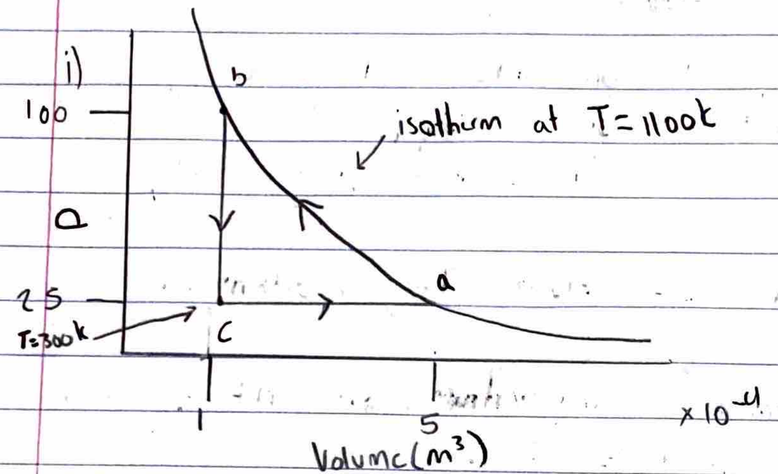
1.)

(a+b) The pressure in all three cylinders are the same. We don't know the mass of each gas in each cylinder, and at equilibrium the pressure in each cylinder is singularly dependant on the atmospheric pressure outside of the "frictionless piston".

344 Q2 2.1.6 $a \to b$ given in (f(2.2))

i)



* amount of helium is constant

$V_a = 0.5 \times 10^{-3}\ m^3$
$P_b = 1.0 \times 10^5$ Pa
$R = 8.31446262\ m^2 kg s^{-2} K^{-1} mol^{-1}$

if molar mass of He ≈ 4g/mol
then
$m = 4 \cdot 0.0015 \approx 6.1$ mg
$a \to b$: shrinks volume P↑ T stays same
$b \to c$: T↓ thus P↓ V stays same
$c \to a$: T↑ V↑ keeps same P

What is the mass of helium ?

what we know mass is constant
$V_a = 0.5 \times 10^{-3}\ m^3$ $P_b = 1.0 \times 10^5$ Pa
at C T = 300k $P_c = 50$ kPA $P_a = 50$ kPA
Points A and B are isothermal meaning at both Points T=1100 k , n is constant

So at Point a $PV = nRT \Rightarrow n_1 = \frac{PV}{RT} = \frac{P(0.5 \times 10^{-3} m^3)}{R\ 1100k}$
at Point b $n_2 = \frac{1.0 \times 10^5 Pa (V_b)}{R\ 1100k}$

because $n_1 = n_2$ $P^a (0.5 \times 10^{-3} m^3) = 1.0 \times 10^5 Pa (V_b) \Rightarrow P^a = \frac{1.0 \times 10^5 P_a (V_b)}{0.5 \times 10^{-3} m^3}$

at Point C the Volume is equivalent to point A and the pressure is equivalent to point b

$n_3 = \frac{P^a \cdot V_b}{R\ 300}$ where $P^a = \frac{1.0 \times 10^5 P_a (V_B)}{0.5 \times 10^{-3} m^3}$

$n_3 = \left(\frac{1.0 \times 10^5 P_a (V_B)^2}{0.5 \times 10^{-3} m} \right) / \cancel{R}\ 300k = \frac{1.0 \times 10^5 P_a (V_b)}{\cancel{R}\ 1100k}$

$\frac{V_B^2}{300(0.5 \times 10^{-3})} = \frac{V_B}{1000} \Rightarrow V_B = \frac{300(0.5 \times 10^{-3} m^3)}{1000} = 1.4 \times 10^{-4}\ m^3$

Then $n = \frac{1.0 \times 10^5 Pa (1.4 \times 10^{-4}) m^3}{R\ 1100} \approx 0.0015$ mol

2.1.8 Parts a → d

3)

a) Suppose the atmosphere has a mixture of ideal gasses having molar mass $M_g\,kg\,mol^{-1}$

Show that

$$\frac{dP}{dh} = -\rho g$$ -g = accelleration of gravity ρ = density

* the negative sign is arbitrary it just assigns a direction

if we know that $P(h) = \rho g h$ then

$$P'(h) = \frac{dP}{dh} = \lim_{\Delta h \to 0} \frac{P(h+\Delta h) - P(h)}{\Delta h}$$

$$\lim_{\Delta \to 0} = \frac{\rho g(h + \Delta h - h)}{\Delta h} = \frac{\rho g \cancel{\Delta h}}{\cancel{\Delta h}} = \rho g$$

b) Use ideal gas equation to show $\left[\frac{d\rho}{\partial P}\right]_T = \frac{M_g}{RT}$

$PV = nRT$ $n = \frac{m}{M_g}$ $\rho = \frac{m}{V}$

$PV = \frac{m}{M_g} RT \Rightarrow \frac{M_g}{RT} = \frac{m}{PV} \Rightarrow P\frac{M_g}{RT} = \rho$ Thus if it is isothermal

$$\frac{\partial \rho}{\partial P} = \frac{M_g}{RT}$$

c) Using $\frac{\partial P}{\partial h} = -\rho g$ and $\frac{\partial \rho}{\partial P} = \frac{M_g}{RT}$

$$\rho(h) = \frac{M_g}{RT} P(h) \Rightarrow \frac{dP}{\partial h} = -\frac{M_g}{RT} P g \Rightarrow \int \frac{dP}{P} = -\int \frac{M_g}{RT} g \, dh$$

