Last Lecture Review

- Canonical Form of Boolean Function?
  - Sum of Minters
  - Product of Maxterms
- Minterms?
  - AND terms containing all variables, for input value=1 the variable is unprimed and for input value=0 the variable is primed
- Maxterms?
  - OR terms containing all variables, for input value=1 the variable is primed and for input value=0 the variable is unprimed
- Sum of minterms?
  - Take those minterms where the function is 1
- Product of Maxterms?
  - Take those maxterms where the function is 0

Canonical Form Conversion

- A function represented as Sum of minterms can be represented as the Product of maxterms of the remaining terms.

- The complement of a function expressed in sum of minterms equals the sum of minterms missing from the original function
  - $F(A, B, C) = \sum(0, 3,4) = m_0+m_3+m_4$
  - $F'(A, B, C) = \sum(1,2,5,6,7) = m_1+m_2+m_5+m_6+m_7$

- Now if we take the complement of F' using DeMorgan's theorem, we obtain F in the product of maxterms form:
  - $(F')' = (m_1+m_2+m_5+m_6+m_7)'$
  - $F = m_1' . m_2' . m_5' . m_6' . m_7'$ [Complement of minterms]
  - $= M_1M_2M_5M_6M_7$ [maxterms]
  - $= \prod(1,2, 5, 6, 7)$

- This implies the following relation:
  - $m'j = Mj$
- So sum of minterms: $\sum(0,3,4)$ = product of maxterms: $\prod(1,2, 5, 6, 7)$

Table A: Conversion of Forms

		Desired Form			
Given Form		Minterm Expansion of F	Maxterm Expansion of F	Minterm Expansion of F′	Maxterm Expansion of F′
	Minterm Expansion of F	-	maxterm nos are those nos, not on the minterm list of F	List minterms not present in F	Maxterm nos are same as minterm nos of F
	Maxterm Expansion of F	minterm nos are those nos, not on the maxterm list of F	-	minterm nos are same as maxterm nos of F	List maxterms not present in F

Table B: Application of Table A

Desired Form					
Given Form		Minterm Expansion of F	Maxterm Expansion of F	Minterm Expansion of F′	Maxterm Expansion of F′
	F=∑(3,4,5,6,7)	-	F=π(0,1,2)	∑(0,1,2)	π(3,4,5,6,7)
	F=π(0,1,2)	∑(3,4,5,6,7)	-	∑(0,1,2)	π(3,4,5,6,7)

Conversion to Sum of minterms : By algebraic method

- Each term is inspected to see if it contains all variables. If it misses one or more can be ANDed with an expression such as x+x’, where x is missing
- Example
  - Express F in Sum of Minterms

$F(A,B,C) = A + B'C$

$A = A(B+B') = AB + AB'$

$= AB(C+C') + AB'(C+C')$

$= ABC + ABC' + AB'C + AB'C'$

$B'C = B'C(A+A')$

$= AB'C + A'B'C$

$F = ABC + ABC' + AB'C + AB'C' + AB'C + A'B'C$

$= A'B'C + AB'C' + AB'C + ABC' + ABC$

$m1 + m4 + m5 + m6 + m7$

$F(A,B,C) = \Sigma(1, 4, 5, 6, 7)$

Conversion to Product of maxterms : By algebraic method

- The function can be expressed as Product of Sum by bringing into OR form by distributive law
  - $x + yz = (x+y)(x+z)$

  any missing term Ored with xx'

Example

$F = xy + x'z$

$= (xy + x')(xy + z)$

$= (x' + x)(x' + y)(x + z)(y + z)$

$= (x' + y + zz')(x + yy' + z)(xx' + y + z)$

$= (x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)$ $(x + y + z)(x' + y + z)$

