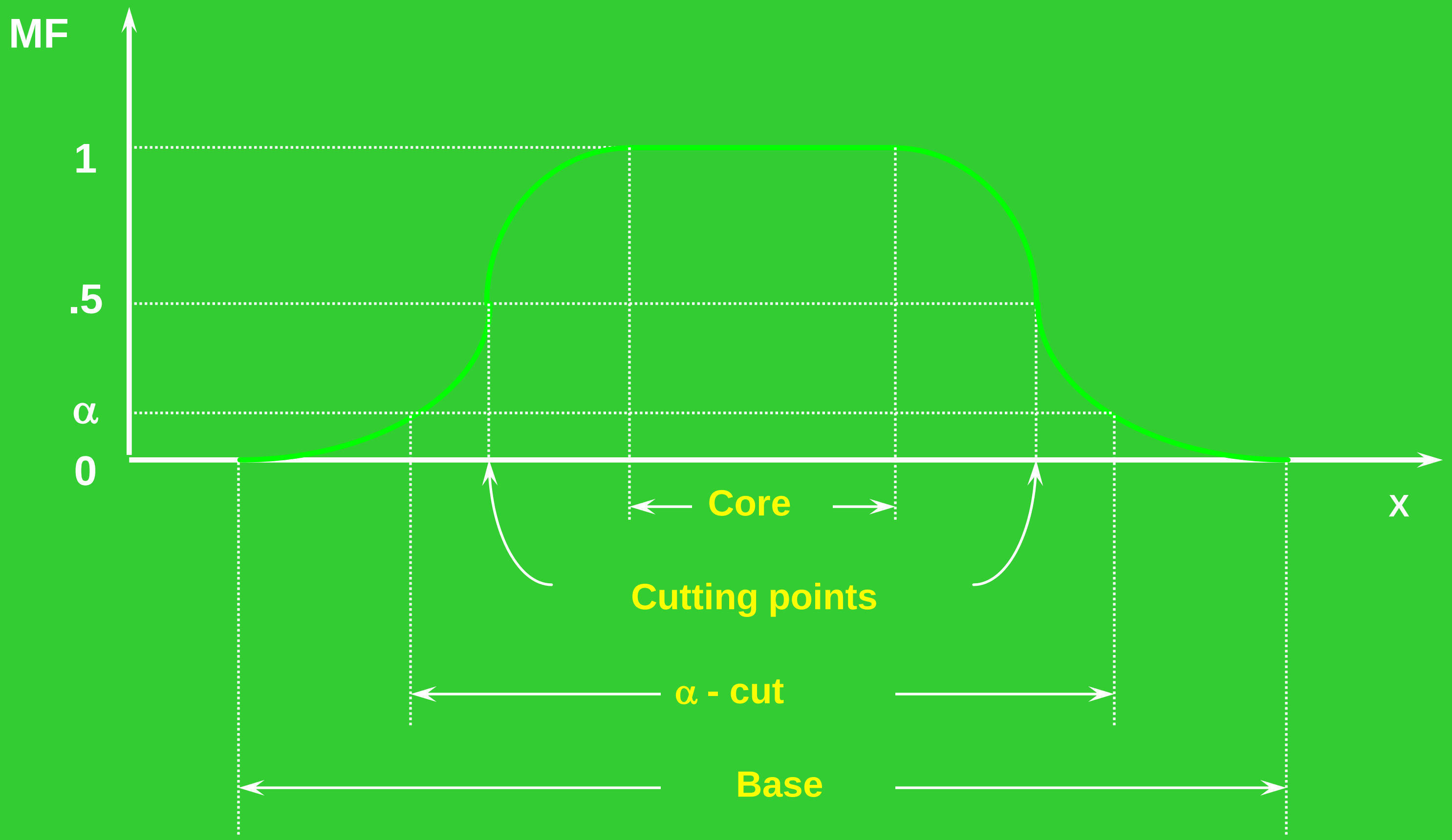
$A_\alpha = \{ x \in X : \mu_A(x) > \alpha \}$



Height= $\max_x \mu_A(x) \leq 1$

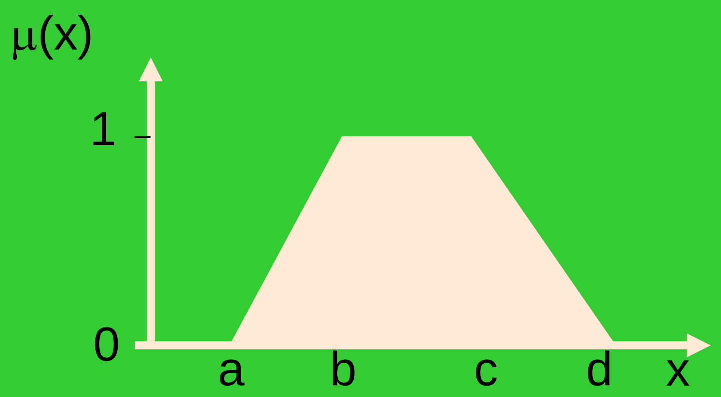
Normal fuzzy set : $\sup_{x \in X} \mu_A(x) = 1$

Fuzzy logic. Terminology.

