

1. (a) i) $a_1 x_1 + a_2 x_2 \xrightarrow{\text{sys}} |a_1 x_1 + a_2 x_2| + a_1 x_1 + a_2 x_2$

$|x_1| + x_1 \rightarrow a_1$
 $|x_2| + x_2 \rightarrow a_2$ \oplus

$|x_1| a_1 + x_1 a_1 + a_2 |x_2| + a_2 x_2$

not equal so not linear

(ii) $x(t-\tau) \xrightarrow{\text{sys}} |x(t-\tau)| + x(t-\tau)$

$|x(t)| + x(t) \xrightarrow{\text{sys}} |x(t-\tau)| + x(t-\tau)$

equal \therefore time invariant

(b) $y(t) = \int_{t-1}^t x(\tau) d\tau$

$a_1 x_1(t) + a_2 x_2(t) \xrightarrow{\text{sys}} \int_{t-1}^t a_1 x_1(\tau) + a_2 x_2(\tau) d\tau$

$\int_{t-1}^t x_1(\tau) d\tau \rightarrow a_1$
 $\int_{t-1}^t x_2(\tau) d\tau \rightarrow a_2$ \oplus

$a_1 \int_{t-1}^t x_1(\tau) d\tau + a_2 \int_{t-1}^t x_2(\tau) d\tau$

not equal so not linear

$x(\tau-t) \xrightarrow{\text{sys}} \int_{t-1}^t x(\tau-t) d\tau$

$\int_{t-1}^t x(\tau) d\tau \rightarrow \int_{t-1}^t x(\tau-t) d\tau$

equal \therefore time invariant

$$(c) \quad y(t) = [x(t) + x(t-2)] m(t)$$

$$a_1 x_1 + a_2 x_2 \xrightarrow{\text{sys}} (a_1 x_1(t) + a_2 x_2(t)) + (a_1 x_1(t-2) + a_2 x_2(t-2))$$

$$\begin{aligned} a_1 (x_1(t) + x_1(t-2)) \\ a_2 (x_2(t) + x_2(t-2)) \end{aligned} \xrightarrow{\oplus} a_1 x_1(t) + a_1 x_1(t-2) + a_2 x_2(t) + a_2 x_2(t-2) \xrightarrow{\text{equal}} \therefore \text{linear}$$

$$x(t-\tau) \xrightarrow{\text{sys}} [x(t-\tau) + x(t-\tau-2)] m(t) \xrightarrow{\text{equal}} \therefore \text{time invariant}$$

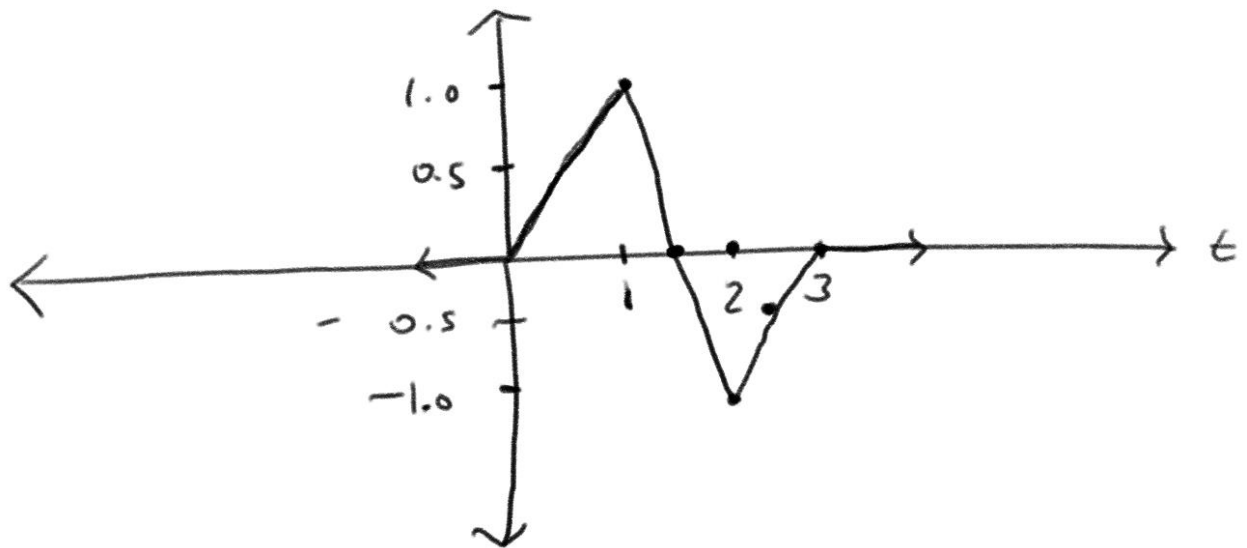
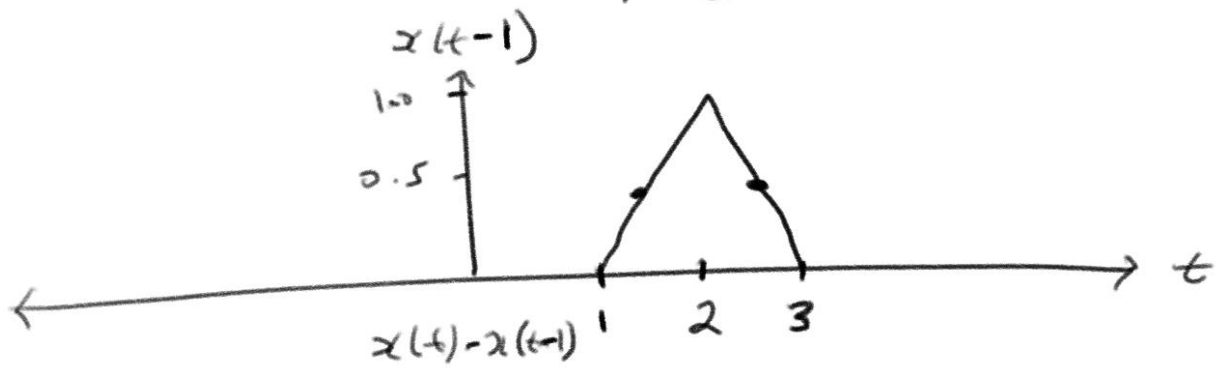
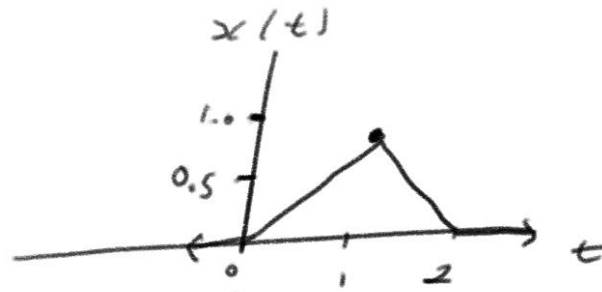
$$[x(t) + x(t-2)] \rightarrow (x(t-\tau) + x(t-2-\tau))$$

Impulse Response and Convolution :

