1. (a)

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

$$X(\omega) = \int_{0}^{\tau} A e^{-j\omega t} \, dt$$

$$A \left[\frac{e^{-j\omega t}}{-j\omega} \Big|_{0}^{\tau} \right]$$

$$X(\omega) = \frac{A e^{-j\omega \tau}}{-j\omega} + \frac{A}{j\omega}$$

$$= \frac{A\cos(\omega\tau)}{-j\omega} + \frac{\cancel{j} A\sin(\omega\tau)}{\cancel{j}\omega} + Aj\omega$$

$$\boxed{X(\omega) = \frac{A\cos(\omega\tau)}{-j\omega} + A\tau\,\mathrm{sinc}(\omega\tau) + Aj\omega}$$

(b)

$$A\,\Pi\left(\frac{t}{\tau}\right) \xrightarrow{FT} A\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2\pi}\right)$$

2. (a)

$$x(t) = A\,\Lambda\left(\frac{t}{\tau}\right)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \begin{cases} \frac{A}{\tau}(t+\tau) & -\tau < t < 0 \\ A\left(\frac{-t}{\tau}+1\right) & 0 < t < \tau \end{cases}$$

$$X(\omega) = \int_{-\tau}^{0} \frac{A}{\tau}(t+\tau)\, e^{-j\omega t}\, dt + \int_{0}^{\tau} -A\left(\frac{t}{\tau}-1\right) e^{-j\omega t}$$

$$u = \left(1+\frac{t}{\tau}\right) \quad \frac{du}{dt} = \frac{1}{\tau}$$

$$dv = e^{-j\omega t} \quad v = \frac{j}{\omega} e^{-j\omega t}$$

$$A\left[\frac{j(1+\frac{t}{\tau})}{\omega} e^{-j\omega t}\right]_{-\tau}^{0} - A\frac{j}{\omega\tau}\int_{-\tau}^{0} e^{-j\omega t}\, dt$$

$$= A\left[\frac{j}{\omega} - \frac{j}{\omega\tau}\cdot\frac{j}{\omega}(1-e^{j\omega\tau})\right]$$

$$= A\left[\frac{j}{\omega} - \frac{j^2}{\omega^2\tau} + \frac{j^2 e^{j\omega\tau}}{\omega^2\tau}\right]$$

$$\int_{0}^{\tau} A\left(1-\frac{t}{\tau}\right) e^{-j\omega t}$$

$$u = 1-\frac{t}{\tau} \quad \frac{du}{dt} = \frac{-1}{\tau}$$

$$dv = e^{-j\omega t} \rightarrow v = \frac{j}{\omega} e^{-j\omega t}$$

$$A\left[\frac{j(1-\frac{t}{\tau})e^{-j\omega t}}{\omega}\right]_{0}^{\tau} + \frac{Aj}{\omega}\int_{0}^{\tau} e^{-j\omega t}\, dt$$

$$= A\left[-\frac{j}{\omega} + \frac{j}{\omega\tau}\cdot\frac{j}{\omega}\left[e^{-j\omega\tau}-1\right]\right]$$

$$= A\left[-\frac{j}{\omega} + \frac{j^2}{\omega^2\tau}e^{-j\omega\tau} - \frac{j^2}{\omega^2\tau}\right]$$

$(+)$

$$X(\omega) = A\left(\frac{2}{\omega^2\tau} - \frac{e^{j\omega\tau}+e^{-j\omega\tau}}{\omega^2\tau}\right]$$

$$= A\left[\frac{2 - 2\cos(\omega\tau)}{\omega^2\tau}\right]$$

$$= A\left[\frac{2[1 - \cos(\omega\tau)]}{\omega^2\tau}\right]$$

$$= A\left[\frac{4\sin^2\left(\frac{\omega\tau}{2}\right)}{\omega^2\tau}\right]\frac{\left(\frac{\tau}{4}\right)}{\left(\frac{\tau}{4}\right)}$$

$$= A\tau\left[\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\frac{\omega\tau}{2}}\right]^2$$

$$\boxed{X(\omega) = A\tau\,\mathrm{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)}$$

(b).

