

## 3.2: Velocity and acceleration in the complex representation

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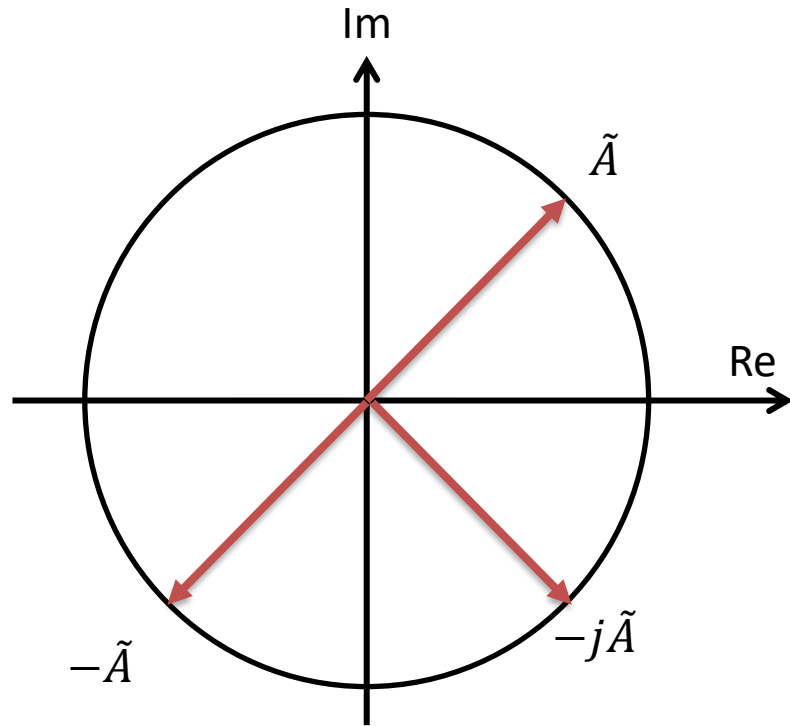
# Complex representation of velocity and acceleration

$$\tilde{y}(x, t) = \tilde{A}e^{j(kx - \omega t)}$$

$$v_y = \frac{\partial \tilde{y}(x, t)}{\partial t} = -j\omega \tilde{A}e^{j(kx - \omega t)}$$

$$a_y = \frac{\partial^2 \tilde{y}(x, t)}{\partial t^2} = -\omega^2 \tilde{A}e^{j(kx - \omega t)}$$

# Where did that $-j$ come from on $\tilde{A}$ ?



- Multiplication by  $-1$  results in a  $180^\circ$  rotation in the complex plane
- Multiplication by  $-j$  results in a  $-90^\circ$  rotation
- Differentiation leads to a  $-90^\circ$  rotation
  - $\cos() \rightarrow \sin()$