1. (a) i) $a_1x_1 + a_2x_2 \xrightarrow{sys} |a_1x_1 + a_2x_2| + a_1x_1 + a_2x_2$

$|x_1| + x_1 \longrightarrow a_1$

$|x_2| + x_2 \longrightarrow a_2$ $+$

$|x_1|a_1 + x_1a_1 + a_2|x_2| + a_2x_2$

not equal so not linear

(ii) $x(t-\tau) \xrightarrow{sys} |x(t-\tau)| + x(t-\tau)$

$|x(t)| + x(t) \xrightarrow{sys} |x(t-\tau)| + x(t-\tau)$

equal ∴ time invariant

(b) $y(t) = \int_{t-1}^{t} x(\tau)\,d\tau$

$a_1x_1(\tau) + a_2x_2(\tau) \xrightarrow{sys} \int_{t-1}^{t} a_1x_1(\tau) + a_2x_2(\tau)$

$\int_{t-1}^{t} x_1(\tau) \longrightarrow a_1$

$\int_{t-1}^{t} x_2(\tau) \longrightarrow a_2$ $+$

$a_1\int_{t-1}^{t} x_1(\tau) + a_2\int_{t-1}^{t} x_2(\tau)$

not equal so not linear

$x(\tau - t) \xrightarrow{sys} \int_{t-1}^{t} x(\tau - t)$

$\int_{t-1}^{t} x(\tau)$ $\quad\int_{t-1}^{t} x(\tau - t)$

equal ∴ time invariant

(c) $y(t) = [x(t) + x(t-2)]\, u(t)$

$$a_1x_1 + a_2x_2 \xrightarrow{sys} (a_1x_1(t) + a_2x_2(t)) + (a_1x_1(t-2) + a_2x_2(t-2))$$

$$a_1(x_1(t) + x_1(t-2))$$
$$a_2(x_2(t) + x_2(t-2))$$
$$\oplus \quad a_1x_1(t) + a_1x_1(t-2) + a_2x_2(t) + x_2(t-2)$$

equal ∴ linear

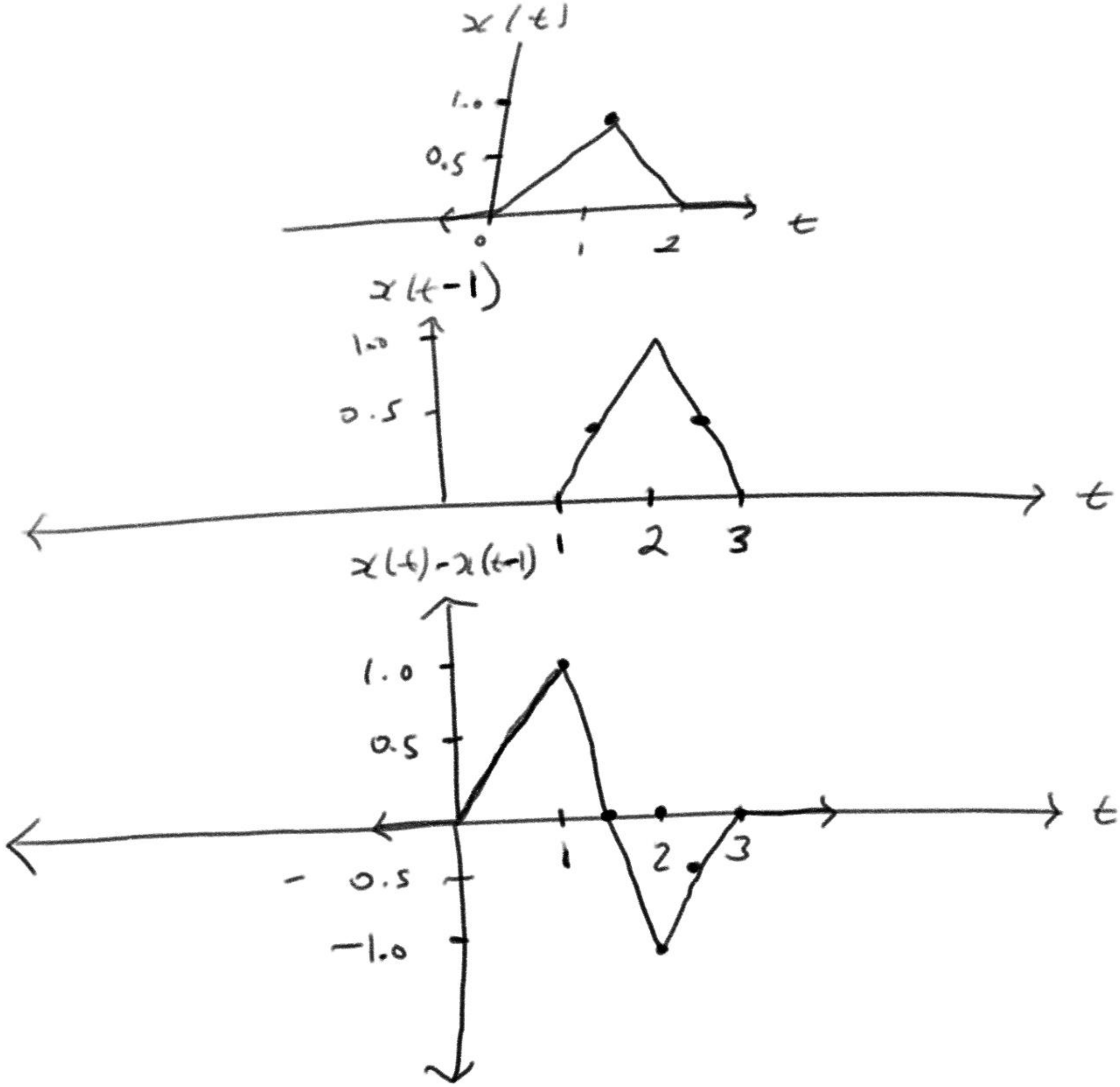
$$x(t-\tau) \xrightarrow{sys} [x(t-\tau) + x(t-\tau-2)]\, u(t)$$

$$[x(t) + x(t-2)] \longrightarrow (x(t-\tau) + x(t-2-\tau))$$

equal ∴ time invariant

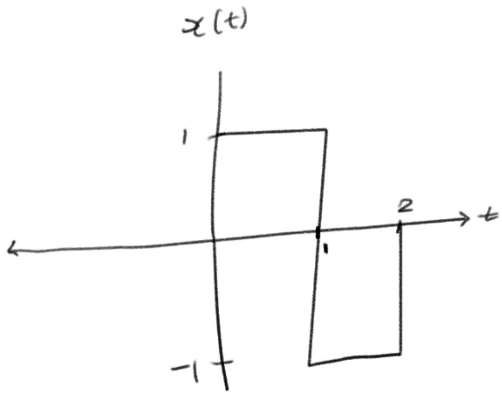
Impulse Response and Convolution:

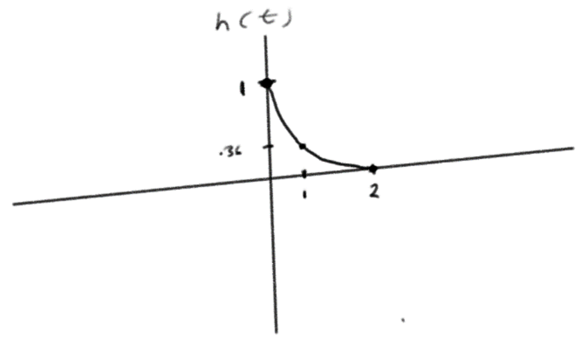
- linear / time invariant
- $h(t) = \delta(t) - \delta(t-1)$
- $x(t) = \begin{cases} t & 0 \le t \le 1 \\ -t & 1 \le t \le 2 \end{cases}$

