Postulates and Theorems

Postulates and Theorems of Boolean Algebra

Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Theorem 4, associative	(a)	$x + (y + z) = (x + y)$	(b)	$x(yz) = (xy)z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

Your Turn

$A + A\overline{B} = ?$

- Solution = A

(A+B) . (A+C) = ?

- Solution = A + BC

(A+A'B) ?

- Solution = A+B

Today's Lecture Outline

- Boolean Function
- Gate Implementation
- Minimization of function
  - Algebraic Manipulations
- Complement of a Boolean function

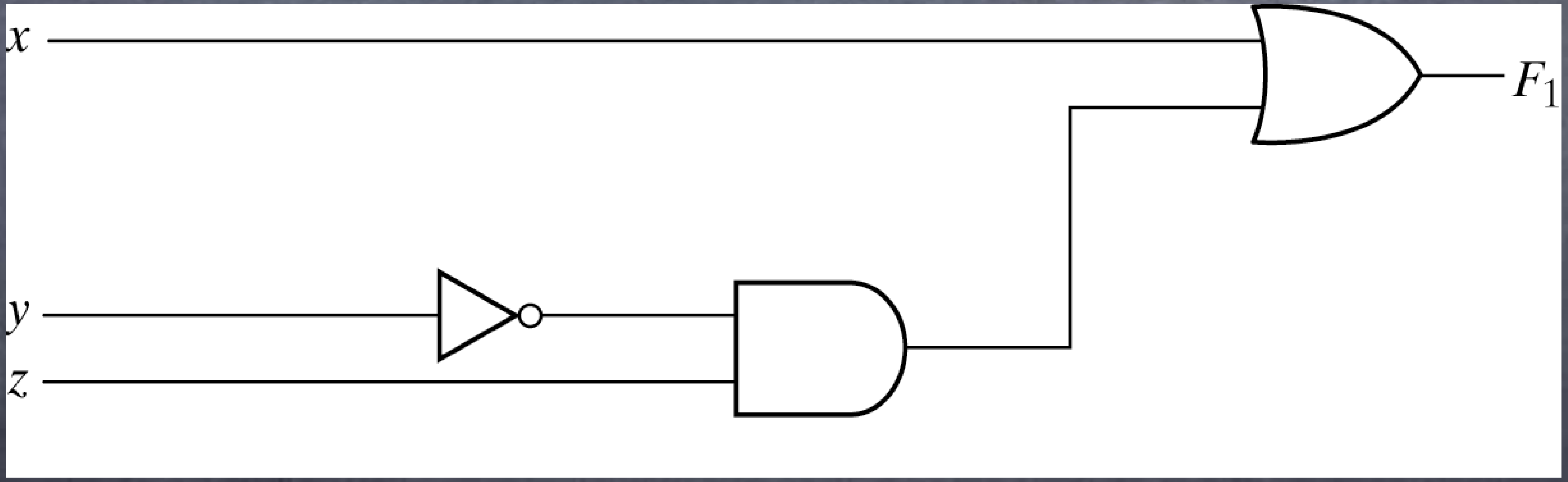
Boolean Function

Boolean algebra is an algebra that deals with the binary variables and logic operations