- linear / time invariant

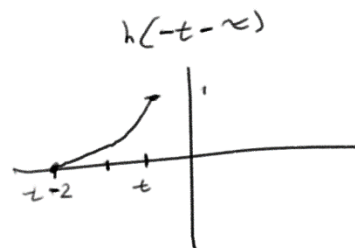
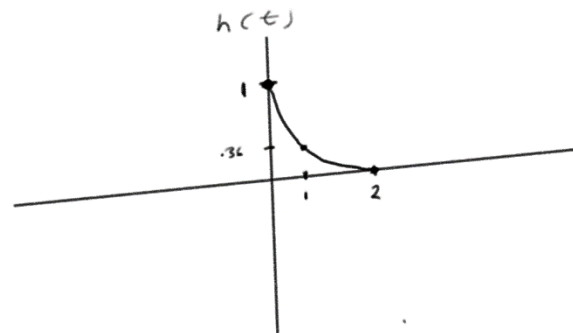
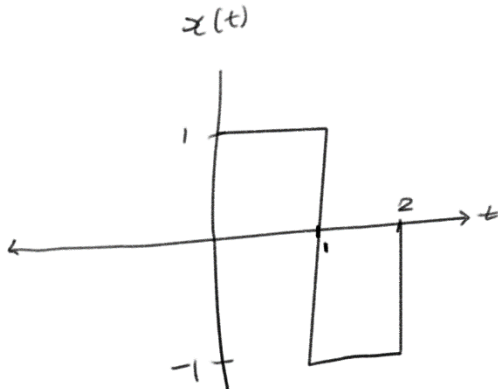
$$- h(t) = \delta(t) - \delta(t-1)$$

$$- x(t) = \begin{cases} t & 0 \leq t \leq 1 \\ -t & 1 \leq t \leq 2 \end{cases}$$

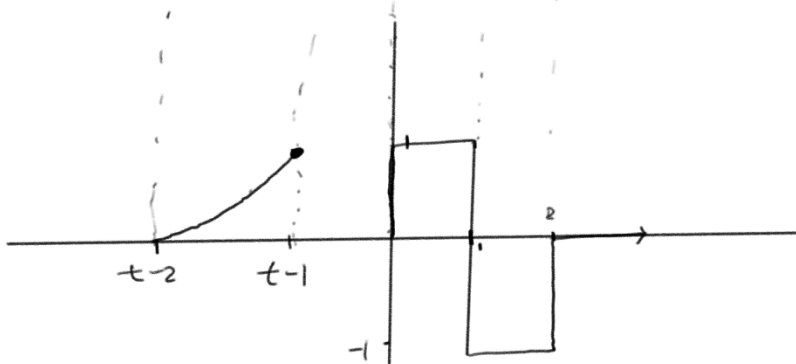


2. $h(t) = e^{-t} [m(t) - m(t-2)]$

$$x(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$

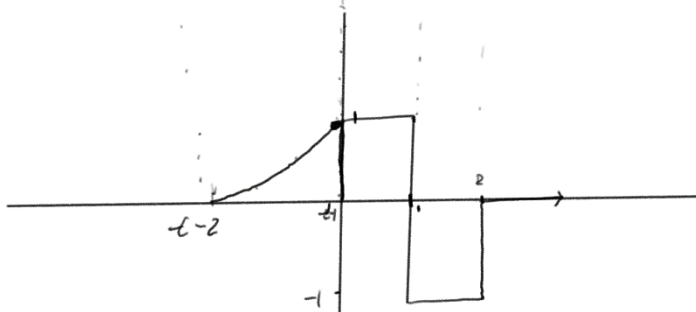


	0	1	0	0	0	0
$x(t) \wedge (-t-\infty)$	0	1	0	0	0	0
$e^{-t} [m(t) - m(t-2)]$	0	0	0	0	0	0
$h(-t-\infty)$	0	1	-1	0		
$x(t)$						



$y(t) = 0$

0	0	0	0	$x(t) \delta(t-2)$
$e^{-t}[m(t)-m(t-2)]$	0	0	0	$h(-t-2)$
0	1	-1	0	$x(t)$

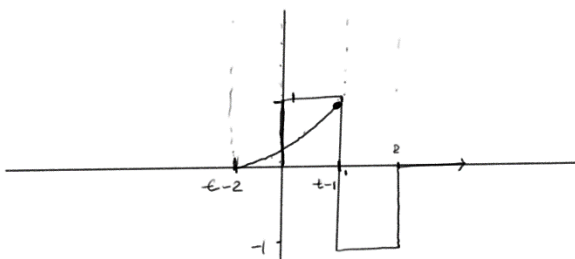


$$t-1=0$$

$$t=1$$

$$t \leq 1 \quad y(t) = 0$$

0	$e^{-t}[m(t)-m(t-2)]$	0	0	$x(t) \delta(t-2)$
$e^{-t}[m(t)-m(t-2)]$	$e^{-t}[m(t)-m(t-2)]$	0	0	$h(-t-2)$
0	1	-1	0	$x(t)$



$$t-1=1$$

$$1 \leq t \leq 2$$

$$y(t) = \int_0^{t-1} e^{-t} [m(t) - m(t-2)]$$

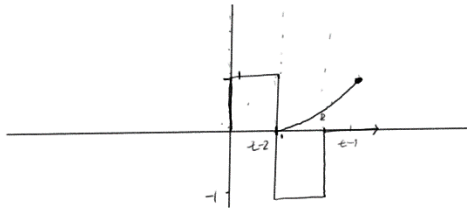
$$y(t) = 2m(t) \int_0^{t-1} e^{-t}$$

$$2m(t) \left[-e^{-t} \right]_0^{t-1}$$

$$y(t) = 2m(t) [-e^{-t+1} - 1]$$

$$1 \leq t \leq 2$$

0	0	0	$x(t) \text{ for } t < -2$
0	0	0	$h(-e^{-t})$
1	-1	0	$x(t)$



$$2 \leq t \leq 3$$

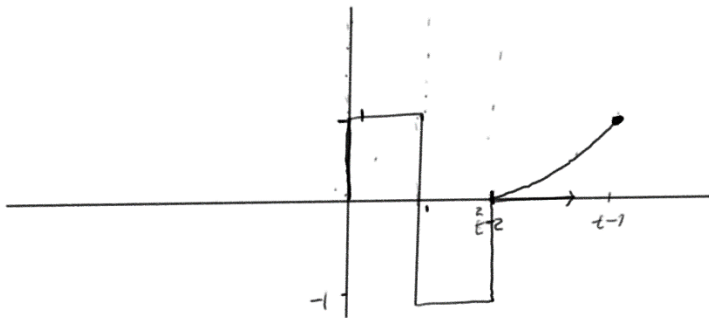
$$y(t) = \int_{t-2}^2 -e^{-t} [u(t) - u(t-2)] dt$$