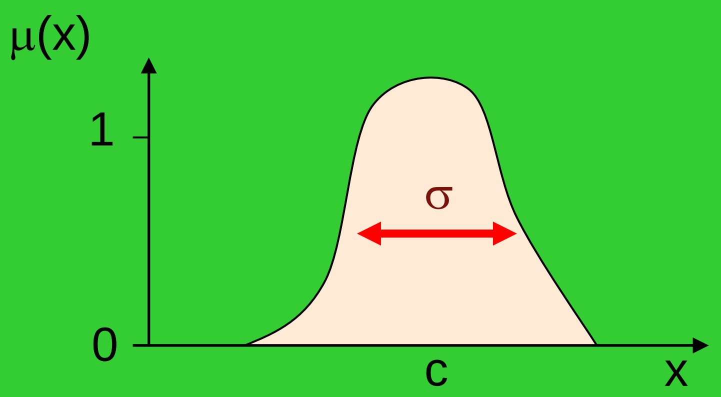


Fuzzy logic. Types of membership functions.

Trapezoid: <a,b,c,d>



Gaus/Bell: N(m,s)



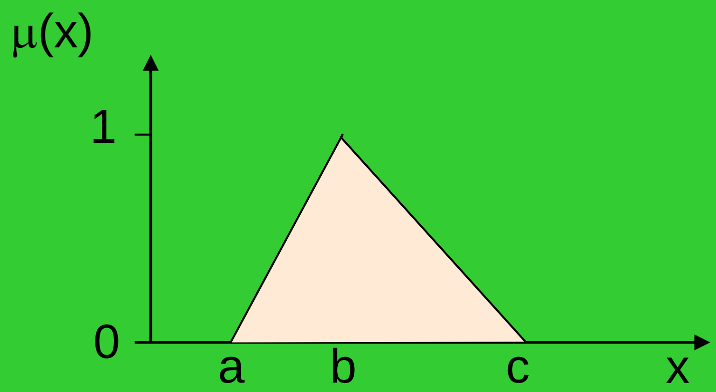
$$Trap(x;a,b,c,d) = \max\left(\min\left(\frac{x-a}{b-a},\frac{d-x}{d-c},1\right),0\right)$$

$$G(x;a) = e^{-(x-a)^2/2\sigma^2}$$

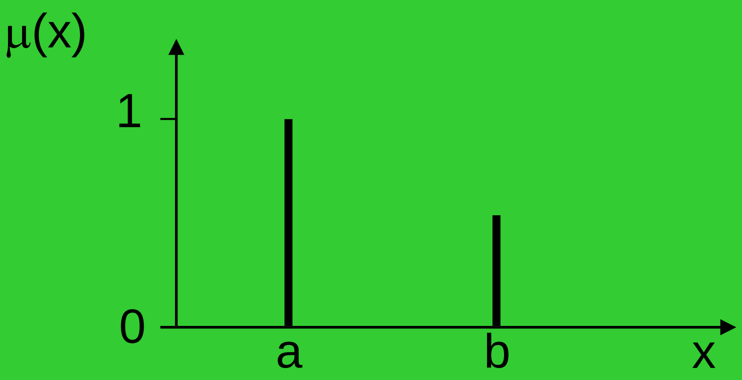
$$B(x;a,b) = \frac{1}{1+\left|\frac{x-a}{b}\right|^{2b}}$$

Fuzzy logic. Types of membership functions.

