1. All Cylinders are identical (except for gas in them)
Same cross-sectional area A.
Pistons each have mass M.
Each Piston is in Mechanical equilibrium.

$$F_{down} = F_{up}$$ pressure of ideal gas.

$$Mg + P_0 A = PA$$

external pressure

In all 3 cylinders, F_{down} is the same

So $P_A = P_B = P_C$ which answers parts (a) & (b)

2. Thermodynamic system : Helium; treat it as ideal gas ($M_g = 4g/mol$)

State a : $T_a = 1100K$ $V_a = 0.5 \times 10^{-3} m^3$
State b : $T_b = 1100K$ $P_b = 1.0 \times 10^5 Pa$
State c : $T_c = 300K$

$a \to b$ isothermal ($T_a = T_b$)
$b \to c$ isochoric ($V_b = V_c$)
$c \to a$ isobaric ($P_c = P_a$)

N = Const - need to find N to determine mass of He present

$$N = P_a V_a / R T_a$$
$$= P_c V_a / R T_a$$
$$= \frac{T_c P_b}{T_b} \frac{V_a}{R T_a}$$

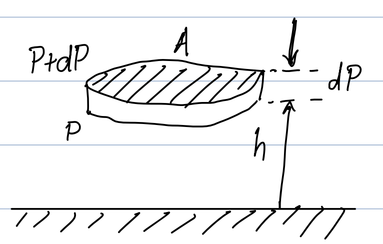
$V_b = V_c$, $\frac{P_c}{T_c} = \frac{P_b}{T_b}$

$$= \frac{300 (1 \times 10^5)(0.5 \times 10^{-3})}{(1100)(8.314)(1100)}$$

$$= 1.49 \times 10^{-3} \text{ moles}$$

$$M_{He} = N \times Mg$$
$$= 1.49 \times 10^{-3} \text{ moles } (4 g/mol)$$
$$= 5.96 \times 10^{-3} g$$
$$\cong 6 \text{ mg.}$$

3. a) Show that $\frac{dP}{dh} = -\rho g$



Mechanical equilibrium $\Rightarrow$ $F_{down} = F_{up}$

$$F_{down} = dm\, g + (P + dP) A$$
$$= (\rho A\, dh) g + (P + dP) A$$
$$F_{up} = PA$$

$$\Rightarrow \quad (\rho A\,dh)g + \cancel{PA} + dP\cancel{A} = \cancel{PA}$$

$$\Rightarrow \quad \boxed{\frac{dP}{dh} = -\rho g} \quad - - - - (1)$$

b) Assume isothermal ideal gas to show $\left(\frac{\partial \rho}{\partial P}\right)_T = {}^{M_g}/_{RT}$

Since $N = {}^{m}/_{M_g}$ & $\rho = \frac{m}{V}$

$PV = NRT$

$$\Rightarrow \quad P = \frac{\rho RT}{M_g}$$

$$\Rightarrow \quad dP = \frac{RT}{M_g}\,d\rho + \frac{\rho R}{M_g}\,dT$$

Const $T \Rightarrow dT = 0$

by definition of partial diffs

$$dP = \left(\frac{\partial P}{\partial \rho}\right) d\rho + \left(\frac{\partial P}{\partial T}\right) dT$$

$$\Rightarrow \quad \boxed{\left(\frac{\partial \rho}{\partial P}\right)_T = \frac{M_g}{RT}} \quad \text{for Const } T \quad - - - (2)$$

c) from (1) & (2) obtain $\left(\frac{\partial \rho}{\partial h}\right)_T$

$$\frac{d\rho}{dh} = \frac{d\rho}{dP}\frac{dP}{dh}$$

at const T

$$\left(\frac{\partial\rho}{\partial h}\right)_T = \frac{dP}{dh}\left(\frac{\partial\rho}{\partial P}\right)_T$$

$$= -\frac{\rho g M_g}{RT}$$

Since g & T are constant

$$\frac{d\rho}{\rho} = -\frac{gM_g}{RT}\,dh$$

Integrate both sides

$$\int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \int_0^h -\frac{gM_g}{RT}\,dh.$$

$$\Rightarrow \ln\left(\rho/\rho_0\right) = -\frac{gM_g}{RT}\,h \qquad h_0 = 0$$

$$\Rightarrow \rho = \rho_0 \exp\left(-\frac{gM_g h}{RT}\right)$$

d) $h_0 = \frac{RT}{gM_g}$

Taking $T = 293\,K$, $M_g = 28.96\times10^{-3}\,kg/mol$

$$h_0 = \frac{(8.314)(293)}{(9.8)(28.96\times10^{-3})}$$

$$= 8.58\times10^{3}\,m$$

$$\sim 8.6\,km.$$

Sea level : $h_{sl} = 0$ $\quad P_{sl} = 1.01 \times 10^5 \, Pa$ $\quad T_{sl} = 20^\circ C = 293 \, K$

top of Burnaby mountain : $h_B = 1200$ feet (from city of burnaby)

