Lecture 2.6
Waves

University of Calgary

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https://www.guitarworld.com/features/orville-peck-i-learned-to-play-guitar-without-the-high-e-string-i-didnt-know-how-to-change-it-when-it-broke

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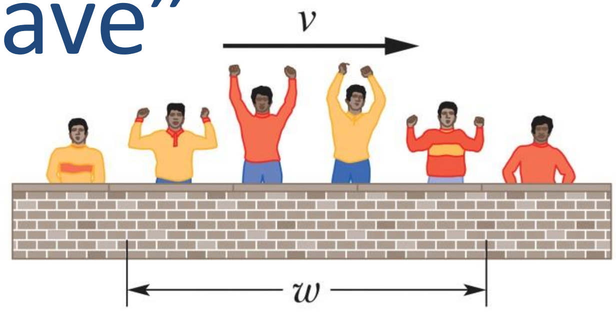
“The wave”



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http://sociologyinfocus.com/tag/sports/

Actually a pretty good way to think about wave motion

- Each person does the same as the person beside them, <u>but at a short time Δt later</u>

The “disturbance” travels at a speed v :

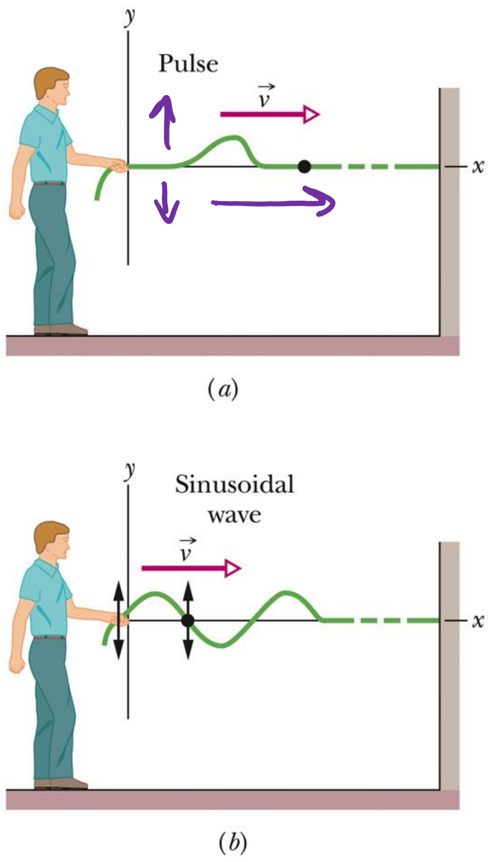
$$v = \frac{d}{\Delta t}$$

with Δt the time lag and d the distance from one seat to the next

Longitudinal and transverse waves

A wave on a string is an example of a **transverse wave**.

As the wave passes, each particle of the string moves up and down, **transversely** to the motion of the wave itself.



