Physics 369: Acoustics, Optics and Radiation

Lecture 6:

Wave Power

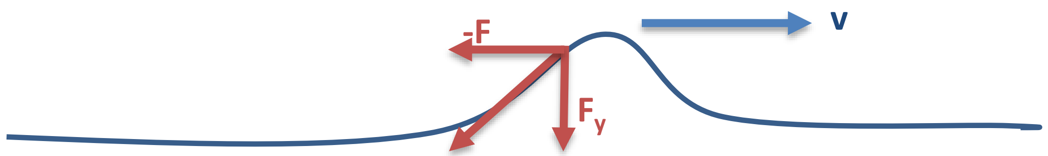
Waves transport energy.

The energy (power) provided by his arms is transported along the heavy rope.

Power transfer is a core concept in engineering, so let’s quantify this power transfer.

Wave Power (transverse wave)

Power = Force x Velocity (Watts)



Power carried by a transverse wave on a piece of string is:

$$P = F_y v_y$$

$$F_y = -F \cdot slope = -F \frac{\partial y}{\partial x}$$

$$v_y = \frac{\partial y}{\partial t}$$

$$P(x,t) = -F \frac{\partial y}{\partial x} \frac{\partial y}{\partial t}$$

Power at a given point and time

For a sinusoidal wave:

$$y(x,t) = A\cos(kx - \omega t + \phi_0)$$

the power $P(x,t) = -F\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$ becomes

$$P(x,t) = -\mathrm{F}(-\mathrm{A}k)(\mathrm{A}\omega)\sin^2(kx - \omega t + \phi_0)$$

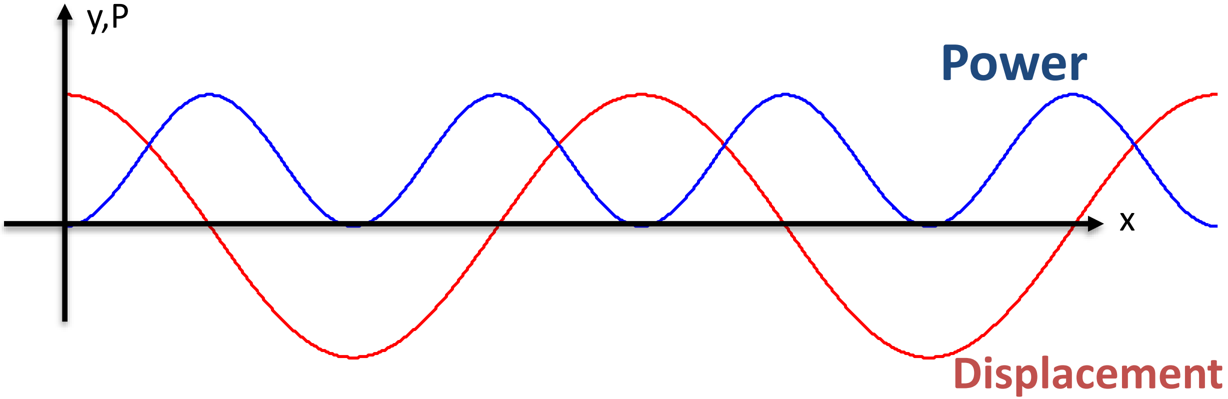
Substituting $\mathrm{k} = \omega/v$:

$$P(x,t) = FA^2\frac{\omega}{v}\omega\sin^2(kx - \omega t + \phi_0)$$

Using $v = \sqrt{F/\mu}$:

$$\boxed{P(x,t) = \sqrt{\mu F}A^2\omega^2\sin^2(kx - \omega t + \phi_0)}$$

μ=mass per unit length (kg/m)
F=tension on string (N)
A=amplitude (m)



Maximum Power: $\mathrm{P_{max}} = \sqrt{\mu F} A^2 \omega^2$

Average Power: $\mathrm{P_{av}} = \sqrt{\mu F} A^2 \omega^2 / 2$

– Reason: average of sin²() over one period is 1/2

Proof: $\sin^2\theta = \frac{1}{2} - \cancel{\frac{\cos(2\theta)}{2}}$

Avg. = 0 over one period

Example

The guy in this YouTube video creates 40 wave peaks in 20 seconds (per hand). The wavelength of the resulting wave is about 2 meters and the amplitude is about 40 cm. Assuming the rope has a mass density of 1.5 kg/m (typical for this sort of rope), calculate his average power output.

http://www.youtube.com/watch?v=DwGbg4P0M3k

$$v^2 = \frac{F}{\mu} \qquad v = f\lambda$$

$$\mathrm{P_{av}} = \sqrt{\mu F}\,A^2\omega^2/2$$

Goal: Find power $\mathrm{P}_{\mathrm{av}} = \sqrt{\mu F} A^2 \omega^2 / 2$

Given: 40 cycles per 20 seconds, A=40 cm, μ=1.5 kg/m, λ=2 m

First, find the angular frequency ω

$$f = \frac{40\ cycles}{20\ s} = 2\ Hz$$
$$\omega = 2\pi f = 4\pi\ Hz$$

Next, need tension F.

$$F = v^2 \mu$$

But the wave speed can be written in terms of the frequency and wavelength: $v = f\lambda$. So:

$$F = (f\lambda)^2 \mu = (2\ s^{-1} \cdot 2m)^2 \cdot 1.5\ kg/m$$
$$\mathrm{F} = 24\ N$$

So:

$$\mathrm{P_{av}} = 0.5\sqrt{\mu F}A^2\omega^2$$

$$= 0.5\sqrt{24\ N \cdot 1.5kg/m}(0.4m)^2(4\pi\ s^{-1})^2 = 75.8\ \mathrm{W}$$

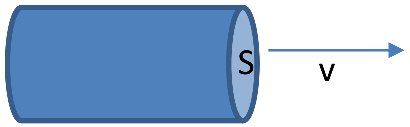
This is per arm, so the total power he is providing to the ropes is about **152 Watts**.

For comparison, 152 Watts on a stationary bike is moderately hard for most people. But that's using legs and not arms.

Power in a sound wave

- Intensity I is average power transported by a wave <u>per unit area</u>:

$$I = P_{av}/S$$



- More useful than the average power when talking about sound waves
- Intensity for a sound wave:

$$\mathrm{I} = \sqrt{\rho B}\ A^2\omega^2/2$$

- Relation to pressure amplitude:

$$I = \frac{P_{max}^2}{2\rho v} = \frac{P_{max}^2}{2\sqrt{\rho B}}$$