UNIVERSITY OF CALGARY

Lecture 2.7
Sinusoidal waves

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Sinusoidal waves

A <u>sinusoidal wave</u> is a special case where the function $f(x \pm vt)$ is a sinusoid:

initial phase

$$\boxed{y(x,t) = A\cos(kx \pm \omega t + \phi_0)}$$

amplitude

"wave number"

$k = \frac{2\pi}{\lambda}$

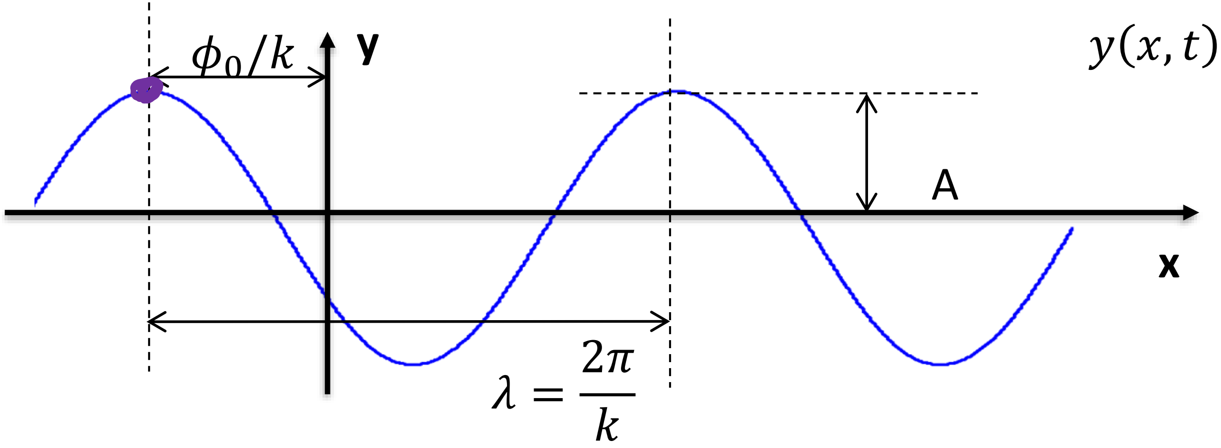
"angular frequency"

$\omega = 2\pi f = \frac{2\pi}{T}$

The wave moves at a speed ("phase speed")

$$v = \frac{\omega}{k} = f\lambda$$

$$y(x,t) = A\cos(kx \pm \omega t + \phi_0)$$



Snapshot at time t=0

At time t=0, the crest of the wave is at the location $x = -\dfrac{\phi_0}{k} = -\dfrac{\phi_0}{2\pi}\lambda$

↳ where $kx + \phi_0 = 0$ ⟶

As t increases, crest is where $kx \pm \omega t + \phi_0 = 0$

$\Rightarrow x_{crest} = -\dfrac{\phi_0}{k} \mp \dfrac{\omega}{k} t$

$\begin{cases} kx + \omega t: \text{Wave moves to left at } v = \omega/k \\ kx - \omega t: \text{Wave moves to right at } v = \omega/k \end{cases}$

Moving left/right

Moves left

Recall: wave function $y(x,t) = \mathrm{f}(\mathrm{x} \pm vt)$

Moves right

These move to the right:

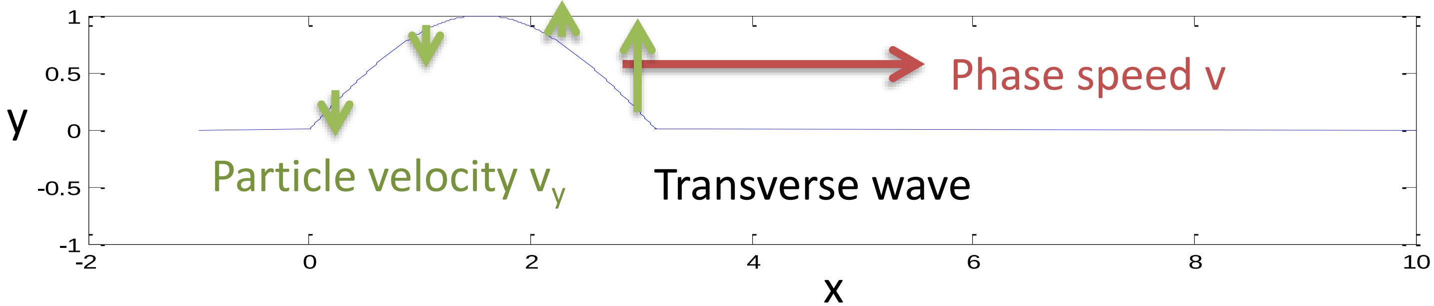
$$y(x,t) = A\cos(kx - \omega t + \phi_0)$$
$$y(x,t) = A\cos(\omega t - kx + \phi_1)$$

These are the same wave if $\phi'_0 = -\phi_1$.

These move to the left:

$$y(x,t) = A\cos(\omega t + kx + \phi_0)$$
$$y(x,t) = A\cos(kx + \omega t + \phi_0)$$

You may encounter any of these forms.



Phase (wave) velocity vs. particle velocity

- Phase velocity (wave velocity) is the velocity at which the wave propagates
- In the x direction above
- Constant – not a function of position: $v = \omega/k$

- Particle velocity (medium velocity) is the velocity of the particles in the medium
- Use the symbol v_y
- Transverse wave: in the y direction

$$v_y = \frac{\partial y(x,t)}{\partial t}$$

- Longitudinal wave: in the same direction as the wave (x)
- Function of position – not constant

Particle (medium) velocity/acceleration

For a sinusoidal wave

$$y(x,t) = A\cos(kx \pm \omega t + \phi_0)$$

$$v_y = \frac{\partial y}{\partial t} = \mp \omega A \sin(kx \pm \omega t + \phi_0)$$

$$a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A \cos(kx \pm \omega t + \phi_0)$$

Expressing in terms of sin()

Can rewrite

$$y(x,t) = A\cos(\omega t \pm kx + \phi_0)$$

as

$$y(x,t) = A\sin(\omega t \pm kx + \phi_0 + \pi/2)$$

Since $\cos(\theta) = \sin(\theta + \pi/2)$

Also, note that sinusoids flip sign every π radians:

$$A\sin(\omega t \pm kx + \phi) = -A\sin(\omega t \pm kx + \phi + \pi)$$

Example

Re-write

$$y(x,t) = -3\sin(kx + \omega t + 60°)$$

in the form

$$y(x,t) = A\cos(kx + \omega t + \phi_0)\ .$$

Where $A > 0$. What is the phase constant ϕ_0 ?

$$y(x,t) = -3\sin\left(kx + \omega t + \pi/3\right)$$

$$= +3\sin\left(kx + \omega t + \frac{\pi}{3} + \pi\right)$$

$$= 3\sin\left(kx + \omega t + 5\pi/6 + \frac{\pi}{2}\right)$$

$$= 3\cos\left(kx + \omega t + \frac{5\pi}{6}\right)$$

$$\phi_0 = \frac{5\pi}{6}$$