3.1: Superposition and complex representation of waves

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Wave superposition

If $y_1(x,t)$ and $y_2(x,t)$ are both solutions to the wave equation then:

$$y_3(x,t) = y_1(x,t) + y_2(x,t)$$

is also a solution to the wave equation.

Image: http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a@8.8:126/College_Physics

→Waves can pass through each other

→Waves can be constructed in linear combinations

Proof

If $y_1(x,t)$ and $y_2(x,t)$ are both solutions to the wave equations, then they can be written as :

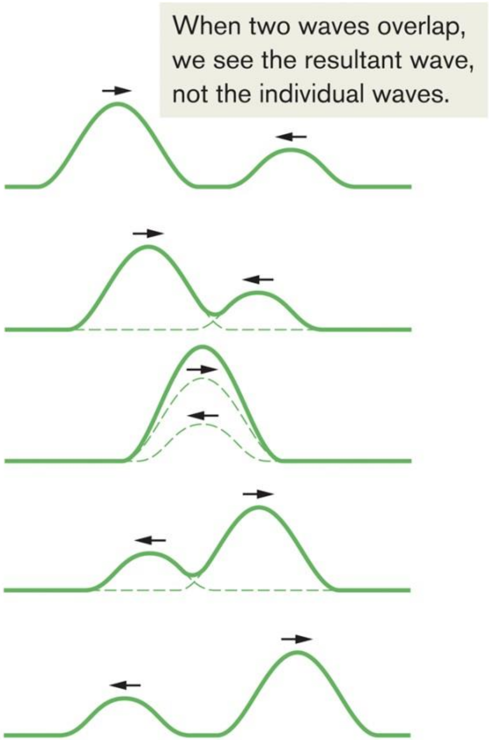
$$\begin{cases} y_1(x,t) = f_1(\mathrm{x} \pm vt) \\ y_2(x,t) = f_2(\mathrm{x} \pm vt) \end{cases}$$

so that

$$\begin{aligned} y_3(x,t) &= y_1(x,t) + y_2(x,t) \\ &= f_1(\mathrm{x} \pm vt) + f_2(\mathrm{x} \pm vt) \end{aligned}$$

is also a function of $\mathrm{x} \pm vt$ and therefore also a solution to the wave equation.

Examples: waves on a string



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Complex representation of a wave

- Superposition of harmonic (sinusoidal) waves requires adding together sinusoidal functions
  - Like adding phasors for SHM
  - Need to extend phasors concept to cover waves

Complex representation of a wave

Start with a wave propagating to the right:

$$y(x,t) = A\cos(kx - \omega t + \phi_0)$$

and write it in terms of the real part of a complex exponential:

$$y(x,t) = A\,\mathrm{Re}\left[e^{j(kx-\omega t+\phi_0)}\right]$$

The Re[] is usually suppressed:

$$\tilde{y}(x,t) = Ae^{j\phi_0}e^{j(kx-\omega t)}$$

Just like for the phasors, we define a complex amplitude

$$\tilde{A} = Ae^{j\phi_0}$$

and then simply write:

$$\tilde{y}(x,t) = \tilde{A}e^{j(kx-\omega t)}$$

The absolute value of $\tilde{A}$ gives the wave amplitude, and the phase of $\tilde{A}$ gives the initial phase ϕ_0.

Example

Express

$$s_1(x,t) = 2.00\cos(kx - \omega t + 60^o)$$

in complex form.

Changing degrees to radians: $60^0(2\pi/360^o) = \pi/3$. So the complex amplitude is

$$\widetilde{s_1} = 2.00e^{j\pi/3}$$

And the wave in complex form is:

$$s_1(x,t) = \mathrm{Re}\left[2.00e^{j\pi/3}e^{j(kx-\omega t)}\right]$$