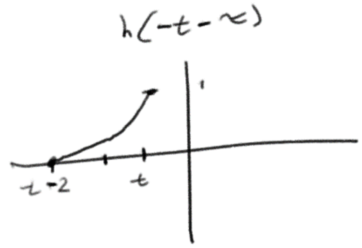

x(t)
1.0
0.5
0
1
2
t
x(t-1)
1.0
0.5
1
2
3
t
x(t)-x(t-1)
1.0
0.5
- 0.5
-1.0
1
2
3
t

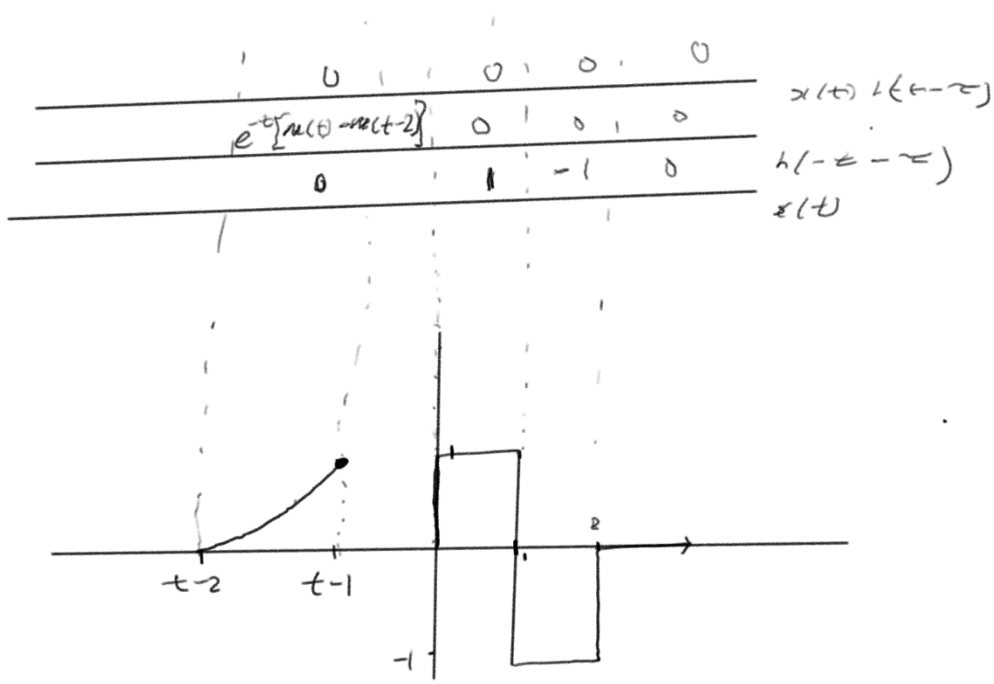
2. $$h(t) = e^{-t}\left[u(t) - u(t-2)\right]$$

$$x(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$

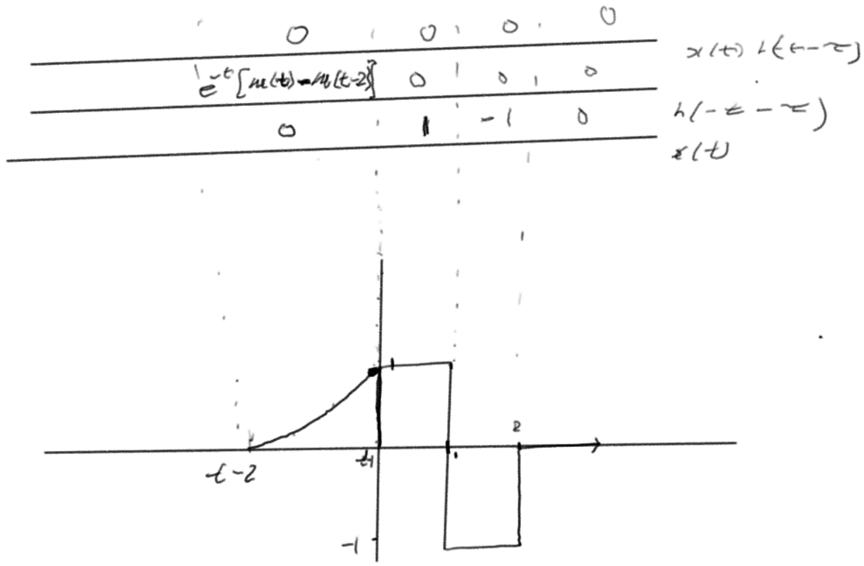








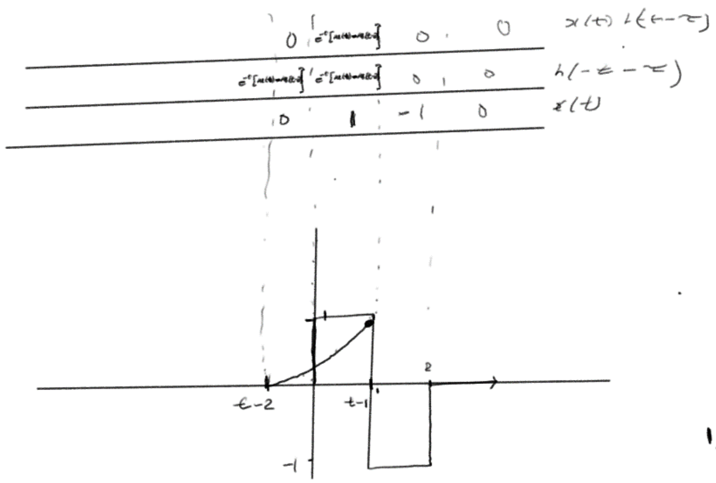
$$y(t) = 0$$



$t - 1 = 0$

$t = 1$

$t \leq 1 \qquad y(t) = 0$



$t - 1 = 1$

$1 \leq t \leq 2$

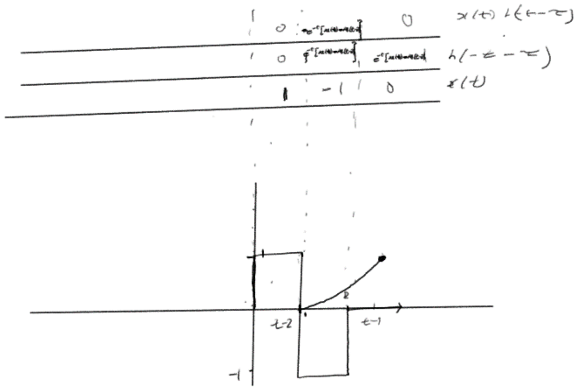
$$y(t) = \int_0^{t-1} e^{-t}[u(t) - u(t-2)]$$