Traingular: <a,b,c>



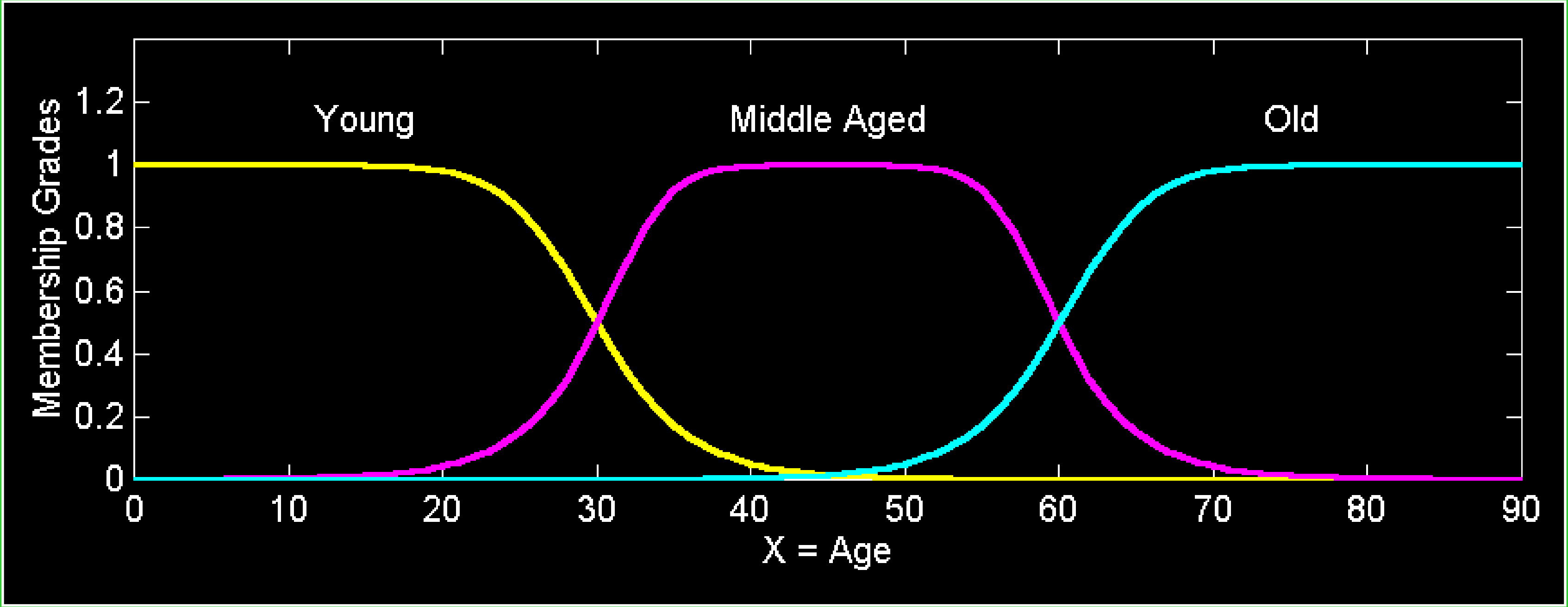
Singleton: (a,1) i (b,0.5)



$$T\left(x;a,b,c\right)=\max\left(\min\left(\frac{x-a}{b-a},\frac{c-x}{c-b}\right),0\right)$$

Fuzzy logic. Linguistic variables.

W=20 => Age=young.
Linguistic variable = linguistic value.
Linguistic variable: temperature
therms (fuzzy sets) : {cold, warm, hot}

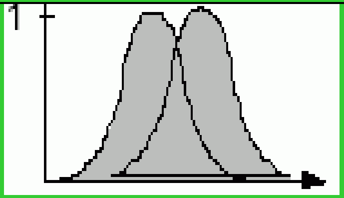


Fuzzy logic. Sum and product of sets.

A, *B* – fuzzy sets.

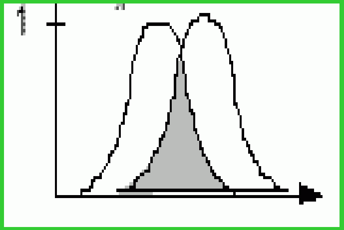
Sum A∪B is a set with a membership function :

$$\mu_{A\cup B}(x) = \max\left(\mu_A(x), \mu_B(x)\right)$$



The product A∩B is a set with a membership function :

$$\mu_{A\cap B}(x) = \min\left(\mu_A(x), \mu_B(x)\right)$$

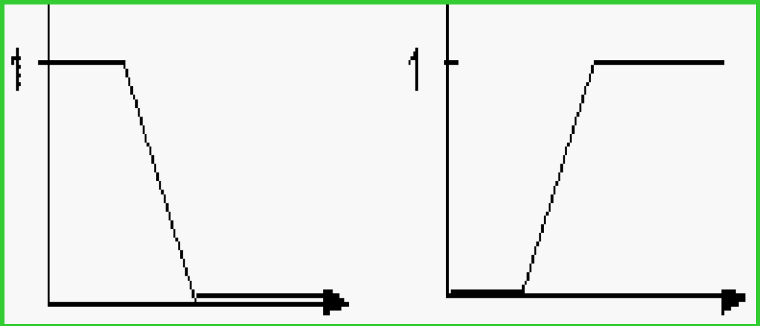


Fuzzy logic. Complement.

