

Lecture 2.1:

Velocity and Acceleration in SHM

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Velocity and acceleration in SHM

- Start with $x(t)$:

$$x(t) = A \cos(\omega t + \phi_0)$$

- Differentiate once to get $v(t)$:

$$\begin{aligned} v(t) &= \frac{dx}{dt} = A[-\omega \sin(\omega t + \phi_0)] \\ &= -A\omega \sin(\omega t + \phi_0) \end{aligned}$$

- Maximum speed is:

$$v_{max} = A\omega$$

- Differentiate again to get $a(t)$:

$$\begin{aligned}a(t) &= \frac{d^2x}{dt^2} = \frac{dv}{dt} \\&= -A\omega[\cos(\omega t + \phi_0)\omega] \\&= -A\omega^2 \cos(\omega t + \phi_0)\end{aligned}$$

- Maximum acceleration is:

$$a_{max} = A\omega^2$$

$$x(t) = A \cos(\omega t + \phi_0)$$

$$v(t) = -A\omega \sin(\omega t + \phi_0)$$

$$a(t) = -A\omega^2 \cos(\omega t + \phi_0)$$

At the equilibrium point ($x = 0$) :

- $\cos(\omega t + \phi_0) = 0$
- Maximum speed $v = \pm v_{max} = \pm A\omega$
- Acceleration $a = 0$

At the min/max displacement ($x = \pm A$):

- $\cos(\omega t + \phi_0) = \pm 1$
- Speed $v = 0$
- Acceleration is $a = \pm a_{max} = \pm A\omega^2$