$$y(t) = -2u(t) \int_{t-2}^2 e^{-t} dt$$

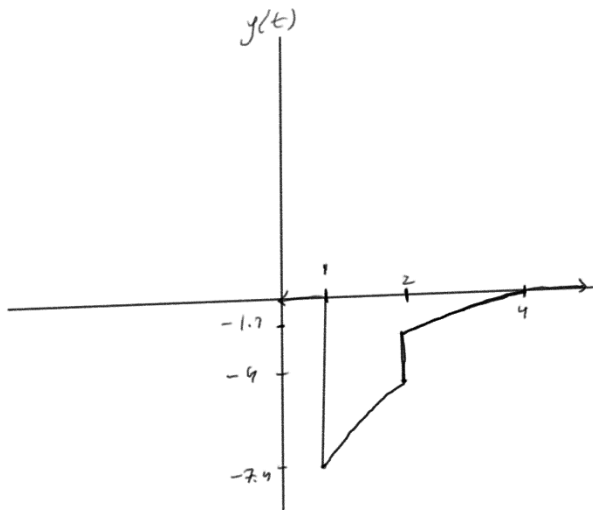
$$y(t) = -2u(t) [-e^{-t}]_{t-2}^2$$

$$y(t) = 2u(t) [e^{-2} - e^{-t+2}] \quad 2 \leq t \leq 3$$

0	0	0	$x(t) \text{ for } t < -2$
0	0	0	$h(-e^{-t})$
1	-1	0	$x(t)$



$$t \leq 2 \quad y(t) = 0$$

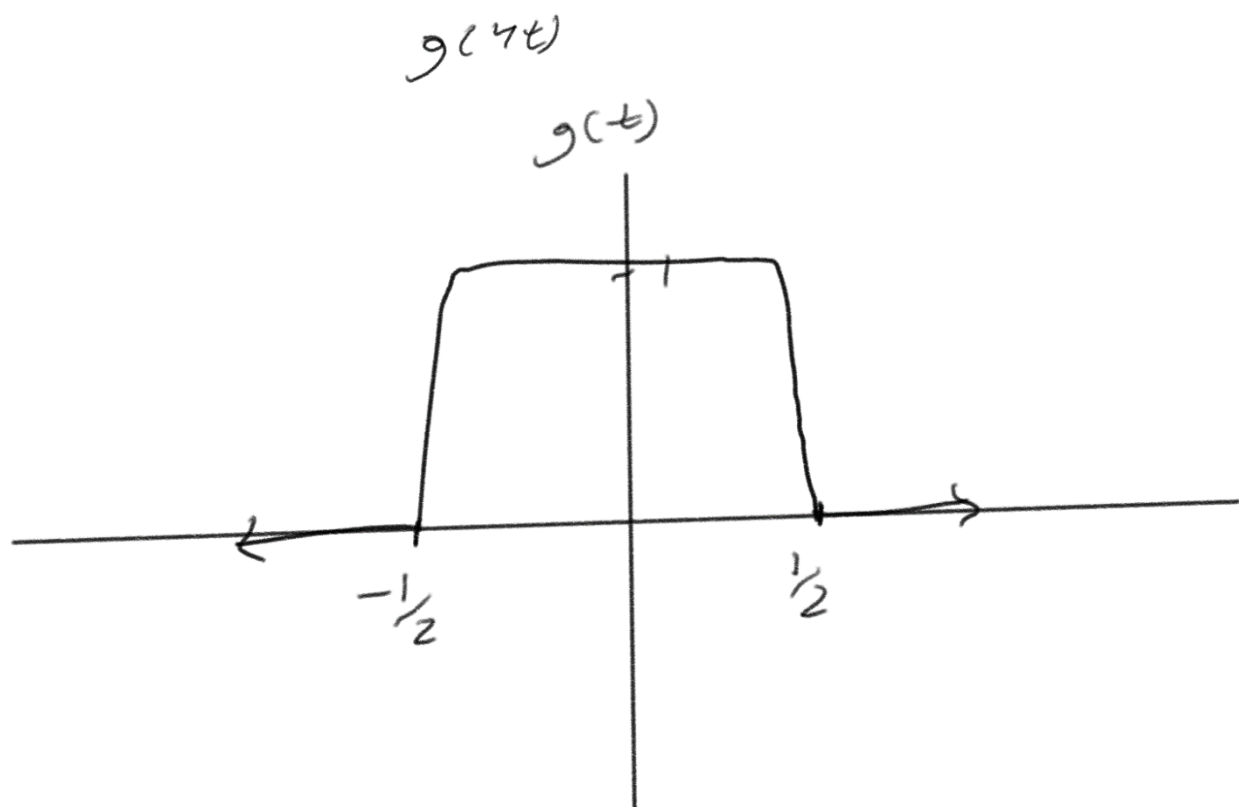


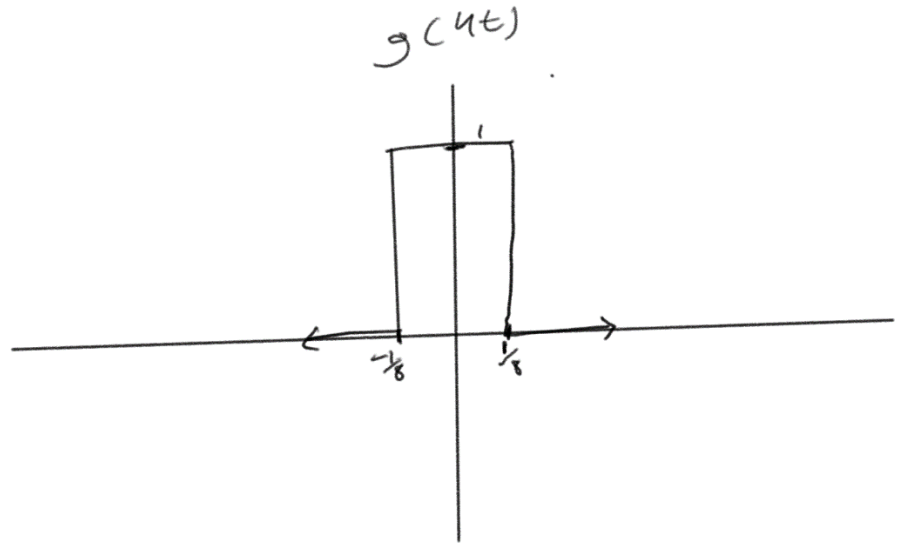
$$y(t) = \begin{cases} 0 & t \leq 1 \\ 2u(t) [-e^{-t+1} - 1] & 1 \leq t \leq 2 \\ 2u(t) [e^{-2} - e^{-t+2}] & 2 \leq t \leq 3 \\ 0 & t \geq 4 \end{cases}$$

3.

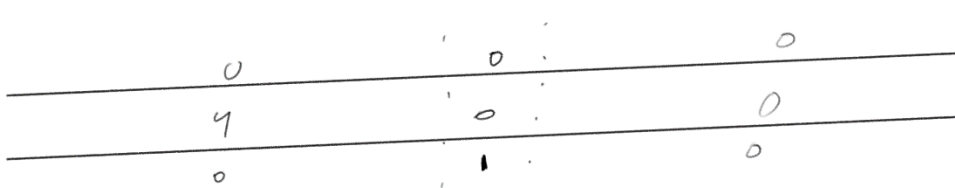
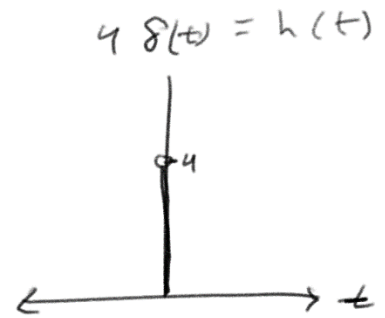
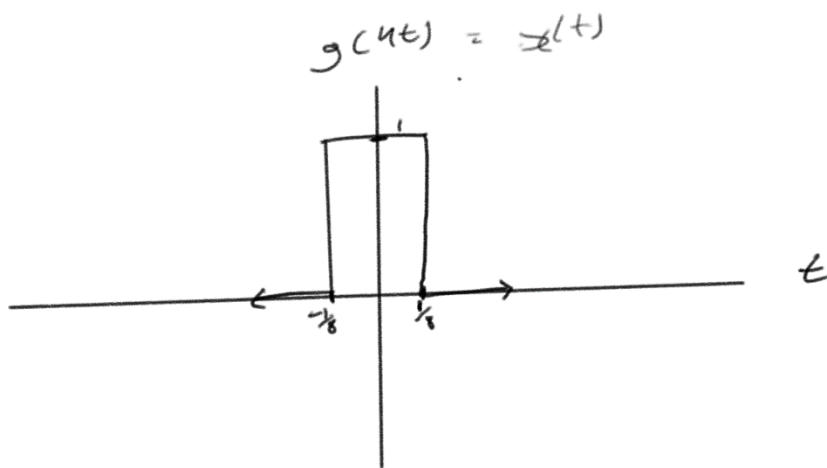
(a)

$$g(t) = \begin{cases} 0 & t < -\frac{1}{2} \\ 1 & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0 & t > \frac{1}{2} \end{cases}$$

