Complement of a Function

- The complement of a function F is F'.
  It is obtained by interchanging 0's for 1's and 1's for 0's in the value of F.

- The complement of a function may be derived algebraically through DeMorgan's theorem.

  Theorem 5(a) (DeMorgan): $(x + y)' = (x' \cdot y')$
  Theorem 5(b) (DeMorgan): $(x \cdot y)' = (x' + y')$

- Example:

  $F_1 = x'yz' + x'y'z$
  $F_1' = (x'yz' + x'y'z)'$
  $= (x + y' + z)(x + y + z')$

Complement of a Function (Example)

If $F_1 = A+B+C$

Then F_1'

$=(A+B+C)'$

$= (A+X)'$ let $B+C = X$

$= A'X'$ by DeMorgan's

$= A'(B+C)'$

$= A'(B'C')$ by DeMorgan's

$= A'B'C'$ associative

Complement of a Function (More Examples)

(x'yz' + x'y'z)'
= (x'yz')' (x‘y'z)'
= (x+y'+z) (x+y+z')

[x(y'z'+yz)]'
= x' + (y'z'+yz)'
= x' + (y'z')' (yz)'
= x' + (y+z) (y'+z')

A simpler procedure

take the dual of the function (interchanging AND and OR operators and 1’s and 0’s) and complement each literal. {DeMorgan’s Theorem}

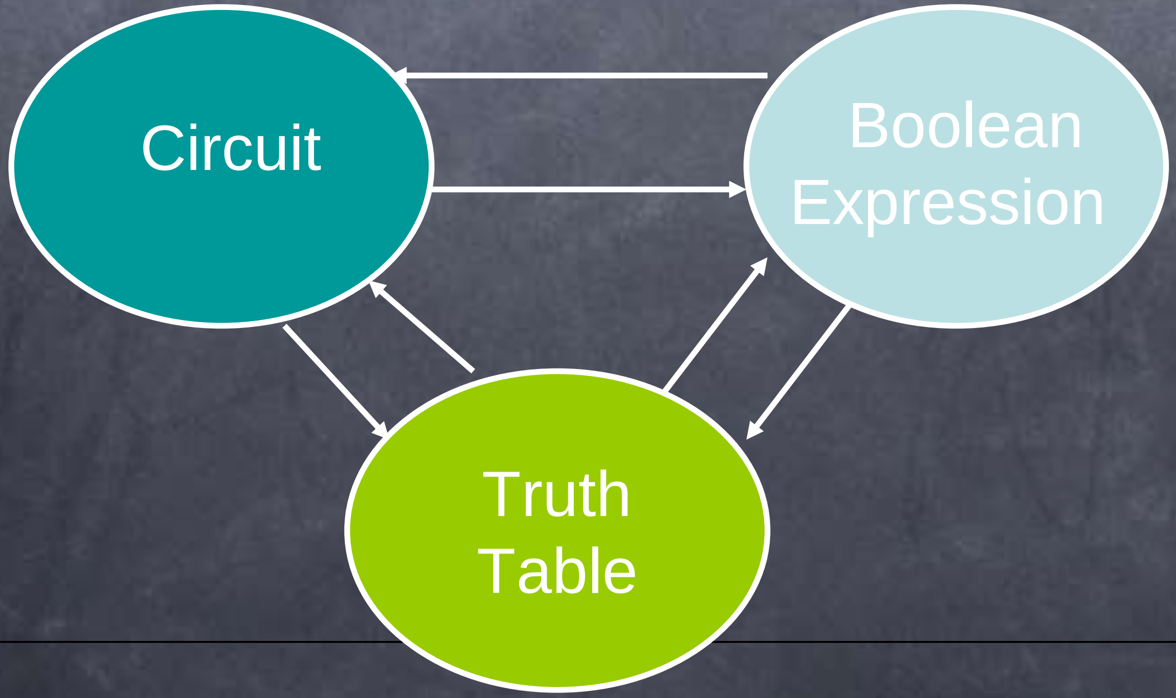
x'yz' + x'y'z

The dual of function is (x'+y+z') (x'+y'+z)

Complement of each literal: (x+y'+z)(x+y+z')

Representation Conversion

- Need to transition between boolean expression, truth table, and circuit (symbols).
- Converting between truth table and expression is easy.
- Converting between expression and circuit is easy.
- More difficult to convert to truth table.



Your Turn

Minimize the following Boolean Function

$A' B (D' + C'D) + B (A + A' C D)$?

Answer = ?

$(x' y' + z)' + z + x y + w z$?

Answer = ?