$$y(t) = 2u(t)\int_0^{t-1} e^{-t}$$

$$2u(t)\left[-e^{-t}\Big|_0^{t-1}\right]$$

$1 \leq t \leq 2$

$$y(t) = 2u(t)\left[-e^{-t+1} - 1\right]$$



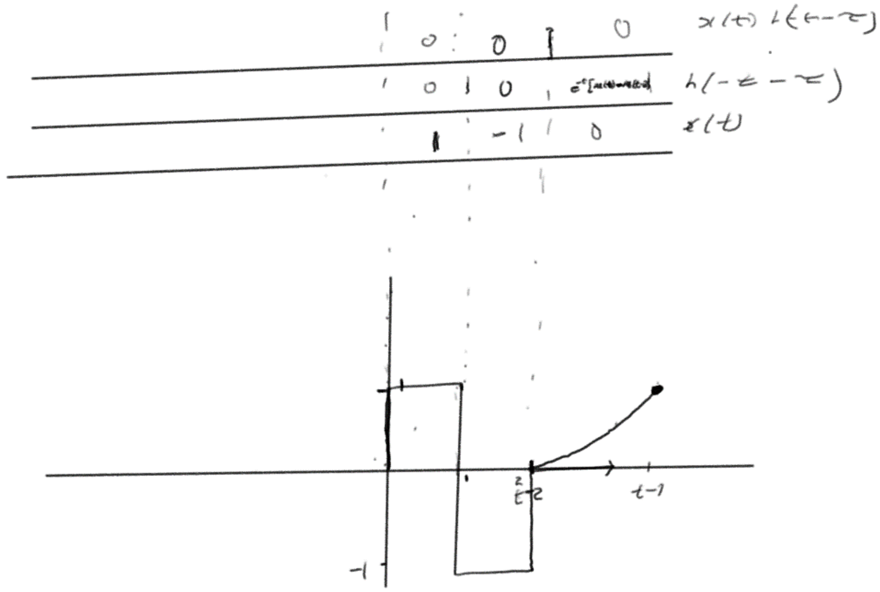
$2 \leq t \leq 3$

$$y(t) = \int_{t-2}^{2} -e^{-t}\left[u(t) - u(t-2)\right] dt$$

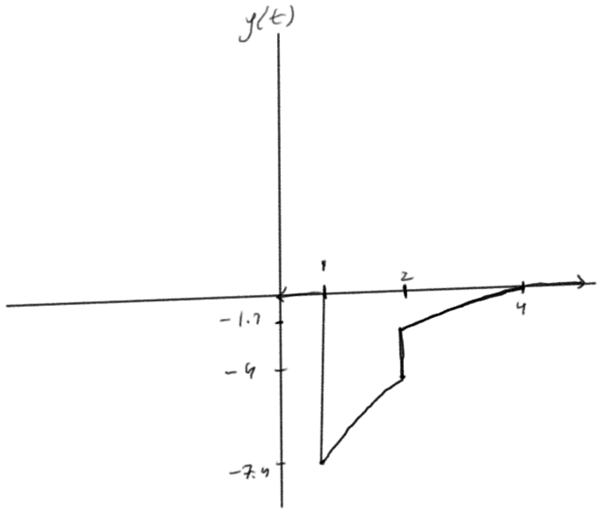
$$y(t) = -2u(t)\int_{t-2}^{2} e^{-t}\, dt$$

$$y(t) = -2u(t)\left[-e^{-t}\Big|_{t-2}^{2}\right]$$

$$y(t) = 2u(t)\left[e^{-2} - e^{-t+2}\right] \quad 2 \leq t \leq 3$$



$4 \leq t \quad y(t) = 0$

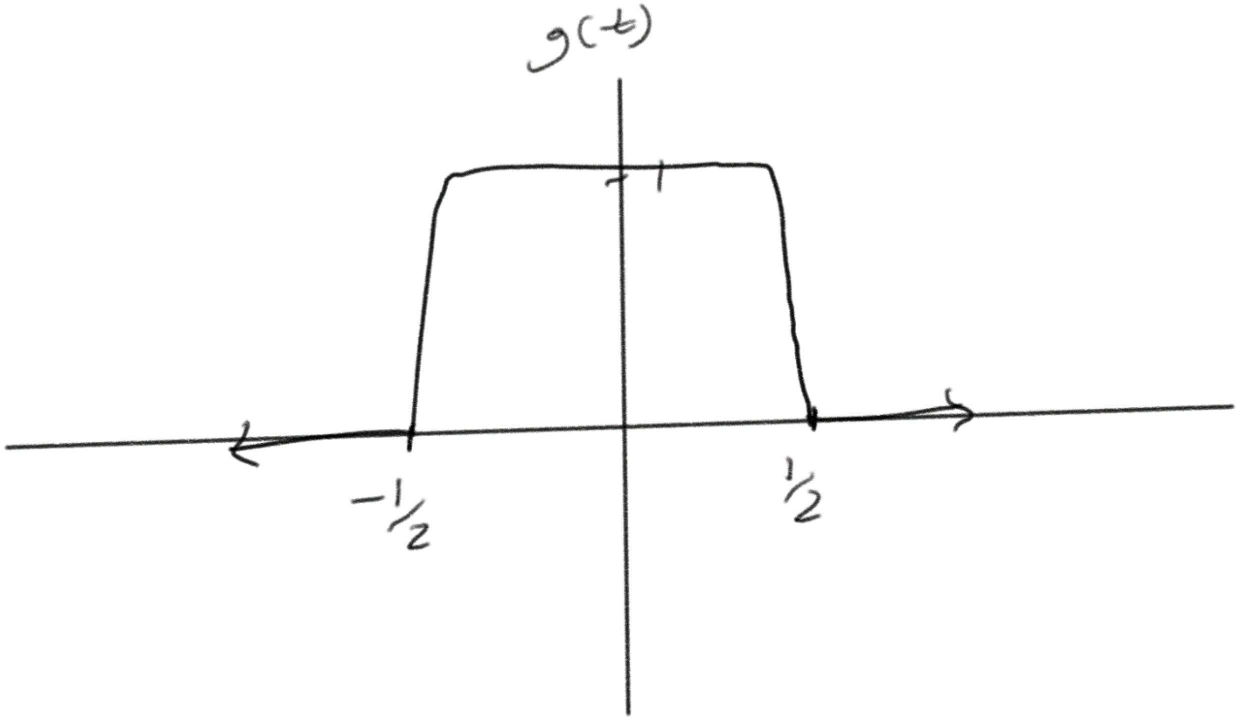


$$y(t) = \begin{cases} 0 & t \leq 1 \\ 2u(t)\left[-e^{-t+1} - 1\right] & 1 \leq t \leq 2 \\ 2u(t)\left[e^{-2} - e^{-t+2}\right] & 2 \leq t \leq 3 \\ 0 & 4 \leq t \end{cases}$$

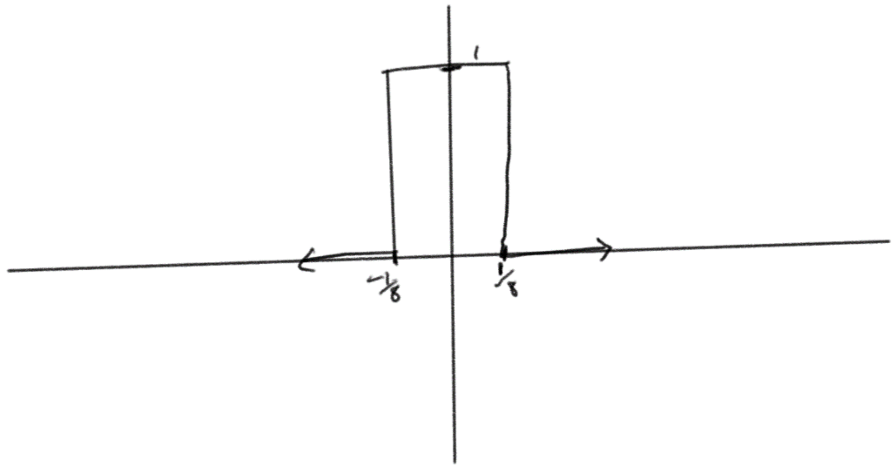
3. (a)

$$g(t) = \begin{cases} 0 & t < -\frac{1}{2} \\ 1 & -\frac{1}{2} < t < \frac{1}{2} \\ 0 & t > \frac{1}{2} \end{cases}$$

g(4t)



