3.3: Longitudinal Waves

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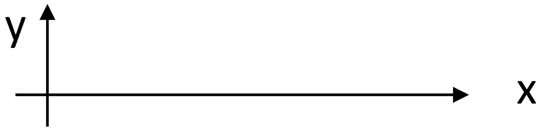
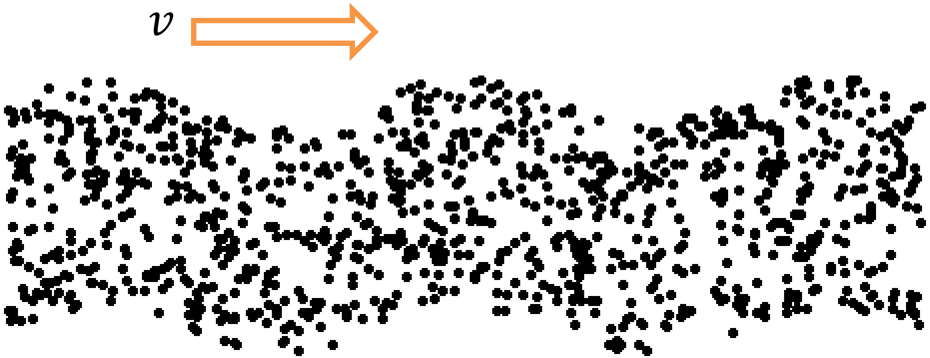
Phys 365/369

Transverse Waves



Particles in the medium move **perpendicular (transverse)** to the motion of the wave itself.



$$y(x,t) = A\cos(kx - \omega t + \phi_0)$$

e.g. wave on a string

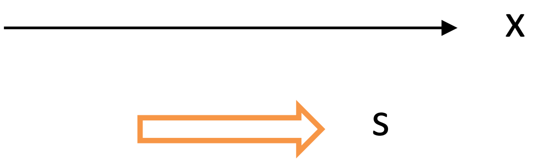
Animation: Dr. Dan Russell, Grad. Prog. Acoustics, Penn State: http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

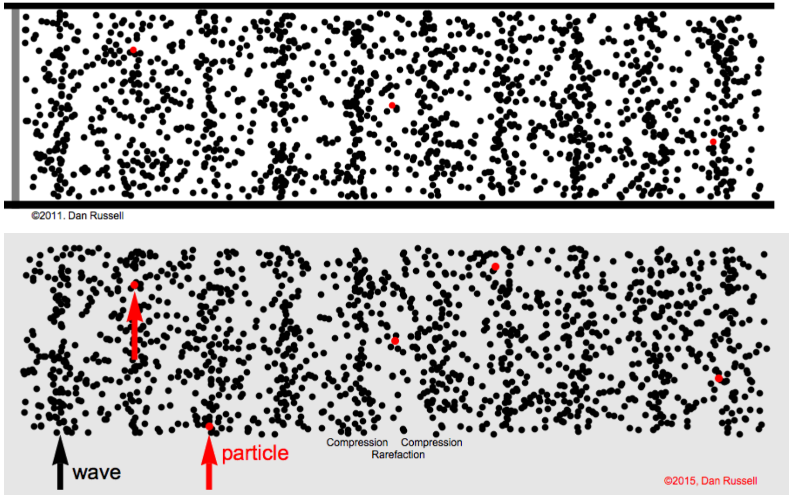
Longitudinal waves

Particles in the medium move forward and back, **parallel** to the motion of the wave itself.

$$s(x,t) = A\cos(kx - \omega t + \phi_0)$$

Displacement in x
(sometimes written Δx)





Sound waves are longitudinal waves.

Longitudinal waves are also largely responsible for the transmission of forces through solid objects.
e.g.
https://www.youtube.com/watch?v=00I2uXDxbaE

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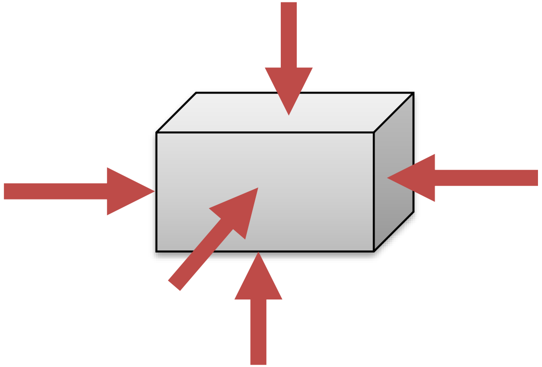
Wave speeds for longitudinal waves

- Mechanical waves are caused by a restoring force
- On a string, tension is the restoring force
- In longitudinal waves, the medium provides the restoring force through its resistance to compression

Bulk Elastic Properties

Bulk Modulus



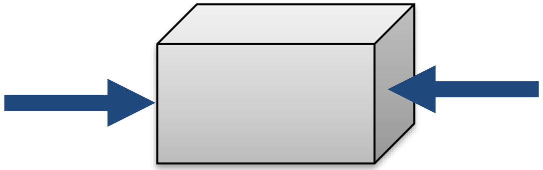
Measure of resistance to uniform compression (~1/"how squishy is it?")