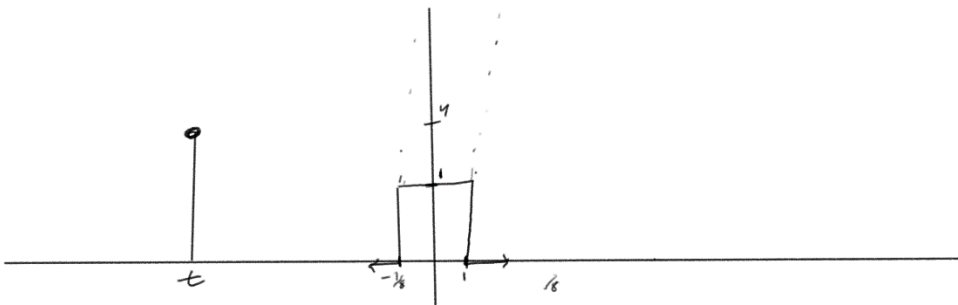


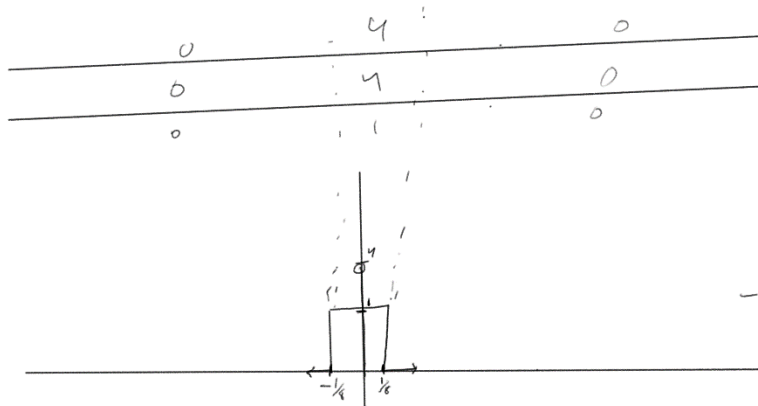


(b) $g(4t) * 4\delta(t)$



$x(t) h(-t-\tau)$
 $h(-t-\tau)$
 $x(t)$





$$x(t) h(-t-2)$$

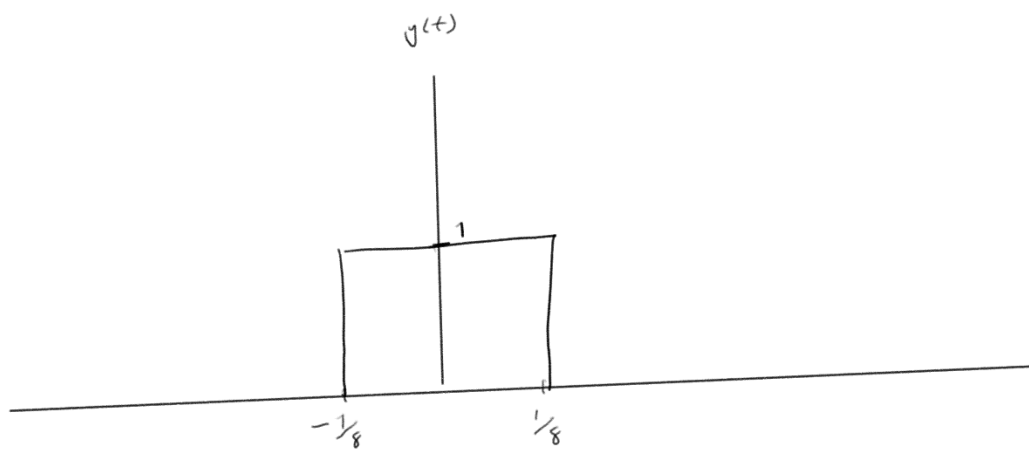
$$h(-t-2)$$

$$x(t)$$

$$t = -\frac{1}{8}$$

$$-\frac{1}{8} < t < \frac{1}{8}$$

$$\begin{aligned} & \int_{-\frac{1}{8}}^{\frac{1}{8}} 4 \, dt \\ &= 4t \Big|_{-\frac{1}{8}}^{\frac{1}{8}} \\ &= \frac{4}{8} + \frac{4}{8} \\ &= 1 \end{aligned}$$

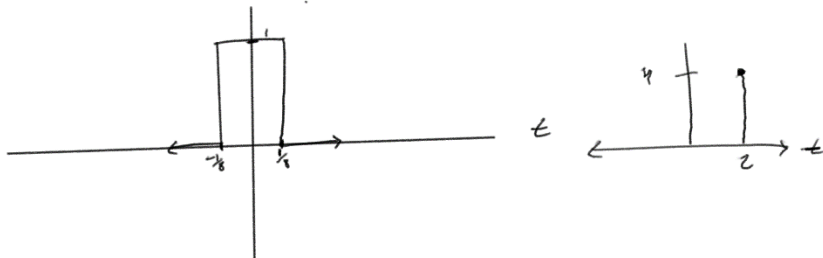


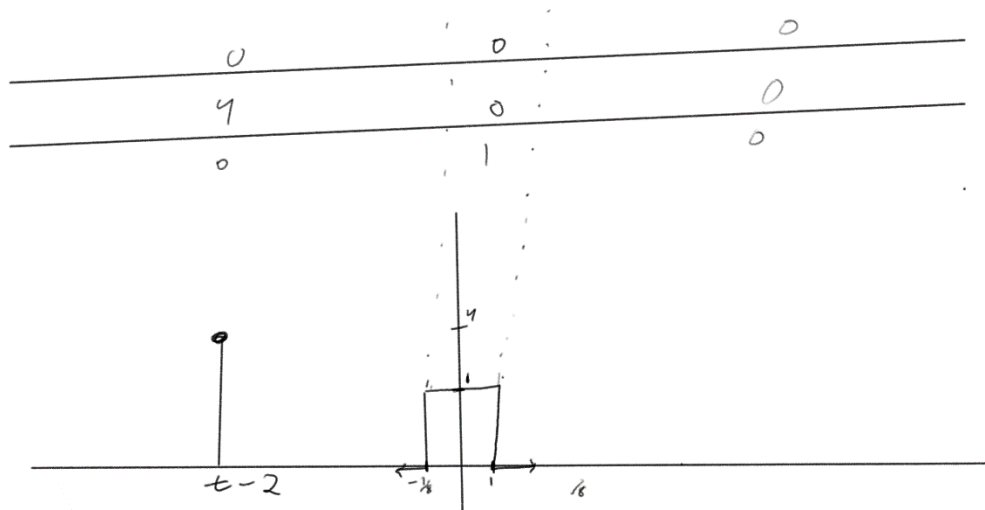
$$g(4t) * \delta(t-2)$$

(C)