Your Turn Solution

A’ B (D’+ C’D) + B (A + A’ C D) ?

$$A'B(D'+C'D) + B(A+A'CD)$$

$$A'B(D'+C') + B(\overset{1}{\cancel{(A+A')}}(A+CD))$$

$$A'BD' + A'BC' + B(A+CD).$$

$$A'BD' + A'BC' + AB + BCD.$$

$$= B(A'B' + A'B + A) + BCD.$$

$$= B(A'B' + A + B)$$

$$B(\cancel{(A+A')}(A+B') + B)$$

$$= B(1) + BCD.$$

$$= B(1+CD)$$

$$= B.$$

Your Turn Solution Contd..

(x' y' + z)' + z + x y + w z ?

$$(x'y' + z)' + z + xy + wz.$$

$$(x+y)\cdot z' + z(\overset{1}{\cancel{1+w}}) + xy$$

$$(\overset{1}{\cancel{z+z'}})(z+x+y) + xy$$

$$(x+y+z) + xy$$

$$(x+y+z+x)(x+y+z+y)$$

$$(x+y+z)(x+y+z)$$

$$= (x+y+z) \quad \text{Ans.}$$

Canonical Forms

- A canonical form is a standard method for representing Boolean functions.
- The two canonical forms that are used are:
  - Sum of Minterms
  - Product of Maxterms
- These forms are sometimes considered the “brute force” method of representing functions as they seldom represent a function in a minimized form.

Minterms

- Any given binary variable can be represented in two forms:
  - x, its normal form, and
  - x', its complement
- If we consider two binary variables and the AND operation, there are four combinations of the variables:
  - xy
  - xy'
  - x'y
  - x'y'
- Each of the above four AND terms is called a minterm or a standard product.
- n variables can be combined to form 2^n minterms.

Minterms Expressed
Two Variables

X	Y	Minterm	Notation
0	0	x’ Y’	m_0
0	1	x’ Y	m_1
1	0	x Y’	m_2
1	1	x Y	m_3

Minterms Expressed Three Variables

				Minterms	
x	y	z		Term	Designation
0	0	0		$x'y'z'$	m_0
0	0	1		$x'y'z$	m_1
0	1	0		$x'yz'$	m_2
0	1	1		$x'yz$	m_3
1	0	0		$xy'z'$	m_4
1	0	1		$x'yz'$	m_5
1	1	0		xyz'	m_6
1	1	1		xyz	m_7

Maxterms

- Any given binary variable can be represented in two forms:
  - x, its normal form, and
  - x', its complement
- If we consider two binary variables and the OR operation, there are four combinations of the variables:
  - $x + y$
  - $x + y'$
  - $x' + y$
  - $x' + y'$
- Each of the above four OR terms is called a maxterm or a standard sum.
- n variables can be combined to form 2^n maxterms.
- Each maxterm is the complement of its corresponding minterm and vice-versa.

Maxterms Expressed

Three Variables

			Maxterms	
x	y	z	Term	Designation
0	0	0	$x + y + z$	M_0
0	0	1	$x + y + z'$	M_1
0	1	0	$x + y' + z$	M_2
0	1	1	$x + y' + z'$	M_3
1	0	0	$x' + y + z$	M_4
1	0	1	$x' + y + z'$	M_5
1	1	0	$x' + y' + z$	M_6
1	1	1	$x' + y' + z'$	M_7

Truth Table to Expression (Sum of Minterms)

- Any Boolean function can be expressed as a sum of minterms or sum of products (i.e. the ORing of terms).
  - You can form the function algebraically by forming a minterm for each combination of the variables that produces a 1 in the function. (Each row with output of 1 becomes a product term)
  - Sum (OR) product terms together.

x	y	z	G
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

xyz + xyz' + x'yz

Sum of Minterms Example

x	y	z		Function F_1	Required Minterms
0	0	0		1	$x'y'z'$
0	0	1		0	
0	1	0		0	
0	1	1		1	$x'yz$
1	0	0		1	$xy'z'$
1	0	1		0	
1	1	0		0	
1	1	1		0	