M_4 M_5 M_0 M_2

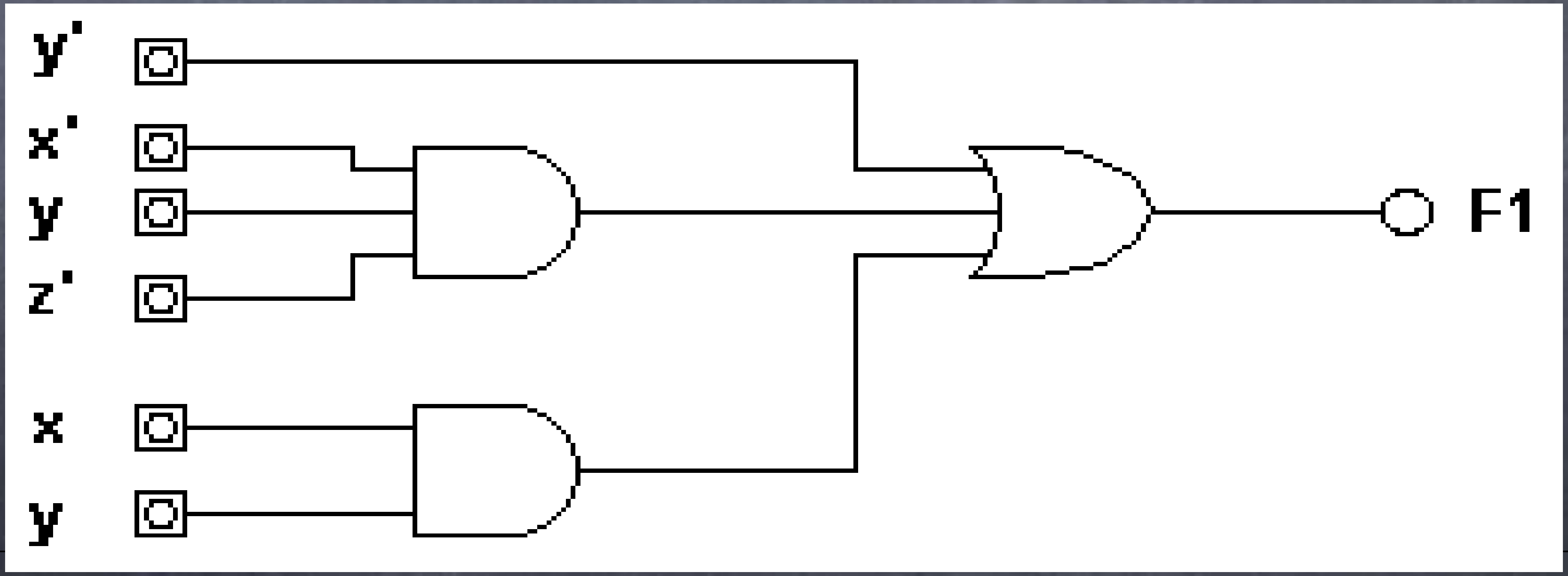
$F(A,B,C) = \prod(0, 2, 4, 5)$

Standard Forms

- Standard forms are those forms that allow the terms forming the function to consist of any number of the variables.
- There are two standard forms:
  - sum of products (SOP)
  - product of sums (POS)

