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**Question 1:**

Let  $A$  be the set containing all the pairs of  $L_r$  and  $L_b$  from one greedy algorithm,  $A = \{(a_{r1}, a_{b1}), (a_{r2}, a_{b2}), \dots, (a_{rn}, a_{bn})\}$ . Also let  $K$  the set containing pairs from the optimal solution,  $K = \{(k_{r1}, k_{b1}), (k_{r2}, k_{b2}), \dots, (k_{rn}, k_{bn})\}$

By exchange argument:

If  $A$  is not same as  $K$ , then here is an  $(k_{rj}, k_{bj})$  belongs to  $K$  that does not belong to  $A$ . Instead  $A$  contains  $(k_{rj}, k_{bj'})$  and  $(k_{rj''}, k_{bj})$ , such that we have two cases:

1. Difference of  $(k_{rj}, k_{bj})$  is greater than or equal to both difference of  $(k_{rj}, k_{bj'})$  and  $(k_{rj''}, k_{bj})$ , In this case our algorithm is performing better than optimal solution already.
2. Difference of  $(k_{rj}, k_{bj'})$  is less than one of the difference of  $(k_{rj}, k_{bj})$  and  $(k_{rj''}, k_{bj})$ , Lets say  $(k_{rj}, k_{bj})$  is less than  $(k_{rj}, k_{bj'})$ , then  $k_{bj'}$ 's pair in  $K$  would either
  - Produce a difference greater than  $(k_{rj}, k_{bj})$  in which case our algorithm's cumulative answer will be less than equal to optimal solution's algorithm.
  - Or  $k_{bj'}$ 's pair say  $(k_{rj'''}, k_{bj'})$  in  $K$  could be a part of chain such that,  $k_{rj'''}$  or any consequent pair in chain if we continue on this sequence would produce a cumulative sum of differences greater than or equal to greedy solution.

This proves that greedy approach produces the same result as our optimal solution does.

**Took help from friend (batch mate).**