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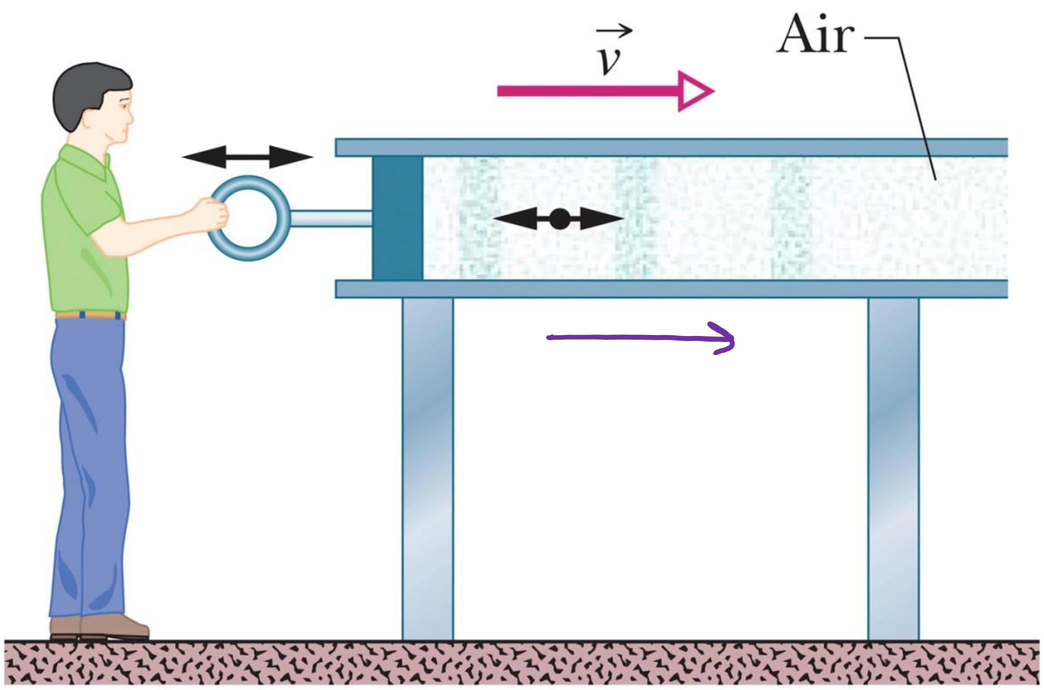
Longitudinal and transverse waves

A sound wave is an example of a **longitudinal wave**.

As the wave passes, each particle of air moves forward and back, **parallel** to the motion of the wave itself.



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Travelling waves

A fixed pattern ("wave function") moving at a velocity v ("phase velocity") is written like this:

any function (describes the shape of the wave)

$$y(x,t) = \mathrm{f}(\mathrm{x} \pm \underbrace{vt}_{v:\ \text{phase velocity}})$$

displacement of medium (height of string, movement of air, ...)

v: phase velocity

+: wave travels to left

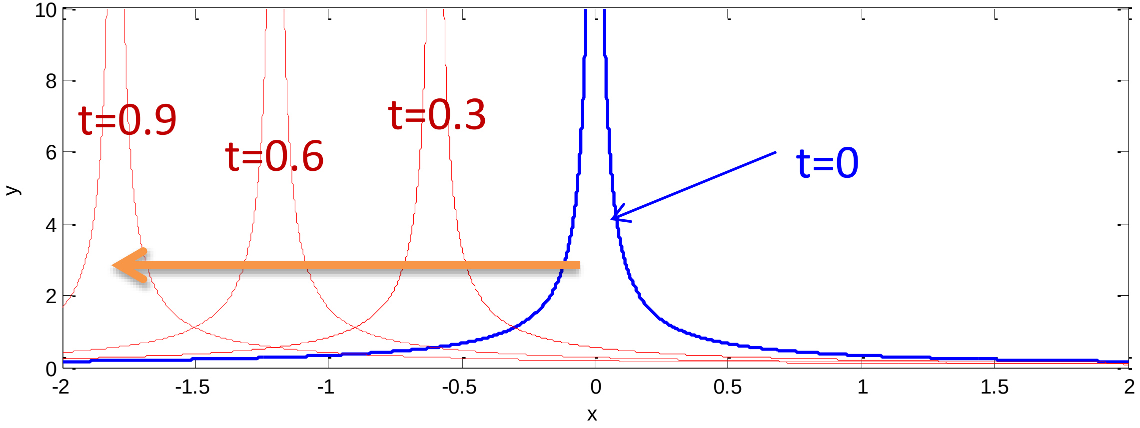
−: wave travels to right

Example

$$y(x,t) = \left|\frac{1}{-3x - 6t}\right|$$

$$y(x,t) = \left|\frac{1}{-3}\right|\left|\frac{1}{x+2t}\right| = \frac{1}{3}\left|\frac{1}{x+2t}\right| = f(x+2t)$$

$\Rightarrow$ wave moving left with speed 2 m/s



$$y(x,0) = \left|\frac{1}{3x}\right|$$

Test yourself

At what speed does the travelling wave $y = \sin(8x + 5t)$ move?