$$B \equiv \frac{-\Delta P}{\Delta V/V}$$

$$= \frac{\text{pressure change}}{\text{fractional volume change}}$$

Units: N/m^2 = Pa

Young's Modulus



Measure of resistance to axial compression (~1/"how stretchy is it?")

$$Y \equiv \frac{F/A}{\Delta l/l}$$

F=force

A=X-sectional area

$$= \frac{\text{tensile stress}}{\text{tensile strain}}$$

Units: N/m^2 = Pa

Speed of sound in a fluid

- In a sound wave, the restoring force is <u>pressure</u>
- The speed of sound in a fluid is

$$v = \sqrt{\frac{B}{\rho}}$$

B=bulk modulus (N/m^2)

ρ =Mass density (kg/m^3)

Compare to $v = \sqrt{F/\mu}$ for transverse wave on a string.

Speed of sound in an ideal gas

Using the ideal gas law $PV = nRT$

and the adiabatic equation $PV^{\gamma} = \text{constant}$

(for thermodynamic processes not involving heat exchange):

$$v = \sqrt{\frac{\gamma RT}{M}}$$

γ =5/3 for monatomic gases (He, Ar, etc)
γ =7/5 for diatomic gases (H_2, O_2, N_2, etc.)
R=ideal gas constant=8.314 J/mol.K
T=temp (K)
M=mean molecular mass

Example

Speed of sound in air (80% N_2, 20% O_2) at 20°C:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Atomic mass of N=14.0 u

Atomic mass of O=16.0 u

M=0.2(2*16.0g/mol)+0.8(2*14.0g/mol)

= 28.8 g/mol

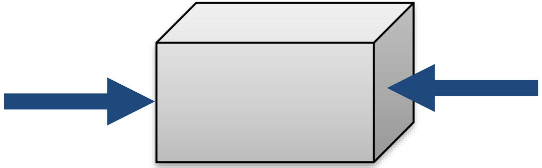
γ=7/5 (diatomic)

T=273+20=293 K

→v=344 m/s

Longitudinal waves in a solid rod

- Consider a long rod struck at one end, causing a longitudinal wave to pass down the rod
- Restoring force depends on Young's modulus *Y* (resistance to axial compression)



$$v = \sqrt{\frac{Y}{\rho}}$$

Y=Young's modulus (N/m^2)

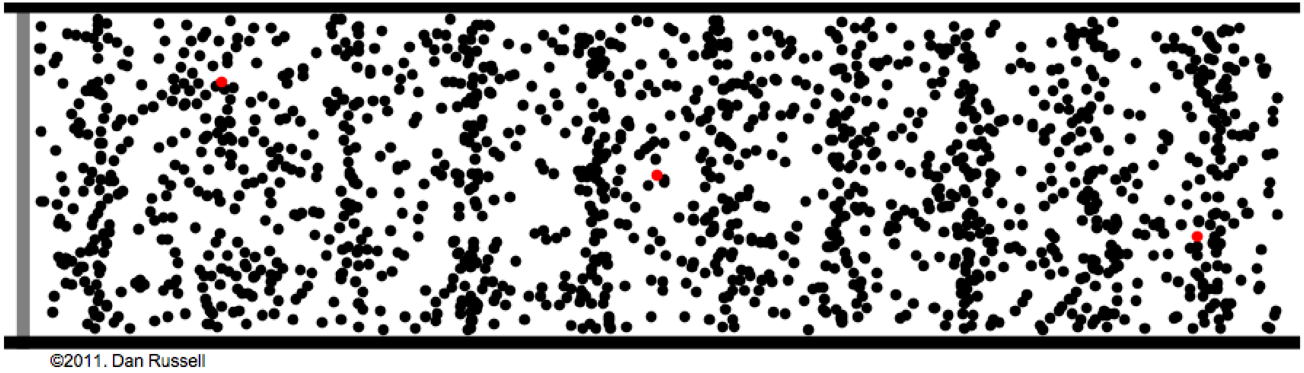
ρ =Mass density (kg/m^3)

Compare to

$v = \sqrt{F/\mu}$ (string)

or $v = \sqrt{B/\rho}$ (sound)

Pressure vs. amplitude displacement



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- Particle <u>displacement</u> is sinusoidal and parallel to the wave motion (longitudinal wave)
- Where the particles bunch together, the density and pressure are greater

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- Pressure change is related to the compression via the bulk modulus:

$$\Delta P = -B\frac{\Delta V}{V}$$

- For a plane wave travelling in x:

$$\Delta p(x,t) = -B\frac{\partial s}{\partial x}$$

- Using $s(x,t) = A\cos(kx - \omega t)$ gives:

$$\boxed{\Delta p(x,t) = BkA\sin(kx - \omega t)}$$

B=Bulk modulus
A=displacement amplitude
k=wave number

$$\Delta p(x,t) = BkA\sin(kx - \omega t)$$

- Larger bulk modulus (less squishy) → larger pressure
- Shorter wave → larger pressure
- Pressure is 90 degrees out of phase with amplitude
  - Max./min. pressure is where $s = 0$
- Maximum pressure is

$$P_{max} = BkA$$