Complement *A'* of set *A* is a set with membership function:

$$\mu_{A'}(x) = 1 - \mu_A(x)$$

$$\mu_{A' \cup A}(x) \le 1; \quad \mu_{A' \cap A}(x) \le 1$$



Fuzzy logic. Fuzzy relationships.

- Classical relations

$R \subset X \times Y$ def: $\mu_R(x,y) = \begin{cases} 1 & \text{if } (x,y) \in R \\ 0 & \text{if } (x,y) \notin R \end{cases}$

- Fuzzy relations

$R \subset X \times Y$ def: $\mu_R(x,y) \in [0,1]$

$\mu_R(x,y)$ describes the degree of linkage between x and y

Fuzzy logic. Examples of fuzzy relations.

Close: X ≈Y; X depends on Y; X similar to Y ...

X = {rainy, cloudy, sunny}

Y = {sunbathing, roller skating, camping, reading}

X/Y	sunbathing	roller skatingi	camping	reading
rainy	0.0	0.2	0.0	1.0
cludy	0.0	0.8	0.3	0.3
sunny	1.0	0.2	0.7	0.0

Here it is more describing probabilities or correlations.

Fuzzy logic. Fuzzy rules.

Common knowledge can often be written naturally using fuzzy rules.

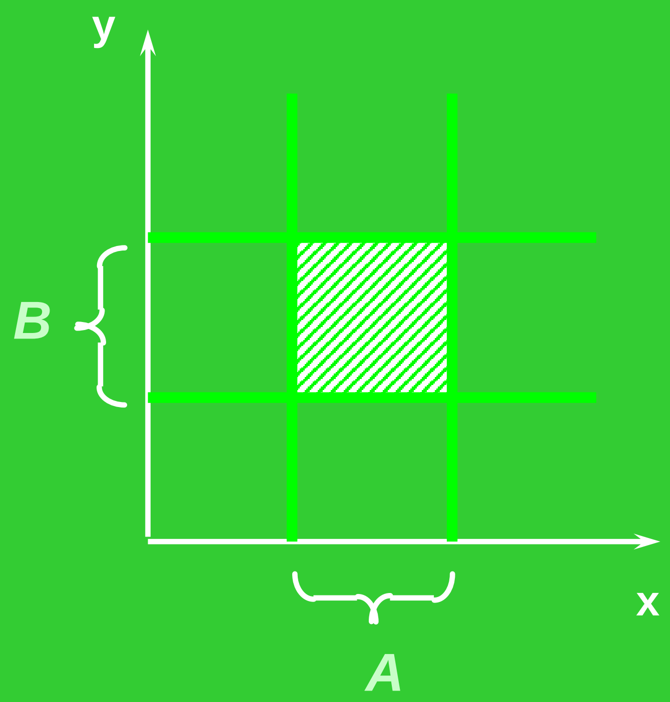
If lingw var-1 = term-1 and lingw var-2 = term-2
then lingw var-3 = term-3

If Temperature = cold and heating price = low
then heating = strong

What does the fuzzy rule mean:
If *x* jest A then *y* is B ?

Logika rozmyta. Reguły rozmyte.

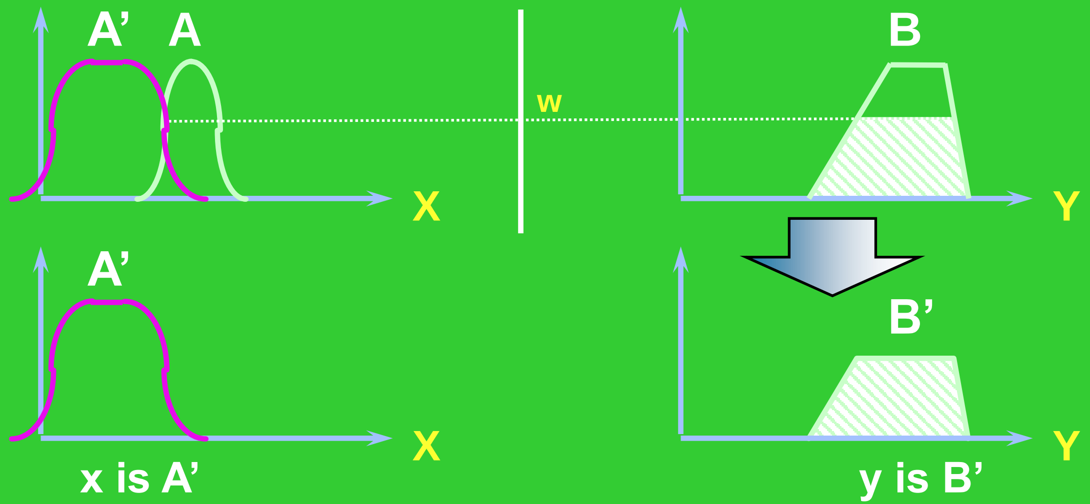
Jeśli *x* jest A to *y* jest B: korelacja lub implikacja.