$$\ln(P) = \frac{-M_g g}{RT} h + C$$

$$P = \exp\left[\frac{-M_g g}{RT} h + C\right] = e^{C} e^{\frac{-M_g}{RT} gh}$$ when $h = 0$ $P = \exp(C) = P_0$

$$P = P_0 e^{\left[\frac{-M_g}{RT} gh\right]}$$

d) $h_o = RT/(M_g g)$ Show that: $h_o = 8.6$ km for earths atmosphere. $T = 293$ k $M_g = 28.96 \times 10^{-3}$ kg

$$h_o = 8.31(293k)/(28.96 \times 10^{-3})(9.81) \approx 8.6 \text{ km}$$

Extra: Assumming $P = 1$ atm $T = 293$ k what is the Pressure at each mountain we can use

$$P = P_o \exp\left[-M_g g h/RT\right]$$ when

$M_g = 28.96 \times 10^{-3}$ kg $\cdot$ mol^{-1}
$g = 9.81$ m/s^2
$R = 8.31 \times 10$ m $\cdot$ m $\cdot$ k^{-1} $\cdot$ mol^{-1}
$T = 293$ K
1 atm $= 101325$ N/m^2

at sea level the entire weight of atmosphere is pressing down on the point $h_o = 8.6$ km $= 8600$ m

$P = 1\text{atm} = \rho g h_o$ $\rho = 1\text{atm}/8600\text{m}(9.81\text{m/s}^2)$

$$\frac{101325 \frac{N}{m^2}}{8600 \, m^2 s^2} = \rho = 1.2 \frac{kg}{m^3}$$

	ρ	P
Sea	1.2 kg/m^3	1 atm
Burnaby	1.149 kg/m^3	92803.7 N/m^2 = 0.916 atm
Whistler	0.931 kg/m^3	0.57 atm *

* obviously the farther h↑ T↓ but we are only considering $T_1 = T_2$

however in the equation

$$P = P_o \exp\left[-h/h_o\right]$$ $P_o = \rho = 1.2$ kg m/s

Then the altitude of SFU and Whistler are given by $h_{SFU} = 370$ m and $h_{whis} = 2182$ metres

$\rho_{SFU} = 1.2 \text{kg/m}^3 \exp\left[-370/h_o\right] = 1.149$ kg/m^3 $P = \rho g(8600-370) = 92803.7 \frac{N}{m^2}$

$\rho_{whistler} = 1.2 \text{ kg/m}^3 \exp\left[2182/h_o\right] = 0.931$ kg/m^3. $P = \rho g(8600-2182)/101325 = 0.57$ atm

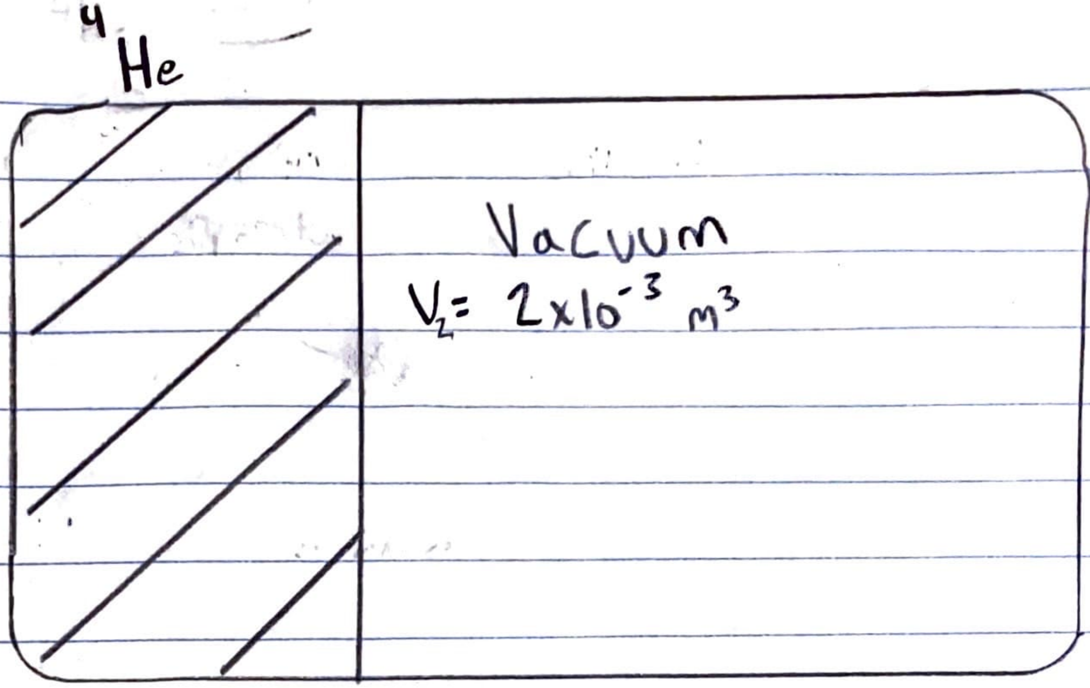
4) 2.3.3

a)

$T = 300\ K$

$V_1 = 1 \times 10^{-3}\ m^3$

$P = 10^5\ Pa$



$PV = nRT$

$n = P_1 V_1 / R T_1$

$n = 10^5 (1 \times 10^{-3}) / (8.31)(300)$

$n =$

Then $V = V_1 + V_2$ because energy is constant $U_1 = U_2$

$U_1 = n_i R T = U_2$ Thus $T = 300\ K$

b) now $PV = nRT \Rightarrow P = \frac{[10^5 (1 \times 10^{-3}) / (8.31 \cdot 300)] R [300]}{V}$

$P = 33\ kPa$