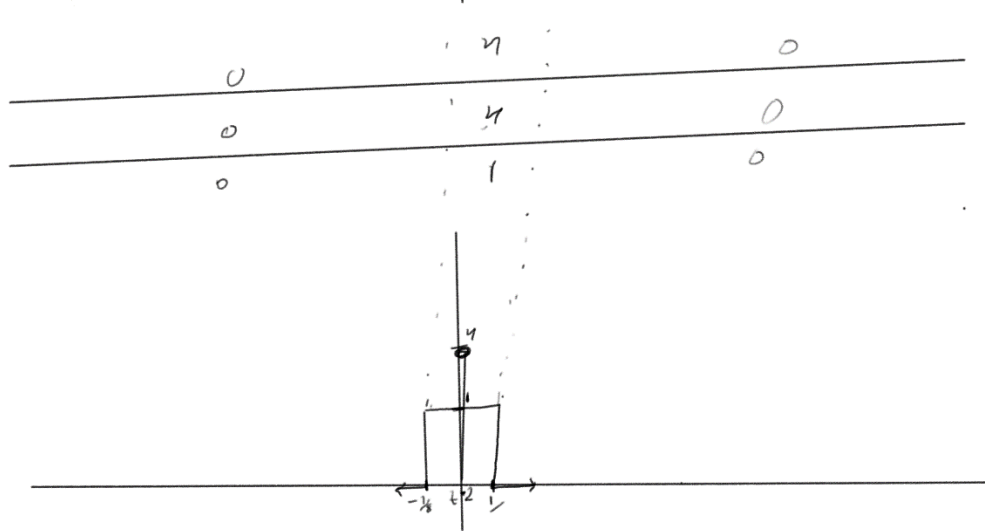
$$g(4t) = x(t)$$

$$\delta(t-2) = h(t)$$





$x(t)h(-t-z)$
 $h(-t-z)$
 $x(t)$



$x(t)h(-t-z)$
 $h(-t-z)$
 $x(t)$

$$t-2 = -\frac{1}{8}$$

$$t = 1.875$$

$$t-2 = \frac{1}{8}$$

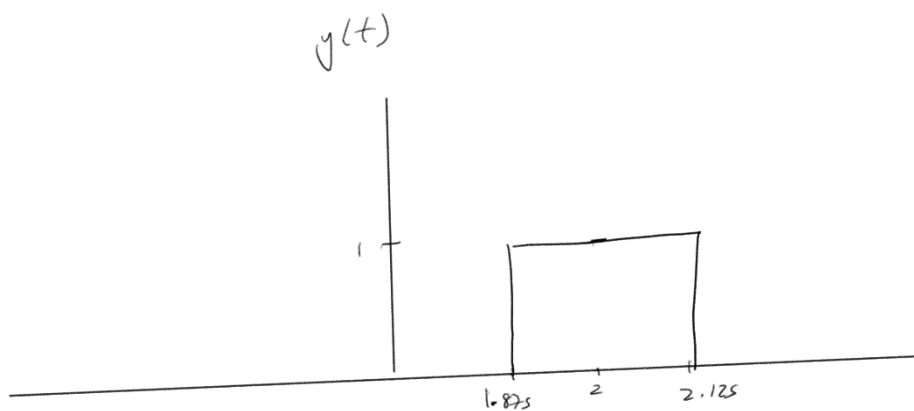
$$t = 2.125$$

$$1.875 < t < 2.125$$

$$y(t) = \int_{-\frac{1}{8}}^{\frac{1}{8}} 4$$

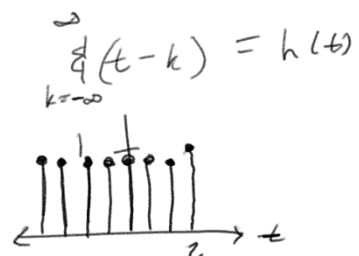
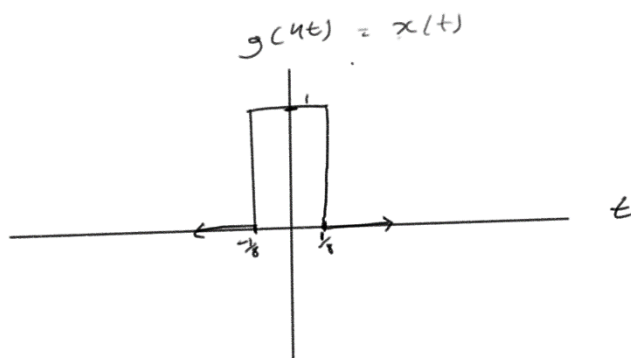
$4t \Big|_{-\frac{1}{8}}^{\frac{1}{8}}$

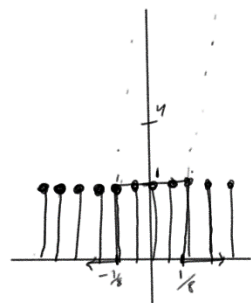
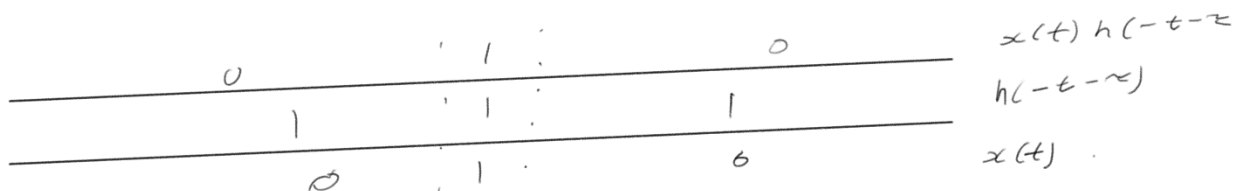
$$\boxed{y(t) = 1}$$



(d)

$$g(4t) * \sum_{k=-\infty}^{\infty} \delta(t-k)$$



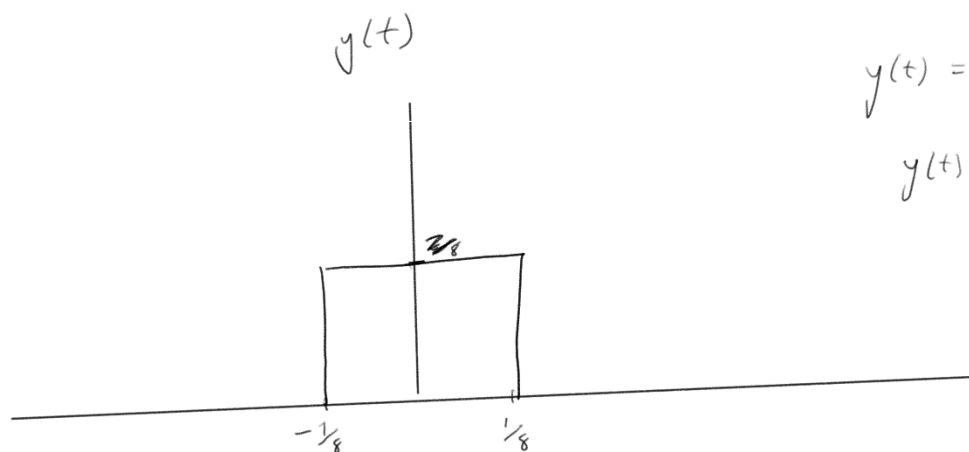


$$y(t) = \int_{-1/8}^{1/8} 1$$

$$y(t) = t \Big|_{-1/8}^{1/8}$$

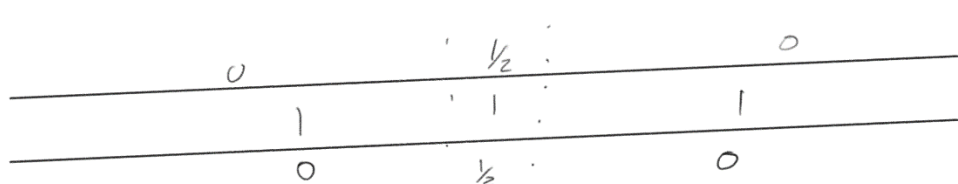
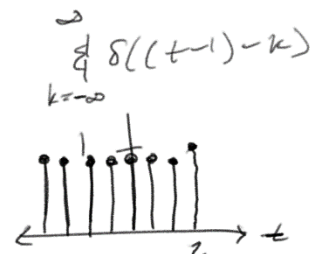
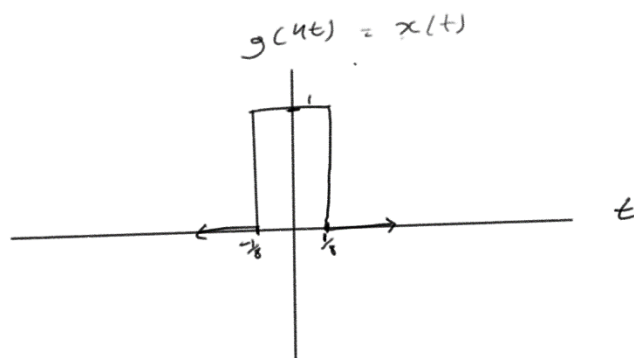
$$y(t) = \frac{1}{8} + \frac{1}{8}$$

$$y(t) = \frac{2}{8}$$



(e)