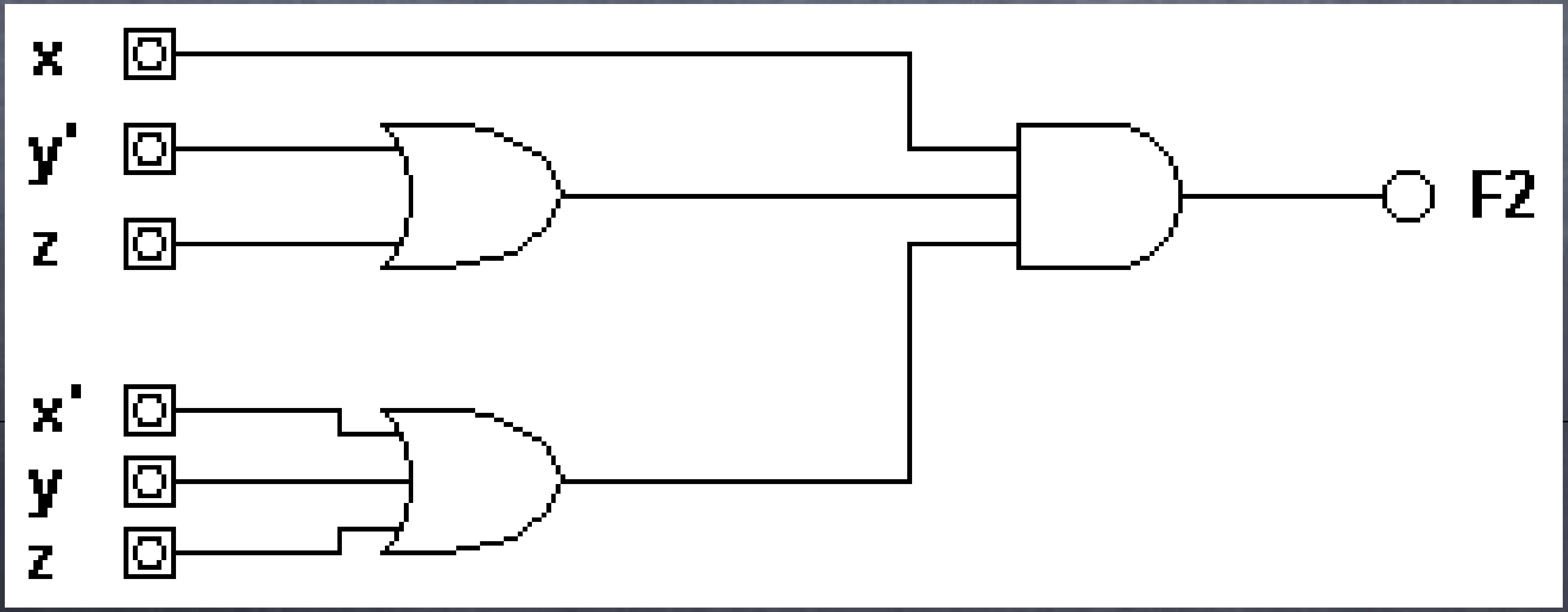
Sum of Products

- The Sum of Products (SOP) is a Boolean expression containing AND terms, called product terms, of one or more literals each.
  - $F_1 = y' + xy + x'yz'$



Product of Sums

- The Product of Sums (POS) is a Boolean expression containing OR terms, called sum terms, of one or more literals each.
  - $F_2 = x(y' + z)(x' + y + z')$

