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Question 2

Let A be the set containing **fuel** filled at each station according to our greedy algorithm, A = $\{a_1,a_2,a_3,\ldots,a_n\}$. Also let K be the set containing fuel filled at each station from the optimal solution, K = $\{k_1,k_2,k_3,\ldots,k_n\}$.

Using Exchange Argument:

If A is not same as K then there exists an station lets say k_i that does not belong to A. At i'th station, our greedy algorithm instead fills a_i amount. Now we replace a_i in A, with k_j. If a_i less than equal k_i , this means A is more economical than K, however if $k_i < a_i$, there are following possibilities:

1. **If we do sum of a's to i'th and its less than the sum of k's then our greedy algorithm is performing better.**
2. **The oher possibilities are that we do sum to i-1 stations and in this case sum of a's is greater or equal to sum of k's which is not possible.**

Hence,

This proves our greedy algorithm works as efficient as the optimal solution