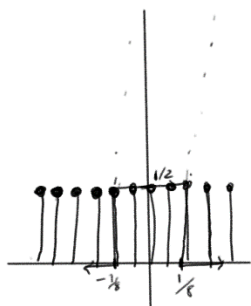
$$g(4t) * \sum_{k=-\infty}^{\infty} \delta((t-1)-k)$$



$$x(t) h(-t-\tau)$$

$$h(-t-\tau)$$

$$x(t)$$



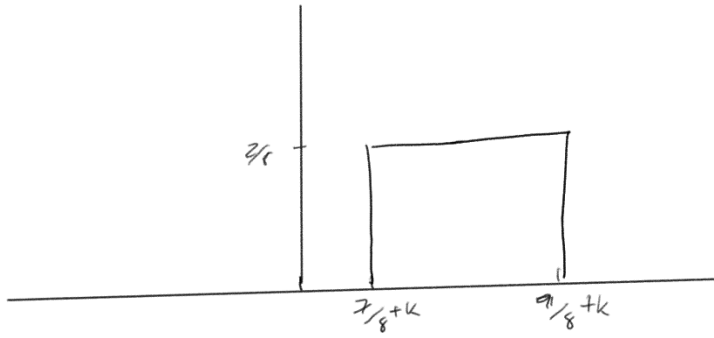
$$y(t) = \int_{-1/4}^{1/4} \frac{1}{2} dt$$

$$y(t) = \frac{1}{2} t \Big|_{-1/4}^{1/4}$$

$$y(t) = \frac{1}{2} \left(\frac{1}{4} \right) + \frac{1}{2} \left(\frac{1}{4} \right)$$

$$y(t) = \frac{1}{8}$$

$$y(t)$$



$$y(t) = \frac{2}{8}$$

$$t - k - 1 = -\frac{1}{8}$$

$$t = \frac{7}{8} + k$$

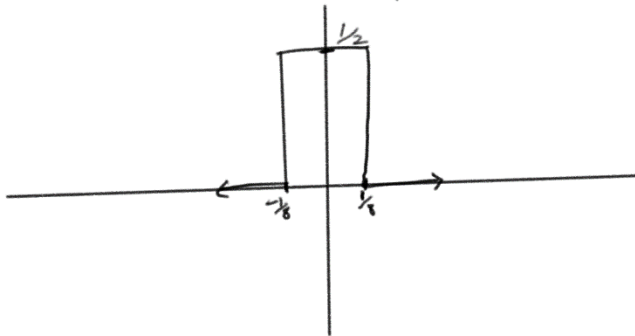
$$t - k - 1 = \frac{1}{8}$$

$$t = \frac{9}{8} + k$$

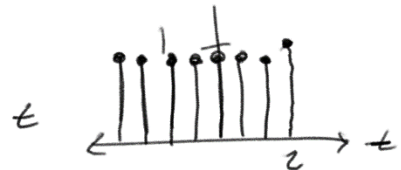
(f)

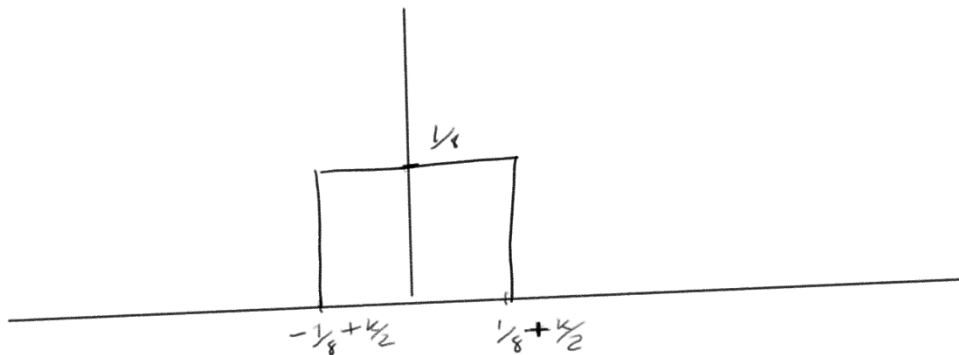
$$\frac{1}{2} g(4t) * \sum_{k=-\infty}^{\infty} \delta(t - \frac{k}{2})$$

$$\frac{1}{2} g(4t) = x(t)$$



$$\sum_{k=-\infty}^{\infty} \delta(t - \frac{k}{2})$$

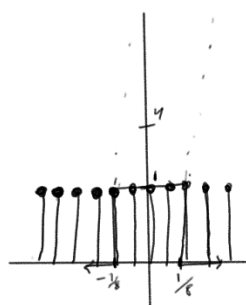
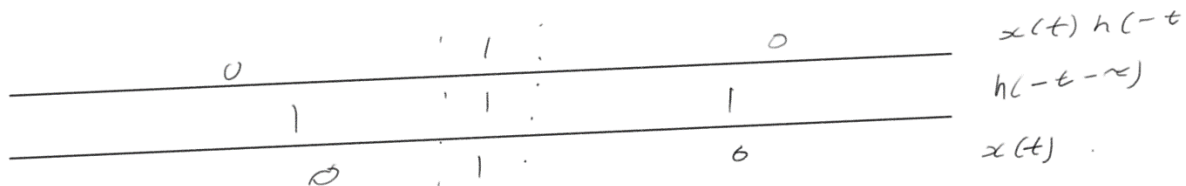




$$t - \frac{k}{2} = -\frac{1}{8}$$

$$t = -\frac{1}{8} + \frac{k}{2}$$

$$t - \frac{k}{2} = \frac{1}{8}$$

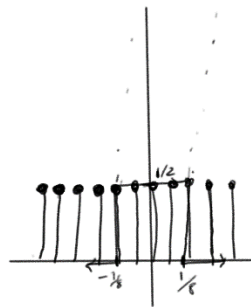
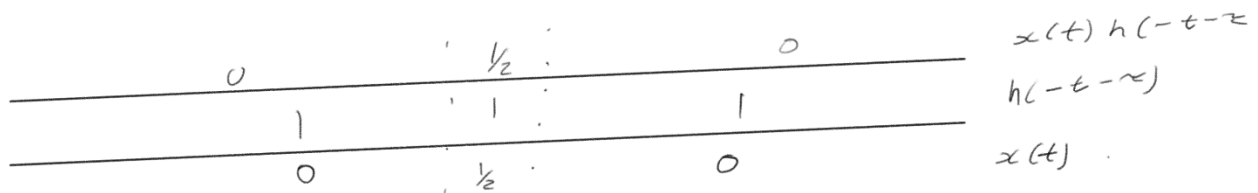