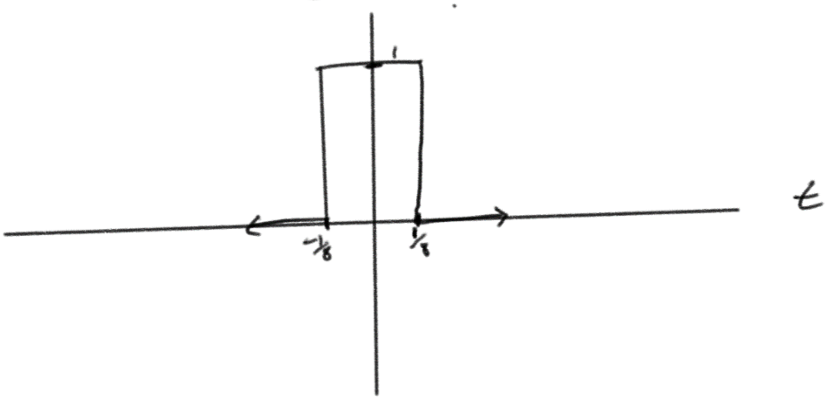
$g(4t)$



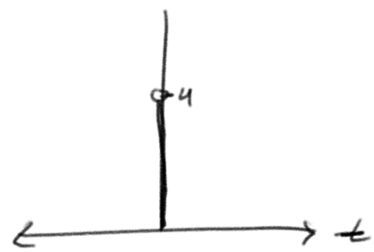
(b)

