EXPERIMENTAL TECHNIQUES IN PHYSICS SUPPORRTED WITH ARTIFICIAL INTELLIGENCE

6 Modeling and simulation of a system with magnetically coupled circuits

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1. Excercise objectives

The purpose of the exercise is to model the phenomenon of magnetic coupling of two circuits using a selected example and to study the effect of its parameters on the energy exchange between the circuits.

2. Theoretical introduction

Inductance, which represents the magnetic field and, in particular, the ability to store energy in this field, is often considered only as a two-terminal element. However, in many cases, the effect of a magnetic field on an electric circuit cannot be modeled only as a two-terminal element. This is the case when the various coils present in the circuit share at least partially a common magnetic field, or in other words, when they are magnetically coupled. In this case, the circuit model of magnetic phenomena is an element with at least two pairs of terminals.

A popular device that uses magnetic coupling is a transformer.

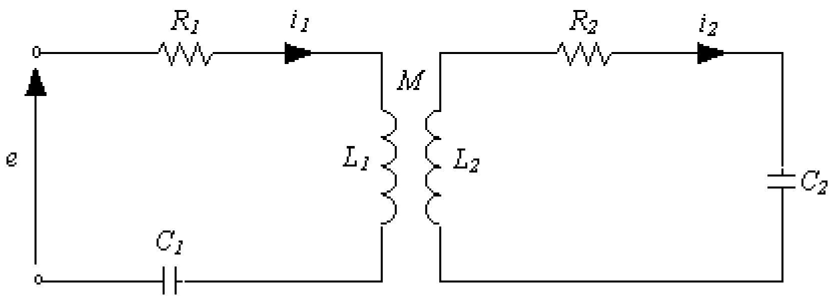

Fig. 1. Electrical circuit with magnetic coupling.

Fig. 1 shows a system consisting of two coupled circuits. This is an example of a model with distributed parameters described by a fourth-order differential equation. Although this is a linear problem, and therefore appropriate in principle for analytical solution methods, it is a relatively too complicated case to consider analytically. It represents a situation where valuable insight into the behavior of the system can be provided by simulation. This problem is particularly interesting because it involves two circuits coupled through a common inductance M. This results in quite complex dynamic behavior, especially in situations where the two circuits have parameter values that produce slightly damped oscillations. Similar properties can be found for mechanical circuits involving the combined motion of mass, elastic and damping elements.

The pair of differential equations describing the circuit can be derived by applying Kirchhoff's law to the left and right circuits, respectively. For the former, the following relation is obtained

$$e(t) = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + \frac{1}{C_1} \int i_1 dt \,, \tag{3.1}$$

while for the latter

$$0 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + \frac{1}{C_2} \int i_2 dt \,. \tag{3.2}$$

These equations can be rewritten as follows:

$$\frac{di_1}{dt} = \frac{e(t)}{L_1} - \frac{1}{T_1} i_1 - \omega_1^2 \int i_1 dt - K_1 \frac{di_2}{dt} \tag{3.3}$$

$$\frac{di_2}{dt} = -\frac{1}{T_2} i_2 - \omega_2^2 \int i_2 dt - K_2 \frac{di_1}{dt} \tag{3.4}$$

where

$$T_1 = \frac{L_1}{R_1} \qquad \omega_1^2 = \frac{1}{L_1 C_1} \qquad K_1 = \frac{M}{L_1}$$

$$T_2 = \frac{L_2}{R_2} \qquad \omega_2^2 = \frac{1}{L_2 C_2} \qquad K_2 = \frac{M}{L_2}$$

By selecting state variables as follows:

$$x_1(t) = \int i_1 dt \qquad x_2(t) = i_1(t)$$

$$x_3(t) = \int i_2 dt \qquad x_4(t) = i_2(t)$$

we obtain four equations of state

$$\dot{x}_1 = x_2 \tag{3.5}$$

$$\dot{x}_2 = -\frac{\omega_1^2}{1-K_1K_2} x_1 - \frac{1}{T_1(1-K_1K_2)} x_2 + \frac{\omega_2^2 K_1}{1-K_1K_2} x_3 + \frac{K_1}{T_2(1-K_1K_2)} x_4 + \frac{e(t)}{L_1(1-K_1K_2)} \tag{3.6}$$

$$\dot{x}_3 = x_4 \tag{3.7}$$

$$\dot{x}_4 = \frac{\omega_1^2 K_2}{1-K_1K_2} x_1 + \frac{K_2}{T_1(1-K_1K_2)} x_2 - \frac{\omega_2^2}{1-K_1K_2} x_3 - \frac{1}{T_2(1-K_1K_2)} x_4 - \frac{K_2 e(t)}{L_1(1-K_1K_2)} . \tag{3.8}$$

The above four equations of state ((3.5)-(3.8)) make it possible to create a structure that allows simulation of the behavior of the system in Fig. 1.

Supplementary literature

[1] D.J. Murray-Smith: Continuous system simulation. Chapman&Hall, London 1995.