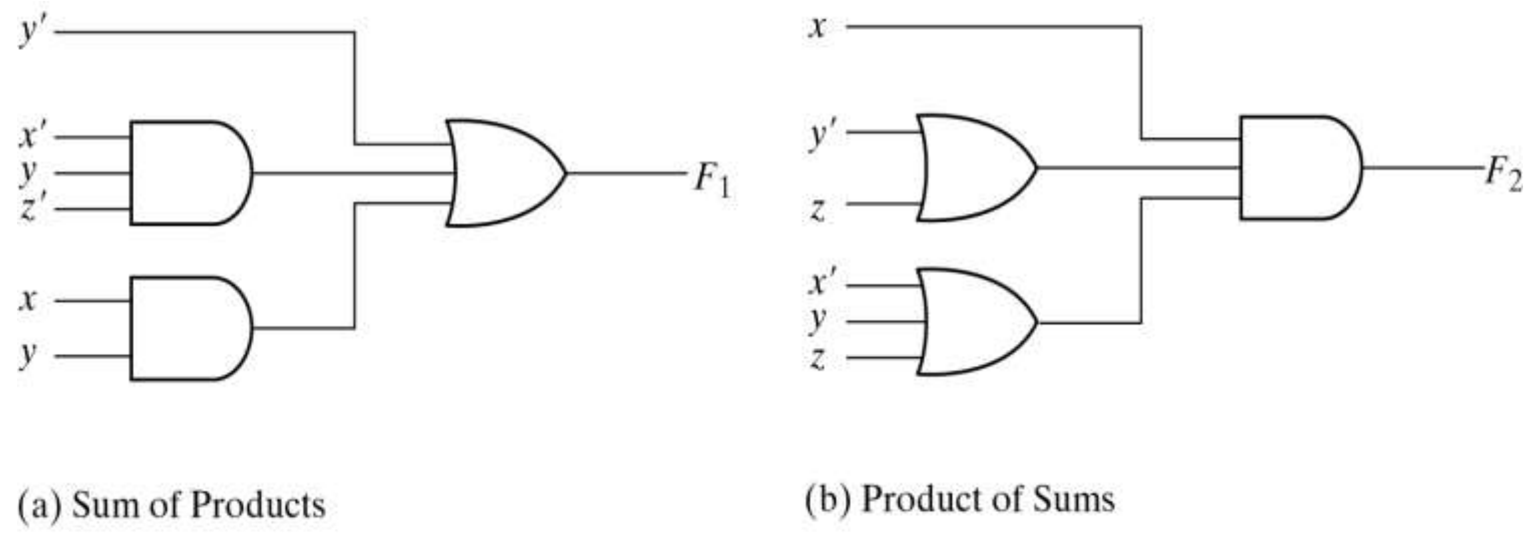


Non Standard Form

F3 = **AB** + **C(D** + **E)**

Two Level Implementation

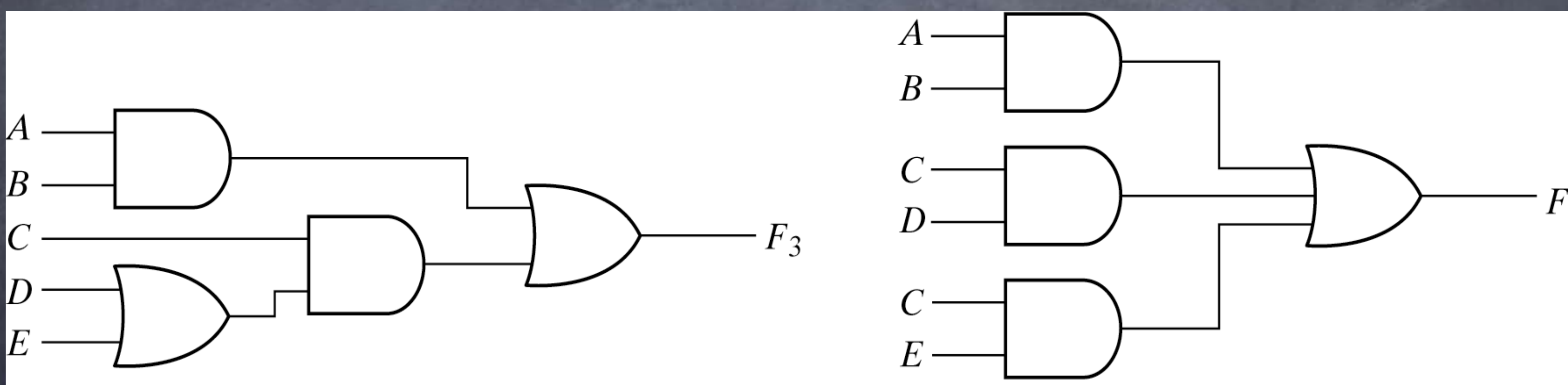
standard type
two-level



Conversion from Nonstandard to Standard Form

A Boolean function may be expressed in a nonstandard form (fig 2.4a shows a function that is neither in sum of products nor in product of sums). It has three levels of gating

It can be converted to a standard form (Sum of product) by using distributive law to remove parenthesis



(a) $AB + C(D + E)$ (b) $AB + CD + CE$

Possible Functions

- n variable can give how many possible functions?
- Answer = 2^{2^n}

Other Logic Operations

- Given two Boolean variables:
  - When binary operators AND and OR are placed between two variables they form two Boolean functions x . y and x + y
  - there are $2^{2^2} = 16$ combinations of the two variables as there are 2^{2^n} possible functions for n binary variables (we will see the details of these 16 functions in next slides)
  - each combination of the variables can result in one of two values, 0 or 1, therefore there are 2^4=16 functions (combinations of 0's and 1's for the four combinations, 00,01,10,11)
- AND and OR represent two of the 16 possible functions.

Function Combinations

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

- F_1 represents the AND Operation
- F_7 represents the OR Operation
- There are 14 other functions

The End