$= 367 \, m$

Peak of Whistler : $h_W = 2182 \, m$ (www. tourism whistler.com)

$$\rho_{sl} = \frac{N}{V} = \frac{P_{sl}}{RT} \qquad N = \frac{m}{M_g}$$

$$= \frac{(1.01 \times 10^5 \, Pa)}{(8.314 \, J/mol\cdot K)(293)}$$

$$= 41.46 \text{ moles}/m^3 \times (28.96 \times 10^{-3} \, kg/mol)$$

$$= 1.2 \, kg/mol$$

$$\rho_B = \rho_{sl} \exp\left(- \frac{g M_g h_B}{RT}\right)$$

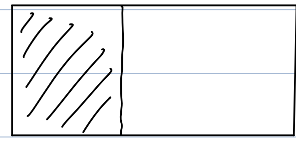
$$= 41.46 \exp\left(- \frac{(9.8)(28.96 \times 10^{-3})(367)}{(8.314)(293)}\right)$$

$$= 40 \text{ moles}/m^3$$

$$\rho_W = 32 \text{ moles}/m^3$$

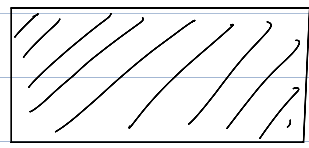
$$P_B = RT \rho_B$$
$$= 9.7 \times 10^4 \, Pa$$
$$= 0.96 \, atm$$

$$P_W = 7.8 \times 10^4 \, Pa$$
$$\sim 0.77 \, atm.$$

4. state a : 

$T_a = 300K \qquad P_a = 10^5 Pa$

$V_a = 1\times10^{-3} m^3$

State b : 

$T_b = ? \qquad P_b = ? \qquad V_b = 3\times10^{-3} m^2$

a) $\Delta U = 0$

Since $\Delta U = \frac{3}{2}NR\Delta T$

$\Delta T = 0$

$T_b = T_a = 300K$

b) $N = \text{Const}$ so

$$\frac{P_a V_a}{T_a} = \frac{P_b V_b}{T_b}$$

$$P_b = \frac{V_a}{V_b} P_a$$

$$= \frac{10^{-3}}{3\times10^{-3}} 10^6$$

$$= 3.3\times10^4 \ Pa$$

$$= 33 \ KPa$$

if you used $V_b = 2\times10^{-3} m^3$ you got $P_b = 50 KPa$.

5. System is an ideal gas
process is quasistatic $\Rightarrow$ quasistatic work.

Fig 3.3 : $W_{ab} = -\int_{V_a}^{V_b} P\,dV$ (isochoric Compression)

I don't know what P is but I know its Constant

$\Rightarrow$ $W_{ab} = -P(V_b - V_a)$

$= P(V_a - V_b)$

$= 300\times10^3\ Pa\ (2 - 0.5)\,m^3$

$= +450\ KJ$

Fig 3.4. : $W_{ab} = -\int_{V_a}^{V_b} P\,dV$

$P = mV + P_0$

$m : \dfrac{(500 - 300)\times10^3}{2 - 0.5} = 1.33\times10^5\ Pa/m^3$

$\dfrac{500\times10^3 - P_0}{2 - 0} = 1.33\times10^5$

$\Rightarrow$ $P_0 = 2.33\times10^5\ Pa$

$P = 1.33\times10^5 V + 2.33\times10^5$

$W_{ab} = -\int_{V_a}^{V_b}\left[1.33\times10^5 V + 2.33\times10^5\right] dV$

$= -\left[1.33\times10^5/2\,(V_b^2 - V_a^2) + 2.33\times10^5(V_b - V_a)\right]$

$= -\frac{1.33\times10^5}{2}(4 - 0.25) - 2.33\times10^5(2 - 0.5)$

$= -598\ KJ$

Fig. 3.5. : $V_a = 3\,m^3$ $\quad V_b = 1\,m^3$

$$W_{ab} = \text{Area under Curve}$$
$$= P\Delta V + \tfrac{1}{2}\pi r^2$$
$$= 2\times10^5(2) + \tfrac{1}{2}\pi\,(100\times10^3)(1)$$
$$= 5.57\times10^5\,J$$

Fig 3.6. $PV^2 = \text{Const} = K$

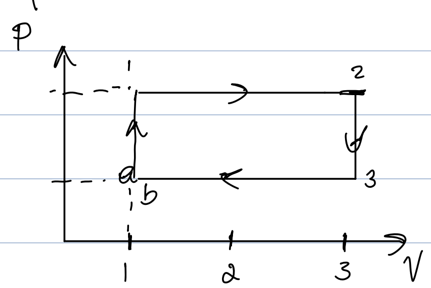
$$P = K/V^2$$
$$W_{ab} = -K\int_a^b \frac{1}{V^2}\,dV$$
$$= K\left(\frac{1}{V_b} - \frac{1}{V_a}\right)$$

$$P_aV_a^2 = K = (50\times10^3\,Pa)(2m^3)^2$$
$$= 200\times10^3\,Pa\,m^6$$

$$W_{ab} = 200\times10^3\,Pa\,m^6\left(\frac{1}{1m^3} - \frac{1}{2m^3}\right)$$
$$= +1\times10^5\,J$$