$$y(t)$$

$$y(t) = \int_{-1/8}^{1/8} 1$$

$$y(t) = t \Big|_{-1/8}^{1/8}$$

$$y(t) = \frac{1}{8} + \frac{1}{8}$$

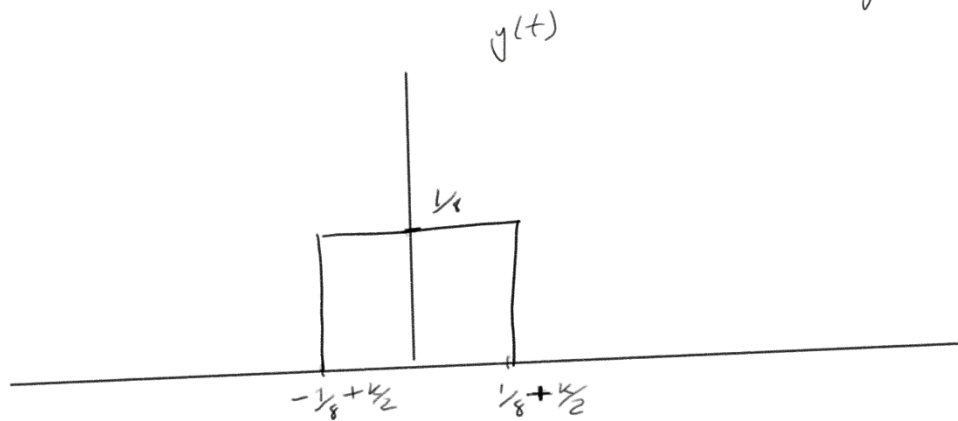


$$y(t) = \int_{-1/8}^{1/8} \frac{1}{2} dt$$

$$y(t) = \frac{1}{2} t \Big|_{-1/8}^{1/8}$$

$$y(t) = \frac{1}{2} \left(\frac{1}{8} \right) + \frac{1}{2} \left(\frac{1}{8} \right)$$

$$y(t) = \frac{1}{8}$$



$$t - \frac{k}{2} = -\frac{1}{8}$$

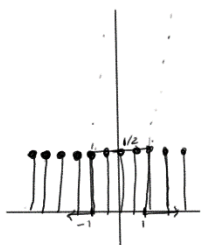
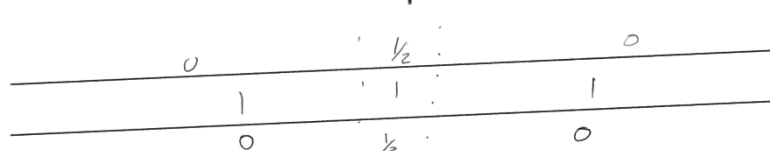
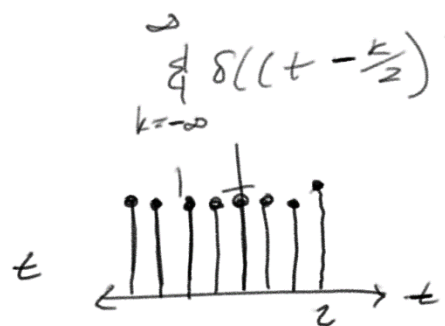
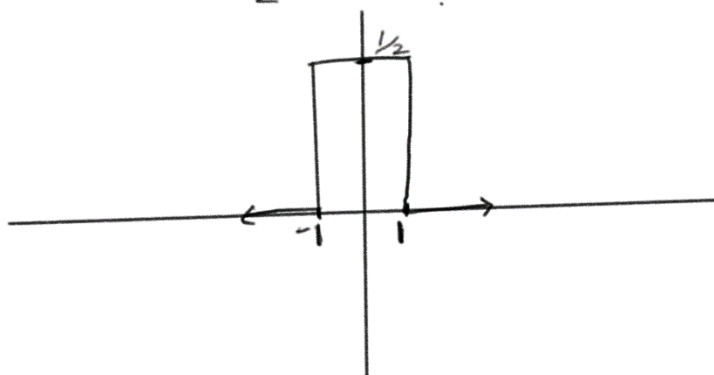
$$t = -\frac{1}{8} + \frac{k}{2}$$

$$t - \frac{k}{2} = \frac{1}{8}$$

(9)