$$g(4t) * 4\delta(t)$$

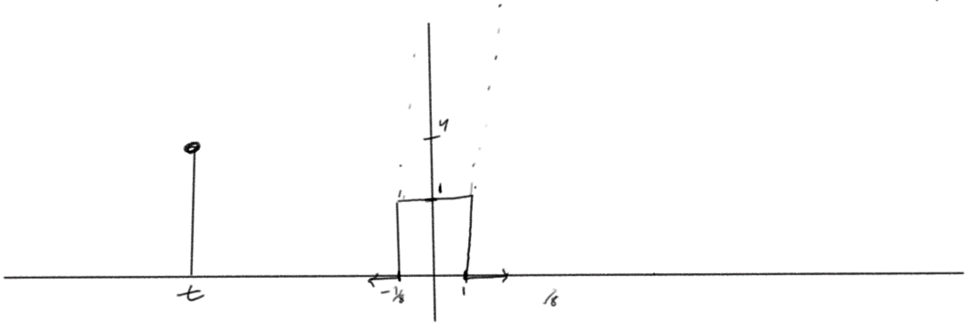
$g(4t) = x(t)$

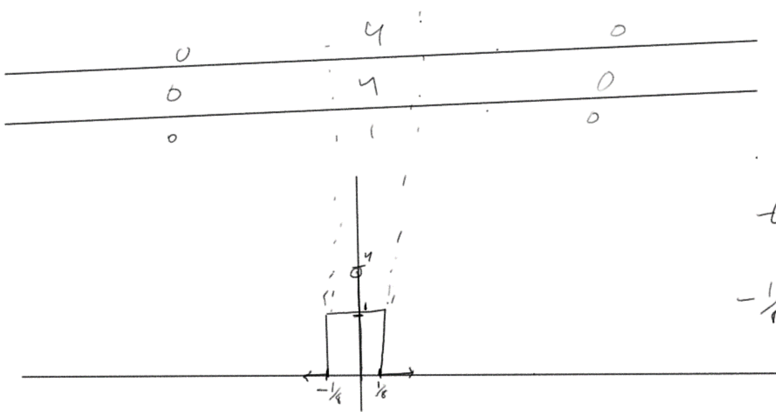


$4\delta(t) = h(t)$



0	0	0	$x(t)\,h(-t-\tau$
4	0	0	$h(-t-\tau)$
0	1	0	$x(t)$





$x(t)\,h(-t-\tau$

$h(-t-\tau)$

$x(t)$

$t = -\frac{1}{8}$

$-\frac{1}{8} < t < \frac{1}{8}$

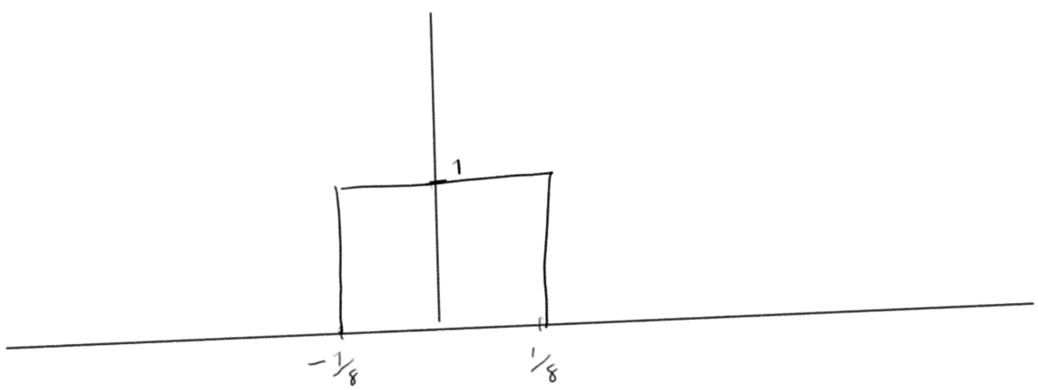
$$\int_{-1/8}^{1/8} 4 \, dt$$

$$= 4t \Big|_{-1/8}^{1/8}$$

$$= \frac{4}{8} + \frac{4}{8}$$

$$= 1$$

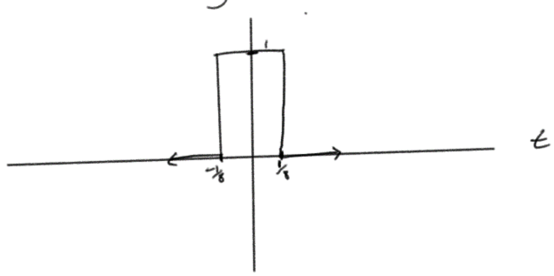
$y(t)$



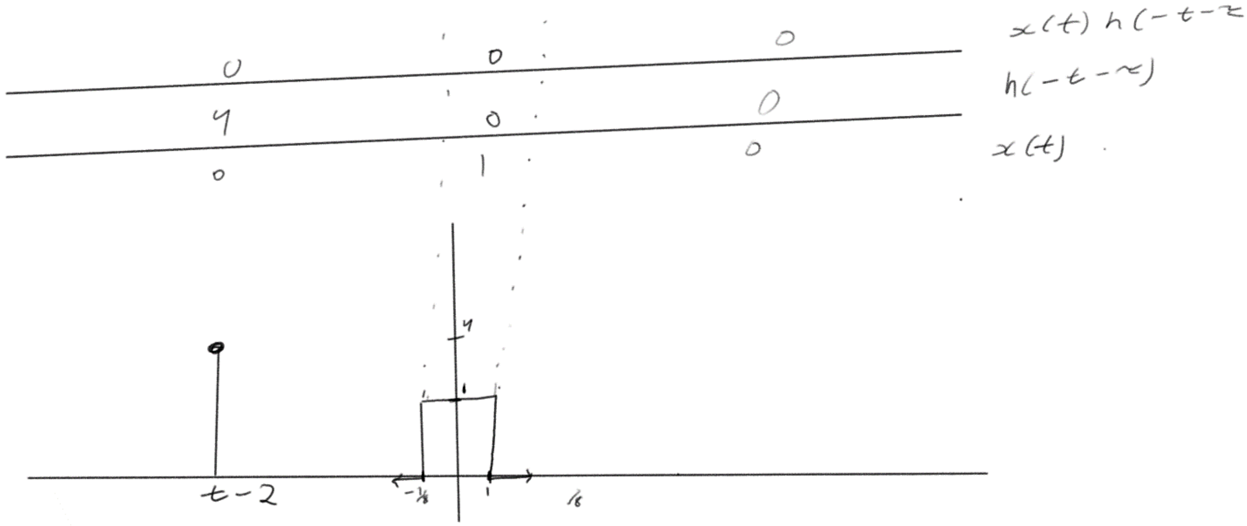
$g(4t) * 4\delta(t-2$

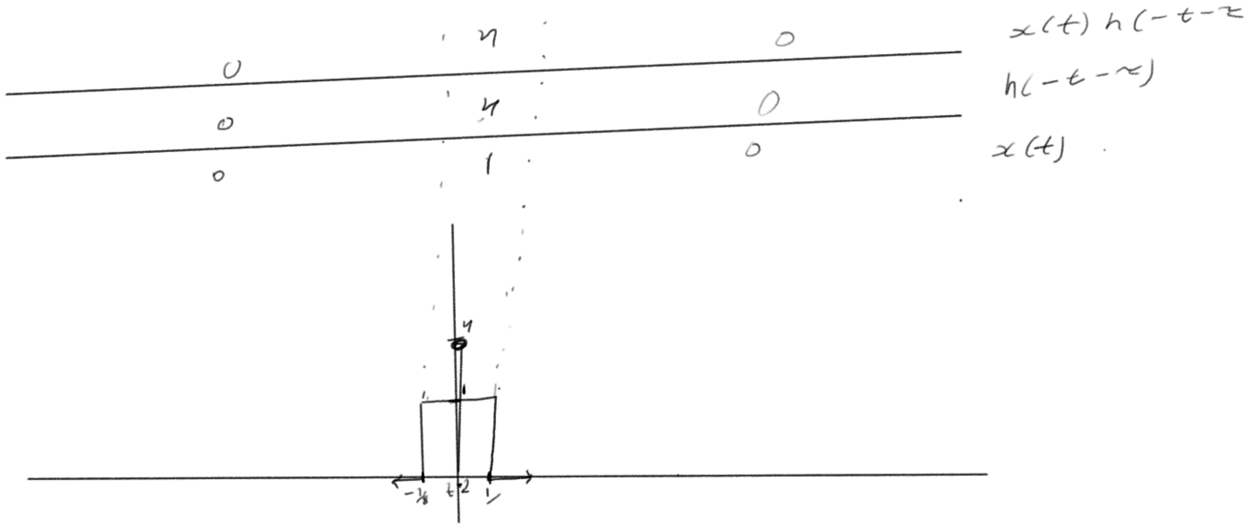
(c)

$g(4t) = x(t)$



$4\delta(t-2) = h(t)$


x(t) h(-t-τ
h(-t-τ)
x(t)
0
0
0
4
0
0
0
1
0
4
1
t-2
-1/8
1
1/8


x(t) h(-t-τ
h(-t-τ)
x(t)
0
4
0
0
4
0
0
1
0
4
1
-1/8
t-2
1

$$t - 2 = -\frac{1}{8}$$

$$t = 1.875$$

$$t - 2 = \frac{1}{8}$$

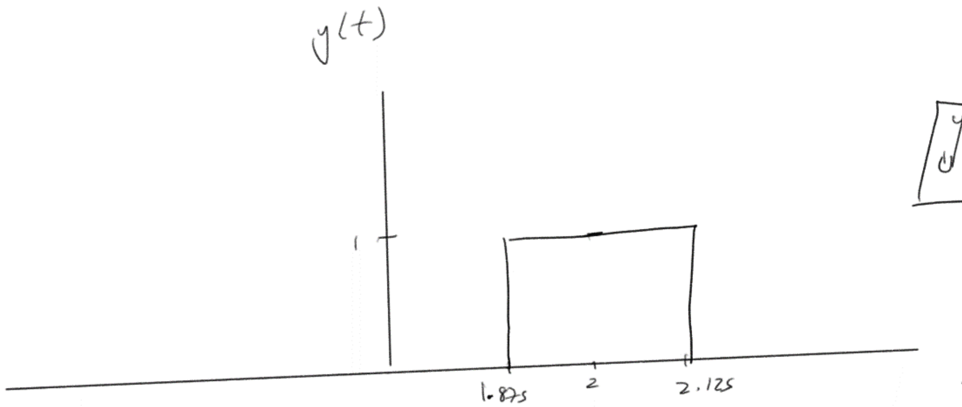
$$t = 2.125$$

$$1.875 < t < 2.125$$

$$y(t) = \int_{-1/8}^{1/8} 4$$

$$4t \Big|_{-1/8}^{1/8}$$

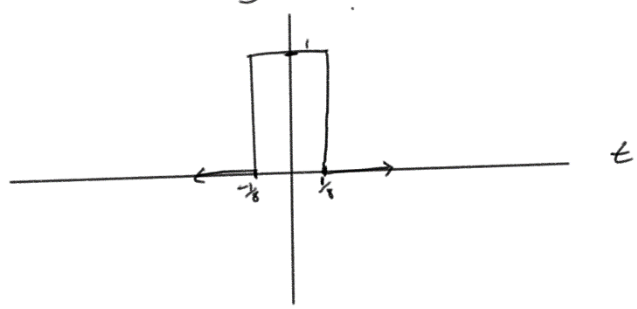
$$\boxed{y(t) = 1}$$



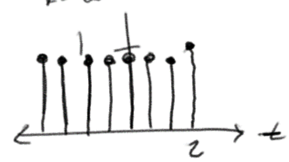
(d)

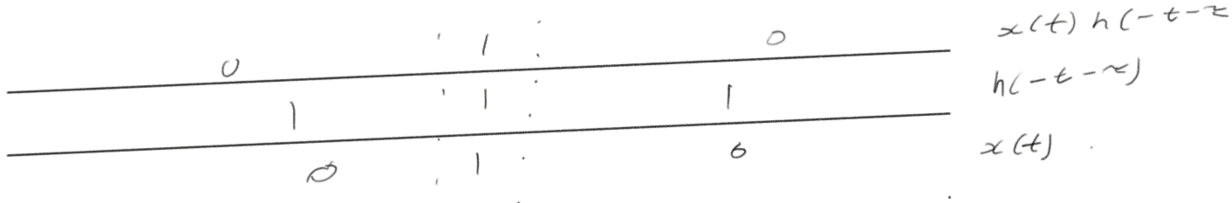
$$g(4t) * \sum_{k=-\infty}^{\infty} \delta(t-k)$$

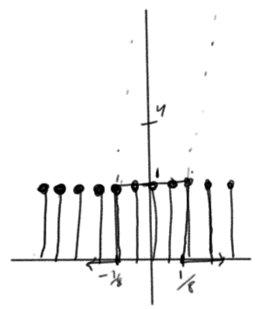
$g(4t) = x(t)$



$\sum_{k=-\infty}^{\infty} \delta(t-k) = h(t)$







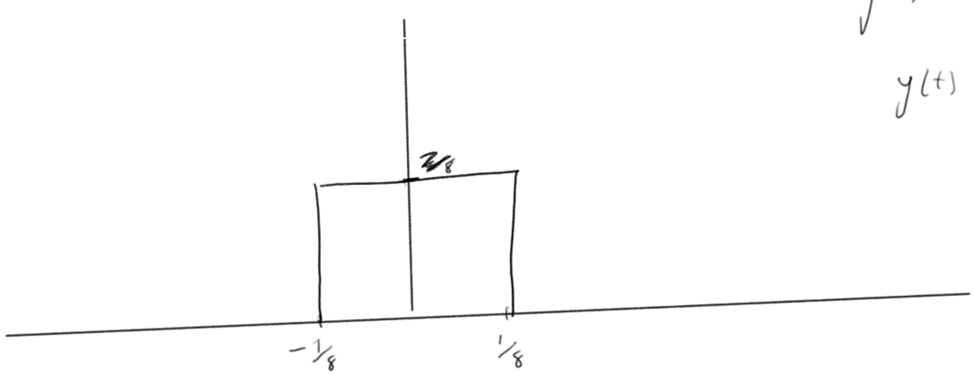
$$y(t) = \int_{-1/8}^{1/8} 1$$

$$y(t) = t \Big|_{-1/8}^{1/8}$$

$$y(t) = \frac{1}{8} + \frac{1}{8}$$

$$y(t) = 2/8$$

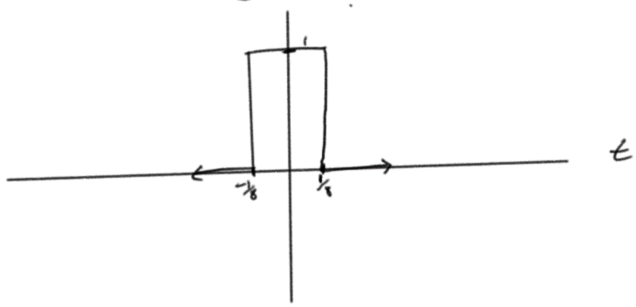
y(t)