$$F_1 = x'y'z' + x'yz + xy'z'$$

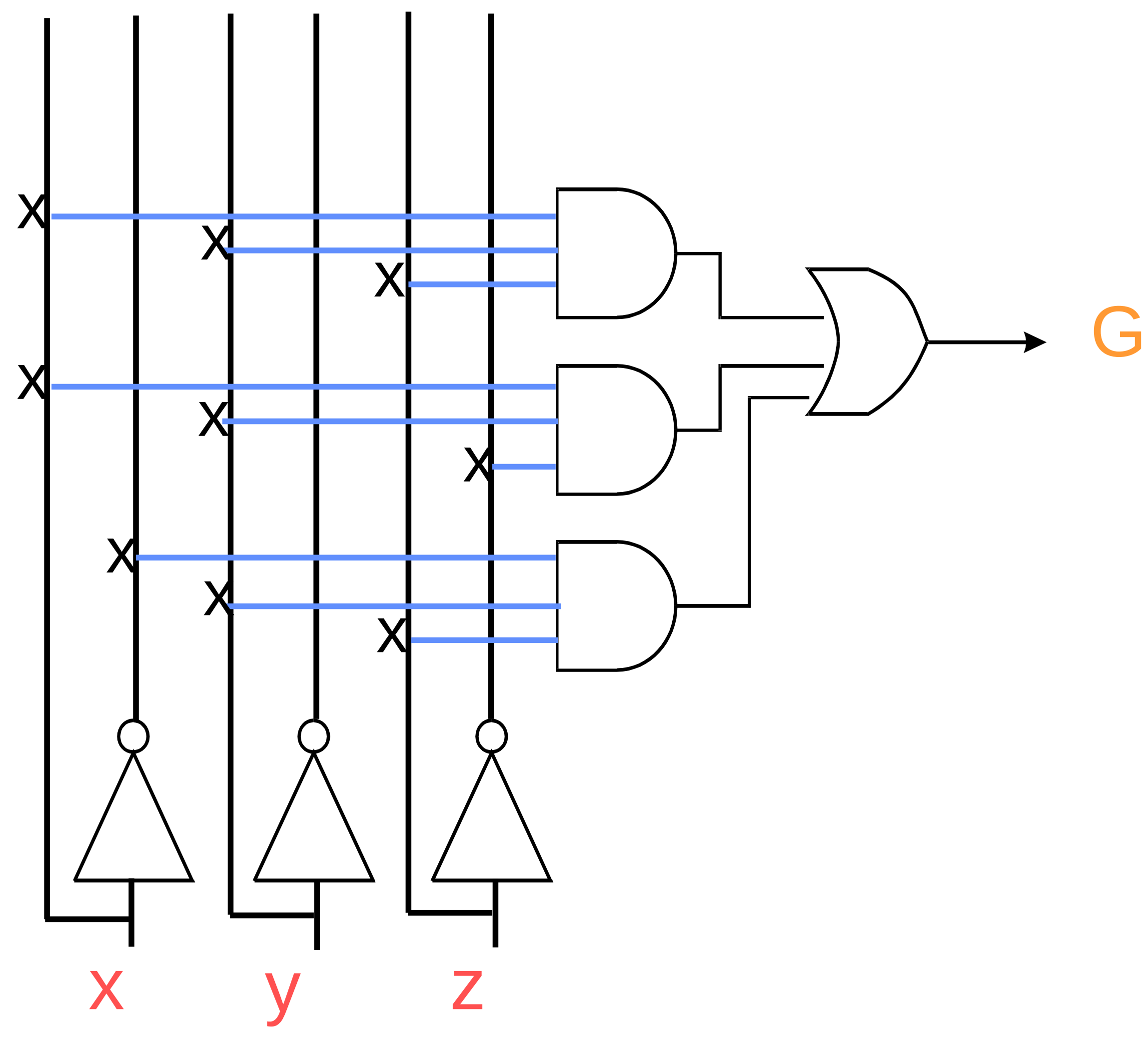
$$= m_0+m_3+m_4$$

$$= \sum(0,3,4)$$

Equivalent Representations of Circuits

- **All three formats are equivalent**
- **Number of 1's in truth table output column equals AND terms for Sum-of-Products (SOP)**

x	y	z	G
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



G = xyz + xyz' + x'yz

Truth Table to Expression (Product of Maxterms)

- Any Boolean function can be expressed as a product of maxterms or product of sums (i.e. the ANDing of terms).
  - You can form the function algebraically by forming a maxterm for each combination of the variables that produces a 0 in the function. (Each row with output of 0 becomes a standard sums)
  - AND these maxterms together.

Product of Maxterms Example

x	y	z		Function F_1	Required Maxterms
0	0	0		1	
0	0	1		0	$x + y + z'$
0	1	0		0	$x + y' + z$
0	1	1		1	
1	0	0		1	
1	0	1		0	$x' + y + z'$
1	1	0		0	$x' + y' + z$
1	1	1		0	$x' + y' + z'$

$$F_1 = (x + y + z')(x + y' + z)(x' + y + z')(x' + y' + z)(x' + y' + z')$$

$$= M_1 M_2 M_5 M_6 M_7$$

$$= \prod(1,2, 5, 6, 7)$$

Summary

- Canonical Form of Boolean Function?
  - Sum of Minters
  - Product of Maxterms
- Minterms?
  - AND terms containing all variables, for input value=1 the variable is unprimed and for input value=0 the variable is primed
- Maxterms?
  - OR terms containing all variables, for input value=1 the variable is primed and for input value=0 the variable is unprimed
- Sum of minterms?
  - Take those minterms where the function is 1
- Product of Maxterms?
  - Take those maxterms where the function is 0

The End