$$x(t) = \begin{cases} \frac{A}{\tau}(t+\tau) & -\tau < t < 0 \\ A\left(-\frac{t}{\tau}+1\right) & 0 < t < \tau \end{cases}$$

$$x(t) = \begin{cases} \frac{At}{\tau} + A & -\tau < t < 0 \\ -\frac{At}{\tau} + A & 0 < t < \tau \end{cases}$$

$$\frac{dx(t)}{dt} = \begin{cases} \frac{A}{\tau} & -\tau < t < 0 \\ -\frac{A}{\tau} & 0 < t < \tau \end{cases}$$ $\rightarrow$ these are two rect pulses

$$\frac{dx(t)}{dt} = \frac{A}{\tau}\left[\Pi\left(\frac{t+\tau/2}{\tau}\right) - \Pi\left(\frac{t-\tau/2}{\tau}\right)\right]$$

$$\Pi\left(\frac{t}{\tau}+\frac{1}{2}\right)$$

$\xrightarrow{FT}$ $$j\omega X(\omega) = A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right) e^{\frac{j\omega\tau}{2}} - A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right) e^{-\frac{j\omega\tau}{2}}$$

$$A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)\left(e^{\frac{j\omega}{2}\tau} - e^{-\frac{j\omega}{2}\tau}\right)$$

$$j\omega X(\omega) = A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)\left(2j\sin\left(\frac{\omega\tau}{2}\right)\right)$$

$$X(\omega) = \frac{2}{\omega} A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)\sin\left(\frac{\omega\tau}{2}\right)$$

$$X(\omega) = A\tau \operatorname{sinc}\left(\frac{\omega\tau}{2\pi}\right)\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\omega/2}$$

$$\boxed{X(\omega) = A\tau \operatorname{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)}$$

3. $$g_1(t) = \begin{cases} -4\sin(1000\pi t) & 0 \le t \le 1ms \\ 0 & \text{otherwise} \end{cases}$$

$$-4\sin(1000\pi t)$$

$$x(t) = \frac{-4}{2j}\left(e^{j2\pi 1000t} - e^{-j2\pi 1000t}\right)$$

$$x(t) = 2je^{j2\pi 1000t} - 2je^{-j2\pi 1000t}$$

$\xrightarrow{FT}$

$$X(\omega) = (j2)2\pi\delta(\omega - 2000\pi) - (j2)2\pi\delta(\omega + 2000\pi)$$

$$\boxed{X(\omega) = (4\pi j)\left[\delta(\omega - 2000\pi) - \delta(\omega + 2000\pi)\right]}$$

4.

$$X(\omega) = \frac{6 + j2\omega}{8 - \omega^2 + j6\omega}$$

(a) $x(2t) \xrightarrow{FT} \frac{1}{2} X\left(\frac{\omega}{2}\right)$

$$= \frac{1}{2}\left[\frac{6 + j2\left(\frac{\omega}{2}\right)}{8 - \left(\frac{\omega}{2}\right)^2 + j6\left(\frac{\omega}{2}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{6 + j\omega}{8 - \frac{\omega^2}{4} + 3j}\right]$$

(b) $x(3t-6) \xleftrightarrow{FT} \frac{1}{3} X\left(\frac{\omega}{3}\right) e^{-\frac{j\omega 6}{3}}$

$$= \frac{1}{3}\left[\frac{6 + j2\left(\frac{\omega}{3}\right)}{8 - \left(\frac{\omega}{3}\right)^2 + 3j}\right] e^{-j2\omega}$$

$$= \frac{1}{3}\left[\frac{6 + \frac{2}{3}\omega j}{8 - \frac{\omega^2}{9} + 3j}\right] e^{-j2\omega}$$

(c) $x(-t) \xrightarrow{FT} X(-\omega)$

$$= \left[\frac{6 - j2\omega}{8 - \omega^2 - j6\omega}\right]$$

(d) $\int_{-\infty}^{t} x(\tau)\, d\tau \xrightarrow{FT} \frac{1}{j\omega} X(\omega)$

$$\frac{1}{j\omega}\left[\frac{6 + j2\omega}{8 - \omega^2 + j6\omega}\right]$$

$$= \left[\frac{2\omega - 6j}{\omega\left(8 - \omega^2 + j6\omega\right)}\right]$$