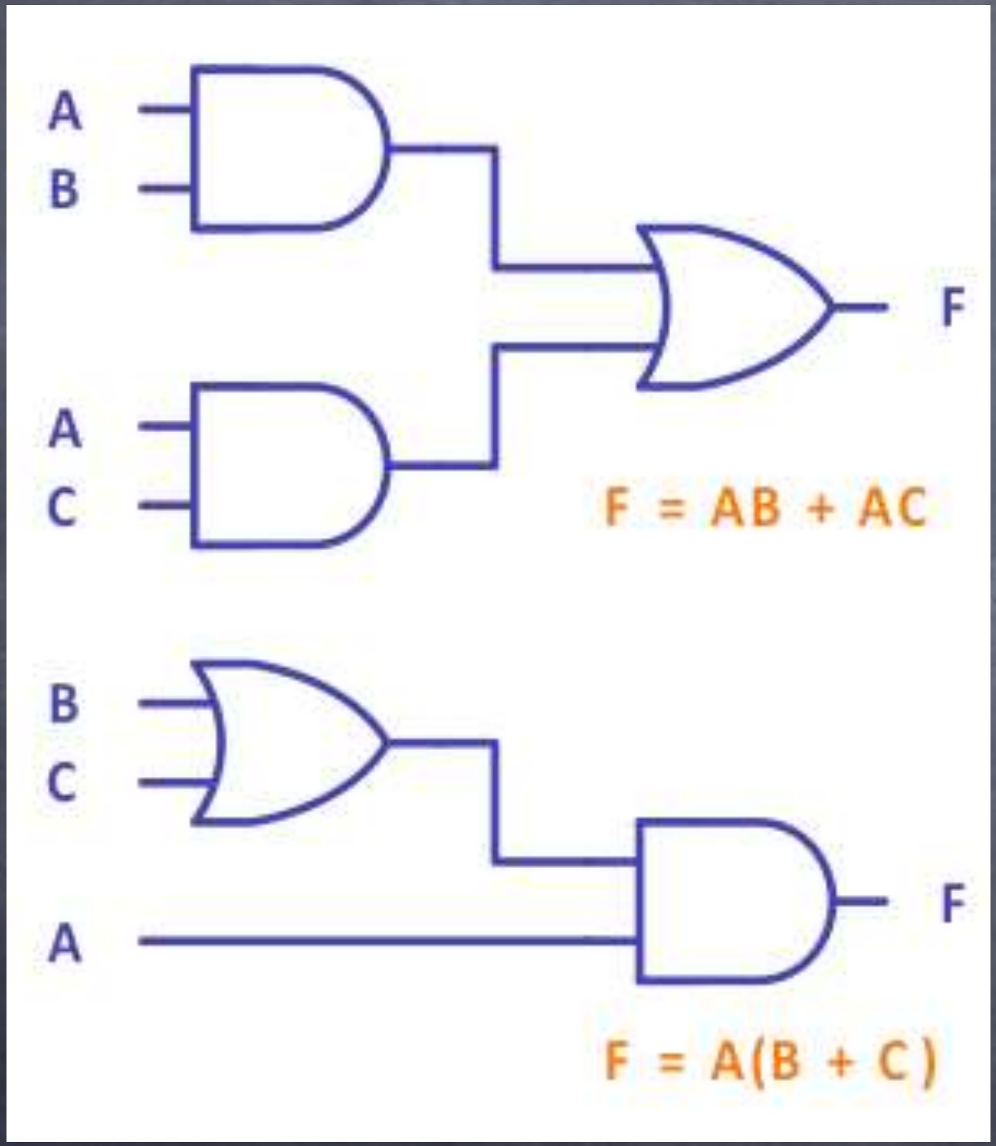
A boolean Function described by an expression. For given value of binary variables the Boolean Function can be 1 or 0

Function as a Gate Implementation

- **A** Boolean function **can be transformed from an algebraic expression into** circuit diagram **composed of** logic gates**.**
  - **–F_1 = x + y'z**
  - **–The logic-circuit diagram for this function is shown below:**