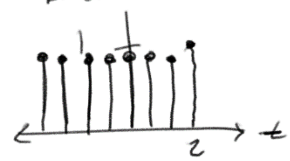
(e)

$$g(4t) * \sum_{k=-\infty}^{\infty} \delta((t-1)-k)$$

$g(4t) = x(t)$



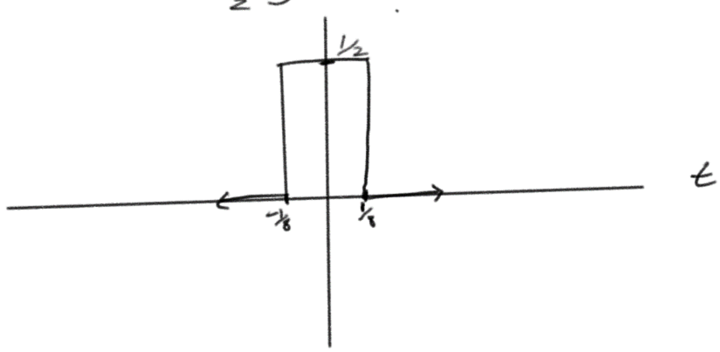
$\sum_{k=-\infty}^{\infty} \delta((t-1)-k)$



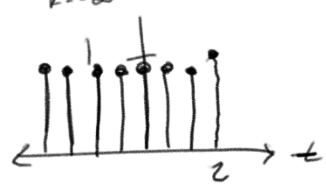
(f)

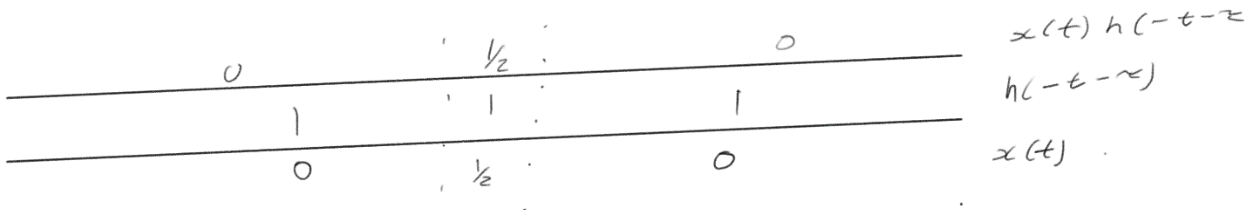
$$\frac{1}{2} g(4t) * \sum_{k=-\infty}^{\infty} \delta(t-k/2)$$

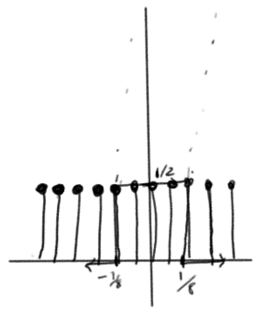
$\frac{1}{2} g(4t) = x(t)$



$\sum_{k=-\infty}^{\infty} \delta((t-k/2)$





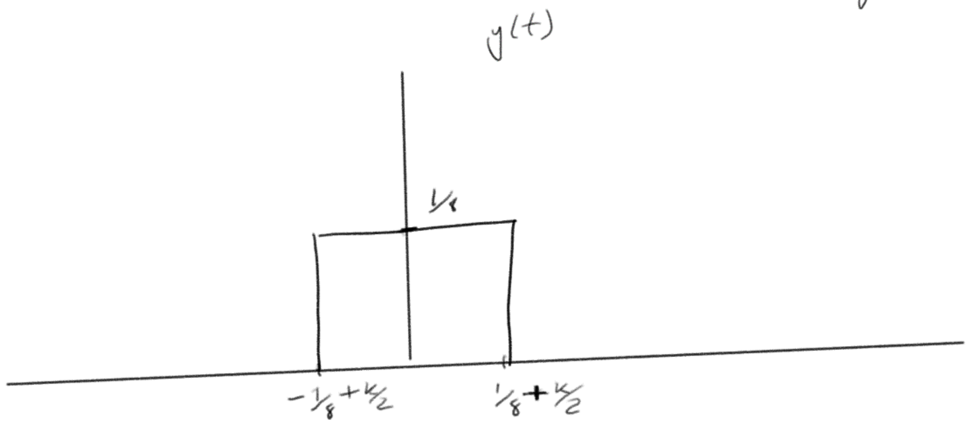


$$y(t) \int_{-1/8}^{1/8} \frac{1}{2}\, dt$$

$$y(t) = \frac{1}{2} t \Big|_{-1/8}^{1/8}$$

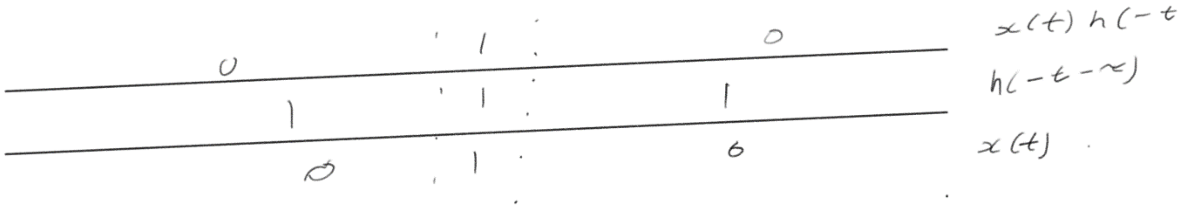
$$y(t) = \frac{1}{2}\left(\frac{1}{8}\right) + \frac{1}{2}\left(\frac{1}{8}\right)$$

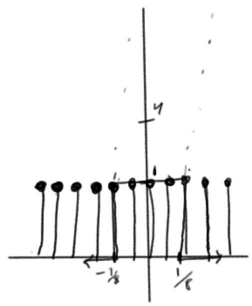
$$y(t) = 1/8$$



$$t - \frac{k}{2} = -1/8 \qquad t - k/2 = 1/8$$

$$t = -1/8 + k/2$$





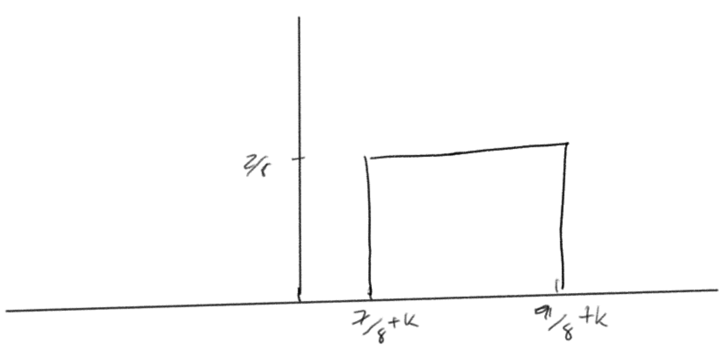
$$y(t) = \int_{-1/8}^{1/8} 1$$

$$y(t) = t \Big|_{-1/8}^{1/8}$$

$$y(t) = \frac{1}{8} + \frac{1}{8}$$

$$y(t) = 2/8$$

$y(t)$



$$t - k - 1 = -\frac{1}{8}$$

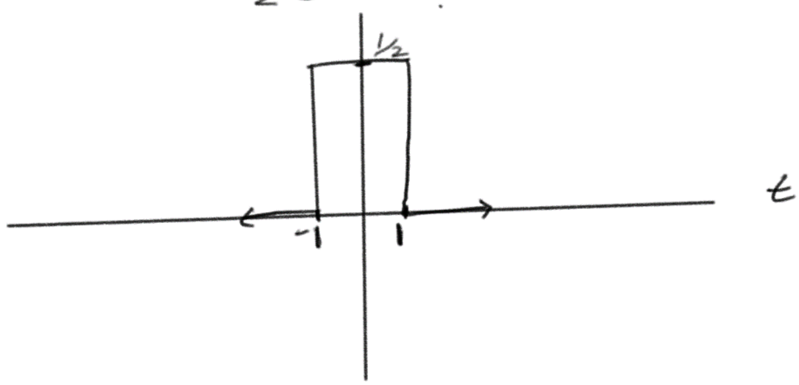
$$t = \frac{7}{8} + k$$

$$t - k - 1 = \frac{1}{8}$$

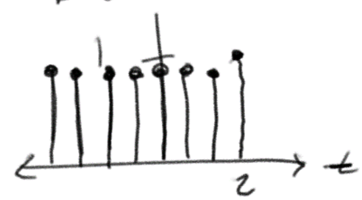
$$t = \frac{9}{8} + k$$

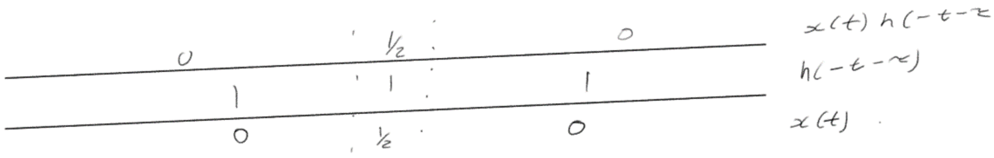
(g) $\frac{1}{2} g(t) * \sum_{k=-\infty}^{\infty} \delta(t - k/2)$