Fig 3.7 $W_{ab} = -$ area of rectangle

best way to Calculate this.



$$W_{ab} = W_{a1} + W_{12} + W_{23} + W_{3b}$$

$$W_{a1} = -\int_a P\,dV \qquad dV = 0$$

$$W_{12} = -\int_1^2 P\,dV = -P\Delta V$$

$$= -(400\times 10^3)(3-1)$$

$$= -800\times 10^3 \text{ J}$$

$$W_{23} = 0$$

$$W_{3b} = -P\Delta V$$

$$= -(200\times 10^3)(1-3)$$

$$= 400\times 10^3 \text{ J}$$

$$W_{ab} = (-800 + 400)\times 10^3$$

$$= -400\times 10^3 \text{ J}$$

Fig 3.8 : $W_{ab} = -\int_a^b P\,dV$

$$P = \frac{NRT}{V}$$

$$T = mV + T_0$$

$$m = \frac{300-460}{1-0.2} = -200$$

$$\frac{300 - T_0}{1-0} = -200 \Rightarrow T_0 = 500$$

$$T = -200V + 500$$

$$P = NR\left[-200 + \frac{500}{V}\right]$$

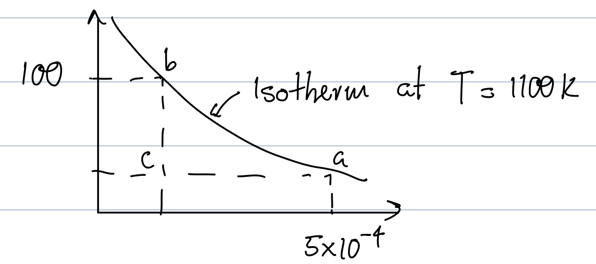
$$W_{ab} = -NR\int_{V_a}^{V_b}\left(-200 + \frac{500}{V}\right)dV$$

$N = 1$

$$= -8.314\left[-200(0.2-1) + 500\ln\left(\frac{0.2}{1.0}\right)\right]$$

$= 5.36 \times 10^3 J$

6.



$T_c = 300 K$

$V_a = 5\times10^{-4} m^3$

$P_b = 100 kPa$

a) W_{ab}, W_{bc}, W_{ca} assuming quasistatic processes

$$W_{ab} = -\int_a^b P\,dV$$

$$= -\int_a^b \frac{NRT}{V}\,dV$$

$$= NRT \ln\left(\frac{V_a}{V_b}\right)$$

from problem 2.1.6 $M_{He} = 6mg$.

$$N = 6\times10^{-3} g / 4 g/mol$$
$$= 1.5\times10^{-3} mol$$

$$V_b = \frac{NRT}{P_b} = \frac{1.5\times10^{-3} \times 8.314 \times 1100}{10^5}$$
$$= 1.37\times10^{-4} m^3$$

$$W_{ab} = 1.5\times10^{-3}(8.314)(1100)\ln\left(\frac{5\times10^{-4}}{1.37\times10^{-4}}\right)$$
$$\approx 17.76 \text{ J}$$

$W_{bc} = 0$ since $dV = 0$

$$P_a = \frac{NRT}{V_a} = 2.74\times10^{4}\ \text{Pa}$$

$$W_{ca} = -\int_c^a P\,dV$$
$$= -2.74\times10^{4}(5\times10^{-4} - 1.37\times10^{-4})$$
$$= -9.95 \text{ J}$$

b) $U_{ab} = U_b - U_a$
$$= \tfrac{3}{2}NR\Delta T$$
$$= 0$$

$$U_{bc} = \tfrac{3}{2}NR(T_c - T_b)$$
$$= \tfrac{3}{2}(P_cV_c - P_bV_b)$$
$$= -\tfrac{3}{2}(2.74\times10^{4}\times1.37\times10^{-4} - 10^{5}\times1.37\times10^{-4})$$
$$= -14.9 \text{ J}$$

$$U_{ca} = \tfrac{3}{2}P_a(V_a - V_c)$$
$$= 14.9 \text{ J}$$

ab : $W_{ab} > U_{ab}$

bc : $W_{bc} > U_{bc}$

ca : $W_{ca} < U_{ca}$ .. Not possible, ∴ there must be some other energy exchange process.

Total work input $W_{ab} + W_{bc} + W_{ca} = 17.76 - 9.95$

$= 7.81\,J$

Total energy change $U_{ab} + U_{bc} + U_{ca} = 0$