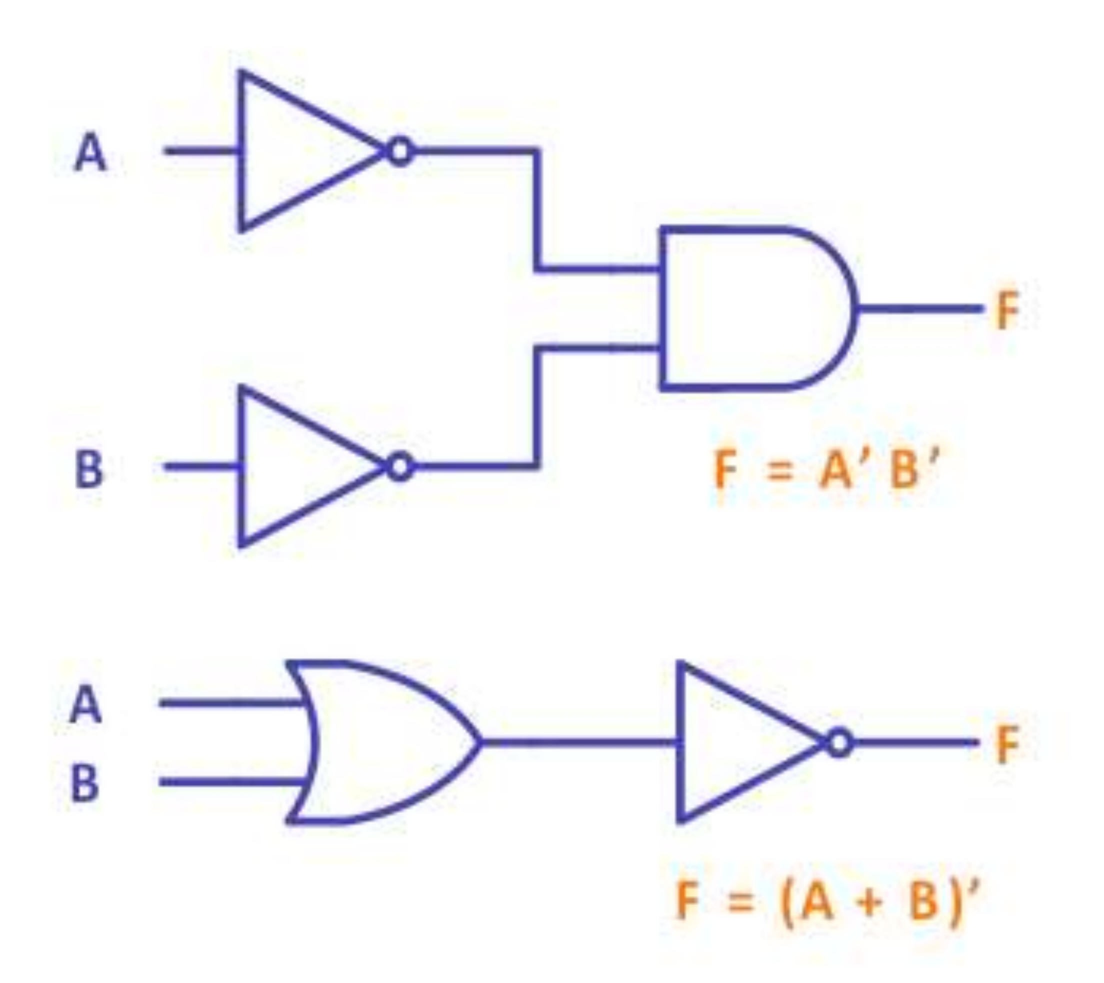


Gate Implementation (Examples)



A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Gate Implementation (Examples)