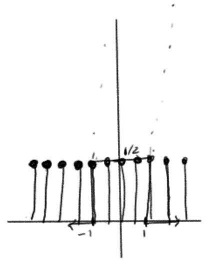
$\frac{1}{2} g(t) = x(t)$



$\sum_{k=-\infty}^{\infty} \delta((t - k/2)$



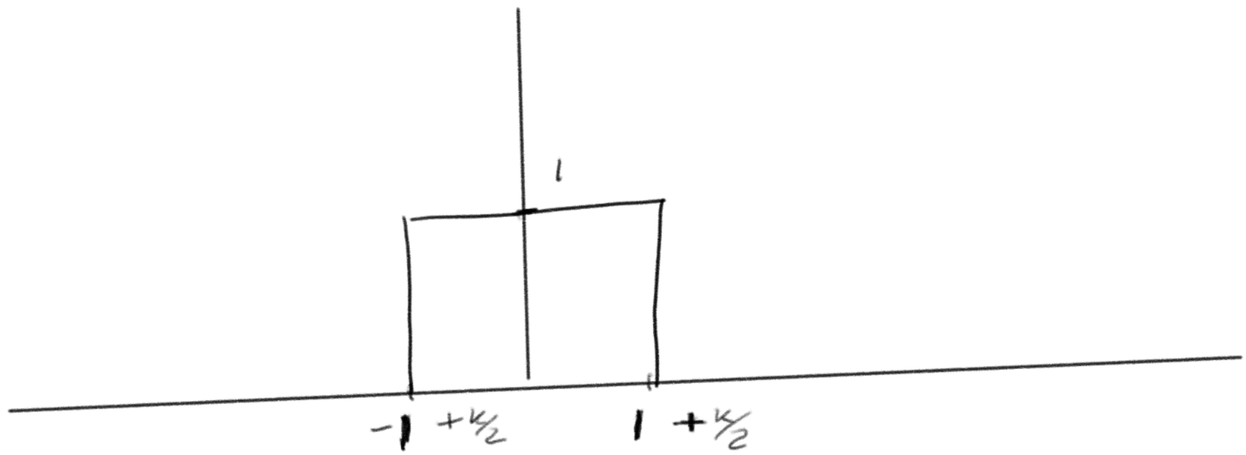




y(t)

$y(t) = \frac{1}{2} \int_{-1}^{1} 1 \, d\tau$

$y(t) = 1$



4.

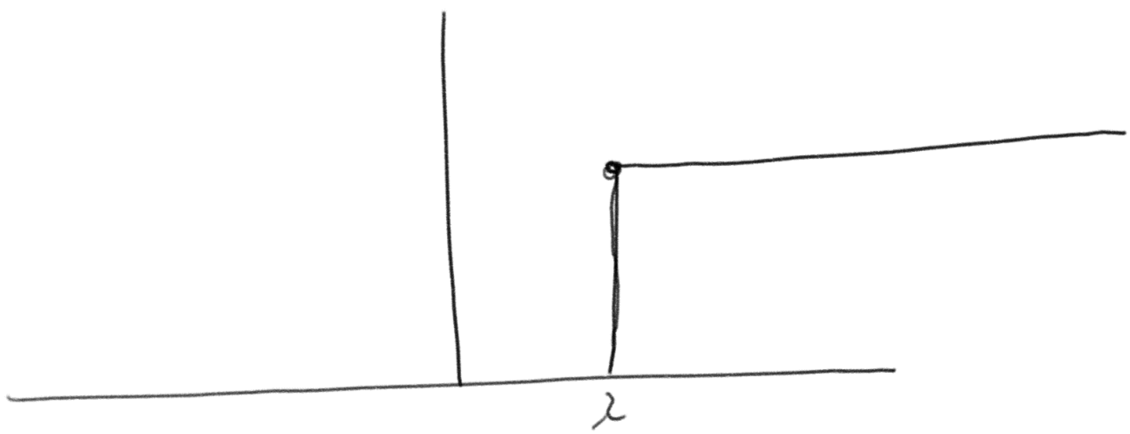
(a) $h_{eq} = h_1(t) + h_3(t) + h_4(t)$

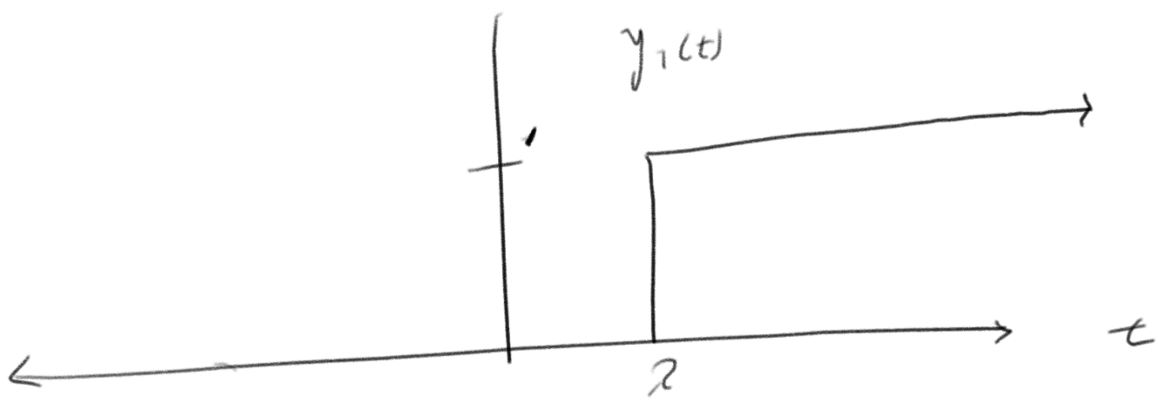
$h_2(t) = 2\delta(t)$

(b) $h(t) = u(t) + 2u(t) + u(t)$

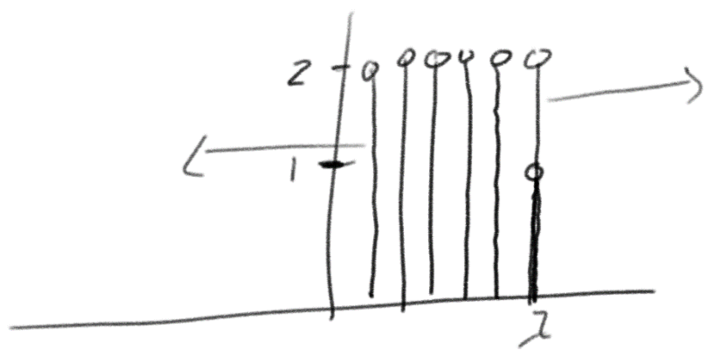
(c) $x(t) = \delta(t)$ $\quad h_1(t) = u(t)$

