UNIVERSITY OF CALGARY

Lecture 1.1
Simple Harmonic Motion

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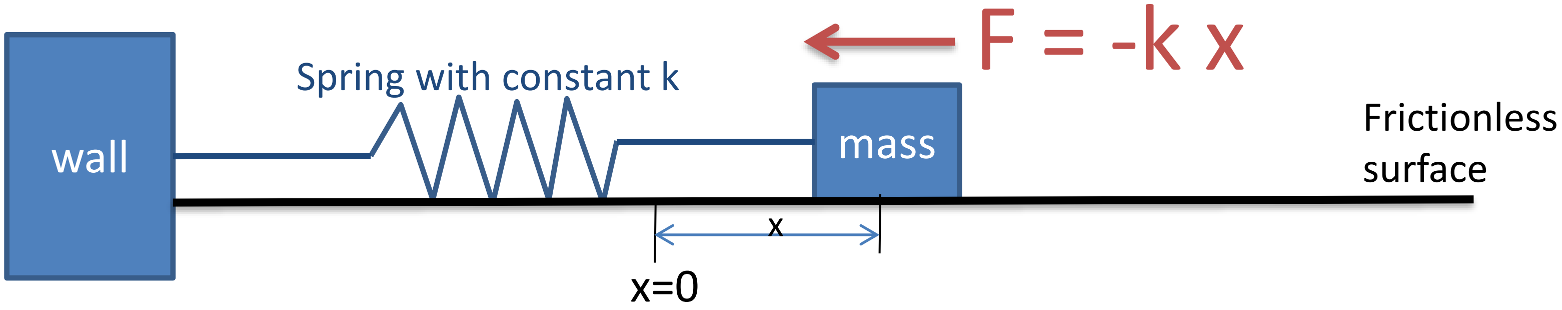
Physics 365/369

Hooke's Law



- As you push a spring farther from equilibrium, it exerts a greater force.
- Letting $x = 0$ at equilibrium (spring unstretched),

$$\boxed{F = -kx}$$

- Typical for many other systems, such as rigid beams, chemical bonds, voltage in an LC resonant circuit,…

Simple Harmonic Motion

Hooke's Law: $\vec{F} = -k\vec{x}$

Plus Newton's 2nd law: $\vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2}$

Gives a 2nd order Ordinary Differential Equation (ODE):

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

NOTE: You CANNOT use formulas like $v = at$ or $x = \frac{1}{2}at^2$ for SHM

These are valid ONLY for constant acceleration!

- Solution depends on the sign of k/m
    - For a mass on a spring, k and m are positive
  - Restrict to case k/m>0 by writing $\boxed{\omega^2 = k/m}$
- $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$ has a general solution:

$$\boxed{x(t) = A\cos(\omega t + \phi_0)}$$

  - This can be verified by substitution
- 2nd order ODE, so two arbitrary constants A, ϕ_0

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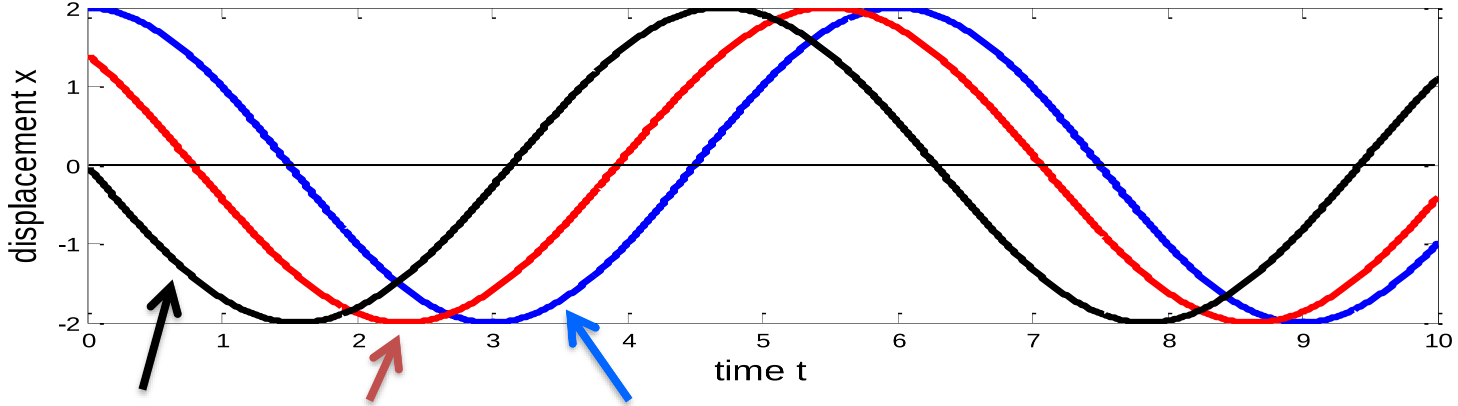
- A is a constant → **amplitude**
- ω ("omega") is a constant → **angular frequency**
  - Units: radians per second
  - Related to the frequency f (in Hz) as:
  - Related to the period T (in s) as:
    - So that $\omega = 2\pi/T$

