UNIVERSITY OF CALGARY

Lecture 2.3: Example: Initial value problems in SHM

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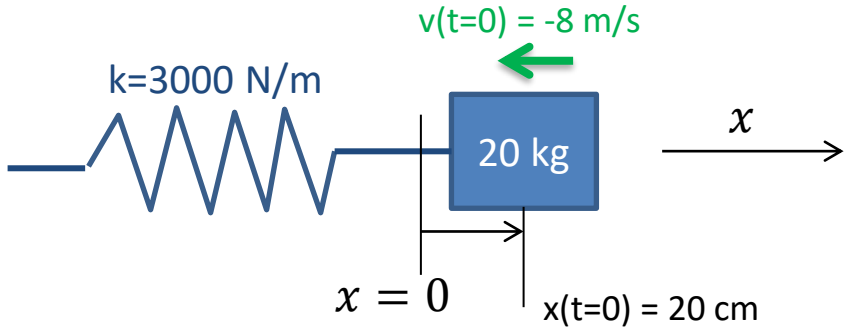
Example

A mass m=20.0 kg is attached to a spring with constant k=3000 N/m.

At t=0, x=+20 cm (measured from equilibrium) and v=-8 m/s.

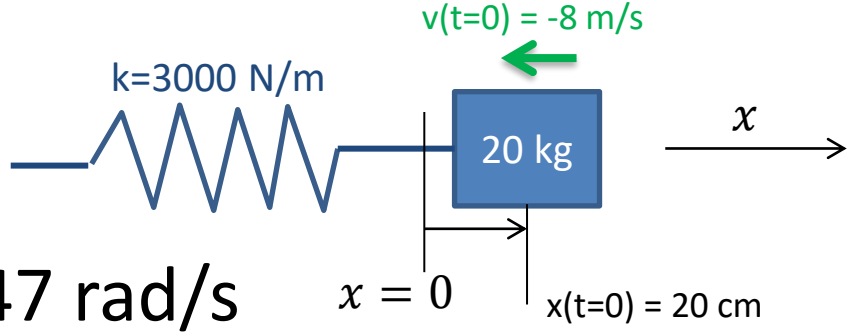
Find: ω, ϕ_0, A



Find: ω.

Use $\omega^2 = k/m$.

$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{3000\ N/m}{20\ kg}} = 12.247\ \text{rad/s}$$



Find: ϕ_0

Start with basic equations for y,v:

$$\begin{cases} x(t) = A\cos(\omega t + \phi_0) \\ v(t) = -A\omega\sin(\omega t + \phi_0) \end{cases}$$

Use the initial condition at $t = 0$:

$$\begin{cases} x(0) = A\cos(\phi_0) \\ v(0) = -A\omega\sin(\phi_0) \end{cases}$$

Divide the two equations to get:

$$\frac{-A\omega \sin\phi_0}{A\cos\phi_0} = \frac{v(0)}{x(0)} = \frac{-8\ m/s}{20\ cm}$$

$$\omega \tan\phi_0 = 40\ s^{-1}$$

$$\phi_0 = \tan^{-1}\left[\frac{40 s^{-1}}{12.247\ s^{-1}}\right]$$

$$\phi_0 = \tan^{-1}(3.27)$$

But **BE CAREFUL HERE**:

$$\tan^{-1}(3.27) = 1.27 + \boldsymbol{n\pi}$$

How to choose n?

Only one solution fits the signs on the initial conditions. We want:

$$x(0) = A\cos(\phi_0) = +20\ \text{cm}$$

BUT

$$\cos(1.27) = 0.296$$
$$\cos(1.27 \pm \pi) = -0.296$$

The initial position should be positive, so choose:

$$\boxed{\phi_0 = 1.27\ \text{rad} = 73°}$$

Find: A.

Back-substitute

$$x(0) = A\cos(\phi_0)$$

$$A = \frac{x(0)}{\cos(\phi_0)} = \frac{20\ cm}{\cos(1.27)}$$

A = 68 cm