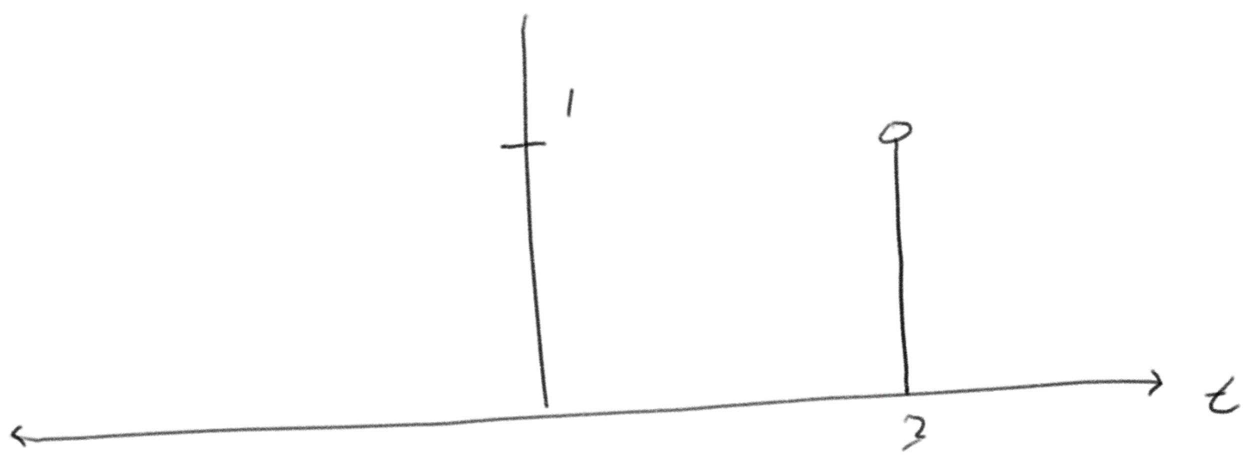
$$y_1(t) = \int_{-\infty}^{\infty} \delta(\lambda)\, u(t-\lambda)\, d\lambda = 1$$



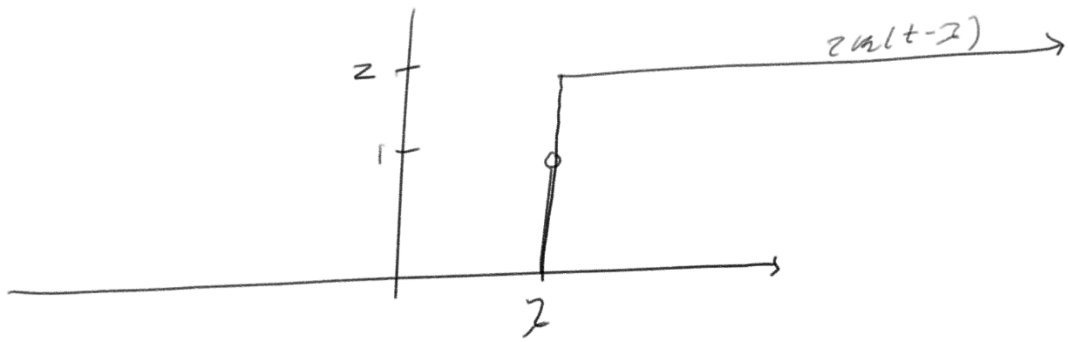


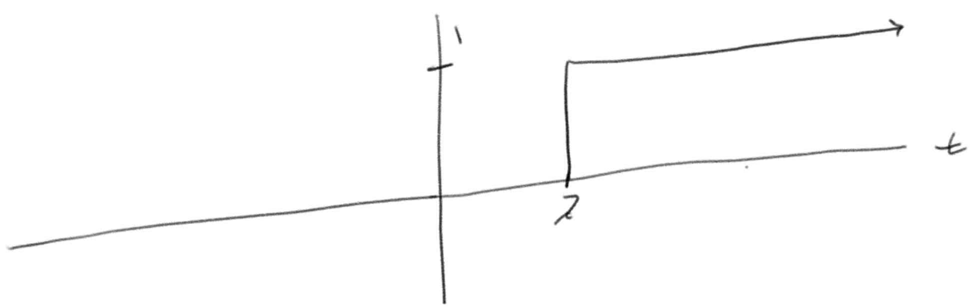
$$w(t) = \int_{-\infty}^{\infty} \delta(\lambda)\, 2\, \delta(t-\lambda)\, dt = 1$$



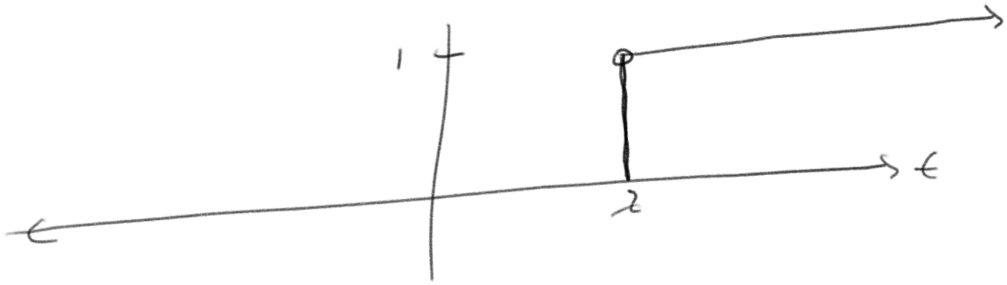


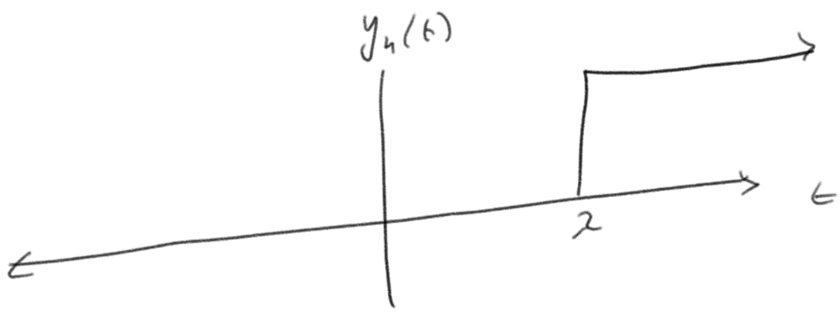
$$y_3(t) = \int_{-\infty}^{\infty} \delta(\lambda)\, 2u(t-\lambda)$$





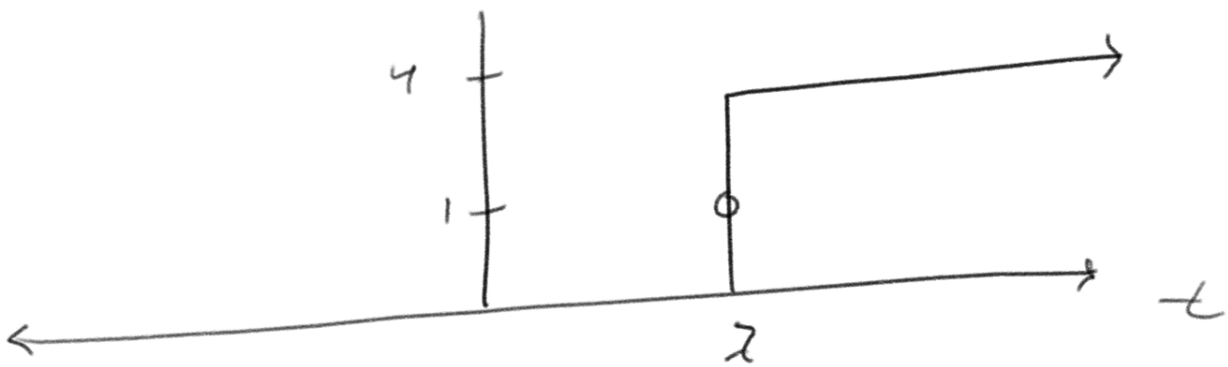
$$y_u(t) = \int_{-\infty}^{\infty} \delta(\lambda)\, u(t-\lambda) =$$





$$u(t) + 2u(t) + u(t) = 4u(t)$$

$$y(t) = \int_{-\infty}^{\infty} \delta(\tau)\, 4u(t-\tau)$$



y(t)