$$\omega = 2\pi f$$
$$f = 1/T$$

- ϕ_0 ("phi naught") is a constant →**phase angle**
  - Initial condition – shifts curve left/right

$$x(t) = A\cos(\omega t + \phi_0)$$

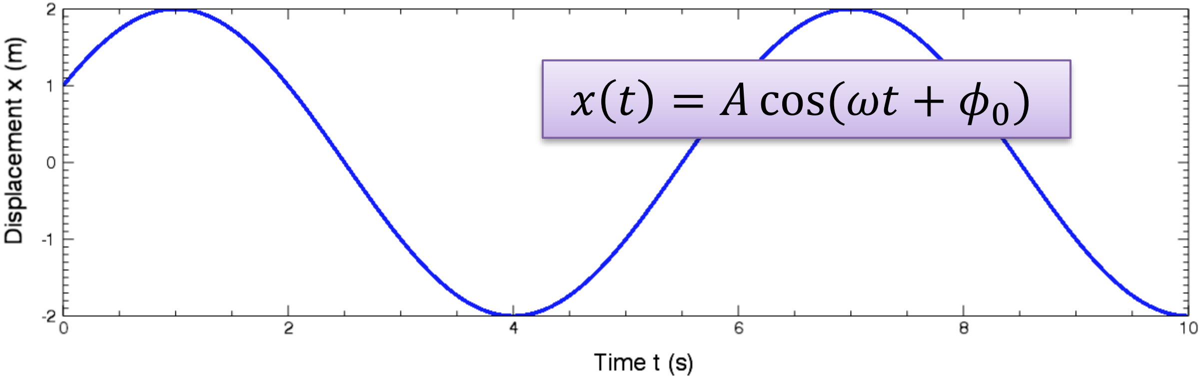


$\phi_0 = \pi/2$ $\phi_0 = \pi/4$ $\phi_0 = 0$

- ϕ_0>0 shifts curve to the left
  - ϕ_0 <0 shifts curve to the right
- $\cos\left(\omega t + \frac{\pi}{2}\right) = -\sin(\omega t)$

EXAMPLE: ϕ_0

Find A, ω, ϕ_0.



Amplitude: $\boldsymbol{A}$**= 2 m**

Angular frequency: Period T = 6 s. Using $\omega = \frac{2\pi}{T}$: $\boldsymbol{\omega = \pi/3}$ **radians/s**

Phase angle: First maximum is at $t_0 = 1\text{s}$.

Maximum in $\cos\theta$ is at $\theta = 2\pi n$ where n is an integer. So:

$$\omega t_0 + \phi_0 = 2\pi n$$

$$\phi_0 = 2\pi n - \omega t_0 = 2\pi n - \left(\frac{\pi}{3}\right)\frac{\text{rad}}{\text{s}}(1.0\text{ s})$$

$$\boldsymbol{\phi_0 = 2\pi n - \pi/3}$$

$$x(t) = (2\text{ m})\cos\left(\frac{\pi}{3}t - \frac{\pi}{3} + 2\pi n\right)$$

There is an infinite set of solutions for ϕ_0