

1. (a)

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(\omega) = \int_0^{\infty} A e^{-j\omega t} dt$$

$$A \left[\frac{e^{-j\omega t}}{-j\omega} \right]_0^{\infty}$$

$$X(\omega) = \frac{A e^{-j\omega \infty}}{-j\omega} + \frac{A}{j\omega}$$

$$= \frac{A \cos(\omega \infty) + j A \sin(\omega \infty)}{-j\omega} + \frac{A}{j\omega}$$

$$X(\omega) = \frac{A \cos(\omega \infty)}{-j\omega} + A \infty \operatorname{sinc}(\omega \infty) + \frac{A}{j\omega}$$

2. (a)

$$x(t) = A \Delta\left(\frac{t}{\tau}\right)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \begin{cases} \frac{A}{\tau}(t+\tau) & -\tau \leq t \leq 0 \\ A\left(\frac{-t}{\tau} + 1\right) & 0 \leq t \leq \tau \end{cases}$$

$$X(\omega) = \int_{-\tau}^0 \frac{A}{\tau}(t+\tau) e^{-j\omega t} dt + \int_0^{\tau} A\left(\frac{-t}{\tau} + 1\right) e^{-j\omega t} dt$$

$$v = \left(1 + \frac{t}{\tau}\right) \frac{dv}{dt} = \frac{1}{\tau}$$

$$dv = e^{-j\omega t} \quad v = \frac{j}{\omega} e^{-j\omega t}$$

$$\int_0^{\tau} A\left(1 - \frac{t}{\tau}\right) e^{-j\omega t} dt$$

$$u = 1 - \frac{t}{\tau} \quad \frac{du}{dt} = -\frac{1}{\tau}$$

$$dv = e^{-j\omega t} \rightarrow v = \frac{j}{\omega} e^{-j\omega t}$$

$$A \left[\frac{j(1+\frac{t}{\tau})}{\omega} e^{-j\omega t} \right]_{-\tau}^0 = A \frac{j}{\omega \tau} \int_{-\tau}^0 e^{-j\omega t} dt$$

$$= A \left[\frac{j}{\omega} - \frac{j}{\omega \tau} \cdot \frac{j}{\omega} (1 - e^{-j\omega \tau}) \right]$$

$$= A \left[\frac{j}{\omega} - \frac{j^2}{\omega^2 \tau} + \frac{j^2 e^{-j\omega \tau}}{\omega^2 \tau} \right]$$

$$A \left[\frac{j(1-\frac{t}{\tau})}{\omega} e^{-j\omega t} \right]_0^{\tau} + \frac{A j}{\omega} \int_0^{\tau} e^{-j\omega t} dt$$

$$= A \left[-\frac{j}{\omega} + \frac{j}{\omega \tau} \cdot \frac{j}{\omega} [e^{-j\omega \tau} - 1] \right]$$

$$= A \left[-\frac{j}{\omega} + \frac{j^2}{\omega^2 \tau} e^{-j\omega \tau} - \frac{j^2}{\omega^2 \tau} \right]$$

→ (+) ←

$$X(\omega) = A \left(\frac{2}{\omega^2 \tau} - \frac{e^{j\omega \tau} + e^{-j\omega \tau}}{\omega^2 \tau} \right)$$

$$= A \left[\frac{2 - 2 \cos(\omega \tau)}{\omega^2 \tau} \right]$$

$$= A \left[\frac{2 [1 - \cos(\omega \tau)]}{\omega^2 \tau} \right]$$

$$= A \left[\frac{4 \sin^2\left(\frac{\omega \tau}{2}\right)}{\omega^2 \tau} \right] \begin{pmatrix} \approx \\ \frac{\pi}{4} \end{pmatrix}$$

$$= A \tau \left[\frac{\sin\left(\frac{\omega \tau}{2}\right)}{\frac{\omega \tau}{2}} \right]^2$$

$$\boxed{X(\omega) = A \tau \operatorname{sinc}^2\left(\frac{\omega \tau}{2\pi}\right)}$$

(b).