A) To the right at 5 m/s
B) To the right at 5/8 m/s.
C) To the right at 8/5 m/s
D) To the left at 5 m/s
E) To the left at 5/8 m/s
F) To the left at 8/5 m/s

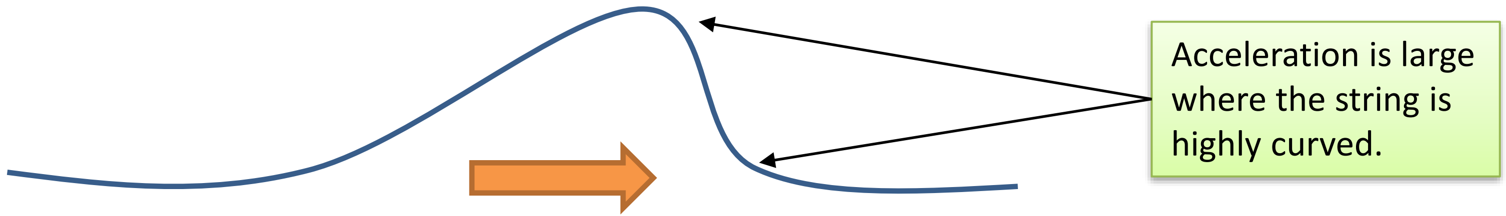
$\Rightarrow y = \sin\left(8\left(x + \frac{5}{8}t\right)\right)$

$= f\left(x + \frac{5}{8}t\right)$

v

The wave equation

Acceleration and spatial structure are linked.



Any wave function $y(x,t) = \mathrm{f}(\mathrm{x} \pm vt)$ is a solution of the **wave equation**:

acceleration

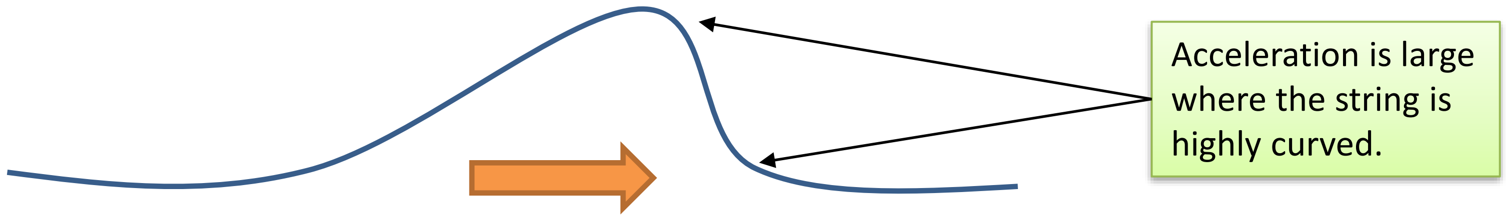
spatial structure

$$\boxed{\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}}$$

(Proof in slides)

Proof: Solutions of the wave equation

Acceleration and spatial structure are linked.



Any wave function $y(x,t) = \mathrm{f}(\mathrm{x} \pm vt)$ is a solution of the **wave equation**:

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We can verify this using the chain rule.

Trial solution is:

$$y(x,t) = \mathrm{f}(\mathrm{x} \pm vt)$$

Let $u = \mathrm{x} \pm vt$ and apply the chain rule:

$$\frac{\partial f(u)}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial t}$$

$$= \frac{\partial f}{\partial u}(\pm v) = \pm v\, f'(u)$$

$$\frac{\partial^2 f(u)}{\partial t^2} = \pm v \frac{\partial}{\partial t} f'(u)$$

$$= \pm v \frac{\partial f'}{\partial u}\frac{\partial u}{\partial t}$$

$$= (\pm v)^2 \frac{\partial^2 f}{\partial u^2} = v^2 f''(u)$$

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Proceeding similarly for the partial derivatives with respect to x:

$$\frac{\partial^2 f(u)}{\partial x^2} = f''(u)$$

So that:

$$f''(u) = \frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}$$

as required.