Probability Question

Answer – 1

Let's consider:
A is an unbiased die
B is the biased die

Sample Space:

A/B	2	3	4	5
1	3	4	5	6
2	4	5	6	7
3	5	6	7	8
4	6	7	8	9
5	7	8	9	10
6	8	9	10	11

Probability for sum 2-12

Sum	Frequency	Probability
2	0	0
3	1	0.04166667
4	2	0.08333333
5	3	0.125
6	4	0.16666667
7	4	0.16666667
8	4	0.16666667
9	3	0.125
10	2	0.08333333
11	1	0.04166667
12	0	0

Since die B is biased, and it won't roll 1 or 6, therefore, the sum can never be 2 or 12.

Answer – 2

Probability of winning on first try: P(7) + P(11) = 0.1667 + 0.041667 = ***0.20833333***

Probability of losing on first try: P(2) + P(3) + P(12) = ***0.04166667***

Answer – 3

Now that we have reached to the second roll, we need to figure out the probability of winning given, we have landed sum X with a probability of p. The probability of reaching second roll is

P(2^{nd} roll) = 1- [P(winning first roll) + P(losing first roll)] = 0.76

In this case, we will continue to roll the die until we get X(when we will win) or 7(we will lose).

Hence, we want to determine the probability of getting X before 7. This is the same as saying if we will land X or 7.

Given the conditional probability formula:

P(A|B) = P(A and B)/P(B)

Let A be the event that sum of die is X and B be the event that the sum of the die is either X or 7.

Since P(A and B) = P(A) = p and P(B) = p + P(7)

P(A) = p
P(B) = p + 4/24

P (Roll an X| Roll X or 7) = p/(p+1/6)

Hence, the probability of winning when first roll is X of probability p is:

P(X) * P (Roll an X| Roll X or 7) = **p *(p/(p+1/6)**

Answer – 4

As they have return on the dollar, that means amount invested is $1.
Return: Value of Winning * Probability of Winning + Value of Losing * Probability of Losing

Value of Winning: $1
Value of Losing: -$1

Probability of Winning:

1. You can win on the first try. That has a 20.83% chance (Question 2a).
2. You can take 2 turns to win:
   2b: You roll a 4 twice, chances of that happening are 8.33%^2 = 0.00694444 = 0.694444%
   2c: You roll a 5 twice, chances of that happening are 12.5%%^2 = 0.015625 = 1.5625%
   2d: You roll a 6 twice, chances of that happening are 16.67%^2 = 0.02777778 = 2.777778%
   2e: You roll a 8 twice, chances of that happening are 16.67%^2 = 0.02777778 = 2.777778%
   2f: You roll a 9 twice, chances of that happening are 12.5%%^2 = 0.015625 = 1.5625%
   2g: You roll a 10 twice, chances of that happening are 8.33%^2 = 0.00694444 = 0.694444%

Total chances of winning = Probability of winning in first try + Probability of taking 2 turns
= 20.83% + 10.066% = 30.89%

Probability of Losing:

1. You can lose on the first try. 4.16% chance
2. You can take 2 turns to lose: If we know that Probability of winning in the 2nd turn is 10.243055%, then probability of losing = 100% - 10.243055% = 89.93%

Hence total chance of losing = 4.16+89.93 = 94.093%
Hence return is: 1*30.89% - 1*93.916945%= -0.632

That means on an average you'll lose ***$0.632*** for every dollar spent in this game.