Fuzzy logic. Fuzzy rules.

If *x* is A then *y* is B.

Fact: *x* is A’, conclusion: *y* is B’

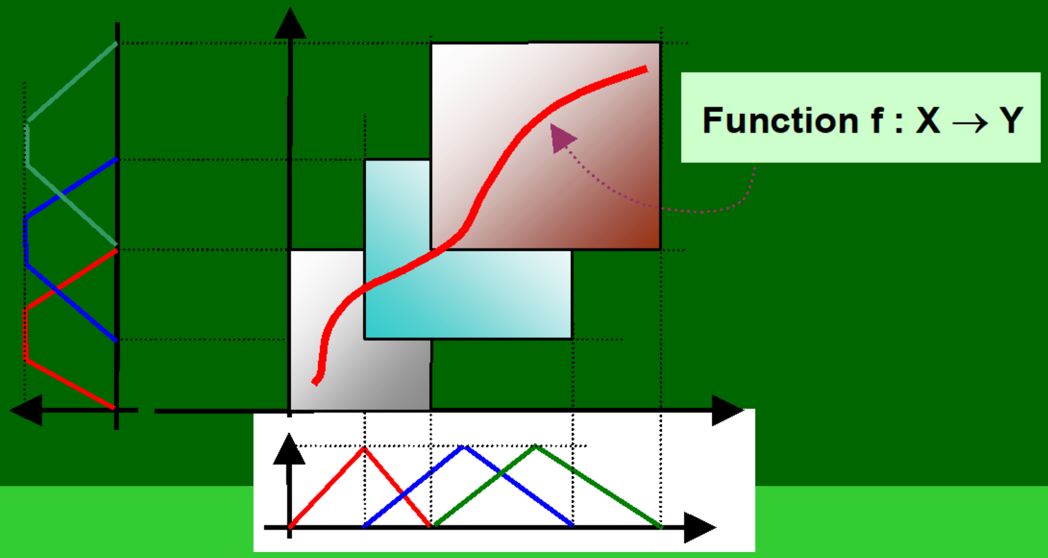


It is easy to generalize for many conditions:

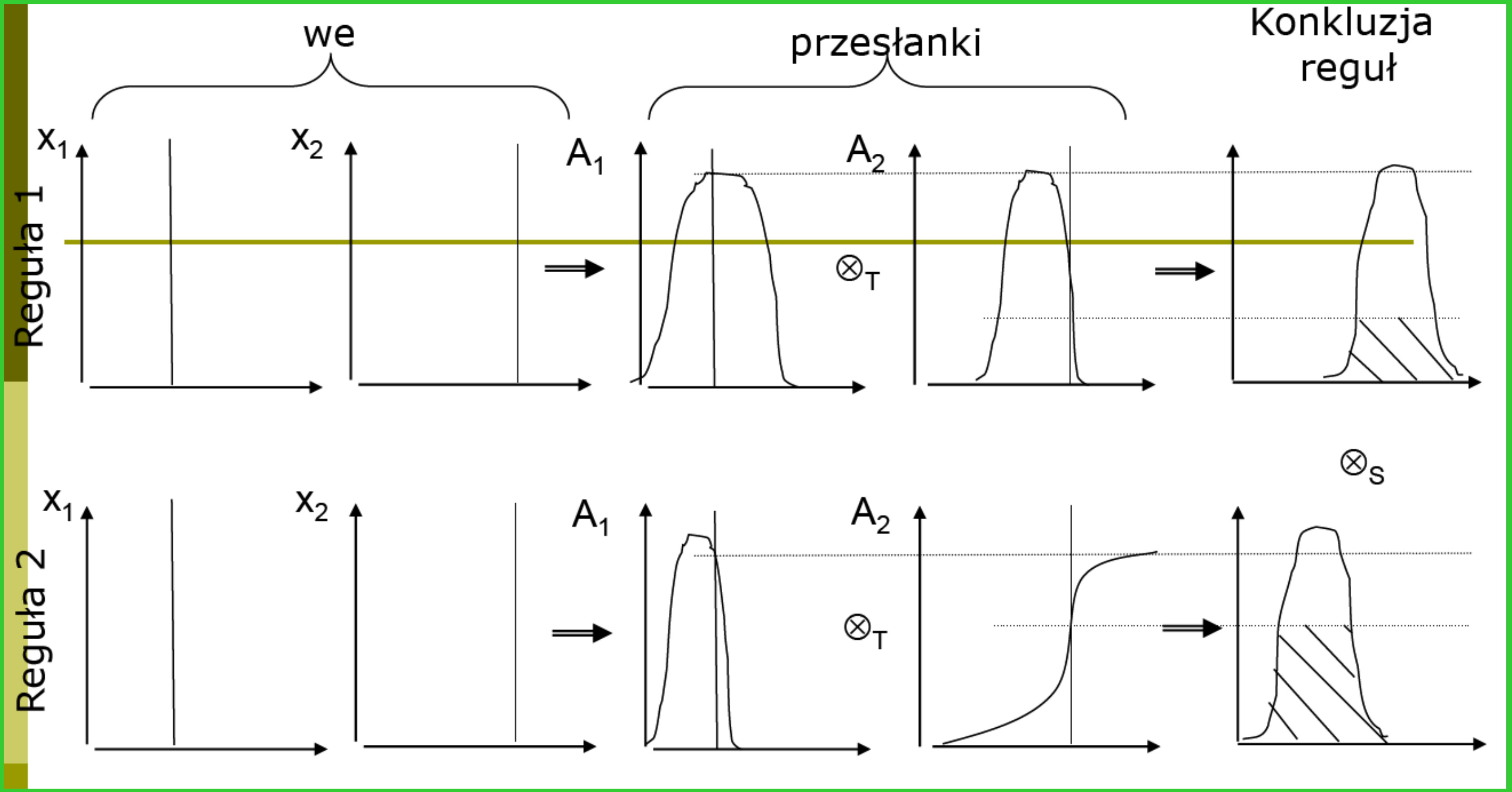
If *x* is A and *y* is B then *z* is C

Fuzzy logic. Fuzzy approximation.

- Fuzzy systems $F: \Re^n \rightarrow \Re^p$ uses **m** rules to map a vector **x** to an output **F(x)**, vector or scalar.



Fuzzy logic. Fuzzy approximation.



Fuzzy logic. Implications.

