Brandenburg
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EXPERIMENTAL TECHNIQUES IN PHYSICS SUPPORRTED WITH ARTIFICIAL INTELLIGENCE

LABORATORY

9 Modeling and simulation of blood sugar and insulin changes

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1. Excercise objectives

The purpose of the exercise is to model the phenomenon of change in blood sugar and insulin content of a healthy and diabetic person after eating a meal and, in the case of a diabetic, after administering a dose of insulin. This is an example of compartmental modeling of a nonlinear system.

2. Theoretical introduction

The problem of modeling changes in blood sugar and insulin is among the very serious ones for diabetics, in whom the concentration of these two components must be controlled. A mathematical model of these changes allows us to better understand the mechanism governing them, as well as to build devices that automatically dispense medication.

Let us denote by g and i the relative contents of sugar and insulin, respectively, in a person's blood. Let $u_1(t)$ be the measure of ingested food (converted to sugar) permeating into the blood at time t, and $u_2(t)$ be the measure of externally injected insulin permeating into the blood at time t.

Biochemical observations show that for healthy people, in the long-term absence of food and without insulin administration ($u_1 = 0$, $u_2 = 0$), a state of equilibrium is produced at which $g = M_1$, and $i = 0$, meaning that there is a certain minimum sugar content in the blood and no insulin. The value of M_1 can be determined for each individual by appropriate testing.

It was found that the presence of insulin in the blood causes a reduction in sugar content, with the degree of reduction being proportional to the levels of both insulin and sugar. In the model, it was assumed to be proportional to the product of gi. If the sugar level falls below the equilibrium state of M_1, sugar is given up by the liver, with the amount of sugar produced being proportional to $M_1 - g(t)$. The liver does not produce sugar if its level is higher than M_1. In addition, it should be noted that sugar levels increase after eating a meal due to penetration into the blood (as a result of digestion of food). As a result, the equation of sugar state can be written in the form of

$$\frac{dg}{dt} = \begin{cases} -a_1 ig + a_2(M_1 - g) + b_1 u_1 & \text{dla} \quad g(t) \le M_1 \\ -a_1 ig + b_1 u_1 & \text{dla} \quad g(t) > M_1 \end{cases} \tag{3.1}$$

where a_1, a_2 and b_1 are constant numerical coefficients with values specific to each person.

An equation characterizing the dynamic changes in blood insulin content can be written on a similar basis. As a result of observations, it was found that if the sugar level exceeds the minimum level M_1, $g(t) > M_1$, then insulin is produced in the pancreas and released into the bloodstream, with its production proportional to $g(t) - M_1$. If $g(t) \le M_1$ then the body does not produce insulin. Another property of this process is that insulin breaks down on its own at a rate proportional to its level in the body. Finally, in the case of diabetics, insulin levels can increase due to external dosing and this increase is proportional to $u_2(t)$. Taking the above into account, the equation of state for insulin can be written in the form of

$$\frac{di}{dt} = \begin{cases} -a_3(g - M_1) - a_4 i + b_2 u_2 & \text{dla} \quad g(t) > M_1 \\ -a_4 i + b_2 u_2 & \text{dla} \quad g(t) \le M_1 \end{cases} \tag{3.2}$$

where a_3, a_4 and b_2 are constant numerical coefficients with values specific to each person.

Both equations (3.1) and (3.2) can be written in simplified form

$$\frac{dg}{dt} = -a_1 ig - a_2(g - M_1)[1 - \mathrm{sgn}(g - M_1)] + b_1 u_1 \tag{3.3}$$

$$\frac{di}{dt} = a_3(g - M_1)\,\mathrm{sgn}(g - M_1) - a_4 i + b_2 u_2 \tag{3.4}$$

where the function sgn($g(t)$ - M_1) is defined as follows

$$\mathrm{sgn}(g - M_1) \;=\; \begin{cases} 0 & \text{dla} \quad i(t) \le M_1 \\ 1 & \text{dla} \quad g(t) > M_1 \end{cases} \tag{3.5}$$

Equations (3.1)-(3.5) are a set of 2 coupled differential equations. At steady state, in the absence of external forcing (u_1 = 0, u_2 = 0), the solution i_{ust} = 0 and g_{ust} = M_1 is consistent with the behavior of the real object.

The main difference between healthy people and those with diabetes is the value of the a_3 coefficient. This is a coefficient that relates the change in insulin levels to the increase in blood sugar. For sick people, its value is close to zero and an increase in sugar has no effect on insulin levels. Therefore, with a zero initial state of insulin in the blood, the solution of the insulin state equation is zero. This state can only be changed by injecting a dose of insulin (u_2 > 0) directly into the blood.