A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

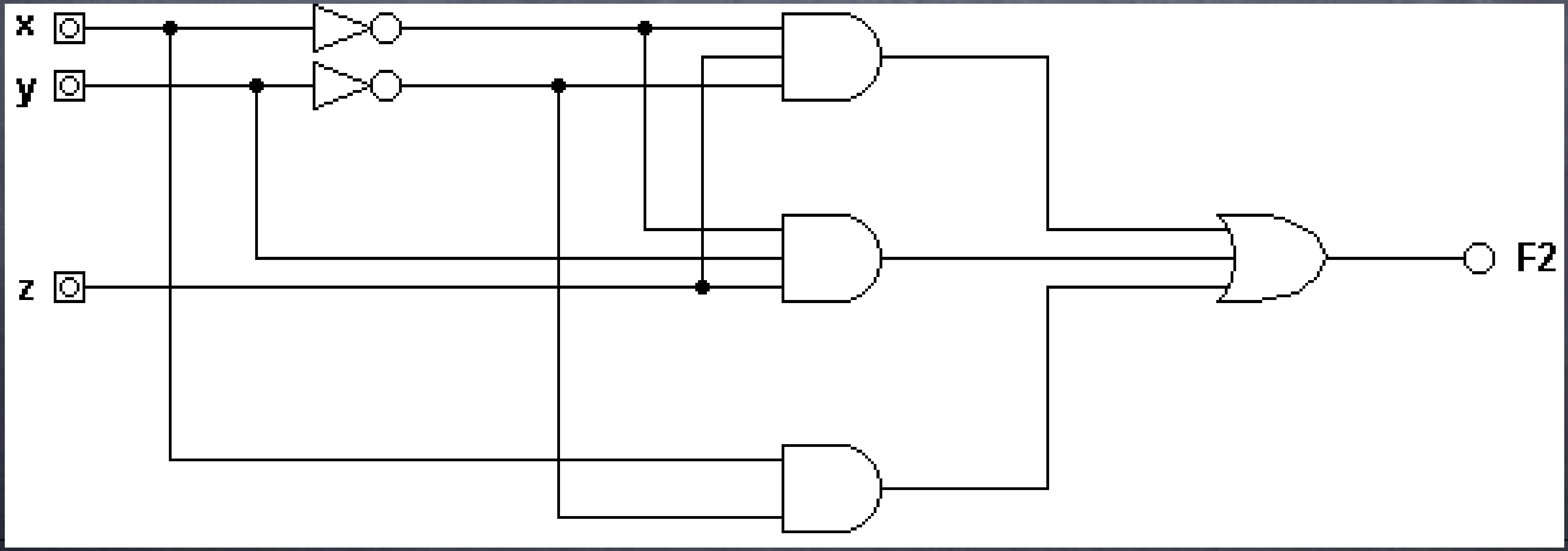
Minimization

- **Functions in algebraic form can be represented in various ways.**
  - **–Remember the postulates and theorems that allows us to represent a function in various ways.**
- **We must keep in mind that the algebraic expression is representative of the gates and circuitry used in a hardware piece.**
  - **–We want to be able to minimize circuit design to reduce cost, power consumption, and package count, and to increase speed.**
- **By manipulating a function using the postulates and theorems, we may be able to minimize an expression.**

Minimization of the F_2

The following is an example of a non-minimized function:

$$F_2 = x'y'z + x'yz + xy'$$

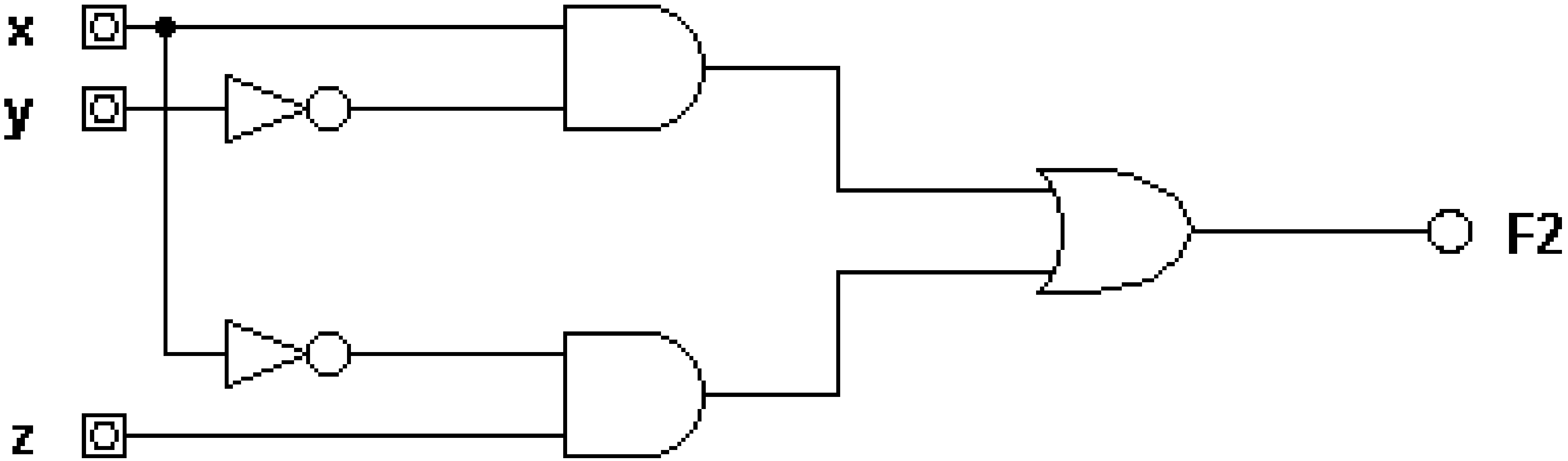


Minimization of the F_2

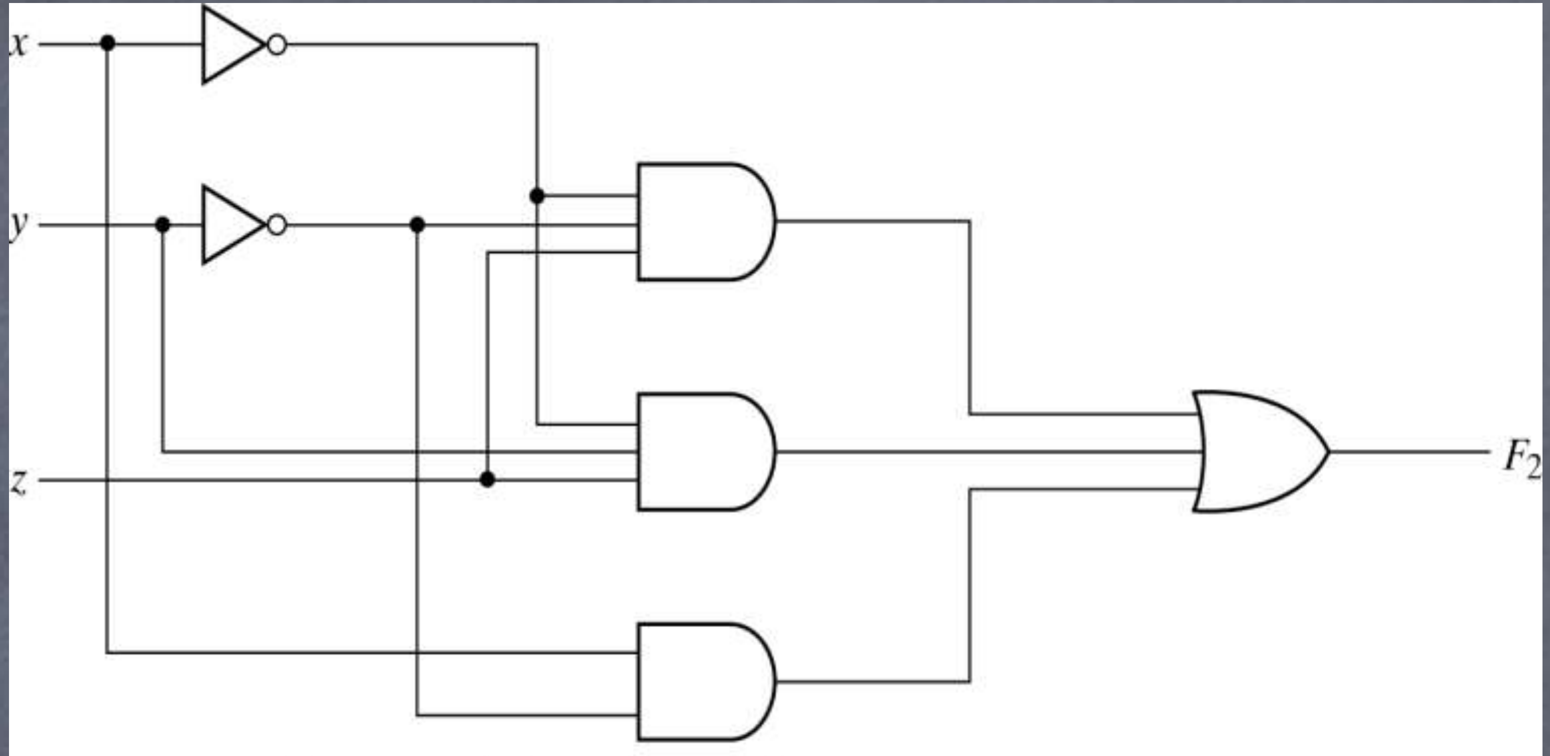
The function can be minimized as follows:

$x'y'z + x'yz + xy' =$

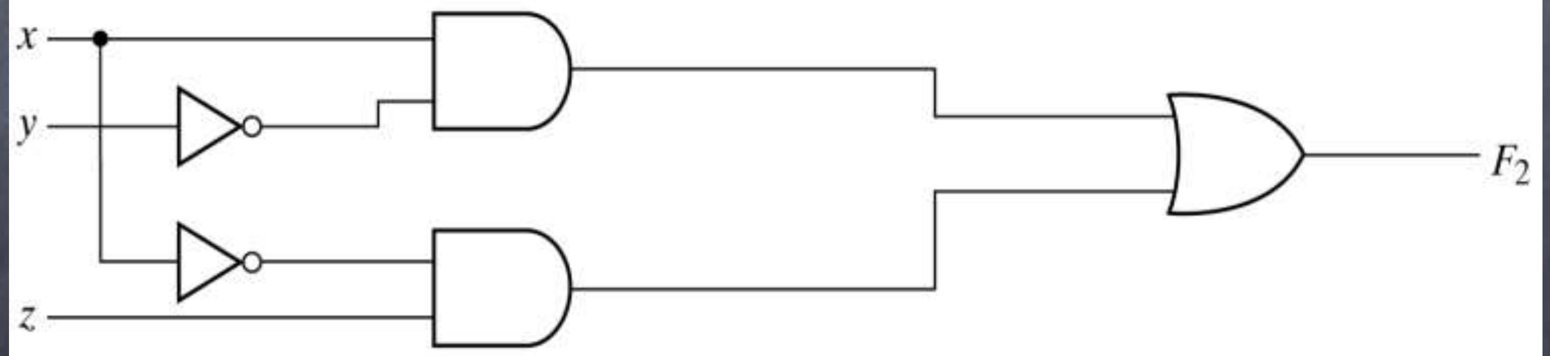
$= x'z \cdot (y' + y) + xy'$	by postulate:	4(a)
$= x'z \cdot 1 + xy'$		5(a)
$= x'z + xy'$		2(b)



Implementation of Boolean Function



(a) $F_2 = x'y'z + x'yz + xy'$



(b) $F_2 = xy' + x'z$

Fig. 2-2 Implementation of Boolean function F_2 with gates

Algebraic Manipulation

- By reducing the number of terms, the number of literals (single variable) or both in a Boolean function, it is possible to obtain a simpler circuit, as each term requires a gate and each variable within the term designates an input to the gate

$F_1 = x'y'z + x'yz + xy'$ contains 3 terms and 8 literals
$F_2 = x'z + xy'$ contains 2 terms and 4 literals.

Example Manipulations

- The following are some example manipulations:
  - $x(x' + y) = ?$
    $x(x' + y) = xx' + xy = 0 + xy = xy$
  - $x + x'y = ?$
    $x + x'y = (x + x')(x + y) = 1(x + y) = x + y$
  - $(x + y)(x + y') = ?$
    $(x + y)(x + y') = x + xy + xy' + yy' = x(1 + y + y') = x$
  - $xy + x'z + yz = ?$
    $= xy + x'z + yz(x + x')$
    $= xy + x'z + xyz + x'yz$
    $= xy(1 + z) + x'z(1 + y)$
    $= xy + x'z$ Consensus Theorem

Consensus Theorem

The End