Mamdani

$$\mu_{A\to B}(x,y)=\mu_A(x)\wedge\mu_B(y)=\min\left(\mu_A(x),\mu_B(y)\right)$$

Larsen

$$\mu_{A\to B}(x,y)=\mu_A(x)\mu_B(y)$$

Łukasiewicz

$$\mu_{A\to B}(x,y)=\min\left(1,1-\mu_A(x)+\mu_B(y)\right)$$

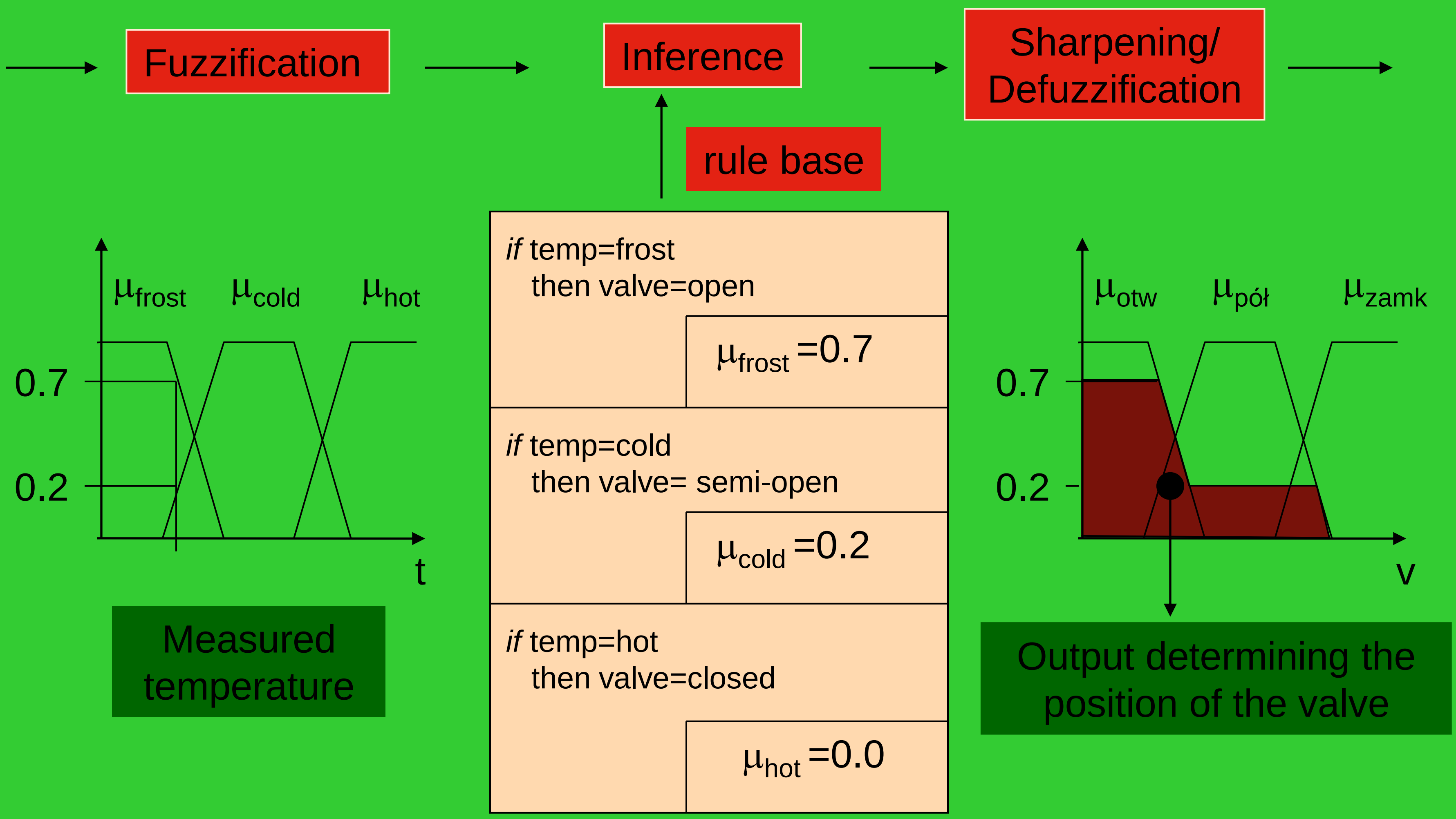
Zadeh

$$\mu_{A\to B}(x,y)=\max\left(\min\left(\mu_A(x),\mu_B(y)\right),1-\mu_A(x)\right)$$

Kleen-Dienes, Goguen, Sharp, limited sum, probabilistic...

Relationships can be derived from Lukasiewicz's multivalued logic

Fuzzy logic. Diagram of a fuzzy system.



Fuzzy logic. Fuzzification block.

Defuzzification block has the task of converting sharp x values, mostly obtained from measurements, into fuzzy sets A'. One of the popular methods of defuzzification is the singleton defuzzification operation.

Fuzzy logic. Inference block.

This block performs fuzzy inference using a rule base of typeu:

If x1 is $A^1{}_1$ and x2 is $A^1{}_2$ and ... i xn is $A^1{}_n$ then y is B^1
...
If x1 is $A^M{}_1$ and x2 is $A^M{}_2$ and ... and xn is $A^M{}_n$ then y is B^M

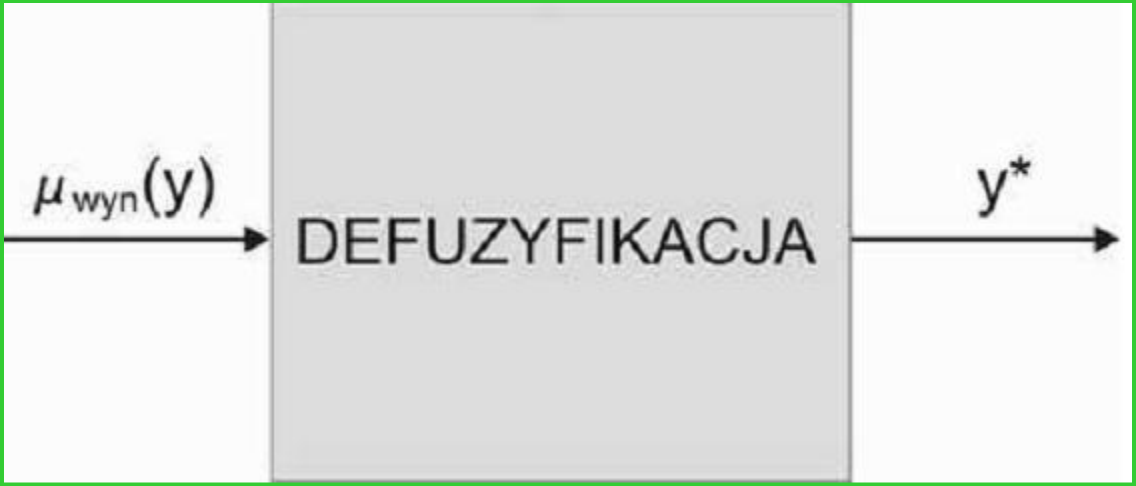
The task of this block is to check the degree of fulfillment of the premise of each rule and determine the response of each rule, i.e. fuzzy sets B'^i.

