Lecture 2.4
Complex numbers (review)

University of Calgary

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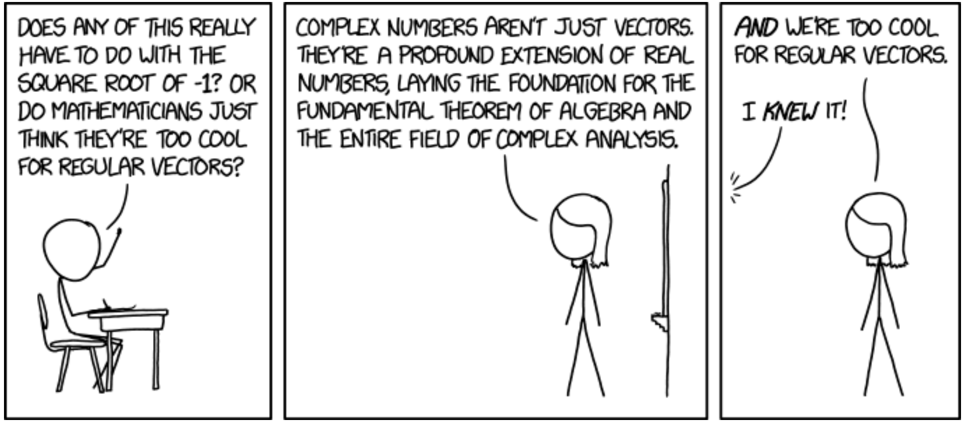


https://xkcd.com/2028/

Phys 365/369

Complex math

We'll use j instead of i for the imaginary unit $\sqrt{-1}$. Blame the electrical engineers.

$j^2 = -1$

If this isn't feeling familiar/comfortable, see the review on D2L (under Content → Course information and resources → Extra notes).

$z = x + jy$

real part

imaginary part

$z_1 = x_1 + jy_1$

$z_2 = x_2 + jy_2$

- Addition: $z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$
- Product: $z_1 z_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$

- Complex conjugate:

$$z = x + jy \;;\quad z^* = x - jy$$

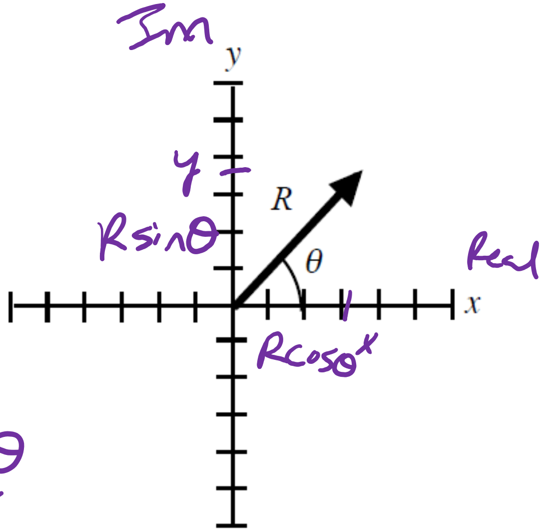
- Magnitude (modulus, absolute value):

$$|z| = \sqrt{zz^*} = \sqrt{x^2 + y^2}$$

- Complex exponential (Euler's relation):

$$e^{j\theta} = \cos\theta + j\sin\theta$$

- Two forms:
  - Rectangular: $z = x + jy$
  - Polar: $z = Re^{j\theta}$



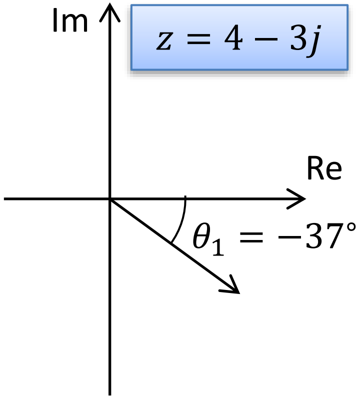
① Polar → rec

$$z = Re^{j\theta} = R(\cos\theta + j\sin\theta) = \underbrace{R\cos\theta}_{x} + j\,\underbrace{R\sin\theta}_{y}$$

② Rec → polar

$$R = \sqrt{x^2+y^2} \;; \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Inverse trig functions: a warning



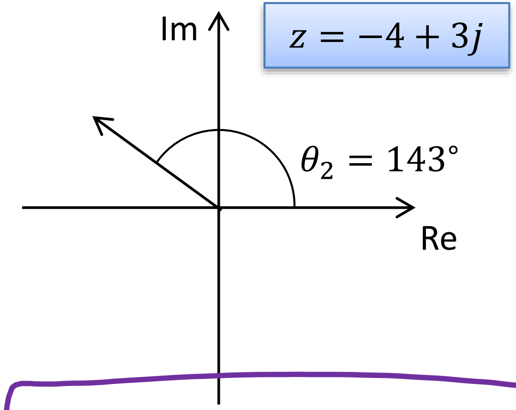
In both of these cases:

$$\frac{y}{x} = -\frac{3}{4}$$

so that

$$\tan^{-1}\left(\frac{y}{x}\right) = -37°$$

But that's not right for θ_2!



HINT: If in doubt, sketch it out.

- Your calculator will always return $\tan^{-1} x$ in the range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ [or in degrees: $(-90°, 90°)$].
  - This will be right about half the time.
- If necessary, add/subtract π $(180°)$