$$\frac{1}{2} g(t) * \sum_{k=-\infty}^{\infty} \delta(t - k/2)$$

$$\frac{1}{2} g(t) = x(t)$$



$y(t)$

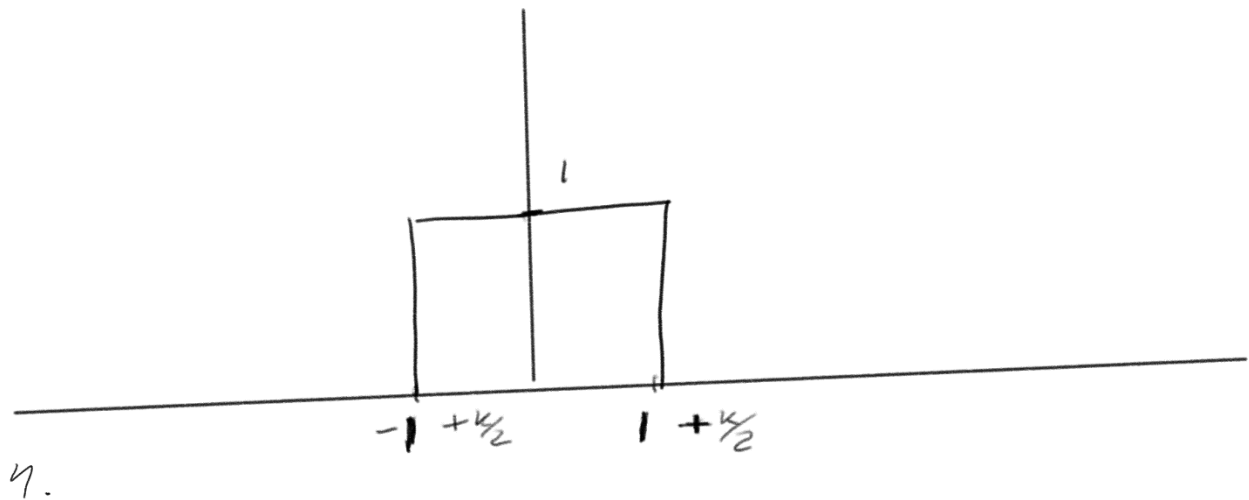
$$x(t) h(-t - \tau)$$

$$h(-t - \tau)$$

$$x(t)$$

$$y(t) = \frac{1}{2} \int_{-1}^1 1 dt$$

$$y(t) = 1$$



$$(a) \quad h_{eq} = h_1(t) + h_3(t) + h_4(t)$$

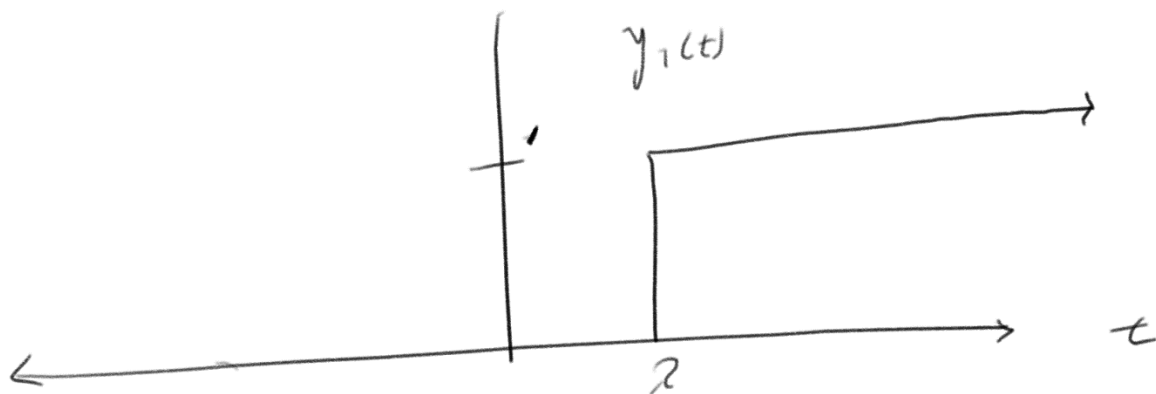
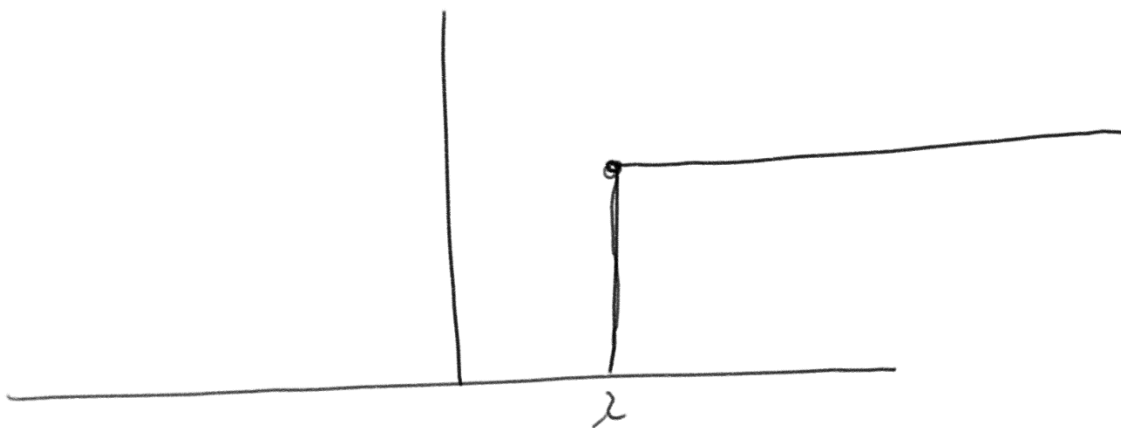
(b)

$$h(t) = m(t) + 2m(t) + m(t)$$

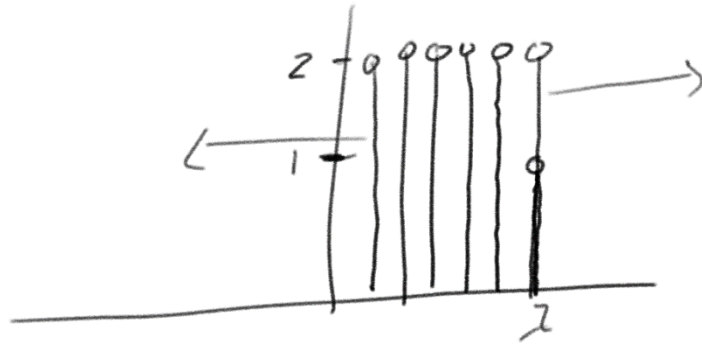
$$h_2(t) = 2\delta(t)$$

$$(c) \quad x(t) = \delta(t) \quad h_1(t) = m(t)$$

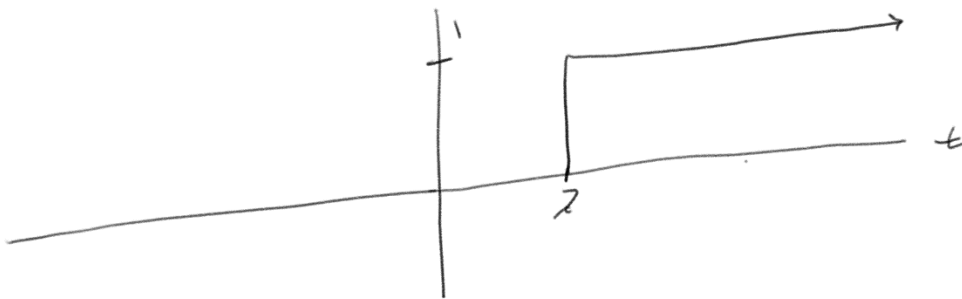
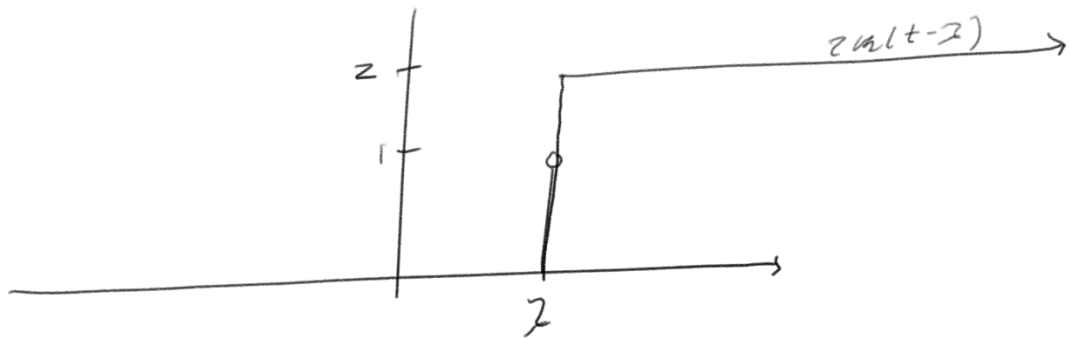
$$y_1(t) = \int_{-\infty}^{\infty} \delta(\tau) u(t-\tau) d\tau = 1$$



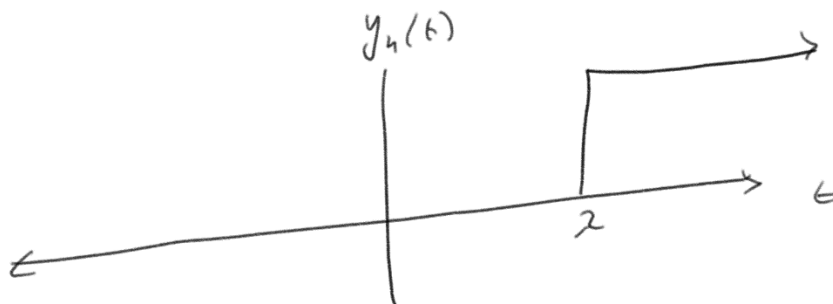
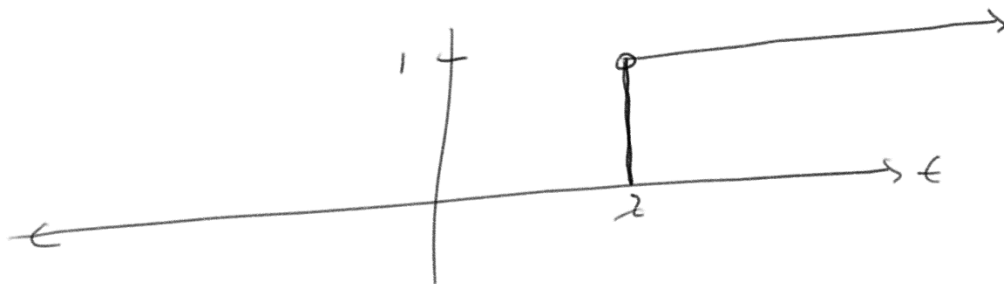
$$\omega(t) = \int_{-\infty}^{\infty} \delta(x) \cdot \delta(t-x) dx = 1$$



$$y_3(t) = \int_{-\infty}^{\infty} \delta(\lambda) 2u(t-2)$$



$$y_4(t) = \int_{-\infty}^{\infty} z(\tau) u(t-\tau) d\tau =$$



$$u(t) + 2u(t) + u(t) = 4u(t)$$

$$y(t) = \int_{-\infty}^{\infty} \delta(\tau) u_m(t-\tau)$$