Fuzzy logic. Defuzzification block.

At the output of the inference block we will get one or many fuzzy sets. However, to control a specific object, we need specific sharp values. It is as if a person, knowing that he must drive slowly, eventually slows down to a specific value of 25 km/h. In a classical fuzzy controller, the sharpening block is responsible for this last step. It can also be implemented in a number of ways.

Fuzzy logic. Defuzzification block.

The task of defuzzification: converting a fuzzy value into a concrete numerical value, passed to the system as input. There are several different methods of defuzzification.



Fuzzy logic. Defuzzification block.

The maximum method is one of the simplest methods for defuzzification. It consists in selecting (depending on the version) the first, last or middle variable of the most activated fuzzy set. It is used in low-cost and slow microprocessors.

Fuzzy logic. Defuzzification block.

The center of gravity method, is a complicated method in which the result is determined using a function of

$$y^* = \frac{\int y \times \mu_{wyn}(y)dy}{\int \mu_{wyn}(y)dy}$$

It takes into account all activated sets (object control is smoother), but it is less frequently used, as it requires more computing power.

Fuzzy logic. Defuzzification block.

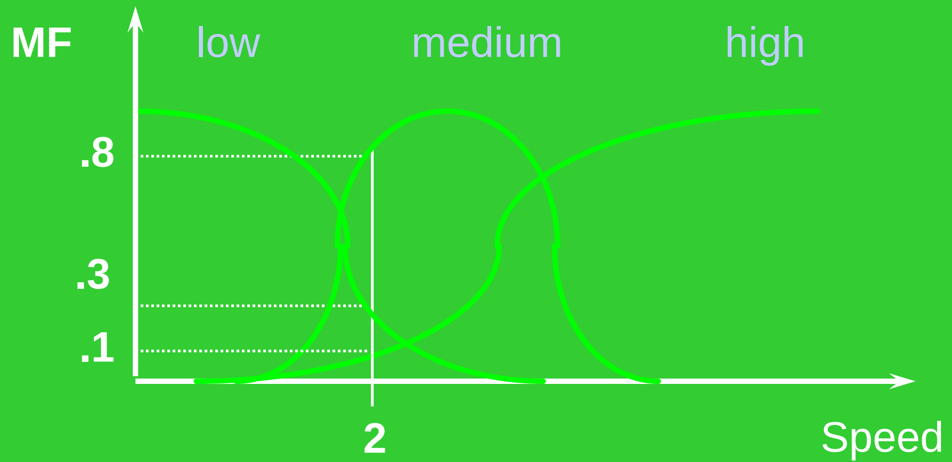
The height method takes into account any active premise. It is definitely simpler to calculate than the center of gravity method (sum instead of integral), while providing smooth control of the object.

$$y = \frac{\sum_i (\mu_i \times y_i)}{\sum_i \mu_i}$$

where *i* is the number of output fuzzy sets, μ_i is the designated degree of activation, and y_i is the representative values of the result for each interval.

Fuzzy logic. MATLAB - rules defining- Fuzzy Inferrence System (FIS).

IF speed is low then braking = 2
IF speed is medium then braking = 4* speed
IF speed is high then braking = 8* speed



R1: $w_1 = .3$; $r_1 = 2$
R2: $w_2 = .8$; $r_2 = 4*2$
R3: $w_3 = 0.1$; $r_3 = 8*2$

Braking = $\Sigma(w_i*r_i) / \Sigma w_i$
= 7.12

Fuzzy logic. MATLAB - The base of rules (FIS).

1. If (temperature is cold) and (oilprice is normal) then (heating is high) (1)
2. If (temperature is cold) and (oilprice is expensive) then (heating is medium) (1)
3. If (temperature is warm) and (oilprice is cheap) then (heating is high) (1)
4. If (temperature is warm) and (oilprice is normal) then (heating is medium) (1)
5. If (temperature is cold) and (oilprice is cheap) then (heating is high) (1)
6. If (temperature is warm) and (oilprice is expensive) then (heating is low) (1)
7. If (temperature is hot) and (oilprice is cheap) then (heating is medium) (1)
8. If (temperature is hot) and (oilprice is normal) then (heating is low) (1)
9. If (temperature is hot) and (oilprice is expensive) then (heating is low) (1)