$$x(t) = \begin{cases} \frac{A}{\tau}(t+\tau) & -\tau < t < 0 \\ A\left(-\frac{t}{\tau}+1\right) & 0 < t < \tau \end{cases}$$

$$x(t) = \begin{cases} \frac{At}{\tau} + A & -\tau < t < 0 \\ -\frac{At}{\tau} + A & 0 < t < \tau \end{cases}$$

$$\frac{dx(t)}{dt} = \begin{cases} \frac{A}{\tau} & -\tau < t < 0 \\ -\frac{A}{\tau} & 0 < t < \tau \end{cases} \rightarrow \text{these are two rect pulses}$$

$$\frac{dx(t)}{dt} = \frac{A}{\tau} \left[\pi\left(\frac{t+\tau}{\tau}\right) - \pi\left(\frac{t-\tau}{\tau}\right) \right]$$

$$\xrightarrow{\text{FT}} j\omega X(\omega) = A\tau \text{sinc}\left(\frac{\omega\tau}{2\pi}\right) e^{\frac{j\omega\tau}{2}} - A\tau \text{sinc}\left(\frac{\omega\tau}{2\pi}\right) e^{-\frac{j\omega\tau}{2}}$$

$$A\tau \text{sinc}\left(\frac{\omega\tau}{2\pi}\right) \left(e^{\frac{j\omega\tau}{2}} - e^{-\frac{j\omega\tau}{2}} \right)$$

$$j\omega X(\omega) = A\tau \text{sinc}\left(\frac{\omega\tau}{2\pi}\right) \left(2j \sin\left(\frac{\omega\tau}{2}\right) \right)$$

$$X(\omega) = \frac{2}{\omega} A\tau \text{sinc}\left(\frac{\omega\tau}{2\pi}\right) \sin\left(\frac{\omega\tau}{2}\right)$$

$$X(\omega) = A\tau \text{sinc}\left(\frac{\omega\tau}{2\pi}\right) \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\omega/2}$$

$$X(\omega) = A \kappa \operatorname{sinc}^2\left(\frac{\omega \kappa}{2\pi}\right)$$

$$3. \quad g_1(t) = \begin{cases} -4 \sin(1000\pi t) & 0 \leq t \leq 1\text{ms} \\ 0 & \text{otherwise} \end{cases}$$

$$-4 \sin(1000\pi t)$$

$$x(t) = \frac{-4}{2j} \left(e^{j2\pi 1000t} - e^{-j2\pi 1000t} \right)$$

$$x(t) = 2j e^{j2\pi 1000t} - 2j e^{-j2\pi 1000t}$$

FT
→

$$X(\omega) = (j2)2\pi \delta(\omega - 2000\pi) - (j2)2\pi \delta(\omega + 2000\pi)$$

$$X(\omega) = (4\pi j) \left[\delta(\omega - 2000\pi) - \delta(\omega + 2000\pi) \right]$$

4.

$$X(\omega) = \frac{6 + j2\omega}{8 - \omega^2 + j6\omega}$$

$$\begin{aligned} (9) \quad x(2t) &\xrightarrow{\text{FT}} \frac{1}{2} X\left(\frac{\omega}{2}\right) \\ &= \frac{1}{2} \left[\frac{6 + j2\left(\frac{\omega}{2}\right)}{8 - \left(\frac{\omega}{2}\right)^2 + j6\left(\frac{\omega}{2}\right)} \right] \\ &= \frac{1}{2} \left[\frac{6 + j\omega}{8 - \frac{\omega^2}{4} + 3j} \right] \end{aligned}$$

$$\begin{aligned} (b) \quad x(3t-6) &\xrightarrow{\text{FT}} \frac{1}{3} X\left(\frac{\omega}{3}\right) e^{-j\omega 6/3} \\ &= \frac{1}{3} \left[\frac{6 + j2\left(\frac{\omega}{3}\right)}{8 - \left(\frac{\omega}{3}\right)^2 + 3j} \right] e^{-j2\omega} \\ &= \frac{1}{3} \left[\frac{6 + \frac{2\omega j}{3}}{8 - \frac{\omega^2}{9} + 3j} \right] e^{-j2\omega} \end{aligned}$$

$$(c) \quad x(-t) \xrightarrow{FT} X(-\omega)$$

$$= \left[\frac{6 - j2\omega}{8 - \omega^2 - j6\omega} \right]$$

$$(d) \quad \int_{-\infty}^{\infty} x(\tau) d\tau \xrightarrow{FT} \frac{1}{j\omega} X(\omega)$$

$$\frac{1}{j\omega} \left[\frac{6 + j2\omega}{8 - \omega^2 + j6\omega} \right]$$

$$= \left[\frac{\frac{6}{j\omega} + 2}{8 - \omega^2 + j6\omega} \right]$$